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Los Angeles

**Essays on Financial Economics and Industrial
Organization**

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Marcus Eduardo Mathias Studart

2015

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ABSTRACT OF THE DISSERTATION

Essays on Financial Economics and Industrial Organization

by

Marcus Eduardo Mathias Studart

Doctor of Philosophy in Economics

University of California, Los Angeles, 2015

Professor Pierre-Olivier Weill, Co-chair

Professor Aaron Tornell, Co-chair

This essay consist of three chapters. In chapter 1, I analyze the equilibrium behavior of asset prices and margins (i.e collateral required to trade shares using debt), when Market Makers smooth out price fluctuations by trading on a margin. I address the questions of 1) whether financial margins can increase in reaction to supply shocks without misinformation about the shocks' nature, 2) when non-fundamental shocks reduce asset prices and increase margins, and 3) how margins and prices react to persistent supply shocks, as opposed to temporary ones.

In the model, price fluctuations are induced by supply shocks, and margins are set to match the price depreciation induced by a future negative tail shock. Temporary shocks (i.e. shocks that fade very soon) are shown to have no effects on prices or margins, when either they are small or Market Makers hold large collateral amounts. If the shock is sufficiently large, some shares will be held by (currently active) risk averse investors, and price falls due to larger risk premium. If, before the shock, Market Makers were holding asset shares, prices fall even more, because Market Makers wealth is marked down, and expected future prices falls, since it becomes

more likely that future shocks will depress prices. Persistent shocks (i.e. shocks that do not fade quickly) reduce current prices, because, in the best scenario, they shift down the future price distribution, and reduce Market Makers' asset valuation. I give conditions for margins to increase with a persistent shock. Falling prices and higher margins are not necessarily the result of a margin spiral (i.e. when margins increase and constrain Market Makers, who are forced to sell, causing prices to fall and margins increase more). Margins can increase because future price variance increases at the same time as Market Makers' asset valuation reduces, and Market Makers may not be financially constrained at all.

In chapter 2, I study repurchase agreements, short-term collateralized loans known as repos, that are commonly used to fund different sorts of assets. Using a dataset of Money Market Mutual Funds (MMF), I find that repos backed by liquid collateral, such as US Treasuries securities, have on average shorter maturities, lower haircut rates and lower interest rates than less liquid collateral, while the average maturities of repos held by MMF are positively correlated with fund size and overall portfolio maturity. Motivated by these evidences, I develop an equilibrium model to price simultaneously assets and repos.

I show that assortative matching between assets and lenders offering different maturities exists in equilibrium. Lenders who offer longer maturities are better suited to finance less liquidity securities, since investors' expected transaction costs are lower, as collateral (to repay debt) is sold long after their debt is considered unworthy. Liquid securities prices increase with repos, in order to make the financing of illiquid securities more attractive to long maturity lenders. Interest rates and haircuts are functions of both the transaction costs and maturities distributions, and are shown to be increasing in illiquidity. Haircuts exceed the securities' transaction costs, in order to cover how much securities depreciate when sold, and to force borrowers to repay the interest on their debt. Moreover, illiquid securities have

higher haircuts, because not only they have larger transaction costs but also because the repos used to finance them pay more interest. I emulate a financial crisis through an increase in the probability of a debt run. As repos are terminated earlier, all asset prices decrease. Illiquid securities prices, however, fall notably more and haircuts and interest rates of repos to fund them increase.

In chapter 3, which is co-authored with Siwei Kwok, we study the interaction of information and competition in incentivizing quality provision. We estimate a discrete quality game with Los Angeles County restaurant hygiene inspection data set, via the two step method of Bajari et al. [2006]. Our results suggest that firm competition incentivizes quality provision, but the effect is non-monotonic. If a market is saturated with a sufficiently large number of firms, an additional firm might actually reduce the likelihood that all others will provide quality. Information, however, enhances the effects of competition and reduces the threshold which additional firms reduce quality provided.

The dissertation of Marcus Eduardo Mathias Studart is approved.

Roger Farmer

Paola Giuliano

Aaron Tornell, Committee Co-chair

Pierre-Olivier Weill, Committee Co-chair

University of California, Los Angeles

2015

To my wife, to my son, to my parents

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VITA

- 2006 B.A. (Economics), UFRJ, Rio de Janeiro, Brazil.
- 2009 M.A. (Economics), EPGE-FGV, Rio de Janeiro, Brazil.
- 2014 M.A. (Economics), University of California, Los Angeles,
California.

CHAPTER 1

Market Making on a Margin

1.1 Introduction

The LTCM crisis is an example of the risks that highly levered financial institutions impose to financial markets stability¹. However, many fail to remember that, before starting to collapse, Long Term Capital Management was actually contributing to the well-functioning of the financial markets. In many markets, some financial institutions act as Market Makers, providing immediacy to investors in need to trade, as they allow prices follow closely the assets' fundamentals. Nonetheless, these institutions tend to operate with relatively small capitalization, with the constant use of short-term collateralized debt, what is seen as dangerous by regulators². Many of the recent regulatory discussions about the Volcker rule, see Duffie [2012], are about keeping Market Makers' (usually proprietary trading desks of banks) ability to provide liquidity services, while avoiding market crashes. If lending is, however, positively correlated to market conditions, as they get worse, credit may constrain

¹Long-term Management Capital was a hedge fund specialized in fixed-income convergence trades. It used high leverage to increase returns. During 1997 and 1998, prices of bonds with similar payoffs were expected converge, what would provide significant profits to LTCM, but instead the price gaps got wider due the financial turmoil caused by the Asian crisis and Russia's debt default. LTCM suffered substantial draws from clients after months of negative returns, and was forced to sell part of their portfolio at disadvantageous terms. The process of selling its portfolio contributed to larger price falls and more market turmoil. Under the supervision of the Federal Reserve Bank, LTCM was acquired by a large group of international banks in 1998.

²Some reasons why Market Making uses leverage to increase returns include: 1) Large operational costs of being actively trading on a market, and 2) the opportunity cost of capital may exceed the unlevered return of making a market.

Market Makers when their services are mostly needed. On that account, it is crucial to understand the relation between margin lending and market making, and how they react to market shocks.

In this paper, I develop a tractable model to analyze market making and collateralized lending reaction to non-fundamental shocks. The model is built on similar grounds of Brunnermeier and Pedersen [2009], which show the existence of destabilizing behavior of margins when lenders cannot observe the true nature of the shocks. In their framework, shocks are either fundamental, which change the intrinsic value of the asset, or non-fundamental, when asset supply increases due to the idiosyncratic need of investors to sell securities³. Because supply shocks occur with small probability in Brunnermeier and Pedersen [2009], they show that the observed price reduction is associated with a change the asset fundamental value, which in the model triggers more volatility of future asset's fundamental value. As a result, lenders cut financing to Market Makers, because the likelihood of larger price changes increase. Therefore, if Market Makers cannot avoid prices to fall when supply shocks arrive, margins increase, Market Makers become more financially constrained and, as a result, the asset price falls more.

This paper is related to three questions. Could financial margins ever increase without misinformation about the true nature of shocks? When do non-fundamental shocks reduce asset prices and increase margins, without fundamental shocks? How does margins and prices react to persistent shocks, as opposed to temporary ones?

One main difference in my framework is that there are no fundamental shocks. Therefore, unlike in Brunnermeier and Pedersen [2009], agents will not mistakenly associate a price fall with changes in asset fundamental value, when in fact the price depreciation was caused by supply shocks. I show that both asset prices and margins reactions depend not only on the shock size, but also on the supply shock

³Non-fundamental shocks are commonly called liquidity shocks.

persistence. I denote by temporary supply shocks, shocks that have no effects on future asset supply distribution, and by permanent supply shocks, the shocks that shift up the future supply distribution. The shock persistence represents a measure of how fast outside investors arrive to purchase the new extra supply. When some investor needs to sell shares immediately, it increases the asset supply to be held by the investors currently active in the market⁴. Outside or "inactive" investors arrive with some delay and purchase back the extra supply. This time delay is associated minimum necessary time for outside investors acquire knowledge to value the securities correctly, or to raise capital to purchase the securities, or even to get informed about the existence of the investment opportunity. Therefore, shock persistence captures implicitly the time length between the shock realization and the arrival of inactive investors⁵.

Margins' reaction to shocks depends on the proximity to dividend payment. While Brunnermeier and Pedersen [2009] focus their analysis on periods near to dividend payment, I focus on the equilibrium response to shocks away from those dates. Supply shocks that are realized in periods adjacent to dividend payment are ineffective to change the next period price distribution, because the following price is determined by the dividends itself. In periods away from dividend payments, the asset price depends both on the distribution of future supply shocks and on Market Makers' debt capacity to hold more shares.

I show that sufficiently small temporary shocks don't have any effects on asset prices or margins. Since Market Makers are risk neutral, the market price is the expected future price, as long as these investors are not financially constrained. The future price distribution is unaffected by temporary shocks, unless Market Makers'

⁴Those are divided in two types, risk neutral Market Makers and risk averse Value Investors.

⁵In the text, I don't mention inactive investors, because they are not modeled explicitly. Their behavior is implicit in the shock structure. Temporary supply shocks assume that someone will buy the shares back soon, while persistent assume that some outside investor will purchase shares later than next period.

wealth is marked down due to losses (since that would reduce their ability obtain debt to buy more shares in the future). But if the shock size doesn't exceed Market Makers' current buying capacity, prices and wealth are constant. Margins are constant for the same reason (if shocks are small), because future price distribution is fixed, and therefore, price depreciation under the tail event is constant. The shock threshold is straightforward to calculate. If the margin is positive, there is a maximum number of shares that Market Makers can buy with credit (the shock threshold). Any additional share above that level will be held by risk averse investors currently available to trade, so that the risk premium increases to incentivize them to hold asset shares. As long as Market Makers were not holding shares before shock, prices fall exclusively due to the increase in risk premium. However, if they had shares in the portfolio, a lower price will make them loose money, reducing the collateral available to hold against future shocks. Since Market Makers will be more likely to be financially constrained in the future, expected prices fall, which, in turn, causes current prices fall more, amplifying the initial shock. The more shares that they initially hold, the larger will be the overall price fall caused by a shock above that threshold.

When shocks are temporary, I show that equilibrium margins are equal to zero if Market Makers' initial wealth is above some threshold. Therefore, they have unlimited access to credit and can hold against any large temporary shock, as long as the shock is temporary. This result is a consequence of how margins are set in the model. If future collateral is fixed and sufficient large, the future price distribution is not only fixed, but the price under the tail event is larger than the mean. Since the current price is the mean future price, a price depreciation between consecutive trading dates cannot occur even after a tail shock arrives, and hence the margin is set equal to zero. Thus, prices never fall when the margin is zero and the shock is temporary.

If shocks are permanent, asset prices and margins behave differently. A persistent shock shifts the future supply distribution, increasing the likelihood that new shocks will constrain Market Makers. In that case, in more states shares will be held by risk averse investors and prices will be lower. As a result, the expected price falls immediately with the tiniest shock. I show that margins can also increase with a permanent shock, and I give sufficient conditions for it. However, although prices can fall at the same time as margins increase, the negative correlation between this variables may not be a result of a margin spiral. The reason is that prices can fall as a result of lower Market Makers valuation (expected price), not because Market Makers are financially constrained. They value less the asset because the average future supply is larger. While margins increase because lenders forecast that prices in the tail scenario is more distant to mean, so that prices depreciate more in the tail event.

If Market Makers hold shares initially, then prices may fall substantially due to feedback between lower wealth (due to price fall) and lower expected price. If Market Makers hold a large number of shares, the total price depreciation due to the permanent shock may be large, although they may not be financially constrained at the current period, but it becomes much more likely that they will be in the future.

The paper is closely related to Brunnermeier and Pedersen [2009]. They develop a model which relates market liquidity, the ease to trade a security close to fundamentals, and funding liquidity, the credit available to Market Makers sustain prices close to asset fundamental values. My paper is different then theirs by not assuming fundamental shocks. In the model here, margins do not increase because lower prices are interpreted as lower fundamental value. Margins increase when the distance between expected and tail event price gets wider. My model is more tractable, since prices and margins can be solved analytically. Unlike their paper, I stress that the persistence and timing of the shock is important for the equilibrium

variables comparative statics.

This paper also related to the classic paper of limits to arbitrage, Shleifer and Vishny [1997], to Grossman and Miller [1988], where market liquidity, demand and supply of immediacy were first formally defined and analyzed, to Grossman and Vila [1992] and Liu and Longstaff [2004], where Market Making trading strategies are analyzed in partial equilibrium setting, to Garleanu and Pedersen [2011], who derive an asset pricing model with heterogeneous-risk-aversion agents facing margin constraints to explain deviations the law of one price, to Chowdhry and Nanda [1998], where there is a discussion of multiple equilibria when margin lending is available, and to Gromb and Vayanos [2002], where welfare implications of arbitrageurs actions was first analyzed.

The paper is organized in the following way. In section 2, I develop the model to analyze asset prices and margins reaction to supply shocks. I define the asset, the shock structure, the three market participants, and the margin setting process. In section 2.1, I analyze a benchmark case, where Market Makers are not present, and risk averse Value Investors hold all the shares. I show that margins are zero, if Market Makers are sufficiently risk averse, because the risk premium is so high that current price is lower than the future tail event price. If they are not so risk averse, margins are shown to be decreasing in asset supply shock, temporary or not, because current prices fall more than the future tail event price.

In section 2.2, I define the market equilibrium with Market Makers participation. I divide the analysis in two parts, in section 2.3 Market Makers don't have initial asset holdings before the shock, and in section 2.4, Market Makers hold assets before period 0. In section 2.3, I solve for asset prices and show uniqueness of equilibrium. In section 2.3.1, I show that future price distribution is independent of temporary shocks in period 1, because Market Makers' wealth is constant. I show that there exists a sufficiently large initial wealth level that makes the margin equal to zero in

period 0, and therefore Market Makers hold against any temporary shock. If Market Makers are not so well capitalized, prices will be constant for shocks below some threshold, but prices will fall with larger shocks. Margins also fall because future price distribution is constant, so that the difference between current and future tail price decreases.

In section 2.3.2, I analyze the effects permanent shocks. I show that prices always fall with persistent shocks, as the future supply distribution shifts up, causing expected prices to fall. I give sufficient conditions for margins to increase, as future tail event price needs to fall more than the expected price. In section 2.4, I show that multiple equilibria can exist when Market Makers have initial asset holdings. One where Market Makers go bankrupt and other where remain in the market. I redo the analysis of sections 2.3.1 and 2.3.2. I show that results are qualitatively similar, but whenever Market Makers lose money, the asset price impact is larger (prices fall more), since there is a feedback between Market Makers' wealth and the expected price. In section 3, I conclude the paper.

1.2 Model

The market is composed of 3 agents. Market Makers and Value Investors trade one risky asset during periods 0, 1 and 2, while Financiers provide credit to Market Makers to buy asset shares on margin. During the last period, random normal distributed dividends D , with mean and variance equal to \bar{D} and σ_D^2 , are paid to asset holders. The market participants do not receive news about dividends until they are paid in period 2. This is the sense that there are no fundamental shocks in the model. Thus, price fluctuations are a result of adjustments in the asset supply Z , and not a result of changes in the dividend distribution.

I assume that Z_t follows an AR(1) process, as show in the equation below, and

that the initial supply Z_{-1} is positive.

$$Z_t = \bar{Z} + \rho Z_{t-1} + \alpha_t \quad \alpha_t \sim F(\cdot) \quad (1.1)$$

The parameter ρ is a key parameter to determine how margins and asset prices respond to shocks. It measures the supply shocks' persistence. During next sections, I will either use $\rho = 0$, when shocks are temporary, or $\rho = 1$ when shocks are persistent. Market opens in all three periods, just after the shock is realized, so that all agents know both shock size and the asset supply before trading.

One could interpret the shocks as supply or as negative demand shocks originating from asset holders that are not modeled explicitly. These investors trades are idiosyncratic, unrelated to the asset fundamentals or Market Makers' and Value Investors' valuations. They may need to sell shares, for instance, when cash is necessary to cover losses in other markets which they participate in. Duffie [2010b] studies the effects of slow capital formation into asset prices, providing several examples of events that supply shocks had large price impact because new capital arrived slowly. The parameter ρ could be capturing the market frictions that creates slow capital mobility such as informational or intermediation frictions. For example, it may take some time for nonspecialized investors to acquire knowledge to value correctly the asset, ranging from hours to days or weeks. New capital may also arrive slowly, because it takes time to raise equity and issue bonds in financial markets, or because new investors need to search for intermediaries.

Value Investors

Value Investors are risk averse agents with mean-variance preferences, whose wealth is assumed to be sufficiently large, so that they are never financially constrained. As group, Value Investors are present in all periods, but individually they short-sighted. Starting from period 0, an unit measure of Value Investors arrive in the

market at each date. They stay active for just one period, and are replaced by a new set of Value Investors in the following date. The old group, who arrived in the period before, sell shares at the market price (if they held any), prior to leaving the market.

Value Investors could be investors which are currently attentive to a trading opportunity in one market, but soon leave that market to move capital to another where returns are larger. In the model, Value Investors' asset demands at period t are shown the equation below.

$$D_t(p_t) = \frac{\mathbb{E}_t[P_{t+1}] - P_t}{\gamma \text{Var}[P_{t+1}]} \quad t = 0, 1, 2. \quad (1.2)$$

Market Makers

Market Makers are risk neutral investors who are present during all three periods and can trade the asset on a margin. They consume at the end of period 2 and are initially endowed with wealth equal to W_{-1} and Z_{-1} asset shares. By buying or selling when prices deviate from fundamentals, they reduce the asset price volatility generated by the supply shocks. They face, however, collateral constraints imposed by Financiers, which may limit the maximum amount of shares that they can buy. In order to purchase x_t asset shares, Market Makers need to provide $m_t x_t$ units of collateral for a long position, where m_t is the per unit asset margin. On each date, Market Makers choose a number of shares to hold until market reopens in the next period⁶. Their choice is feasible, as long as they have sufficient collateral to purchase the desired quantity x_t . That is, x_t is feasible if it belongs to B_t , in equation (1.3).

$$B_t = \{x_t : m_t x_t \leq w_{t-1}\} \quad (1.3)$$

⁶It is implicit here that $P_t > m_t^+$. This fact is true as long as next period tail price is not negative. When Market Makers are present, I show that prices are bounded below by \hat{D} , which is a positive number.

Market Makers' collateral (their wealth) is marked-to-market as in equation (1.4). After market is closed and the asset price is realized, Financiers set their marked-to-market wealth as the change in portfolio value plus Market Makers' wealth in the previous period⁷. Throughout the paper, I denote by w_t a individual Market Marker's wealth, while by W_t , the aggregate wealth of all Market Makers.

$$w_t = (P_t - P_{t-1})x_{t-1} + w_{t-1} \quad (1.4)$$

Financiers

Financiers lend resources to Market Makers on a margin account, requesting collateral in exchange for credit. Given Market Makers' desired position x_t , Financiers demand a minimum collateral of $m_t x_t$. The interest rate charged on debt balances is normalized to zero, since it avoids carrying one more parameter that doesn't enhance the comprehension of the results. All results would be similar, if instead the interest rate was positive.

The process of setting margins follows a Value at Risk rule, which determines that the collateral per asset share at t is equal to the resulting price depreciation (ΔP) when a negative tail event is realized in $t+1$. When setting margins, Financiers look for tail events that occur with some specific small probability π (1% for instance). Moreover, the tail event is time specific. If $t+1$ is period 2, a tail event is a shock with $\hat{\alpha}$, which the $1-\pi$ percentile of the supply innovation distribution. Otherwise, if $t+1$ is period 1, the tail event is a low dividend \hat{D} , which is the π percentile of the dividend distribution. Period 0 and 1 margins are shown in equations (1.5) and

⁷Here is one example of how the mark-to-market equation works. Suppose that investor A deposits \$5,000 in cash into a margin account. The broker allows the investor to buy as much as \$10,000 in securities value. The price of one security at time zero is \$100. Suppose that the investor buys 100 units of that security and in time 1 the security price changes to \$50. The investor wealth will be marked-to-market in the following way. At time zero, bond holdings are -\$5,000, and securities holdings, \$10,000. The mark-to-market wealth at time 1 is the value of securities minus the debt, $50 \cdot 100 - 5,000 = 0$. That is the same as the price change (-50) times the amount of shares held (100), plus the initial wealth (+5,000).

(1.6) below.

$$m_0 \equiv \max \{P_0 - P_1(\hat{\alpha}, \alpha_0, W_0), 0\} \quad (1.5)$$

$$m_1 \equiv \max \left\{ P_1 - P_2(\hat{D}, W_1, \alpha_1), 0 \right\} = \max \left\{ P_1 - \hat{D}, 0 \right\}. \quad (1.6)$$

The last equality of equation (1.6) result from the fact that market opens after dividends are realized. Notice also the stabilizing behavior of margins. When period 1 price falls below \hat{D} , the margin must fall in the same period, allowing Market Makers to increase leverage and avoid a crash. In Brunnermeier and Pedersen [2009] benchmark, margins behave similarly, and in order to achieve instability, they need uninformed financiers to interpret a price drop as an increase in dividend risk⁸, when, in fact, it was induced by an increase in asset supply.

Asset Price

Define X_t as the aggregate number of shares that market makers hold in period t ⁹. After applying the market clearing condition, we obtain that the asset price is linear on the excess of supply to Market Makers' aggregate demand.

$$D_t(p_t) + X_t = Z_t, \quad P_t = \mathbb{E}_t[P_{t+1}] - \gamma \text{Var}[P_{t+1}] \underbrace{(Z_t - X_t)}_{\text{excess of supply}} \quad (1.7)$$

1.2.1 Margins and Prices without Market Makers

Before moving to the full equilibrium analysis, it is worth analyzing asset prices and margins behavior with no Market Makers as a benchmark case, since it helps

⁸I further discuss this issue during the next section.

⁹All uppercase variables denote aggregate variables.

understand the paper's main results. In that case, margins are functions of the stochastic supply and Value Investors' preferences only. I show that when Market Makers are not present, margins are stable in all periods, and never increase when the asset price falls.

Definition 1.2.1. *Margin m_t is stable (unstable) when the margin is a nonincreasing (increasing) function of the asset supply.*

The equilibrium prices with no Market Makers are given by equations (1.8) and (1.9), which result from the Value Investors' optimal demands and the market clearing condition (1.7)¹⁰.

$$P_1 = \bar{D} - \gamma\sigma_D^2(Z_{-1} + \rho\alpha_0 + \alpha_1) \quad (1.8)$$

$$P_0 = \bar{D} - \gamma\sigma_D^2(Z_{-1} + \rho\alpha_0) - \gamma^3\sigma_D^4\sigma_\alpha^2(Z_{-1} + \alpha_0) \quad (1.9)$$

Since Value Investors are risk averse, period 0 prices must be lower than the average price in period 1, due to a positive risk premium. Asset prices drop in response to a shock in period 0, persistent or not, because Value Investors demand larger risk premium to hold more shares. When shocks are persistent, not only the risk premium increases but the expected price falls, due to an increase in expected risk premium, represented by the term $\gamma\sigma_D^2(Z_{-1} + \rho\alpha_0)$.

Using the margins' definitions and the market clearing prices above, one can find the equilibrium margins without Market Makers, which are shown in equations (1.10) and (1.11).

$$m_0^+ = \max \{ \gamma\sigma_D^2 [\hat{\alpha} - \gamma^2\sigma_D^2\sigma_\alpha^2(Z_{-1} + \alpha_0)], 0 \} \quad (1.10)$$

$$m_1^+ = \max \left\{ (\bar{D} - \hat{D}) - \gamma\sigma_D^2(Z_{-1} + \rho\alpha_0 + \alpha_1), 0 \right\} \quad (1.11)$$

¹⁰The asset price in period 2 is equal to the realized dividend D , because market opens after dividends are realized, and therefore the asset becomes risk free.

Note that if asset supply has any effect on margins, the effect is negative. In period 0, margins are positive only when value investors' risk aversion is not greater than a threshold $\hat{\gamma}(Z_{-1}, \sigma_D, \sigma_\alpha, \pi)$, which is an decreasing function of the asset supply, supply shock and dividend variances. Otherwise, if Value Investors' risk aversion parameter is above that threshold, the resulting risk premium is sufficiently large to make P_0 lower than the next period tail event price. As a result, Financiers set the initial margin equal to zero.

Lemma 1.2.1. *If risk aversion parameter is larger than $\hat{\gamma}(Z_{-1}, \sigma_D, \sigma_\alpha, \pi)$ then margin $t = 0$ is zero for any positive shock. If Value Investors' risk aversion is strictly lower than $\hat{\gamma}(Z_{-1}, \sigma_D, \sigma_\alpha, \pi)$, margin is strictly positive but strictly decreasing in positive supply shock. Margins are not unstable.*

Even when margins are not zero (when Value Investors' risk aversion is less than $\hat{\gamma}$), margins are stable. Since temporary shocks only reduce the current price and not future price, margin falls. If shocks are persistent, current prices fall more than the next period asset price under a tail event, due to Value Investors' risk aversion. While a marginal supply shock at $t = 0$ decreases prices by $\gamma\sigma_D^2\rho$ in period 1, it decreases prices by $\gamma\sigma_D^2\rho + \gamma^3\sigma_D^4\sigma_\alpha^2$ in $t = 0$. The propensity of margins to fall when prices drop is a desirable feature, since it would allow market makers to increase leverage to absorb supply shocks and avoid price crashes.

In Brunnermeier and Pedersen [2009], margins respond to supply shocks in a similar way. In their benchmark, Financiers know the exact reason why prices have fallen. Since temporary supply shocks don't change future price distribution, larger price depreciation become less likely after current price has dropped. Therefore, Financiers set margins lower than before.

In order to get the destabilizing behavior of margins, Brunnermeier and Pedersen [2009] need three assumptions: stochastic expected dividends, ARCH fundamental

volatility, and Financiers do not observe the shocks and asset demands¹¹¹². If a shock occurs, Financiers will filter out the probability that the shock was a fundamental one. Furthermore, when supply shocks occur with small probability, as assumed in Brunnermeier and Pedersen [2009], Financiers will infer that prices have fallen due to fundamental shock. Therefore, they set higher margins, because larger price depreciation is more likely with the resulting higher fundamental volatility.

In the model presented here, there are no fundamental shocks, and period 1 margin always decrease with supply shock. The margin m_1 is stable even when Market Makers are present, because the following period price distribution is given by the dividend distribution, which is fixed. I show, however, that margin can increase in period 0, since, under some circumstances, supply shocks can change the price distribution in period 1.

1.2.2 Continuum of Market Makers

Equilibrium margins and asset prices can respond differently to shocks when Market Makers are present. Their aggregate wealth becomes a state variable to the conditional price distribution, because Market Makers purchase the asset with borrowed capital backed by their wealth. When wealth is low, the asset price mean falls and the asset price distribution becomes riskier, since the new supply will be more likely to be held by Value Investors, who value less the asset, making shocks have larger price impact.

I proceed by solving the model backwards, starting from period 1, since in period 2, the asset price is equal to the realized dividend. The Market Makers' problem is

¹¹If the asset's fundamental value (expected dividends) changes, the future volatility of asset's fundamental value increases.

¹²The agents only observe asset prices.

straightforward and their optimal choices are shown below.

$$X_0 = \begin{cases} \frac{W_{-1}}{m_0} & \text{if } \mathbb{E}[P_1] > P_0 \\ x \in [0, \frac{W_{-1}}{m_0}] & \text{if } \mathbb{E}[P_1] = P_0 \end{cases} \quad X_1 = \begin{cases} \frac{W_0}{m_1^+} & \text{if } \bar{D} > P_1 \\ x \in [0, \frac{W_0}{m_1}] & \text{if } \bar{D} = P_1 \end{cases} \quad (1.12)$$

Market Makers will buy as many shares as possible if the expect price next period is greater than the current one. If current price is equal to expected price, then they are indifferent between any feasible quantity. A consequence of their behavior is that prices are equal to the expect price unless they are financially constrained¹³.

Definition 1.2.2. An equilibrium are prices and margin functions $\{P_0 : \mathbb{R} \rightarrow \mathbb{R}, P_1 : \mathbb{R}^3 \rightarrow \mathbb{R}, m_0 : \mathbb{R} \rightarrow \mathbb{R}^+, m_1 : \mathbb{R}^3 \rightarrow \mathbb{R}^+\}$, Market Makers' and Value Investors' demands and collateral W_0 such that:

1. Given $W_0, (\alpha_0, \alpha_1)$ and P_1 , Financiers set m_1 equal to $\max\{P_1 - \hat{D}, 0\}$. Market Makers and Value investors form demands according to equation (1.12). The resulting price is given by equation (1.7) and equal to P_1 .
2. Given α_0, P_0 and $P_1(\alpha_0, \cdot)$, W_0 is determined by the mark-to-market equation (1.4). Market Makers and Value Investors form price expectation according to

$$\mathbb{E}[P_1] = \int P_1(\alpha_0, \alpha_1, W_0) dF(\alpha_1). \quad (1.13)$$

Financiers set $m_0 = \max\{P_0 - P_1(\alpha_0, \hat{\alpha}, W_0), 0\}$ and Market Makers' and Value Investors' demands are determined by equations (1.12) and (1.2). The resulting price is determined by equation (1.7) and is equal to P_0 .

¹³Market Makers crowd-out completely Value Investors when their financial constraints don't bind.

1.2.3 Market Makers have no initial asset holdings

In this subsection, I analyze the simplest case when Market Makers don't hold shares before period 0. Since there is no feedback between the asset price P_0 and the marked-to-market wealth available in period 1, I show that there is only one equilibrium. The reason is that the collateral available to Market Makers is independent of the realized asset price in period 0, that is, $W_0 = W_{-1}$. In the next subsection, I show that multiple equilibria is a possibility due the feedback between lower price, lower future collateral, lower expected price, which in turn causes asset prices to fall more.

When Market Makers are not financially constrained in period t , they hold all the available supply, and, because they are risk neutral, prices are equal to the expected price that prevails next period. When Market Makers are constrained in period t , current price is lower than expected price. The proposition below establishes the equilibrium asset price in period 1, as a function of the wealth and asset supply, and the uniqueness of the market clearing price.

Proposition 1.2.2. *Given the collateral available and the asset supply on period 1, W_0 and Z_1 , respectively,*

$$P_1 = \begin{cases} \bar{D} & \text{if } W_0 > (\bar{D} - \hat{D})Z_1 \\ \frac{P_1^* + \hat{D} + \sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0}}{2} & \text{otherwise} \end{cases} \quad (1.14)$$

where

$$P_1^* = \bar{D} - \gamma\sigma_D^2 Z_1. \quad (1.15)$$

P_1 is unique given (W_0, Z_1) and bounded below by \hat{D} . P_1 is increasing in W_0 , and strictly increasing if $W_0 < (\bar{D} - \hat{D})Z_1$. P_1 is decreasing in Z_1 , and strictly decreasing if $Z_1 > \frac{W_0}{\bar{D} - \hat{D}}$.

The proposition is formally proved in the appendix. Note that if Market Makers are not financially constrained, the asset price equals expected dividends, and as a result margin equals $\bar{D} - \hat{D}$. Therefore, Market Makers are unconstrained when asset supply is less than the maximum number of shares that they can buy, $W_0/(\bar{D} - \hat{D})$. If the asset supply is larger than that threshold, some shares will be held by Value Investors, and the equilibrium price is the solution to a fix-point problem with a unique solution (equation (1.16)). The fix-point problem exist, because margin is set according to the market price, which determines the maximum number of shares that Market Makers can buy, and thus the equilibrium price.

$$P_1 = \bar{D} - \gamma\sigma_D^2 \left(Z_1 - \frac{W_{-1}}{P_1 - \hat{D}} \right) \quad (1.16)$$

More collateral will not increase the asset price when Market Makers buying power is sufficient to purchase all available securities. The same logic follows when increasing the asset supply. Marginally larger supply doesn't reduce the asset price, if Market Makers buying power is sufficient to purchase all the available securities. However, the equilibrium asset price is strictly increasing in the collateral available (W_0) and strictly decreasing in the asset supply, when the supply exceeds $W_0/(\bar{D} - \hat{D})$. The asset price P_0 is bounded above by \bar{D} , since neither Market Makers nor Value Investors will purchase securities with price larger than the expected dividends. It is bounded below by \hat{D} when the collateral available to Market Makers is not zero. The reason is that m_1 is zero when the price P_1 is below \hat{D} , and as result, Market Makers could buy unlimited number shares and bring prices to \bar{D} .

The next proposition shows that the period 1 margin is never destabilizing, that is, it never increases in response to positive supply shocks.

Proposition 1.2.3. *Margin m_1 is decreasing in the asset supply in period 1, and strictly decreasing when Market Makers are financially constrained.*

The proof is straightforward. Since \hat{P}_2 is fixed and equal to \hat{D} , the difference $P_1 -$

\hat{D} reduces in response to larger asset supply. When Market Makers are financially constrained, the effects of supply increase are partially smoothed because the margin falls, allowing these investors to hold more securities than they would if margin was kept fixed.

Before analyzing the margin and asset price reactions to shocks, I proceed with the following proposition that calculates the equilibrium price in period 1. It shows the uniqueness of P_1 and, thus, the uniqueness of the equilibrium.

Proposition 1.2.4. *P_0 is given by*

$$P_0 = \begin{cases} \mathbb{E}[P_1|W_{-1}] & \text{if } W_{-1} > (\mathbb{E}[P_1|W_{-1}] - P_1(\alpha_0, \hat{\alpha}, W_{-1}))\alpha_0 \\ \frac{P_0^* + \hat{P}_1 + \sqrt{(P_0^* - \hat{P}_1)^2 + 4\gamma \text{Var}(P_1)W_{-1}}}{2} & \text{otherwise} \end{cases} \quad (1.17)$$

where

$$P_0^* = \mathbb{E}[P_1|W_{-1}] - \gamma\sigma_D^2 Z_0. \quad (1.18)$$

and the equilibrium is unique.

If Market Makers are unconstrained, then price equals the expected price. They are not financially constrained when the supply is below the maximum number of shares that they can buy, $W_{-1}/(\mathbb{E}[P_1|W_{-1}] - P_1(\alpha_0, \hat{\alpha}, W_{-1}))$. When the supply exceeds that number, Value Investors will hold shares, and the equilibrium price is the unique solution to a fix-point problem (equation (1.19)), similar to Proposition 1.2.2.

$$P_0 = \mathbb{E}[P_1|W_{-1}] - \gamma \text{Var}(P_1) \left(Z_0 - \frac{W_{-1}}{P_0 - P_1(\alpha_0, \hat{\alpha}, W_{-1})} \right) \quad (1.19)$$

If Market Makers' collateral is positive, note that P_0 must be bounded below by \hat{D} . The argument is that P_1 is greater than \hat{D} , and therefore if the asset price in period 0 is lower than \hat{D} , the margin is set equal to zero. As a result, Market

Makers would purchase unlimited number of shares, and would bring the price back to the expected price, which is greater than \hat{D} .

1.2.3.1 Asset Price and Margin Responses to Temporary Shocks

Suppose that a temporary shock $\alpha_0 > 0$ arrives during the first period. The next proposition states that if Market Makers hold capital above some threshold, they will never financially constrained in period 1 and neither the asset price will change in response to α_0 shocks. In that sense, the market price will be immune to supply shocks in period 0.

Proposition 1.2.5. *If $W_{-1} \geq (\bar{D} - \hat{D})(\hat{\alpha})$ then Market Makers are never financially constrained and P_0 is independent of the shock realized at $t = 0$.*

If $W_{-1} \geq (\bar{D} - \hat{D})(\hat{\alpha})$, Market Makers hold enough collateral to sustain P_1 equal to \bar{D} , when shocks as large as $\hat{\alpha}$ arrive at $t = 1$. In that case, m_0 must be zero, because P_0 is bounded above by \bar{D} . Since a temporary shock cannot change $P_1(\hat{\alpha}, W_{-1})$, the margin will be always equal to zero. As a result, the asset price will be constant and equal to $\mathbb{E}[P_1|W_{-1}]$. If the initial collateral is sufficiently low, so that $P_1(\hat{\alpha}, W_{-1})$ is below $\mathbb{E}[P_1|W_{-1}]$, margin will be positive. Since these two statistics depend only on W_{-1} and not on the temporary shock, the margin will be also constant. But as before, the margin neither increase or reduce with larger supply in period 0. Since the margin is positive, there exists a large enough shock that will constrain Market Makers and cause the asset price to drop below expected price. The shock, however, will not be amplified, because it doesn't have any effect on the next period wealth or on the next period asset supply.

1.2.3.2 Asset Price and Margin Response to Persistent Shocks

The next proposition shows that permanent shocks necessarily reduce the asset price in period 0, while proposition 1.2.7 gives sufficient conditions for margins increase in response to larger supply in period 0.

Proposition 1.2.6. *Any permanent shock decreases asset prices at $t = 0$.*

I prove the proposition in the appendix, but the result is straightforward. Larger persistent shocks shift the supply distribution in period 1, increasing the interval of shocks that constrain Market Makers, and reducing prices in states where Market Makers were already financially constrained. Therefore, the expected price falls and, as a result, the asset price drops even when Market Makers are not financially constrained. Otherwise if they become financially constrained, price falls more because, in addition, the risk premium increases in order to satisfy Value Investors' demands.

However, margins not necessarily increase with persistent shocks. If prices are already close to the lower bound, the margin will be close zero, and an increase in asset supply can actually reduce the margin. It is straightforward to notice that as supply goes to infinity, prices in all period 1 states goes to the lower bound \hat{D} . The next proposition gives sufficient conditions for margins to increase after the arrival of a permanent supply shock. It uses assumption 1.2.1, which states that the tail event probability is small.

Assumption 1.2.1. *π is approximately equal to 0.*

If Market Makers are not financially constrained, then the margin m_0 equals $\mathbb{E}[P_1|W_{-1}] - P_1(\hat{\alpha}, W_{-1})$, if this difference is positive. In order to increase margins at $t = 0$, a persistent shock must reduce $P_1(\hat{\alpha}, W_{-1})$ more than the mean. A sufficient condition for margins to be increasing is that $\hat{\alpha}$ is a low probability event, and wealth is sufficiently high to keep Market Makers unconstrained for new shocks close and

below $\hat{\alpha}$ (but not $\hat{\alpha}$). As a result, the permanent shock decreases $P_1(\hat{\alpha}, W_{-1})$ without changing much the mean price, because the set of new shocks where Market Makers are constrained has small probability. Proposition 1.2.7 formally states this result.

Proposition 1.2.7. *When $W_{-1} \geq (\bar{D} - \hat{D})\hat{\alpha}$ and shocks are persistent, there exist a neighborhood $[\check{\alpha}, \bar{\alpha}]$ that m_0 is strictly increasing in the period 0 shock. The margin response to shocks is larger when Value Investors' risk aversion is larger. No shocks below $\hat{\alpha}$ can financially constrain Market Makers.*

When Value Investors' risk aversion is larger, the impact of permanent shocks on m_0 is larger, because tail event price falls more relative to the expected price. The positive correlation between between asset price and margin may be deceptive to an external observer. Although prices drop and margins increase in reaction to a persistent shock, Market Makers may not be financially constrained. The asset price falls as a result of Market Makers' lower valuation, because the expected price falls, since the persistent shock shifts the future supply distribution. Not necessarily due to larger risk premium. While the margin increases when the next period price distribution has become riskier, more disperse, Market Makers may not hit their constraint, since they value less the asset, given the lower capacity of Market Makers to smooth future shocks.

In fact, it is still hard to get financially constrained with persistent shocks (eventually harder than temporary shocks), if initial wealth is as high as $(\bar{D} - \hat{D})\hat{\alpha}$. Note that the last part of Proposition 1.2.7 states that shocks need to be at least as large as $\hat{\alpha}$, in order to constrain Market Makers. Given Assumption 1.2.1, this event has small probability. The reason is that Market Makers are financially constrained when the supply exceeds the maximum number of shares that they can buy, W_{-1}/m_0 . However, since $W_{-1}/(\bar{D} - \hat{D})$ is larger than $\hat{\alpha}$, as assumed, and m_0 is bounded above by $\bar{D} - \hat{D}$, the shock must still be at least as large as $\hat{\alpha}$. Nonetheless,

although they are not financially constrained in period 1, the initial shock increases the likelihood that they will be financially constrained in period 1.

1.2.4 Market Makers have initial asset holdings

In the last subsection, I analyzed asset prices and margin reactions to shocks when Market Makers have no initial asset holdings. If they hold shares before $t = 0$, that is, Z_{-1} is strictly positive, the shock will have larger price impacts, since the losses reduce the collateral available to smooth future shocks. Furthermore, it creates the possibility of multiple equilibria, which I analyze next.

1.2.4.1 Possibility of Multiple Equilibria

The next proposition shows that P_0 is continuous for strictly positive levels of Market Makers collateral, but not at zero.

Proposition 1.2.8. *P_0 is continuous in W_0 at the neighborhood of $(0, \infty)$, but discontinuous at $W_0 = 0$.*

The first part of the proposition is proven in the appendix. The discontinuity at $W_0 = 0$ exists due to the fact that, as long Market Makers hold arbitrarily small collateral amounts, they can sustain asset price above \hat{D} . But if they have zero collateral, all shares will be held by Value Investors, and future prices will be lower than \hat{D} if supply is sufficiently large. As result, period one prices can be much lower if Market Makers capital is fully depleted, as opposed to having arbitrarily small collateral W_0 .

Definition 1.2.3. Define $\tilde{W}_{-1} = -(\bar{D} - \gamma\sigma_D^2\mathbb{E}[Z_1] - P_{-1})Z_{-1}$ and $\check{W}_{-1} = (\bar{D} - \hat{D})Z_1$

Proposition 1.2.9. *There exist one equilibrium where Market Makers are wiped-out of the market, if initial wealth is below \tilde{W}_{-1} . If initial wealth is above \check{W}_{-1} and*

$P_{-1} \leq \bar{D}$ there exist one equilibrium where Market Makers stay active in the market.

The intuition is that \tilde{W}_{-1} is the minimum initial wealth to avoid bankruptcy given the initial supply, supply shock and future expected supply. It is the value that exactly matches the wealth reduction, if the realized P_0 is the expected price with zero future collateral¹⁴. Since P_0 cannot be greater than the expected price, the initial wealth below \tilde{W}_{-1} guarantees the future wealth is zero, when agents expect that the future wealth is zero. Any initial wealth equal or below \tilde{W}_{-1} , thus guarantees the existence of this type of equilibrium, which I will call bankruptcy equilibrium. Note that this threshold increases with initial asset holdings, Value Investors' risk aversion and initial price P_{-1} . The other threshold \check{W}_{-1} is the one that guarantees the existence of one (just one) equilibrium where Market Makers are not wiped-out. I prove that this equilibrium is unique among the ones where Market Makers are not bankrupt.

In the remaining subsections, I analyze the equilibrium where Market Makers are not bankrupt, by assuming initial wealth is at least equal to \check{W}_{-1} . I will refer to this equilibrium as the non-bankruptcy equilibrium. I redo the analysis of how the equilibrium variables respond to temporary and persistent supply shocks in period 0, and compare to the results of when Market Makers have no asset holdings.

1.2.4.2 Asset Price and Margin Responses to Temporary Shocks

The proposition below is very similar to proposition 1.2.5. The only difference is the existence of one more condition, that the initial price is below some threshold.

Proposition 1.2.10. *If $W_{-1} \geq (\bar{D} - \hat{D})(Z_{-1} + \hat{\alpha})$ and $P_{-1} \leq \mathbb{E}_{\alpha_1}[P_1(Z_{-1} + \alpha_1, W_{-1})]$, then, in the non-bankruptcy equilibrium, Market Makers are never financially constrained, and P_0 is independent of the shock realized at $t = 0$.*

¹⁴Which is happens when Market Makers loose all capital.

Analogous to the case where Market Makers had no initial asset holdings, the temporary shock doesn't have any direct effect on the future asset supply distribution. If the wealth available in period 1 is at least equal to $(\bar{D} - \hat{D})(\hat{\alpha})$, then Market Makers would not be financially constrained, when a shock as large as $\hat{\alpha}$ arrives in $t = 0$. Therefore $P_1(\hat{\alpha}, W_{-1})$ would be equal to \bar{D} and the margin m_0 would be zero. Then, the asset price would be greater or equal to $\mathbb{E}[P_1(Z_{-1} + \alpha_1, W_{-1})]$. However, there is one extra condition to guarantee that the asset price in period 0 is invariable to temporary shocks: Market Makers must not lose money in $t = 0$. If $P_{-1} \leq \mathbb{E}_{\alpha_0}[P_1(Z_{-1} + \alpha_0, W_{-1})]$, future wealth will never drop below W_{-1} and, therefore, the asset price will be independent of the supply shock at period 0. In that sense, the asset price is completely immune to supply shocks in period 1, because Market Makers hold sufficiently large collateral¹⁵.

Conditions on Market Makers collateral in Proposition 1.2.10 may be restrictive. That doesn't need to be the case. If wealth is sufficiently low such that m_0 is positive, note that margin will not change unless future wealth changes, because both the expected price and $P_1(\hat{\alpha}, W_0)$ do not depend directly on the period 0 shock¹⁶. The future price distribution will not shift down, unless wealth available in period 1 falls. However, if the shock is sufficiently large to constrain Market Makers, then both P_0 and Market Makers' future collateral fall, since their wealth is marked down after losses. As a result, the future price's distribution would shift down. There exists an amplification mechanism, because when price falls, Market Makers' ability to sustain high prices in the next period reduces, causing asset price to fall more in the current period.

Proposition 1.2.11. *There exist a sufficiently low W_{-1} and $\alpha^{**}(W_{-1})$ such that:*

¹⁵Note that I am restricting the analysis here to the non-bankruptcy equilibrium. If initial asset holdings are sufficiently large, a bankruptcy equilibrium exists.

¹⁶ W_0 is the only state to those variables.

1. *Temporary shocks below $\alpha^{**}(W_{-1})$ have no effects on P_0 and on P_1 distribution.*
2. *Any temporary shock above $\alpha^{**}(W_{-1})$ will depress price P_0 and will reduce the expected period 1 price.*

The proof is the appendix. The major step is to show the existence of a initial wealth that makes m_0 strictly positive. If future collateral is constant, the margin is constant, therefore, the maximum number of shares that they can buy is finite and constant. As result, there is a large enough temporary shock that will constrain Market Makers, and will make the price fall below the expected price. When the collateral is marked down, the expected price also drops.

The asset price P_0 and P_1 distributions don't change if period 1 shocks are below α^{**} , because Market Makers can buy all the extra shares, but a shock slightly larger than α^{**} can cause large price drop, increase price variance and reduce the expected price. That occurs because the shares held by Value Investors amplify losses, and less collateral is available to smooth price fluctuations on period 1.

1.2.4.3 Asset Price and Margin Response to Persistent Shocks

I continue the analysis of the non-bankruptcy equilibrium with persistent shocks. The following proposition is analogous to Proposition 1.2.6. Just like before, permanent shocks initially shift down the future price distribution, changing the expected future price. The logic is unchanged: it increases the set of new shocks where Market Makers are financially constrained, and decreases prices in the set of new shocks that Market Makers were already financially constrained. The novelty here, in comparison to the case where Market Makers don't hold shares, is the feedback mechanism between P_0 and the collateral available in period 1, W_0 .

Note that period 1 equilibrium collateral, W_0 , is fix-point solution of the equation

below.

$$W_0^* = (P_0(\alpha_0, W_0^*) - P_{-1})Z_{-1} + W_{-1} \quad (1.20)$$

If a persistent supply shock arrives, the direct effect is the reduction in expected price, and therefore the reduction in period 0 asset price. The indirect effect occurs due to the fact that a lower price reduces the collateral available in period 1, reducing further the expected price, and reducing even more the collateral. The price impact induced by the supply shock is larger when Market Makers hold more shares, as the feedback gets stronger.

Proposition 1.2.12. *Any permanent shock decreases asset prices at $t = 0$. Price changes are amplified when asset holdings are positive.*

Assuming that \check{W}_0 is the equilibrium wealth associated with a supply shock $\check{\alpha}$. The total derivative of P_0 in the neighborhood of $\check{\alpha}$ is show in equation (1.21). The derivative exists as long as $\frac{\partial P_0(\check{\alpha}, \check{W}_0)}{\partial \alpha_0} Z_{-1}$ is less then one, otherwise it would be infinite. Note that, ceteris paribus, positive asset holdings induces larger price changes, due to the term in the denominator being less than one.

$$\frac{dP_0}{d\alpha_0} = \frac{\frac{\partial P_0(\check{\alpha}, \check{W}_0)}{\partial \alpha_0}}{1 - \frac{\partial P_0(\check{\alpha}, \check{W}_0)}{\partial \alpha_0} Z_{-1}} \quad (1.21)$$

The combination of large asset holdings and permanent shock can induce a large price fall. However the price fall may be not associated with Market Makers being financially constrained, but due to the fact that Market Makers initially value the asset less, what causes more losses as they hold shares. With less collateral, Market Makers will value even less the asset, inducing more losses. It is possible that Market Makers are never financially constrained, although prices could fall substantially. The reason is that the difference between expected price and the $P_1(\hat{\alpha})$ decrease when the shock is sufficiently large or wealth is sufficiently low. That happens because

P_1 is bounded below by \hat{D} and, therefore, as the shock in period 0 approaches infinity, both $\mathbb{E}[P_1]$ and $P_1(\hat{\alpha})$ approach \bar{D} . Therefore margin m_0 approaches zero. It is possible that Market Makers will not be financially constrained at the highest margin, in that case there isn't a shock that will ever constrain Market Makers.

In that sense, the equilibrium reaction to persistent shocks is very different than temporary shocks. If the temporary shock is below some threshold, it will have no effect on P_0 , but sufficient large shocks will constrain Market Makers, and will cause prices to fall due to an initial increase risk premium, followed by an amplification mechanism caused by lower wealth. Persistent shocks always reduce the asset price in period 0, initially due to lower Market Makers' asset valuation, causing price and future wealth to fall, forcing Market Makers to lower even more their valuations. It is possible that risk premium will never increase, and the resulting price drop is caused by Market Makers lower valuation.

1.3 Conclusion

In a model with no fundamental shocks, I analyze how asset prices and margin constraints respond to persistent and temporary supply shocks. Market Makers smooth out asset price fluctuations by buying on a margin the extra shares supplied to the market, but, in order to do so, they must provide enough collateral to cover the portfolio depreciation under a tail event, next period.

Because margins limit Market Makers access to credit, their wealth is a state to the next period price distribution. When not financially constrained, the asset price equals the expected price. If Market Makers wealth is sufficient low given the current asset supply, risk premium increases, as Value Investors need to hold part of the asset supply. The more wealth is available in future periods, the more valuable is the asset to Market Makers, since more collateral reduces the chances that they will

be financially constrained. As a result, more wealth increases the expected price, because it is more likely that Market Makers will hold all supply, keeping prices higher than if Value Investors were the marginal buyers.

In the model, shocks are implicitly capturing the entry and exit of outside investors. Persistency is related to the length between the time when some investors sell shares and the time when other investors arrive to purchase them. In the meantime, Market Makers or Value Investors are the ones who hold shares. More persistent shocks is analogous to more time for capital to arrive. Therefore, in the model, persistent shocks shift the subsequent shock distribution, while temporary shocks have no effects on it.

I analyze four different cases, which are combination of whether Market Makers have initial asset holdings or not, and whether the shock is persistent or not. I have shown that without initial holdings the equilibrium is unique, and that there exist a initial Market Makers' wealth which makes the initial margin equal to zero. In that case, Market Makers can absorb any temporary shock, and the asset price is constant. If Market Makers are not so well capitalized, I show the existence of a shock threshold, that larger shocks constrains Market Makers and price falls. Prices fall because the risk premium increases, although the next period price distribution remains unchanged, since neither the shock distribution or Market Makers' wealth change with a temporary shock.

A permanent shock is shown to always reduce the asset price, even when Market Makers are not financially constrained. Permanent shocks shift up the next period shock distribution, and as result, they shift down the next period price distribution. States where Market Makers are financially constrained (and prices are lower) are now more likely, reducing Market Makers' current asset valuation. I also give conditions for period 0 margin to increase with the permanent shock. Margins increase when the tail event price drops more than expected price (which is the current

price when Market Makers are unconstrained). An external observer might associate margin increase and price depreciation with Market Makers being financially constrained. While that is a possibility, that may not be true in the model. Asset prices can fall, because future supply distribution shifts and Market Makers value less the asset today, not necessarily because they are financially constrained.

I also show that multiple equilibria can exist when Market Makers initially hold shares. A bankruptcy equilibrium occurs when low prices wipe out Market Makers, when low prices are the result of Market Makers being expected to be wiped out. This equilibrium exists when wealth is sufficiently low given initial shares held by Market Makers, and Value investors risk aversion.

Because I don't provide equilibrium selection mechanism, I analyze only the non-bankruptcy equilibrium, where Market Makers remain actively trading in future periods. When Market Makers hold sufficiently large initial wealth and initial prices are not sufficiently large, initial margin is zero, so that Market Markers can absorb any temporary shock. If Market Makers hold less capital, I show the existence of a shock threshold, which Market Makers get financially constrained when larger shocks arrive. Below this shock threshold, the asset price is constant. Beyond it, the asset price falls initially due to the increase in risk premium, and after due to an amplification mechanism. As price falls, Market Makers future wealth drops, since they were initially holding asset shares, resulting in lower expected price, and in prices to fall even more. In summary, prices are constant for shocks within an interval. Outside that interval there is a market crash.

Permanent shocks, on contrary, cause different dynamics. Initially they reduce prices because Market Makers value the asset less, as the shock will increase the likelihood that in next period Market Makers will be financially constrained. With positive asset holdings, prices drop more because Market Makers' wealth falls whenever the asset price drops. Lower wealth feedbacks into lower valuation and into

lower prices again. If Market Makers hold large number of shares, the asset price can fall substantially in response to permanent shocks, even when they are not financially constrained.

1.4 Appendix

1.4.1 Proof of Propositions

1.4.1.1 Proof of Lemma 1.2.1

Note that since

$$m_0^+ = \max \{ \gamma \sigma_D^2 [\hat{\alpha} - \gamma^2 \sigma_D^2 \sigma_\alpha^2 (Z_{-1} + \alpha_0)], 0 \} \quad (1.22)$$

the margin is equal to zero when

$$\hat{\alpha} - \gamma^2 \sigma_D^2 \sigma_\alpha^2 (Z_{-1} + \alpha_0) = 0. \quad (1.23)$$

As a result, the risk aversion threshold is shown below.

$$\hat{\gamma} = \sqrt{\frac{\hat{\alpha}}{\sigma_D^2 \sigma_\alpha^2 (Z_{-1} + \alpha_0)}}. \quad (1.24)$$

The threshold $\hat{\gamma}$ is decreasing dividend variance and initial supply, as I wanted to prove.

1.4.1.2 Proof of Proposition 1.2.2

I denote Z_1 as the asset supply in period 1 after the shock is realized. Note first that if W_0 is sufficiently high, Market Makers can drive excess of supply to zero and sustain the price equal to \bar{D} . In this case, the margin is $\bar{D} - \hat{D}$, therefore, we conclude W_0 must be at least as high as $(\bar{D} - \hat{D})Z_1$, in order to avoid being financially constrained.

If $W_0 < (\bar{D} - \hat{D})Z_1$, Market Makers will be financially constrained and price will be lower than \bar{D} . In order to solve for the asset price, one needs to solve the following fix point:

$$P_1 = \bar{D} - \gamma\sigma^2\left(Z_1 - \frac{W_0}{P_1 - \hat{D}}\right). \quad (1.25)$$

The feasible set of solution is between $\max\{\hat{D}, P_1^*(Z_1)\}$ and \bar{D} . Where $P_1^*(Z_1) = \bar{D} - \gamma\sigma_D^2 Z_1$.

The solutions to the fix-point problem are:

$$\frac{P_1^* + \hat{D} \pm \sqrt{(P_1^* + \hat{D})^2 + 4\gamma\sigma_D^2 W_0}}{2}. \quad (1.26)$$

Next, I show that the negative root is unfeasible. Note that if $P_1^* > \hat{D}$ then

$$\hat{D} - P_1^* - \sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0} < 0 \quad (1.27)$$

and that

$$\frac{P_1^* + \hat{D} - \sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0}}{2} < P_1^* \quad (1.28)$$

which is a contraction. If $P_1^* < \hat{D}$. Then note that

$$P_1^* - \hat{D} - \sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0} < 0 \quad (1.29)$$

and, as a result,

$$\frac{P_1^* + \hat{D} - \sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0}}{2} < \hat{D} \quad (1.30)$$

which is also a contraction. Therefore, only the positive root is feasible, and uniqueness of the solution is proved. The other results are straightforward. The asset price is strictly increasing in wealth and strictly decreasing in asset supply when Market Makers are financially constrained. Asset price must be bounded below by \hat{D} , note

that if price is below the expected dividend is because Market Makers are financially constrained. If price is below \hat{D} , margin would be zero and Market Makers could buy an unlimited amount of shares, bringing asset price equal to the expected dividend, what contradicts the fact that they were financially constrained in the first place.

1.4.1.3 Proof of Proposition 1.2.3

Since the margin in period 1 equals $\max\{P_1 - \hat{D}, 0\}$ and the asset price is decreasing in the supply, the margin must be decreasing in asset supply. When Market Makers are financially constrained, the asset price is strictly decreasing in supply and, thus, the margin is strictly decreasing in Z_1 .

Lemma 1.4.1. *P_1 is decreasing in Value Investors risk aversion and is strictly decreasing when supply $W_0/(\bar{D} - \hat{D})$. When risk aversion goes to ∞ , P_1 goes to \hat{D} .*

When $Z_1 > W_0/(\bar{D} - \hat{D})$, Market Makers are financially constrained. Then the price derivative with respect to risk aversion is negative given that market price is greater than P_1^* .

$$\frac{\partial P_1}{\partial \gamma} = -\sigma_D^2 Z_1 \frac{[P_1 - P_1^* + W_0/Z_1]}{\sqrt{(P_1^* - \hat{D})^2 + 4\gamma\sigma_D^2 W_0}} < 0 \quad (1.31)$$

The limit result is due to the monotone convergence theorem. Given any sequence $\{\gamma_n\}$, $\{P_1(\gamma_n)\}$ is strictly decreasing sequence bounded below by \hat{D} . Since \hat{D} is the $\inf\{\gamma_n\}$, when risk aversion goes to infinity, price goes to \hat{D} .

1.4.1.4 Proof of Proposition 1.2.4

The proof is similar to the last proposition. If wealth is greater or equal to $(\mathbb{E}[P_1|W_0] - \hat{P}_1)Z_0$, then Market Makers are not constrained. In this case, the current price is

equal to the next period expected price. If initial wealth is below $(\mathbb{E}[P_1|W_0] - \hat{P}_1)Z_0$, then Market Makers are financially constrained. The fixed-point problem to be solved is:

$$P_0 = \mathbb{E}[P_1|W_0] - \gamma \text{Var}(P_1) \left(Z_0 - \frac{W_{-1}}{P_0 - \hat{P}_1} \right) \quad (1.32)$$

The solution has to be larger than $\max\{\hat{P}_1, P_0^*\}$, because otherwise margins would be zero and Market Makers would not be constrained. The negative root is unfeasible, because that would make $P_0 < \max\{\hat{P}_1, P_0^*\}$, a contradiction.

1.4.1.5 Proof of Proposition 1.2.5

The proof is simple. The wealth available in $t = 1$ is sufficient to sustain the price equal to \hat{D} when shocks at least as large as $\hat{\alpha}$ arrive. That means that

$$m_0 = \max\{P_0 - \bar{D}, 0\} = 0 \quad (1.33)$$

and, therefore, any shock can be absorbed by the Market Makers.

1.4.1.6 Proof of Proposition 1.2.6

Note that, given W_1 expected price can be written as

$$\begin{aligned} \mathbb{E}_{\alpha_1}[P_1(\rho\alpha_0 + \alpha_1)] &= \bar{D}P_r(\alpha_1 < \frac{W_1}{\bar{D} - \hat{D}} - \rho\alpha_0) + \\ &\int_{\frac{W_1}{\bar{D} - \hat{D}} - \rho\alpha_0} P_1(\rho\alpha_0 + \alpha_1) dF(\alpha_1) \end{aligned} \quad (1.34)$$

and the partial derivative as below.

$$\frac{\partial \mathbb{E}_{\alpha_1}[P_1(\rho\alpha_0 + \alpha_1)]}{\partial \alpha_0} = \int_{\frac{W_1}{\bar{D} - \hat{D}} - \rho\alpha_0} \frac{\partial P_1(\rho\alpha_0 + \alpha_1)}{\partial \alpha_0} dF(\alpha_1) \quad (1.35)$$

Since the asset price at $t = 1$ is strictly decreasing in the asset supply, for shocks above $\frac{W_1}{\bar{D} - \hat{D}} - \rho\alpha_0$, it proves that expected price strictly increases with a persistent shock at $t = 0$.

1.4.1.7 Proof of Proposition 1.2.7

First assume that α_0 is large enough to make Market Makers financially constrained in period 1, when $\hat{\alpha}$ arrives. Therefore, $\frac{W_0}{\bar{D}-\hat{D}} - \rho\alpha_0 < \hat{\alpha}$.

$$\frac{\partial m_0}{\partial \alpha_0} = \int_{\frac{W_0}{\bar{D}-\hat{D}} - \rho\alpha_0}^{\infty} \frac{\partial P_1(\alpha_1; \alpha_0)}{\partial \alpha_0} f(\alpha_1) d\alpha_1 - \frac{\partial P_1(\hat{\alpha}; \alpha_0)}{\partial \alpha_0} \quad (1.36)$$

Note that for all $\alpha \geq \hat{\alpha}$, $f(\alpha) \approx 0$. Therefore, as long as $\frac{W_1}{\bar{D}-\hat{D}} - \rho\alpha_0 \approx \hat{\alpha}$, the first term in the last equation is approximately zero. As a result,

$$\frac{\partial m_0}{\partial \alpha_0} = -\frac{\partial P_1(\hat{\alpha}; \alpha_0)}{\partial \alpha_0} > 0, \quad (1.37)$$

since $P_1(\hat{\alpha}; \alpha_0)$ is strictly decreasing the asset supply.

The margin response to the shock is larger with larger γ , since the second derivative is negative when Market Makers are financially constrained.

$$\frac{\partial^2 P_1(\hat{\alpha}; \alpha_0)}{\partial \alpha_0 \partial \gamma} < 0 \quad (1.38)$$

Note that, in order to constrain Market Makers, the supply at time zero has to exceed the buying power W_{-1}/m_0 . Since the margin is bounded above by $\bar{D} - \hat{D}$ and the initial wealth is larger than $(\bar{D} - \hat{D})\hat{\alpha}$, the shock needs to be at least as large as $\hat{\alpha}$.

$$0 < \alpha - \frac{W_{-1}}{m_0} < \alpha - \frac{W_{-1}}{\bar{D} - \hat{D}} = \alpha - \hat{\alpha} \quad (1.39)$$

1.4.1.8 Proof of Proposition 1.2.8

If $W_0 > 0$, P_0 can only be discontinuous at the level of wealth that Market Makers become financially constrained, because both the expected price and price which holds when financially constrained are continuous functions. Define wealth threshold between constrained and not constrained as W_0^* . Therefore, W_0^* must solve $W_{-1} =$

$(\mathbb{E}[P_1(W_0^*)] - P_1(W_0^*))Z_0$. Note that the asset price at W_0^{*+} is given by $\mathbb{E}[P_1(W_0^*)]$, while at W_0^{*-} , the asset price is given by

$$P_0(W_0^{*-}) = \frac{P_0^* + \hat{P}_1 + \sqrt{(P_0^* - \hat{P}_1)^2 + 4\gamma Var(P_1)W_{-1}}}{2}. \quad (1.40)$$

where

$$P_0^* = \mathbb{E}[P_1(W_0^*)] - \gamma Var(P_1)Z_0 \quad (1.41)$$

After substituting both P_0^* and $W_{-1} = (\mathbb{E}[P_1(W_0^*)] - P_1(W_0^*))Z_0$ at the price equation above, we obtain that price is continuous at W_0 .

$$P_0(W_0^{*-}) = \frac{\mathbb{E}[P_1(W_0^*)] - \gamma Var(P_1)Z_0 + \hat{P}_1 + \sqrt{(\mathbb{E}[P_1(W_0^*)] + \gamma Var(P_1)Z_0 - \hat{P}_1)^2}}{2} \quad (1.42)$$

and moreover

$$P_0(W_0^{*-}) = \mathbb{E}[P_1(W_0^*)]. \quad (1.43)$$

1.4.1.9 Proof of Proposition 1.2.9

Assume that $W_{-1} \leq \tilde{W}_{-1} = \max\{\bar{D} - \gamma\sigma_D^2\mathbb{E}[Z_1] - P_{-1}, 0\}Z_{-1}$. Notice that if future wealth is expected to be equal to zero, then the expected price will be equal to $\bar{D} - \gamma\sigma_D^2\mathbb{E}[Z_1]$. Since $P_0 \leq \mathbb{E}[P_1|W_0 = 0]$, Market Makers are wiped out of market when they are expected to lose of all capital in period 0. That is, when

$$W_0 \leq \bar{D} - \gamma\sigma_D^2\mathbb{E}[Z_1] - P_{-1} + \tilde{W}_{-1} = 0. \quad (1.44)$$

Therefore, a equilibrium where Market Makers get bankrupt exists when initial wealth is equal or below \tilde{W}_{-1} .

In order to prove the existence of the non- bankruptcy equilibrium, note that if initial wealth is above \tilde{W}_{-1} , then Market Makers will not be wiped out, when they are not expected to. The lowest price that realize is, thus, \hat{D} , and the lowest future

wealth is still positive.

$$\begin{aligned} W_0 &= (P_0 - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1} \geq \\ &(\hat{D} - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1} = (\bar{D} - P_{-1})Z_{-1} > 0 \end{aligned} \tag{1.45}$$

The next step is to prove uniqueness among the non-bankruptcy equilibria. The equilibrium wealth is the solution to the following equation.

$$W_0 = \underbrace{(P_0(W_0) - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1}}_{h(W_0)} \tag{1.46}$$

Note that the price function is continuous in $W_0 > 0$ and as result $h(W_0)$ is continuous. If W_0 is close to zero, then price will be above \hat{D} and

$$h(0^+) = (\hat{D} - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1} > 0^+. \tag{1.47}$$

The function $h(\cdot)$ is bounded above by $(\bar{D} - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1}$ and strictly increasing in W_0 . Therefore, as W_0 goes to infinity, $h(W_0)$ goes to $(\bar{D} - P_{-1})Z_{-1} + (\bar{D} - \hat{D})Z_{-1}$. Then there exists a unique W_0^* such that $W_0^* = h(W_0^*)$, as I wanted to prove.

1.4.1.10 Proof of Proposition 1.2.11

Define $\tilde{W}_0 = (\bar{D} - \hat{D})(Z_{-1} + \hat{\alpha})$. Notice that there is a positive ϵ that makes $P_1(\hat{\alpha}, W_0 - \epsilon)$ lower than $\mathbb{E}[P_1(\alpha, W_0 - \epsilon)]$ as long as π is close to zero (as assumed in the text). Define $W_0^\epsilon = W_0 - \epsilon$ and W_{-1}^{**} as the unique solution to the equation below.

$$W_0^\epsilon = (\mathbb{E}[P_1(W_0^\epsilon)] - P_{-1})Z_{-1} + W_{-1}^{**} \tag{1.48}$$

Assume that the margin evaluated at this initial wealth level is m_0^ϵ . As shown

above $m_0^\epsilon > 0$. Therefore, the maximum number of shares that Market Makers can buy is W_{-1}^{**}/m_0 , so define $\alpha^{**} = W_{-1}^{**}/m_0$. It is straightforward that any shock above α^{**} will constrain Market Makers.

Note that if shock is below α^{**} the asset price will not change, as price P_0 will be equal to $\mathbb{E}[P_1(W_0^\epsilon)]$. Shocks above α^{**} will cause the price to fall below $\mathbb{E}[P_1(W_0^\epsilon)]$, since some shares will be held by Value Investors. The new equilibrium price is given by the following equations.

$$P_0(W_0^{***}) = \frac{P_0^* + \hat{P}_1(W_0^{***}) + \sqrt{(P_0^* - \hat{P}_1)^2 + 4\gamma \text{Var}(P_1|W_0^{***})W_{-1}}}{2} \quad (1.49)$$

$$W_0^{***} = (P_0(W_0^{***}) - P_{-1})Z_{-1} + W_{-1}^{**} \quad (1.50)$$

Note that $W_0^{***} < W_0^\epsilon$ because Market Makers loose wealth when Value Investors hold shares. Note also $P_0(W_0^{***}) < \mathbb{E}[P_1(W_0^\epsilon)] = P_0(W_0^\epsilon)$, for two reasons: 1) price is lower due to the increase in risk premium and 2) due to lower future wealth. That proves the results of Proposition 1.2.11.

CHAPTER 2

Repo Financing of Illiquid Securities

2.1 Introduction

Repurchase agreements (repos) are a form of collateralized lending, whereby large financial institutions obtain funds to purchase assets with leverage.¹ This paper develops a new framework to analyze the mechanics of repo financing of illiquid securities, using an equilibrium model that prices both repos and assets. The theory is motivated by data. Using a recent dataset of money market mutual funds (MMF) portfolio holdings, I document empirical facts of how broker-dealers finance assets with repos. First, securities with low transaction costs, such as US Treasury bonds, are financed with repos that have shorter maturities, lower haircuts and lower rates than the repos used to finance less liquid securities, such as corporate bonds and asset backed securities. Second, I find that lending preferences of MMF are heterogeneous, as they offer different mixes of repo maturities depending on fund size and overall portfolio maturity.

Motivated by these new facts and the observation that repo lending is scarce in comparison to the total asset market, I develop a model to address: What assets should be financed through repos? What determines repo terms, maturity choice,

¹Repurchase agreements (repos) are collateralized loans that the borrower agrees to sell securities to the lender at a specific price and commits to repurchase the same securities at a later date for some other price. These contracts are important financing instruments to large financial institutions in the US and in Europe, who use repos to on a daily basis. Figure(1) shows a repo contract.

interest rates and haircuts?² What are the asset pricing effects of repos on liquid versus illiquid securities?

In the model, a small number of patient lenders offer repo financing to asset buyers, who purchase securities with different transaction costs. Due to regulation constraints or preferences, lenders offer different maturities: some roll over debt at short and others at longer maturities. Because lenders are scarce, not all assets are financed with repos, and therefore the allocation of securities to lenders is a matching problem. The main result is that longer maturity lenders have a comparative advantage in financing less liquid securities. The assets with highest transaction costs are left to be financed by buyers' personal funds, as there is less gain to finance these assets. Within the group of securities financed with repos, there is assortative matching in the equilibrium: less liquid securities are financed by long maturity lenders. Consistent with the data, haircuts and interest rates increase with collateral illiquidity while asset prices fall with transaction costs. Moreover, the existence of the repo market increases the relative prices of liquid versus illiquid assets. The greater is the difference between discount rates between lenders and borrowers, the greater are the equilibrium relative prices of liquid versus less liquid securities. In addition, I show that when debt is less likely to be rolled over, the relative prices of liquid to illiquid securities, haircuts and interest rates of illiquid securities increase.

Throughout the paper use the term asset buyer (or just buyer) to refer to borrowers. In repo markets, different jargons are used. Since repo is a lending agreement in which securities are sold and repurchased later, buyer is considered to be the entity which first buys the asset and the seller is the one who first sells the asset. In this paper, the asset buyer is the agent who borrows money and buys an asset with the borrowed capital.

²Haircut is the depreciation rate applied in the security market value to match the price paid by the lender when the repo contract was signed. See figure 2.1.

The match between buyers and lenders generates positive surplus, because lenders discount future cash flows at lower rates. Yet, since there are fewer lenders and free entry of buyers, the surplus is fully appropriated by lenders. Therefore, an impatient buyer can purchase one of the many securities available with credit supplied by a lender, but she can't be better off than financing the asset with her own funds³. The repo terms, interest rate and haircut are set by lenders to maximize their profits, subject to the buyer's participation constraint and with the restriction that the repo must be default-free.

After forming a relationship to buy securities, buyers enjoy the asset's dividends and choose whether to terminate the repo or to roll it over at maturity. But buyers have no reason to end the repo⁴, unless something forces them to do so. In the model, buyers roll over debt multiple times, until all lenders refuse to finance their debt, after a shock arrives. The shock itself is interpreted as a reduction in the buyer's credit ratings, and it is crucial for the assortative matching result. Due to this shock, the buyer-lender match that uses long maturity repos tend to last more than a match using short maturity repos, since the time between the shock and the maturity date is larger. For this reason, some lenders are better suited to finance assets that more costly trade.

All assets pay the same amount of dividends, but have different transaction costs. Since buyers incur these costs whenever they cannot roll over their repo contract, if transaction costs are increased and asset prices kept constant, the buyers' expected utility of holding the repo decreases. As a result, lenders must offer buyers better terms, either by reducing interest, haircut or both to satisfy the buyer's participation constraint. Larger transaction costs also impose more restrictions on the quantity

³MMF returns are close to Fed Funds Rates as these funds benchmarks are short-term interest rates. Therefore one can imagine that required returns of these funds are very low. On the contrary, broker-dealers cost of funds are much larger, in recent years the their weighted average cost of capital was about 5% per year.

⁴Since he will get the same repo term if he change lender.

of credit that can be lent, as repayment must not exceed the security's resale value. Illiquid securities demand larger haircuts than more liquid ones, because assets with higher transaction costs lose more value when sold. The buyer's down payment must be at least as large as the asset trading cost, in order to guarantee that she gets non-negative utility when selling the asset to repay her debt, upon forced termination of the repo. Otherwise, the buyer prefers to default and leave the market, an option that gives zero utility.

Lenders who offer longer maturities are better suited to finance less liquid securities. If illiquid securities were financed with short maturity repos, larger transaction costs would be incurred faster, and in compensation for both larger expected trading costs and the fact that dividend flows stop sooner, the short maturity lender must offer buyers much larger utility. Therefore, financing illiquid assets with short repos requires substantially smaller haircuts (more credit) or lower interest. But illiquid securities cannot be financed with small haircuts, because repos must be default free, thus, lenders' only (unprofitable) option is to offer contracts that charge buyers low interest. In contrast, longer maturity lenders don't face the same issue, since repos that are rolled over at lower frequencies last longer. They allow assets to pay more dividends and to incur transaction costs less often. These lenders obtain larger benefits from financing illiquid securities, because, in addition, long maturity repos exploit more the differences in discount rates between patient lenders and impatient buyers, as payments are made later in the future, so that they can charge more interest from buyers.

I find repo terms and asset prices that sustain the optimal allocation of assets in the market equilibrium. Prices must be such that lenders offering long maturity repos are better off financing illiquid securities. In order to make the repo financing of these assets relatively more profitable, the price of liquid securities must be sufficiently higher than those of illiquid securities. Larger prices will reduce the amount

of interest that can be charged from a buyer and will increase the loan's size in order to keep buyers satisfied. As a result, the repo financing of liquid securities is less profitable to long maturity lenders. Since repo financing of the illiquid securities is unprofitable to short maturity lenders, separation exists, as they still prefer to finance low transaction cost securities, even with higher prices. The greater is the difference in discount rates, the greater is the necessary price increase in liquid securities to achieve this separation. This happens because with lower discount rates, the lenders opportunity cost of investment is reduced, and therefore larger loans to finance liquid securities cost less to lenders. Therefore, these assets must be very expensive to force repo interest rates to be sufficiently low.

If the likelihood that repos are terminated increases, liquid securities must be relatively more expensive to achieve separation. This occurs because expected transaction costs increase, as repos are terminated more quickly and costs are incurred sooner, making the repo financing of illiquid securities less profitable, *ceteris paribus*. Liquid securities must cost relatively more to prevent long maturity lenders from preferring these securities.

This paper contributes to the recent literature on repos that studies the behavior of both asset and credit markets, and to the literature that studies asset pricing with transaction costs. My contribution is both empirical and theoretical. My work is closely related to Amihud and Mendelson [1986], where assets differing in transaction costs are traded by agents with different investment horizons. I add the repo market to the asset pricing analysis, something absent in their paper. My research is also related to Geanakoplos [2010] and more recent papers that study the joint equilibrium in the asset and collateralized credit markets. One main difference is that my model has multiple securities varying in liquidity (transaction costs) and the trading motives exist due to different discount rates, while in Geanakoplos [2010], the gains originate from differences in optimism. The other difference is that my

model allows for maturity choice. He and Xiong [2012a] also study maturity choice and show, using Geanakoplos' framework, that short maturity debt is preferred to long maturities. I show that lenders offering longer maturities have greater utility than ones offering short maturity repos, I explain why liquid and less liquid securities are financed with short and long repos, respectively, and I derive asset pricing implications.

The remainder of the paper proceeds as follows. In the next section, I describe the data and the main empirical facts of repos between MMF and broker dealers. In section 3, I describe the model and find the equilibrium repo terms, maturity choice, haircuts and interest rates, and asset prices. Proposition 3 is the main result: I find assortative matching between assets and maturities. I find asset prices and repo terms that clear both markets. In the following section, I solve one numerical example in order to illustrate the main mechanisms underlying the results.

2.2 Empirical Facts

I present evidence of clienteles in repurchase agreements between broker-dealers and Money Market Mutual Funds(MMF). Broker-Dealers, the borrowers, tend use short maturities to finance treasuries and government agency securities, securities known to have smaller bid-ask spread⁵. Alternatively, broker-dealers use longer maturities to finance other securities such as corporate bonds, equities, asset backed securities and Mortgage Backed Securities. These securities not only tend to have on average larger round-trip costs but also more variation in bid-ask spreads⁶. Clienteles in maturities are also present among lenders. MMF differ in the average maturity

⁵Broker-Dealers in the data are primary dealers with the Federal Reserve which are also prime brokers. They are the largest financial institutions: Morgan Stanley, Goldman Sachs, Deutsche Bank AG, Credit Suisse, Barclays, Citibank, Merrill-Lynch, Wells-Fargo, JP Morgan Chase.

⁶Among those only Corporate Bonds and Equities are publicly traded.

of the repos which they hold. Some Funds, specially the smaller ones, pursue low average portfolio maturity and tend to invest mostly in overnight repos. Larger Funds have longer weighted average portfolio maturities and invest in repos that mature longer.

This new empirical facts are found in SEC filing data for MMF, which are required to declare their portfolio holdings to the SEC every month. I write a python script to download and parse all the N-MFP forms filed from 2011 to 2013 to compile information about fund demographic information and portfolio holdings. Repos compose about 20% of the total securities that MMF held in the period, an average of \$544 billions per month, but not all funds invest in repos. According to the latest calculations from the Federal Reserve Bank, the tri-party repurchase agreement market totals \$1.6 trillions, thus repos in the MMF data are about a third of the total tri-party market⁷.

Each repurchase agreement report contains borrower identity, interest rate, maturity, principal amount, collateral type and market value of the securities used as collateral. I compute haircuts from the principal amount and the total collateral value and observe a large variation in repo's maturities and haircuts, not only among different collateral classes, but also significant variation within particular asset classes. *Treasury Repos*, repurchase agreements backed by treasuries, have on average very low maturities, haircuts close to zero and low variation in both in maturity and haircuts. Similar facts hold for *Government Agency Repos*, since haircuts and maturity distributions have low means and low standard deviations, but still larger than Treasuries. The remaining large class of repurchase agreements, *Other Repos*, are the ones backed by Corporate Bonds, Stocks, Asset Backed Securities, Mortgage Backed Securities and Mixed Securities Pools. Unlike government agency

⁷Tri-Party Repurchase Agreements are repos that clearing agencies participate in the transaction. The two largest clearing institutions are JP Morgan and BNY Mellon.

and treasury repos, these securities are financed through repos with longer average maturities, larger haircuts but substantial variation in both in maturities and haircuts. Details are shown on table (1). The correlation between the haircut rate applied to collateral and repo maturity size is .383 in a sample of 104,000 repurchase agreements.

The Lenders

Money Market Funds (MMF) are always the lenders in the repos reported. Originating in 1970's, MMF are mutual funds that generally provides higher returns than interest-bearing bank accounts. They have grown significantly and nowadays hold about \$2.5 trillion in assets. In response to the 2007-2008 financial crisis, the Security Exchange Commission (SEC) adopted a series of amendments to its rules to make MMF more resilient to crises. Amendments to rule 2a-7 require money market funds to restrict their underlying holdings to investments that have shorter maturities and higher credit ratings than those previously permitted to be held. The average dollar-weighted portfolio maturity of investments held in a MMF cannot exceed 60 days. No more than 3% of assets can be invested in securities that do not fall within the first or second-highest tier of the top credit rating agencies. Taxable funds must hold at least 10% of their assets in investments that can be converted into cash within one day. At least 30% of assets must be in investments that can be converted into cash within five business days. No more than 5% of assets can be held in investments that take more than a week to convert into cash.

MMF invest on repos as way to diversify their portfolio with securities that are considered liquid and safe. But repos are also practical instruments to keep overall portfolio maturity under the limits imposed by the SEC. If some fund holds assets with maturities exceeding 60 days, it needs to invest some other portion of its capital in securities with shorter maturities, in order to keep the overall portfolio maturity

within the allowed limits. While might be not a simple task to find liquid short-term assets being traded in secondary markets, a MMF can lend using a repo customized to satisfy its maturity requirements.

The amount invested in repos as share of portfolio value vary significantly among funds, ranging from close to zero to almost 100%. In general, larger funds hold repos of larger maturities. Funds which do not hold repos regularly invest mostly overnight repos. The correlation between fund size and average maturity of all repos held by a fund is .442, while the correlation between the longest repo maturity held and the fund size is .34.

Motivated by these empirical facts, I construct a model in the next section to explain why lenders and borrowers are matched in manner presented in data: Securities with high haircuts are financed with repos that matures in longer periods, while the opposite occur to securities with low margins. I assume that assets have different transaction costs with a known distribution and lenders, due to preferences or regulatory restrictions, can be sorted by the maturities that they offer. I show that, in equilibrium, there is assortative matching between securities and lenders to finance them. Equilibrium haircuts, interest rates and collateral prices adjust to insure that long maturity lenders, who have better conditions to finance illiquid securities, prefer to finance the securities with higher transaction costs.

2.3 Model

Securities In a given competitive market, a continuum of riskless securities are traded. Each security is identified by its transaction cost $c \in [c_l, c_h]$, with $0 < c_l < c_h < 1$. The cumulative measure of securities with transaction cost below c is denoted by $S(c)$, a continuous differentiable function. Whenever security c is traded, the buyer as well the seller must pay a proportional cost c . That is, if security c 's

price is $p(c)$, the buyer pays $p(c)(1 + c)$ and the seller receives $p(c)(1 - c)$ in order to complete the transaction. Aside from transaction costs, securities are identical and all pay a continuous flow of dividends normalized to one, until infinity.

Investors The market has *2 types of participants*: a large number of identical *buyers* and heterogeneous *lenders*. Lenders are long lived and are risk neutral, while buyers are also risk neutral, but their investment horizon is on average shorter. They remain active in the market for an average of $1/\kappa$ years, leaving the market after the arrival of a liquidity shock, which is distributed exponentially with parameter κ . While buyers discount future utility at rate r , lenders discount at rate \hat{r} , which is strictly less than r . Lenders are not allowed to buy securities but can lend cash to buyers to purchase assets through repos. Each lender is identified by ϵ , the maturity intensity parameter, distributed according to F_ϵ in a positive interval $[\epsilon_l, \epsilon_s]$. A type- ϵ lender offers loans whose maturity is on average $1/\epsilon$, as explained below. Buyers are allowed to purchase at most one security, either financed through their own personal funds, or through a repo with one lender, and lenders can finance at most one buyer.

Repurchase Agreements A repurchase agreement (repo) is a loan which is backed by the security purchased with credit. I define the repo *terms* as the triplet (ϵ, h, w) . It consists of the average *maturity* $1/\epsilon$, the *haircut rate* h , and the *interest payment* w .

Haircut Rate The haircut rate h is the proportion of the security price which is paid with the buyer's personal funds. The buyer makes a down payment of $hp(c)$ and receives a loan of $(1 - h)p(c)$, which I term as *principal amount* of the debt.

$$\mathbb{P} \equiv (1 - h)p(c) \tag{2.1}$$

After being purchased, the security is transferred to the lender as collateral, and it is only returned after the principal amount plus interest is paid. The security's dividends are paid to the buyer as long as the loan remains in good standing.

Maturity Dates The repurchase agreement maturity date, τ_ϵ , is a random variable with exponential distribution, with mean $1/\epsilon$. Assuming a stochastic maturity with an exponential distribution makes the model more tractable: it eliminates one state variable, time to maturity, thus making it easier to keep track of the repo contract distribution. Throughout the text, I use the word maturity to refer to the average time until the next payment is due. A larger parameter ϵ represent repos whose interest payments must be made in (on average) shorter time periods. Buyers can choose from a continuum of maturities, since there is a continuum of lenders types. The shortest available maturity is $1/\epsilon^s$ and the longest is $1/\epsilon^l$.

When the maturity time τ_ϵ arrives, the buyer decides whether to extend (roll over) the repo at same original terms (ϵ, h, w) or to terminate it. Buyers, however, have no reasons to terminate a repo, they prefer to extend (roll over) the repurchase agreement, because it is more profitable to continue receiving dividend flow and avoid paying the asset's transaction cost. If attempting to roll over the repo, the buyer must make the prearranged interest payment w to keep the loan in good standing.

Repo Termination The disadvantage of financing securities with repos is that, at some point, lenders might refuse to refinance the debt. I introduce rollover risk through an exogenous *termination* shock, which prevent repos to be rolled over next maturity date. This shock represents a run on a buyer's debt. As a result, buyer walks away after repo is terminated, since all lenders will refuse to provide credit to

her⁸. I assume that the time until the termination shock arrives, $\tau_\beta - t$, is distributed independently among buyers with exponential distribution and mean $1/\beta$. A larger β increases the repo termination risk, since it reduces the shock's average arrival time. I make the assumption that $\beta \geq \kappa$, that is, the buyers' investment horizon is reduced if she attempts to finance assets with repos. One advantage of longer maturities is that the buyer enjoys the security's dividend stream for longer periods after the shock arrives, since the repo is terminated only at the next maturity date. Therefore, contracts that are rolled over after longer periods of time last on average longer.

In what follows, I will refer to "rollover dates" as dates when the repo is rolled over and "termination dates" as dates when the repo comes to an end.

Assumption 2.3.1 (Default Free Repos). *Lenders only accept riskless repurchase agreements.*

Assumption 2.3.1 is motivated by the MMF's investment purposes and the rules which they must comply with. The main objective of these investment funds is to be a safe investment instrument⁹, since SEC's rules constrain MMF to invest in highly rated debt. The consequence of assumption 1 is that the repo terms (ϵ, h, w) must be such that the buyer never defaults on the scheduled payments. It reduces feasible contracts to non-defaultable ones and generates a lower bound in haircut rates.

Assumption 2.3.2. *The measure of lenders is strictly smaller than the measure of securities, which is strictly smaller than the measure of buyers.*

Assumption 2.3.2 states that the quantity of available financing is smaller than the number of securities, and that there are more buyers than securities. The result

⁸See Duffie(2010) for the mechanics of dealer-banks failure.

⁹MMF can only purchase debt from companies with very high credit ratings from at least two of the three most regarded rating institutions.

is that only a portion of securities will be financed through repos and not all buyers can purchase the asset. This assumption impacts how surplus is shared between buyers and lenders.

Definition 2.3.1. Security c asset resale value is $p(c)(1 - c)$, trading cost is $p(c)c$ and round-trip cost is equal to $p(c)2c$.

2.3.1 Buyers' and Lenders' Value Functions when holding Repos

Define $V_B(c, \epsilon, h, w)$ as the buyer's value of holding security c through a repo (c, ϵ, h, w) with lender ϵ , after she receives the loan. The buyer receives the unity flow of dividends until the maturity date τ_ϵ arrives. If the termination shock has not arrived, that is, if $\tau_\epsilon < \tau_\beta$, the repo is rolled over after the buyer makes the interest payment w , or he defaults and walk away.

$$V_B = \mathbb{E}_t \left[\int_t^{\tau_\epsilon} e^{-r(s-t)} 1 ds + e^{-r(\tau_\epsilon-t)} [\max\{V_B - w, 0\} \mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} + \max\{\mathbb{V}^P, 0\} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}}] \right] \quad (2.2)$$

If the termination shock occurs, the loan is terminated at the maturity date. The buyer has the option to repay the loan, an option valued \mathbb{V}^P , or to default. If she defaults, the collateral is sold by the lender and she is left with zero discounted utility. If the buyer chooses to repay, she sells the securities and pays the principal amount plus the interest with the cash received in the asset market.

$$\mathbb{V}^P \equiv \underbrace{p(c)(1 - c)}_{\text{resale value}} - \underbrace{((1 - h)p(c) + w)}_{\text{principal+interest}} \quad (2.3)$$

Define $V_L(c, \epsilon, h, w)$ as lender ϵ value of a repo (ϵ, h, w) with security c as collateral. I write below lenders' value already assuming repayment at rollover and

termination dates, which is true in the equilibrium, since repos are restricted to be default free.

$$V_L(c, \epsilon, h, w) = \mathbb{E}_t[e^{-\hat{r}(\tau_\epsilon - t)}[(V_L + w)\mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} + (w + (1 - h)p(c))\mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}}]] \quad (2.4)$$

If no shock has arrived between the last rollover date, at maturity, the lender receives the interest payment and extend the repo under the original terms (ϵ, h, w) . Otherwise, the repo must be terminated, the asset must be sold and the lender receives the principal amount plus interest payment. Proposition 2.3.1 provides the solution to the value function equations (2.2) and (2.4).

Proposition 2.3.1. *The values of a repurchase agreement to a buyer and a lender are given by*

$$V_B(c, \epsilon, h, w) = \frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} - \frac{\epsilon}{r + \beta}w + \frac{\epsilon\beta}{(r + \epsilon)(r + \beta)}[p(c)(1 - c) - (1 - h)p(c) - w] \quad (2.5)$$

and

$$V_L(c, \epsilon, h, w) = \frac{\epsilon}{\hat{r} + \beta}w + \frac{\epsilon\beta}{(\hat{r} + \epsilon)(\hat{r} + \beta)}[(1 - h)p(c) + w], \quad (2.6)$$

respectively.

Buyers' and lenders' value functions are derived in the appendix. Notice that, for a buyer, the value of holding a repo is divided in three terms: the discounted value of dividends received until the repo is terminated (first term in equation (2.5)), the discounted cost of interest payment at rollover dates (second term in equation (2.5)), and the discounted value of repayment done in termination dates (third term in equation (2.5)). A lender's value function is analogous, although a lender doesn't receive asset dividends, and she discounts the flow of payments at a lower rate \hat{r} . The first term in equation (2.6) is the discounted value of interest payment at rollover dates, while the second term is the discounted value of repayment in termination dates.

2.3.2 Repurchase Agreement Design

I assume that lenders are able to extract full surplus in a match, because there is free entry of buyers and limited number of lenders. If one buyer had strictly positive surplus, all the others would try to outbid her and offer more surplus to the lender. Therefore, when financing security c through a repo, the buyer is quoted repo terms (ϵ, h, w) that gives the same utility as funding the asset with personal funds.

$$\underbrace{\Pi_B(c, \epsilon, h, w)}_{\text{Profit Buyer}} \equiv \underbrace{V_B(c, \epsilon, h, w)}_{\text{Repo Value Buyer}} - \underbrace{hp(c)}_{\text{Down Payment}} - \underbrace{p(c)c}_{\text{Transaction Cost}} \quad (2.7)$$

I denote by Π_B and by Π_L , the buyers' and lenders' profit functions, respectively. The buyer's profit is defined as the repo value minus the down payment and the security transaction cost, $p(c)c$, while, for a lender ϵ , the profit is defined as the repo value minus the principal amount.

$$\underbrace{\Pi_L(c, \epsilon, h, w)}_{\text{Profit Lender}} \equiv \underbrace{V_L(c, \epsilon, h, w)}_{\text{Repo Value Lender}} - \underbrace{(1-h)p(c)}_{\text{Principal}} \quad (2.8)$$

The optimal repo term must satisfy four constraints. The first two are participation constraints: the repo agreement must be beneficial to both agents, which is the case when the lender's and buyer's profits are positive, since their opportunity cost is zero.

$$\Pi_B(c, \epsilon, h, w) \geq 0 \quad \text{and} \quad \Pi_L(c, \epsilon, m, w) \geq 0 \quad (2.9)$$

The other two are non-default constraints. The repo terms (ϵ, h, w) must be such that it is never optimal for buyers to skip any of the scheduled payments. Non-default constraint [A] exists to avoid default at termination dates, while non-default constraint [B] to rule out default at rollover dates.

$$\underbrace{p(c)(1-c)}_{\text{Resale Value}} - \underbrace{((1-h)p(c)+w)}_{\text{principal+interest}} \geq 0 \quad (\text{Non Default Constraint [A]}) \quad (2.10)$$

$$V_B(c, \epsilon, h, w) - w \geq 0 \quad (\text{Non Default Constraint [B]}) \quad (2.11)$$

2.3.3 Lender's Problem and Repo Term Quotes

A lender ϵ is approached by multiple buyers in search of funding for different securities. I break down the lender's maximization problem in two steps. In the first, the lender quotes haircuts and interest payment to maximize her expected profit subject to the buyer's participation and the non-default constraints. In second step, since the lender can only finance one security at a given time, she chooses to finance the security that gives the largest expected profit, among all possible securities. The next two equations formalize these ideas. Equation (2.12) displays the indirect profit function of a lender ϵ when financing security c , while equation (2.13) gives the maximum expected profit that lender ϵ could achieve. Furthermore, it is useful to define $a(\epsilon)$ as the optimal asset choice of lender ϵ , the arg max of equation (2.13).

$$\Pi_L^*(c, \epsilon) = \max_{h,w} \Pi_L(c, \epsilon, h, w) \quad \text{s.t. to (2.9), (2.10) and (2.11)} \quad (2.12)$$

$$\Pi_L^{**}(\epsilon) = \max\left\{ \max_{c \in [c_l, c_h]} \Pi_L^*(c, \epsilon), 0 \right\} \quad (2.13)$$

I define below a market equilibrium that the asset allocation is determined by a matching function φ . Later, I look for a specific matching function that allocates efficiently assets among lenders, and find asset prices and repo terms associated with that allocation. Since the measure of securities exceed the number of lenders, not all assets are financed by repos, thus some securities will funded with buyers' personal funds. Thus, it is helpful to define a partition of the set of securities. I denote the

set of unmatched securities by \mathbb{C}_u , while the set of securities financed by lenders by \mathbb{C}_m . A matching function φ , therefore, maps \mathbb{C}_m into the set of lenders $[\epsilon_l, \epsilon_s]$.

Definition 2.3.2. An equilibrium is asset prices $p(\cdot)$, lenders' optimal asset choice $a(\cdot)$, an asset set partition $\{\mathbb{C}_m, \mathbb{C}_u\}$, a matching function $\varphi : \mathbb{C}_m \rightarrow [\epsilon_l, \epsilon_s]$, and repo terms $(w^*(c), h^*(c), \varphi(c))$ for each security $c \in \mathbb{C}_m$ such that:

1. Given asset prices $p(\cdot)$, each lender ϵ quotes haircuts $h(c, \epsilon)$ and interest payments $w(c, \epsilon)$ to each security in order to solve (2.12), and chooses the optimal asset to finance $a(\epsilon)$ to maximize (2.13).
2. Lenders' demands are equal to the allocation proposed by φ , that is, $\varphi^{-1}(\epsilon) = a(\epsilon)$ for all ϵ .
3. For each security, $w^*(c) = w(c, \varphi(c))$ and $h^*(c) = h(c, \varphi(c))$.
4. The asset market clears.

I construct the equilibrium in the following steps. First, I obtain the optimal repo quotes by solving each lender's maximization problem (equation (2.12)). Next, I find a partition of the security set and a matching function that allocates efficiently assets to lenders. Finally, I calculate the equilibrium asset price and repo terms that sustain that allocation. Lemma (2.3.2) and proposition (2.3.3) show that buyers' participation constraint and non-default constraint [A] bind in all matches. These two equations allow us to derive the principal amount, haircut and interest payment quotes for any pair (c, ϵ) . As a result of lemma 2.3.2, lenders offer the zero utility to buyers, same utility that they would obtain when financing assets with personal funds, due to the free entry of buyers.

Lemma 2.3.2. *The buyer's participation constraint binds. The buyer indifference*

condition is given by the following equation.

$$\frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} (1 - \epsilon w - rhp(c)) = \left(1 + \frac{\epsilon\beta}{(r + \beta)(r + \epsilon)}\right) p(c)c \quad (2.14)$$

Note that if the buyer participation constraint did not bind, the lender could increase profits with a slightly larger interest payment. Therefore, in order to extract full surplus, the lender chooses w and h so that expected net utility flow, the left side of equation (2.14), is equal to the expected discounted transaction costs, the right side of equation (2.14). Note also that assets with low c allow lenders to offer lower net utility flow to buyers, through larger ϵw and $rhp(c)$, because the expected discounted transaction costs are smaller. For any given asset transaction cost, it is also true that lenders offering longer maturities can give less net utility flow to buyers, because the expected discounted transaction costs is smaller, as these contracts last on average longer.

Proposition (2.3.3) shows that non-default constraint at termination dates bind, and figure (2.2) gives the intuition of the result. The buyer indifference curve is displayed in blue. Above this curve, the haircut and interest payment combination gives negative utility to buyer, while below the curve, the buyer obtains strictly positive utility. The optimal contract is one that gives zero utility to buyer, and is in the feasible set, the light blue area. Larger haircut allows lenders to charge more interest, therefore, the boundary of the set is increasing in the haircut rate. Repo term A is not the optimal, because lenders discounts future payments less than buyers, and, at point B, where both more credit is given and more interest is charged, lenders obtain higher utility while buyers are indifferent. This result is summarized below.

Proposition 2.3.3. *Non-default constraint [A] binds in equilibrium, thus the principal amount plus interest equals the security resale value $p(c)(1 - c)$. The principal*

amount and interest payment as functions of (c, ϵ) are

$$(1 - h(c, \epsilon))p(c) = p(c) \left(1 + \left(\frac{r + \beta - \epsilon}{r + \beta + \epsilon} \right) c \right) - \frac{1}{(r + \epsilon)} \quad (2.15)$$

and

$$w(c, \epsilon) = \frac{1}{(r + \epsilon)} - \left(\frac{r + \beta}{r + \beta + \epsilon} \right) p(c)(2c), \quad (2.16)$$

respectively.

Proof in the appendix. After rearranging the principal amount equation (2.15), one obtains the haircut quote as a function of transaction costs and lenders' maturity intensity. Thus, the repo quotes to finance security c are given by equations (2.16) and (2.17).

$$h(c, \epsilon) = \frac{(r + \epsilon)^{-1}}{p(c)} - \frac{(r + \beta - \epsilon)}{(r + \beta + \epsilon)} c \quad (2.17)$$

2.3.4 Efficient Allocation of Assets to Lenders

I look for an efficient asset allocation among lenders, and later I find asset prices which sustain that allocation as an equilibrium. Using assortative matching insights, I demonstrate that it is efficient to allocate assets with higher trading costs to lenders offering longer maturities. The surplus between a buyer and lender is a function of both the lenders maturity intensity, the security's price and transaction cost, as defined in the equation (2.18) below.

$$\begin{aligned} \Sigma(c, \epsilon) = & \frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} + \frac{\epsilon(r - \hat{r})w}{(\hat{r} + \beta)(r + \beta)} + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} p(c)(1 - c) \\ & - p(c)(1 + c) \end{aligned} \quad (2.18)$$

The surplus of pair (c, ϵ) is divided in four terms: the expected discounted dividends until the repo is terminated, the difference between present values of interest payments discounted at lenders' rate and of interest payments discounted at buyers' rate (DPV), the discounted payment received by the lender when the repo is terminated

and last, the security purchase cost. The second and third terms are decreasing in transaction costs for any fixed maturity, because larger transaction costs limits the repayment value at termination dates and, thus, the credit exchanged. As result, when financing less liquid securities 1) less interest can be charged by the lender and 2) lower payments are received by lender when the repo is terminated.

Proposition 2.3.4. *The surplus function $S(c, \epsilon)$ is submodular when the securities' resale value is strictly decreasing in c and round trip costs are increasing in c .¹⁰*

$$\frac{\partial^2 \Sigma(c, \epsilon)}{\partial c \partial \epsilon} < 0 \quad (2.19)$$

Proposition (2.3.4) (proved in the appendix) has two important consequences. First, it is efficient to allocate assets with high transaction costs to be financed by lenders with longer maturities, and second, when financing less liquid assets, lenders who offer longer maturities increase more their surpluses than lenders offering shorter maturities. These two facts actually have the same meaning and they can be both verified in equations (2.20) and (2.21), which are discrete versions of equation (2.19). If a long maturity lender ϵ' attempts to finance security with larger c , for instance c'' instead of c' ($c'' > c'$), she can get larger increase in surplus than a low maturity lender ϵ'' (equation (2.20))¹¹. Therefore, if the long maturity lender can actually increase more her surplus, therefore, in the efficient allocation, she must be the one who finances the less liquid asset, because the aggregate surplus is larger (equation (2.21)).

$$\Sigma(c'', \epsilon'') - \Sigma(c', \epsilon'') < \Sigma(c'', \epsilon') - \Sigma(c', \epsilon') \quad (2.20)$$

$$\Sigma(c'', \epsilon'') + \Sigma(c', \epsilon') < \Sigma(c', \epsilon'') + \Sigma(c'', \epsilon') \quad (2.21)$$

¹⁰ *Rewriting the Surplus as function of maturity $\mu \equiv 1/\epsilon$ instead, we get $\frac{\partial^2 \Sigma(c, \mu)}{\partial c \partial \mu} > 0$. In that case, the function is supermodular, but the intuition is exactly the same.*

¹¹ Long maturities have low ϵ , while short have high ϵ .

Longer maturities allow buyers to receive on average more dividends, have lower discounted transaction costs, because repos are terminated later after the termination shock, and explore more the gains of trade, since longer horizon exploits more the difference in discount rates. Shorter maturities, on the contrary, offer buyers on average less dividends, larger discounted transaction cost, and thus buyers need to be compensated with lower haircuts and interest payments. But since securities with larger transaction costs command higher haircut rates in order to keep lending risk free, short maturity lenders are forced to charge substantially lower interest from buyers when financing these securities. As a result, long maturity lenders have a comparative advantage in financing illiquid securities. Repo financing of less liquid securities, nonetheless, must be optimal for long maturity lenders in equilibrium. The main reason why they may prefer to fund liquid instead of illiquid securities is that low transaction costs restrict less the credit amount, since liquid securities don't require a large minimum haircut. Later, I show that long maturity lenders obtain larger interest rates when financing illiquid securities, because liquid securities prices increase, reducing the amount of interest that they could otherwise charge buyers. The relative price change is how the economy adjusts to the equilibrium.

Before finding asset prices that clear both credit and asset markets, one needs to find the matching function which allocates assets to lenders.

2.3.5 The Matching Function

In order to calculate the efficient matching function, guess and verify the existence of assortative matching between assets and maturities. I argue that in the equilibrium only assets below some c^* (yet to be found) are financed through repos. The logic behind this argument is that transaction costs are inefficiencies that should be avoided, if possible, because they limit the credit exchanged between buyers and lenders. It can actually be shown that there is no equilibrium where only the most

illiquid assets are funded with repos¹².

Therefore, in the efficient allocation, all securities below some asset c^* are financed with repos, while securities above the threshold are funded with personal funds. In addition the lowest transaction cost asset (c_l) is matched with shortest maturity $1/\epsilon_s$, while securities above c_l are matched sequentially with lenders ranked by maturity, until the longest maturity available $1/\epsilon_l$ is matched with the highest security c^* .

In the equilibrium, each lender ϵ optimal choice, $a(\epsilon)$, is equal to $\varphi^{-1}(\epsilon)$, therefore, one finds lenders' demand for securities below c , $D(c)$, by integrating lenders whose asset choice is below c .

$$D(c) = \int_{\varphi(c)}^{\epsilon_s} f_\epsilon(x) dx, \quad S(c) = \int_{c_l}^c s(x) dx \quad (2.22)$$

Since market clearing requires that the quantity of assets financed is equal to the supply of securities below c^* , I find c^* by setting $D(c^*)$ equal to $S(c^*)$.

$$c^* = S^{-1}(F_\epsilon(\epsilon_s)) \quad (2.23)$$

Note that c^* is a decreasing function of lenders' mass, which, in the model is a measure of available resources to lend. Finally, the matching function can be found, if one sets $D(c)$ equal to $S(c)$ for all securities below c^* . The solution is shown in equation (2.24).

$$\varphi(c) = F_\epsilon^{-1} [F_\epsilon(\epsilon_s) - S(c)] \quad (2.24)$$

The mapping is one to one and strictly decreasing in transaction costs. Having found the function that matches securities to maturities, it remains to solve for the equilibrium prices and repo terms.

¹²If only the most illiquid securities prices are financed with repos, their prices would need to fall below the buyers' valuations, as a result, markets for those securities would not clear, since there is free entry of buyers.

2.3.6 Equilibrium

In our candidate equilibrium allocation, only securities with transaction cost below c^* are financed through repos, that is, \mathbb{C}_m is equal to $[c_l, c^*]$, and liquid assets are matched with short maturities while the less liquid ones with longer maturities. If chosen correctly, the equilibrium asset prices guarantee that each lender is satisfied with matching function allocation. Therefore, the trick to find asset prices is to use the first order condition of lenders, and apply the matching function in it.

$$\frac{\partial \Sigma(c, \varphi(c))}{\partial c} = 0 \quad (2.25)$$

Equation (2.25) implies that each lender ϵ maximum profit is achieved when financing security $\varphi^{-1}(\epsilon)$. Equation (2.25) defines a differential equation from which I can find prices as function of transaction costs. In order to solve for the constant associated with (2.25), I use the fact the marginal buyer of security c^* is a buyer funded with equity. The marginal buyer is indifferent between buying or not these securities as free entry set their profits to zero. Proposition (2.3.5) provides the equilibrium asset prices.

Proposition 2.3.5. *Equilibrium Asset Prices are given by following function*

$$p(c) = \begin{cases} \frac{1}{r+c^*(r+2\kappa)} e^{\int_c^{c^*} \Delta(x) dx} \\ \frac{1}{r+c(r+2\kappa)} \end{cases} \quad \text{if } c > c^* \quad (2.26)$$

where

$$\Delta(x) = \frac{2(\hat{r} + \varphi(x))(r + \beta) - \hat{r}(r + \beta + \varphi(x))}{(2(\hat{r} + \varphi(x))(r + \beta) - \hat{r}(r + \beta + \varphi(x)))x + \hat{r}(r + \beta + \varphi(x))} \quad (2.27)$$

and

1. The asset resale value, $p(c)(1 - c)$ is strictly decreasing in transaction costs.
2. Round-trip costs, $p(c)2c$, are strictly increasing in transaction costs.

The proof is in the appendix. As a result of proposition (2.3.5), asset prices and resale value decrease with transaction costs, while and roundtrip costs increase with transaction costs. Items (2) and (3) of proposition (2.3.5) justifies our initial guess in proposition (2.3.4). The variable $\Delta(x)$ is a quite important one. It measures the percent change in asset prices due to a marginal increase in transaction costs. Proposition (2.3.6) shows that $\Delta(x)$ is decreasing in lenders' discount rate. This fact means that relative prices between low and high transaction cost securities increase when the lenders discount rates are reduced. Lower discount rates creates larger gains of trade a buyer and a lender.

Proposition 2.3.6. *The larger are the differences in discount rates, the greater is the increase in the relative prices of low to high transaction cost securities.*

$$\frac{\partial \Delta(x; \hat{r})}{\partial \hat{r}} < 0 \tag{2.28}$$

After finding equilibrium asset prices, the process to obtain repo terms is straightforward. Note that the equilibrium repo term of security c is the repo quote of the best suited lender to finance c . Thus, one just needs to substitute ϵ by $\varphi(c)$ in equations (2.16) and (2.17). Proceeding this way, we find both haircuts and interest payments as functions of transaction costs. Nonetheless, since it is a common practice to use rates instead of lump-sum payments when comparing investments, I define repo interest rates and calculate the equilibrium terms with interest rates below.

I denote i_ϵ as the annual interest rate associated with maturity intensity ϵ and, therefore, maturity $1/\epsilon$. This variable is defined as the solution to equation (2.29), that is, i_ϵ is the discount rate that equates the expected discounted repo payment to the principal amount.

$$(1 - h(c, \epsilon))p(c) = \mathbb{E}_t [e^{-i(\tau_\epsilon - t)}p(c)(1 - c)] \tag{2.29}$$

Note that equation (2.29) already assumes that non-default constrain [A] binds, as it sets the interest payment plus principal amount equal to the security resale value. As I show later, the last equation is also useful to infer transaction cost from data on repos.

Proposition (2.3.7) states formally the results above and provides the equilibrium repo terms formulas.

Proposition 2.3.7. *Equilibrium haircut rates and repo rates as function of transaction costs are given by*

$$h(c) = \frac{(r + \varphi(c))^{-1}}{p(c)} - \frac{(r + \beta - \varphi(c))}{(r + \beta + \varphi(c))}c. \quad (2.30)$$

and

$$i(c) = \frac{\varphi(c)(h(c) - c)}{1 - h(c)} \quad (2.31)$$

respectively.

In order to complete the model solution, I prove in the appendix (Lemma 2.5.1) that non-default constraint [B], that rules out default in rollover dates, never binds in the equilibrium.

2.3.7 Example

I solve a numerical example in order to illustrate the model main mechanisms. Parameters β and κ are chosen to set buyers' average horizon and the probability of a debt run, that forces repos to be terminated, equal to 20 years and 5 percent, respectively. Buyers' discount rates are chosen to match the weighted average cost of capital of large US banks in recent years, while lenders' discount rates are set equal to .25 percent, the upper bound on fed funds target rates since 2009. This number is consistent with MMF returns during the dataset span, 2010-2013. Assets' transaction costs distribution is assumed to be a truncated normal, bounded between

1 and 18 percent, with mean equal to 4 percent. These numbers were chosen to match the model implied transaction cost distribution, which I describe during the next section. Finally, maturities range between 1 and 100 business days, and are distributed according to a truncated normal with mean equal to 5 business days. The appendix contains one subsection devoted specifically to the matching function calculation used in this example. The equilibrium asset prices, haircuts, interest rates and asset-to-maturity allocation are plotted in figure (2.3).

As expected and consistent with the liquidity based asset pricing theories, asset returns are increasing in transaction costs. The difference here is that repos raise prices of all securities financed with credit, but the effect is asymmetrical, and notably larger among liquid securities. The reason is that liquid securities prices need to be sufficiently high in order to create a separation in equilibrium. The interest rates and haircuts that long maturity lenders obtain when financing liquid assets must provide less profits than financing illiquid securities. As a result, the equilibrium displays haircuts and interest rates which are increasing functions of transaction costs. Low maturity lenders finance liquid securities, they lend more, since haircuts are low and prices are high, and obtain lower interest rates, while lenders who offer longer maturities finance assets with high transaction costs, lend less, as haircuts and securities prices are lower, and earn higher interest rates. Therefore, haircuts and interest rates are linked in equilibrium. Furthermore, repos backed by illiquid securities have larger haircuts, because not only they lose more value when sold, but also because the repos used to fund these securities pay more interest. Thus, higher haircuts are necessary to avoid default when repos are forced to be terminated.

I emulate a financial crisis through an increase in the probability of a debt run. In this new setting, the average time until termination shock arrives is now three times shorter than before, which has the effect of reducing both buyers' and lenders'

horizons. The parameter change reduces, *ceteris paribus*, the relative profits of financing illiquid versus financing liquid securities, since the effects of shorter horizons are more pronounced among higher transaction costs. In the new equilibrium, prices fall more among illiquid securities, so that long maturity lenders obtain larger profits to finance illiquid securities than when funding lower transaction costs securities. The result is that interest rates and haircuts of repos backed by high transaction costs securities increase, as shown in figure (2.4).

2.3.8 Model Implied Transaction Costs

The equation that defines interest rates can also be used to infer the collateral transaction costs from each repo in the MMF data set. Instead of solving for i , if we take h and i and the average maturity as givens in equation (2.29), we obtain an estimate for c . This equation can be interpreted alternatively as equating the share of the security price financed with credit, $1 - h$, to the discounted expected residual value $1 - c$. Therefore, the larger is the repo haircut rate observed, *ceteris paribus*, the larger is implied transaction cost. When the observed interest rate is larger, there is more discounting, and thus the implied transaction cost is lower. A similar effect holds for longer maturities, since longer time to maturity results in the residual share being discounted more, thus the implied transaction is also lower. Equation (2.32) provides a formula to calculate the transaction cost distribution of the whole sample. Furthermore, one can calculate the distribution per security type, as done in table (2.2).

$$c = 1 - (1 + i_d \cdot d)(1 - h) \tag{2.32}$$

As expected treasuries and government agency securities are the asset classes with both the lowest implied transaction costs and dispersion in transaction costs. On the contrary, asset backed securities and corporate bonds are the asset classes with largest dispersion in trading costs. They range in the data set from 1.94 to above

16 percent. This observation is compatible with the empirical literature on corporate bond liquidity, which states substantial variation in round trip costs. Finally, mortgage backed securities have both the highest average implied transaction cost and lowest variation in trading costs.

2.4 Conclusion

In this paper, I develop an asset pricing model motivated by the evidences that repos backed by liquid securities have on average shorter maturities, lower haircuts and lower interest rates than repos backed by less liquid securities, and that MMF, the lenders, have heterogeneous maturity preferences. Both repos and the assets financed with repos are priced in equilibrium, and the model provides formulas for repos' interest rates, haircuts and maturities as functions of the collateral's transaction cost. I show that repos with longer maturities are superior to fund illiquid securities, because they last longer and have smaller expected transaction costs, resulting in surpluses that are larger than shorter maturities. Asset prices adjust to provide incentives to long maturity lenders, who are better in financing illiquid securities, to fund the securities with higher transaction costs. I show that the relative prices of liquid to illiquid securities increase with repos and, thus, repos to fund illiquid assets have higher haircuts (less lending) and higher (annual) interest rates. I also show that these variables are linked in the equilibrium. Haircuts on illiquid securities are larger not only because they have higher transaction costs, but also because in equilibrium they pay higher interest rates and, therefore, the lenders require more protection against default. Furthermore, I simulate a financial crisis via an increase in the likelihood that repos stop being rolled over. When investors have shorter horizons, the simulation shows all asset prices fall, but the reduction is larger among liquid securities. This adjustment is necessary to keep long maturity

lenders willing to fund illiquid securities. That results in higher interest rates and larger haircuts in repos backed by securities with high transaction costs.

2.5 Appendix

2.5.1 Proof of Propositions

2.5.1.1 Proof of Proposition 2.3.1

Note that the expectation term is a linear operator, therefore we can separate it in multiple parts and solve them separately, as I do below.

$$\mathbb{E}_t \left[\int_t^{\tau_\epsilon} e^{-r(s-t)} ds \right] = \int_t^\infty \epsilon e^{-\epsilon(\tau_\epsilon-t)} \left(\int_t^{\tau_\epsilon} e^{-r(s-t)} ds \right) d\tau_\epsilon = \frac{1}{r + \epsilon} \quad (2.33)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} \right] = \int_t^\infty \int_t^\infty \epsilon e^{-\epsilon(\tau_\epsilon-t)} \beta e^{-\beta(\tau_\beta-t)} e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} d\beta d\tau_\epsilon \quad (2.34)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} \right] = \int_t^\infty \int_{\tau_\epsilon}^\infty \epsilon e^{-\epsilon(\tau_\epsilon-t)} \beta e^{-\beta(\tau_\beta-t)} e^{-r(\tau_\epsilon-t)} d\beta d\tau_\epsilon \quad (2.35)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon < \tau_\beta\}} \right] = \int_t^\infty \epsilon e^{-(r+\epsilon+\beta)(\tau_\epsilon-t)} d\tau_\epsilon = \frac{1}{r + \epsilon + \beta} \quad (2.36)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}} \right] = \int_t^\infty \int_t^\infty \epsilon e^{-\epsilon(\tau_\epsilon-t)} \beta e^{-\beta(\tau_\beta-t)} e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}} d\beta d\tau_\epsilon \quad (2.37)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}} \right] = \int_t^\infty \int_t^{\tau_\epsilon} \epsilon e^{-\epsilon(\tau_\epsilon-t)} \beta e^{-\beta(\tau_\beta-t)} e^{-r(\tau_\epsilon-t)} d\beta d\tau_\epsilon \quad (2.38)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}} \right] = \int_t^\infty \epsilon e^{-(r+\epsilon)(\tau_\epsilon-t)} (1 - e^{-\beta(\tau_\epsilon-t)}) d\tau_\epsilon = \quad (2.39)$$

$$\mathbb{E}_t \left[e^{-r(\tau_\epsilon-t)} \mathbb{I}_{\{\tau_\epsilon > \tau_\beta\}} \right] = \frac{\epsilon\beta}{(r + \epsilon)(r + \epsilon + \beta)} \quad (2.40)$$

After substituting equations (2.33), (2.36) and (2.39) in equation (2.2), we obtain:

$$V_B = \frac{1}{r + \epsilon} + \frac{\epsilon(V_B - w)}{r + \epsilon + \beta} + \frac{\epsilon\beta}{(r + \epsilon)(r + \epsilon + \beta)} \max\{p(c)(1 - c) - (1 - h)p(c) - w, 0\} \quad (2.41)$$

and solving for V_B , we obtain the first equation in proposition 2.3.1.

$$V_B = \frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} - \frac{\epsilon}{r + \beta} w + \frac{\epsilon\beta}{(r + \epsilon)(r + \beta)} \max\{p(c)(1 - c) - (1 - h)p(c) - w, 0\} \quad (2.42)$$

The process of finding V_L is analogous. The differences are that the lender doesn't receive dividends and that she discounts payment flows at rate \hat{r} .

$$V_L(c, \epsilon, w, h) = \frac{\epsilon}{\hat{r} + \beta} w + \frac{\epsilon\beta}{(\hat{r} + \epsilon)(\hat{r} + \beta)} ((1 - h)p(c) + w) \quad (2.43)$$

2.5.1.2 Proof of Proposition 2.3.3

Lender ϵ maximizes her profits subject to the buyer's indifference constraint and non-default constraint at termination dates.

$$\max_{w, h} \frac{\epsilon w}{(\hat{r} + \beta)} + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} [w + (1 - h)p(c)] - (1 - h)p(c) \quad (2.44)$$

$$\frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} - \frac{\epsilon w}{(r + \beta)} + \frac{\epsilon\beta}{(r + \beta)(r + \epsilon)} [p(c)(1 - c) \quad (2.45)$$

$$- w - (1 - h)p(c)] - hp(c) - p(c)c = 0$$

$$w \leq (h - c)p(c) \quad (2.46)$$

Kuhn-Tucker Conditions

First order conditions to the problem above is given by the following equations.

$$\frac{\epsilon w}{(\hat{r} + \beta)} + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} + \mu \left(\frac{\epsilon}{(r + \beta)} + \frac{\epsilon\beta}{(r + \beta)(r + \epsilon)} \right) - \lambda = 0 \quad (2.47)$$

$$-\frac{\epsilon w}{(\hat{r} + \beta)} p(c) + p(c) - \mu \left(\frac{\epsilon\beta}{(r + \beta)(r + \epsilon)} p(c) - p(c) \right) + \lambda p(c) = 0 \quad (2.48)$$

$$\lambda (w - (h - c)p(c)) = 0; \lambda \geq 0 \quad (2.49)$$

The variable λ is the Lagrange multiplier associated with the non default constraint and μ is the Lagrange multiplier associated with the buyer's participation constraint. Solving the first two equations for λ , we obtain that λ is strictly positive, as shown in the next equation.

$$\lambda = \frac{\epsilon(r - \hat{r})(\hat{r} + \beta + \epsilon)}{(r + \epsilon)(\hat{r} + \beta)(\hat{r} + \epsilon)} > 0 \quad (2.50)$$

Therefore, we conclude that the non-default constraint must bind. If the non-default constraint binds

$$w + (1 - h)p(c) = p(c)(1 - c) \quad (2.51)$$

and substituting w in the indifference equation below,

$$\underbrace{\frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} - p(c)(1 + c)}_{\text{Net Benefit}} = \underbrace{\frac{\epsilon w}{r + \beta} - (1 - h(c, \epsilon))p(c)}_{\text{Discounted Net Financial Costs}} \quad (2.52)$$

we obtain two equations, wherefrom the repo quotes can be solved. The interest payment quote is therefore

$$w = \left(\frac{r + \beta}{r + \beta + \epsilon} \right) \left(p(c)(1 - c) + \frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} - p(c)(1 + c) \right). \quad (2.53)$$

$$w = \frac{1}{(r + \epsilon)} - \left(\frac{r + \beta}{r + \beta + \epsilon} \right) p(c)(2c) \quad (2.54)$$

and the principal amount is given by the equation below.

$$(1 - h(c, \epsilon))p(c) = \left(\frac{\epsilon}{r + \beta + \epsilon} \right) p(c)(1 - c) + \left(\frac{r + \beta}{r + \beta + \epsilon} \right) \left(p(c)(1 + c) - \frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} \right) \quad (2.55)$$

$$(1 - h(c, \epsilon))p(c) = p(c) \left(1 + \left(\frac{r + \beta - \epsilon}{r + \beta + \epsilon} \right) c \right) - \frac{1}{(r + \epsilon)} \quad (2.56)$$

2.5.1.3 Proof of Proposition 2.3.4

The surplus between a buyer purchasing security c and lender ϵ is the sum of their profits.

$$\Sigma(c, \epsilon) = \Pi_B(c, \epsilon) + \Pi_L(c, \epsilon) \quad (2.57)$$

$$\Sigma(c, \epsilon) = \frac{r + \epsilon + \beta}{(r + \epsilon)(r + \beta)} - \frac{\epsilon w}{r + \beta} + \frac{\epsilon w}{\hat{r} + \beta} + \frac{\epsilon \beta}{(\hat{r} + \epsilon)(\hat{r} + \beta)} (p(c)(1 - c)) - p(c)(1 + c) \quad (2.58)$$

$$\Sigma(c, \epsilon) = \frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} + \frac{\epsilon(r - \hat{r})w}{\hat{r} + \beta} + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \{p(c)(1 - c)\} - p(c)(1 + c) \quad (2.59)$$

Substitute the interest payment and principal amount as functions of (c, ϵ) .

$$\Sigma(c, \epsilon) = \frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} + \frac{\epsilon(r - \hat{r})}{(\hat{r} + \beta)(r + \beta)} \left[\frac{1}{(r + \epsilon)} - \left(\frac{r + \beta}{r + \beta + \epsilon} \right) p(c)(2c) \right] + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \{p(c)(1 - c)\} - p(c)(1 + c) \quad (2.60)$$

$$\Sigma(c, \epsilon) = \frac{r + \beta + \epsilon}{(r + \beta)(r + \epsilon)} + \frac{\epsilon(r - \hat{r})}{(\hat{r} + \beta)(r + \beta)(r + \epsilon)} - \frac{\epsilon(r - \hat{r})}{(\hat{r} + \beta)(r + \beta)} \left(\frac{r + \beta}{r + \beta + \epsilon} \right) p(c)(2c) + \frac{\epsilon\beta(1 - c) - (\hat{r} + \beta)(\hat{r} + \epsilon)(1 + c)}{(\hat{r} + \beta)(\hat{r} + \epsilon)} p(c) \quad (2.61)$$

The cross derivative with respect to c and ϵ is displayed below.

$$\frac{\partial \Sigma(c, \epsilon)}{\partial c \partial \epsilon} = -\frac{(r - \hat{r})}{(\hat{r} + \beta)} \frac{r + \beta}{(r + \beta + \epsilon)^2} \frac{\partial p(c)(2c)}{\partial c} + \frac{\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)^2} \hat{r} \frac{\partial \{p(c)(1 - c)\}}{\partial c} < 0 \quad (2.62)$$

It is negative when $\frac{d\{p(c)(1-c)\}}{dc} < 0$ and when $\frac{\partial p(c)(2c)}{\partial c} > 0$, as we wanted to show.

2.5.1.4 Proof of Proposition 2.3.5

The profit of a lender ϵ , when finance asset c , is shown in the next equation.

$$\Pi_L(c, \epsilon) = \frac{\epsilon w}{(\hat{r} + \beta)} + \frac{\epsilon\beta}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \{p(c)(1 - c)\} - (1 - h)p(c) \quad (2.63)$$

Adding and subtracting w to the last equation and using the fact that

$$\Pi_L(c, \epsilon) = \frac{(\hat{r} + \beta + \epsilon)}{\hat{r} + \beta} w - \frac{(\hat{r} + \beta + \epsilon)}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \hat{r} \{p(c)(1 - c)\} \quad (2.64)$$

after substituting the interest payment quote, we obtain

$$\begin{aligned} \Pi_L(c, \epsilon) = \frac{(\hat{r} + \beta + \epsilon)}{\hat{r} + \beta} \left[\frac{1}{(r + \epsilon)} - \left(\frac{r + \beta}{r + \beta + \epsilon} \right) p(c)(2c) \right] \\ - \frac{(\hat{r} + \beta + \epsilon)}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \hat{r} \{p(c)(1 - c)\}. \end{aligned} \quad (2.65)$$

First Order Conditions

$$\begin{aligned} \frac{\partial \Pi_L(c, \epsilon)}{\partial c} = \frac{(\hat{r} + \beta + \epsilon)(r + \beta)}{(\hat{r} + \beta)(r + \beta + \epsilon)} [p'(c)2c + 2p(c)] \\ + \frac{(\hat{r} + \beta + \epsilon)}{(\hat{r} + \beta)(\hat{r} + \epsilon)} \hat{r} \{p'(c)(1 - c) - p(c)\} \end{aligned} \quad (2.66)$$

Using the matching function, the first order condition defines first order differential equation:

$$\begin{aligned} \frac{\partial \Pi_L(c, \epsilon)}{\partial c} = p'(c) \left[\frac{2(r + \beta)}{(r + \beta + \varphi(c))} c + \frac{\hat{r}(1 - c)}{\hat{r} + \varphi(c)} \right] \\ + p(c) \left[\frac{2(r + \beta)}{(r + \beta + \varphi(c))} - \frac{\hat{r}}{\hat{r} + \varphi(c)} \right] = 0 \end{aligned} \quad (2.67)$$

$$\frac{p'(c)}{p(c)} = - \frac{2(\hat{r} + \varphi(c))(r + \beta) - \hat{r}(r + \beta + \varphi(c))}{2(\hat{r} + \varphi(c))(r + \beta)c + \hat{r}(r + \beta + \varphi(c))(1 - c)} \quad (2.68)$$

$$\frac{p'(c)}{p(c)} = - \frac{2(\hat{r} + \varphi(c))(r + \beta) - \hat{r}(r + \beta + \varphi(c))}{(2(\hat{r} + \varphi(c))(r + \beta) - \hat{r}(r + \beta + \varphi(c)))c + \hat{r}(r + \beta + \varphi(c))} \quad (2.69)$$

Solving the the differential equation above, we obtain

$$p(c) = p(c^*) e^{\int_c^{c^*} \Delta(x) dx} \quad (2.70)$$

for all c between c_l and c^* . where the function $\Delta(\cdot)$ is

$$\Delta(x) = \frac{2(\hat{r} + \varphi(x))(r + \beta) - \hat{r}(r + \beta + \varphi(x))}{(2(\hat{r} + \varphi(x))(r + \beta) - \hat{r}(r + \beta + \varphi(x)))x + \hat{r}(r + \beta + \varphi(x))} \quad (2.71)$$

and $p(c^*)$ is a constant to be determined. Note the investor buying the security c^* will finance it using maturity $1/\epsilon_l$. Note also that she is indifferent between purchasing

the asset through repo and purchasing the asset with their own capital. Since there is a larger number of buyers than assets, the net utility of funding security c^* with personal funds is zero. Define $U(c)$ as the net utility of purchasing a security c with 100% financing from a buyer's own capital.

$$U(c^*) = \frac{1 + \beta p(c^*)(1 - c^*) - (r + \beta) p(c^*)(1 + c^*)}{r + \beta} = 0 \quad (2.72)$$

Solving the last equation for $p(c^*)$, we obtain:

$$p(c^*) = \frac{1}{r + c^*(r + 2k)}. \quad (2.73)$$

Therefore the asset prices in equilibrium are

$$p(c) = \begin{cases} \frac{e^{\int_c^{c^*} \Delta(x) dx}}{r + c^*(r + 2k)} & \text{if } c \leq c^* \\ \frac{1}{r + c(r + 2k)} & \text{if } c > c^* \end{cases} \quad (2.74)$$

In order to find that asset prices are decreasing in transaction costs and roundtrip costs are increasing in transaction costs, note that

$$\frac{\partial p(c)}{\partial c} = p(c^*) \left[e^{\int_c^{c^*} \Delta(x) dx} - c e^{\int_c^{c^*} \Delta(x) dx} \Delta(c) \right] = 2p(c) [1 - c\Delta(c)] > 0, \quad (2.75)$$

because for any c ,

$$c\Delta(c) < 1. \quad (2.76)$$

The asset resale value is decreasing in transaction costs,

$$\frac{\partial p(c)(1 - c)}{\partial c} = p'(c)(1 - c) - p(c) \quad (2.77)$$

$$\frac{\partial p(c)(1 - c)}{\partial c} = -p(c) [1 + \Delta(c)(1 - c)] < 0 \quad (2.78)$$

as we wanted to show.

2.5.1.5 Proposition 2.3.6 Proof

The delta function can be further simplified, as shown below.

$$\Delta(x; \hat{r}) = \frac{(\hat{r} + 2\varphi(x))(r + \beta) - \hat{r}\varphi(x)}{((\hat{r} + 2\epsilon)(r + \beta) - \hat{r}\epsilon)x + \hat{r}(r + \beta + \epsilon)} \quad (2.79)$$

The derivative with respect to the lenders discount rate is strictly negative.

$$\frac{\partial \Delta(x; \hat{r})}{\partial \hat{r}} = -\frac{\varphi(x)(r + \beta + \varphi(x))(r + \beta)}{[(\hat{r} + 2\varphi(x))(r + \beta) - \hat{r}\varphi(x)]x + \hat{r}(r + \beta + \varphi(x))} < 0 \quad (2.80)$$

While the derivative with respect to the intensity that repos are terminated is positive.

$$\frac{\partial \Delta(x; \beta)}{\partial \beta} = \frac{2\varphi(x)\hat{r}(\hat{r} + \epsilon)}{[(\hat{r} + 2\varphi(x))(r + \beta) - \hat{r}\varphi(x)]x + \hat{r}(r + \beta + \varphi(x))} > 0 \quad (2.81)$$

The conclusions are 1) when lenders discount rates decrease, the relative prices of liquid versus illiquid securities increase, and 2) when repos are terminated more quickly, the relative prices of liquid versus illiquid securities increase.

2.5.1.6 Proposition 2.3.7

The interest rate i_ϵ is defined as the rate which equates the expected present value of payment (interest plus principal) at next maturity date to the cash received when repo was initiated.

$$(1 - h(c))p(c) = \mathbb{E} [e^{-i(\tau_\epsilon - t)}(w + (1 - h)p(c))] \quad (2.82)$$

Note that the model predicts that the repo payment is equal to the security resale value. Therefore,

$$(1 - h(c))p(c) = \mathbb{E} [e^{-i(\tau_\epsilon - t)}p(c)(1 - c)] \quad (2.83)$$

$$(1 - h(c))p(c) = \mathbb{E} [e^{-i(\tau_\epsilon - t)}] p(c)(1 - c) \quad (2.84)$$

$$(1 - h(c))p(c) = \frac{\epsilon p(c)(1 - c)}{i + \epsilon}. \quad (2.85)$$

The equilibrium rate is found by solving the last equation for i .

$$i(c) = \frac{\varphi(c)(h(c) - c)}{1 - h(c)}. \quad (2.86)$$

2.5.1.7 Lemma 2.5.1

Lemma 2.5.1. *The non-default constraint at rollover dates does not bind.*

Note that the profit of all buyers are zero in equilibrium then

$$V_B(c, \epsilon, h, w) - hp(c) - p(c)c = 0. \quad (2.87)$$

Substituting V_B in the discounted value of rolling over the repo at maturity dates, we get

$$V_B(c, \epsilon, h, w) - w = hp(c) + p(c)c - w \quad (2.88)$$

which can be simplified to

$$V_B(c, \epsilon, h, w) - w = p(c)(1 + c) - (1 - h)p(c) - w. \quad (2.89)$$

Using the fact that the total repayment equals the security's resale value, we obtain that the option value is equal to purchase cost minus resale value plus the fee.

$$V_B(c, \epsilon, h, w) - w = p(c)(1 + c) - p(c)(1 - c) \quad (2.90)$$

Since the purchase cost is greater than the security resale value, the option is a positive number, as we wanted to show.

$$V_B(c, \epsilon, h, w) - w = p(c)2c \geq 0 \quad (2.91)$$

2.5.2 Matching Function Derivation

The measure of maturities is normalized to unity. In the example, I assume that the maturity distribution is a truncated normal distribution with density f_d . I define f_Z as the standard normal density and Φ as the cumulative standard normal distribution. The variables d_{max} and d_{min} are the maximum and minimum maturities available, where the distribution is truncated between, and μ_d and σ_d are parameters which determine the shape of the maturity distribution.

$$f_d(x) = \frac{\frac{1}{\sigma_d} f_Z\left(\frac{x-\mu_d}{\sigma_d}\right)}{\Phi\left(\frac{d_{max}-\mu_d}{\sigma_d}\right) - \Phi\left(\frac{d_{min}-\mu_d}{\sigma_d}\right)} \quad (2.92)$$

The Maturity intensity F_ϵ is found by changing variables, since the maturity intensity is the inverse of maturity.

$$F_\epsilon(e) = 1 - F_d(1/e) \quad (2.93)$$

The maturity distribution F_d is found by integrating the density f_d .

$$F_d(x) = \frac{\Phi\left(\frac{x-\mu_d}{\sigma_d}\right) - \Phi\left(\frac{d_{min}-\mu_d}{\sigma_d}\right)}{\Phi\left(\frac{d_{max}-\mu_d}{\sigma_d}\right) - \Phi\left(\frac{d_{min}-\mu_d}{\sigma_d}\right)} \quad (2.94)$$

The securities transaction costs distribution is assumed to be normal truncated between 1% and c^* , with density $s(x)$. Parameters μ_c and σ_c determine the shape of the distribution.

$$s(x) = \frac{\frac{1}{\sigma_c} f_Z\left(\frac{x-\mu_c}{\sigma_c}\right)}{\Phi\left(\frac{c^*-\mu_c}{\sigma_c}\right) - \Phi\left(\frac{0-\mu_c}{\sigma_c}\right)} \quad (2.95)$$

The matching function φ is calculated using equation (2.24).

$$\varphi(c) = \frac{1}{F_d^{-1}(S(c))} \quad \text{and} \quad \varphi^{-1}(\epsilon) = S^{-1}(F_d(1/\epsilon)) \quad (2.96)$$

Note that

$$S(c) = \frac{\Phi\left(\frac{c-\mu_c}{\sigma_c}\right) - \Phi\left(\frac{c_l-\mu_c}{\sigma_c}\right)}{\Phi\left(\frac{c^*-\mu_c}{\sigma_c}\right) - \Phi\left(\frac{c_l-\mu_c}{\sigma_c}\right)} \quad (2.97)$$

The inverse of a normal distribution truncated between a and b and parameters μ and σ is given by

$$F^{-1}(y) = \mu + \sigma \Phi^{-1} \left[y \left[\Phi \left(\frac{b - \mu}{\sigma} \right) - \Phi \left(\frac{a - \mu}{\sigma} \right) \right] + \Phi \left(\frac{a - \mu}{\sigma} \right) \right]. \quad (2.98)$$

Therefore we calculate the matching function in the following steps.

$$F_d^{-1}(S(c)) = \mu_d + \sigma_d \Phi^{-1} \left[S(c) \left[\Phi \left(\frac{d_{max} - \mu_d}{\sigma_d} \right) - \Phi \left(\frac{d_{min} - \mu_d}{\sigma_d} \right) \right] + \Phi \left(\frac{d_{min} - \mu_d}{\sigma_d} \right) \right] \quad (2.99)$$

$$\varphi(c) = \frac{1}{\mu_d + \sigma_d \Phi^{-1} \left[S(c) \left[\Phi \left(\frac{d_{max} - \mu_d}{\sigma_d} \right) - \Phi \left(\frac{d_{min} - \mu_d}{\sigma_d} \right) \right] + \Phi \left(\frac{d_{min} - \mu_d}{\sigma_d} \right) \right]} \quad (2.100)$$

$$\varphi^{-1}(\epsilon) = \mu_c + \sigma_c \Phi^{-1} \left[F_d(1/\epsilon) \left[\Phi \left(\frac{c^* - \mu_c}{\sigma_c} \right) - \Phi \left(\frac{c_l - \mu_c}{\sigma_c} \right) \right] + \Phi \left(\frac{c_l - \mu_c}{\sigma_c} \right) \right] \quad (2.101)$$

2.5.3 Figures and Tables

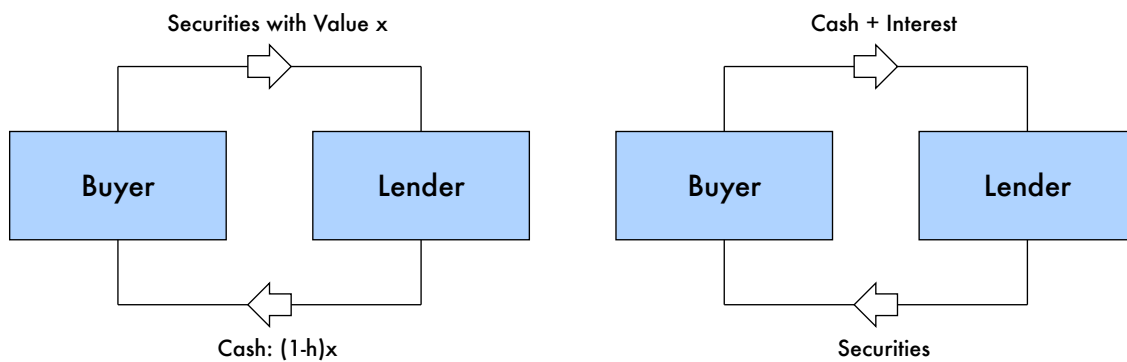


Figure 2.1: Repurchase Agreement Diagram

Notes: A repurchase agreement is a collateralized loan. At date zero (left), buyer sells securities with market value x and receives $(1 - h)x$ in cash. At date 1, buyer repurchases the security by paying the cash received at date zero, $(1 - h)x$, plus interest, and receives the securities back. The variable h is called haircut and is the depreciation rate applied to the security market value to match the transaction price at the initial date.

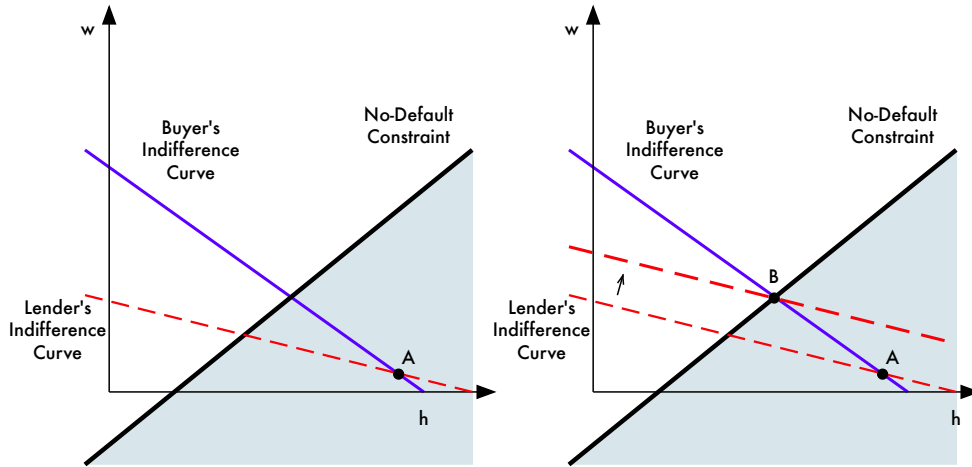


Figure 2.2: Binding Non-Default Constraint

Notes: The buyer indifference curve is displayed in blue. Above this curve, the haircut and interest payment combination gives negative utility to buyer, while below the curve, the buyer obtains strictly positive utility. The optimal contract is one that gives zero utility to buyer, and is in the feasible set, the light blue area. Larger haircut allows lenders to charge more interest, therefore, the boundary of the set is increasing in the haircut rate. Repo term A is not the optimal, because lenders discounts future payments less than buyers, and, at point B, where both more credit is given and more interest is charged, lenders obtain higher utility while buyers are indifferent.

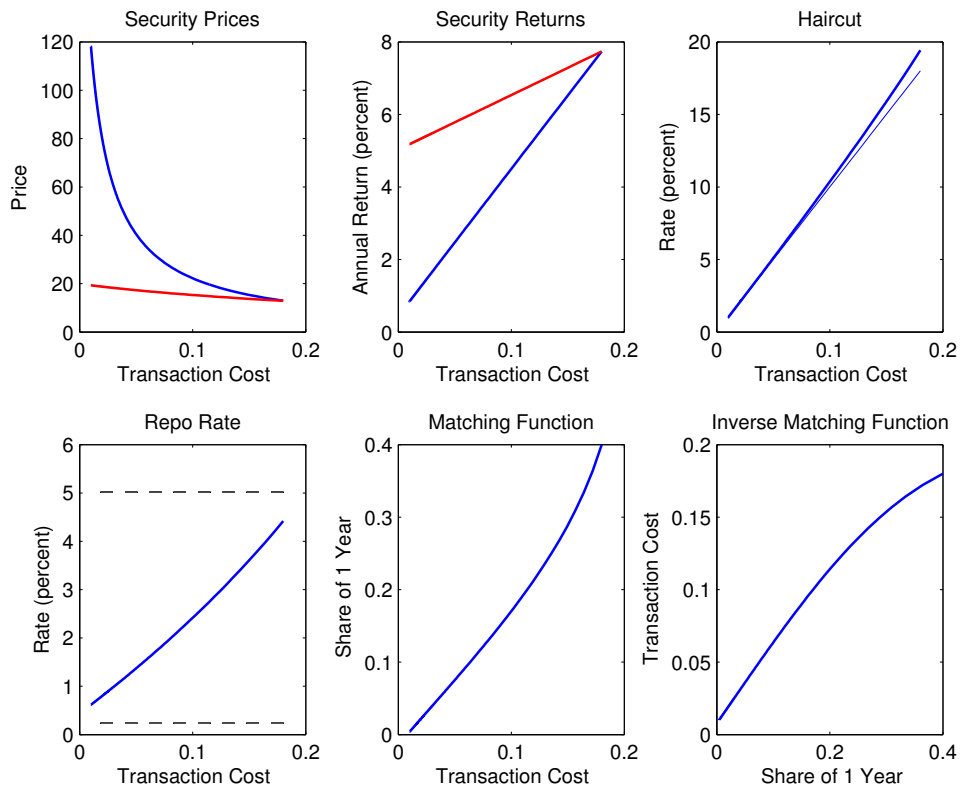


Figure 2.3: Numerical Example - Benchmark

Notes: Equilibrium asset prices and returns are plotted in blue, while asset prices and returns when repo markets are shut down are plotted in red. Haircuts are in percentages and are plotted against a 45° degree line. Interest rates are annualized and displayed in percentages.

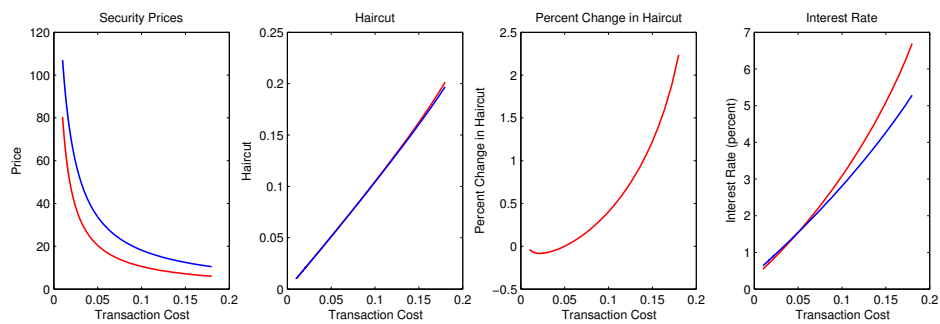
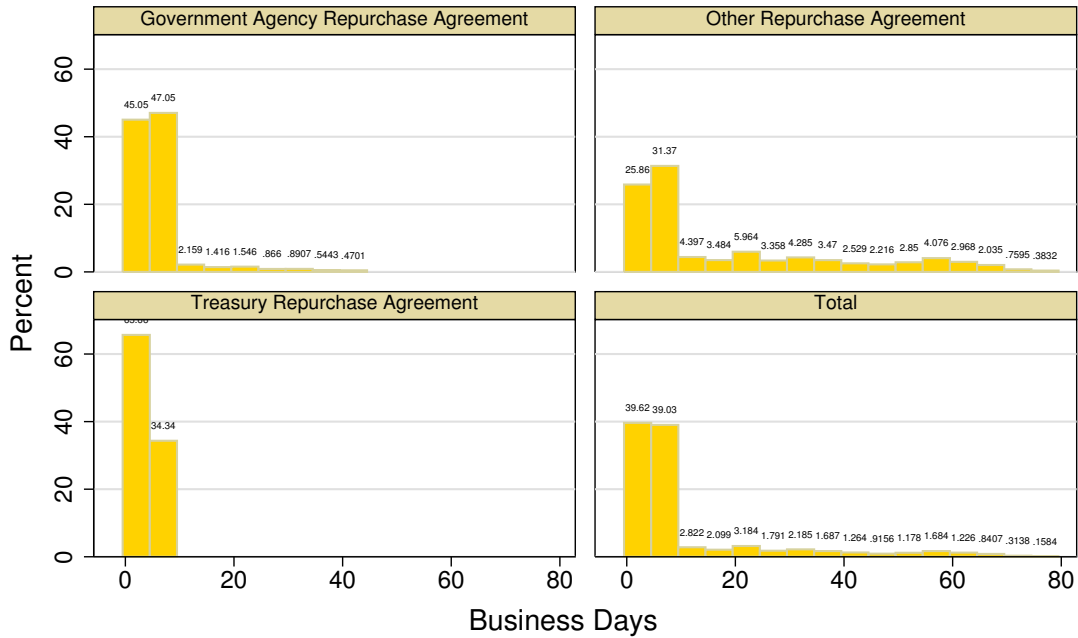


Figure 2.4: Numerical Example - Financial Crisis

Notes: Variables represented in blue are before and in red after the change in parameters. Asset Prices fall while haircuts and interest rates increase.

Maturity Histogram Business Days



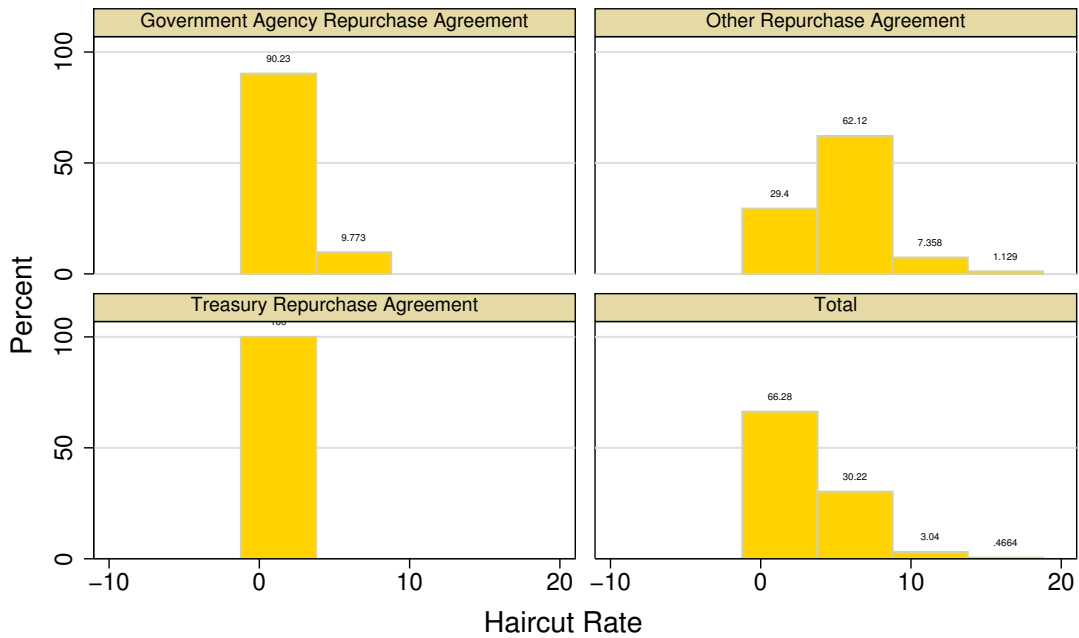
Graphs by Domain

Figure 2.5: Maturity Histogram

Notes: Maturity sizes are in business days. Each bar width represents 5 business days increments. "Government agency repurchase agreement" are repos backed by US government agencies bonds. "Treasury repurchase agreements" are repos back by US Treasury securities. "Other repurchase agreements" are repos backed by the remaining securities types. "Total" denotes the frequency distribution among all three repurchase agreements classes.

Haircut Rate Histogram

Haircut Rate

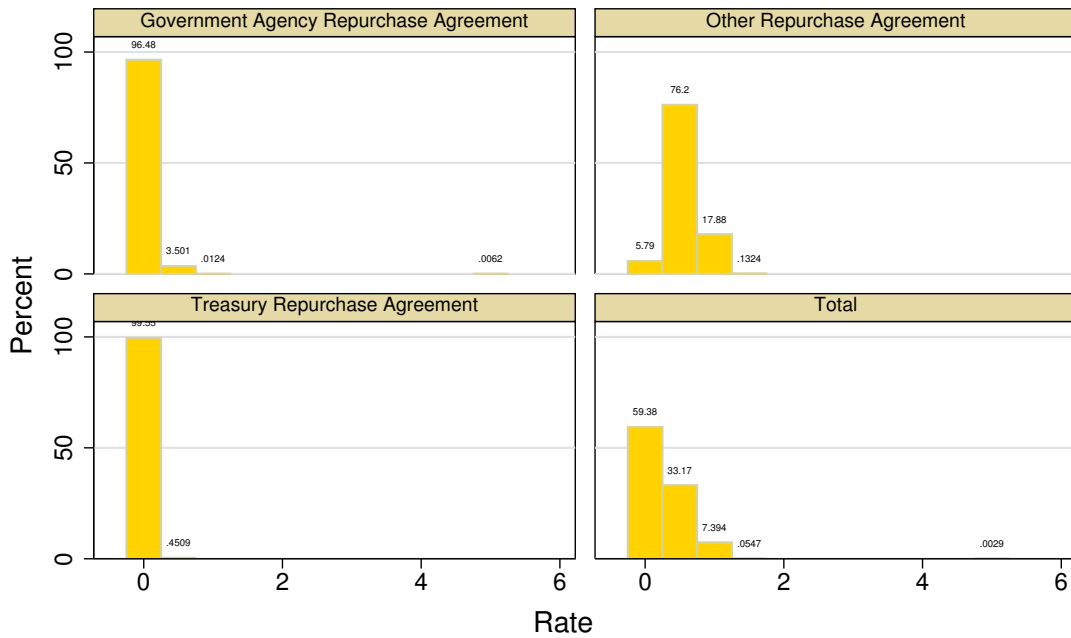


Graphs by Domain

Figure 2.6: Haircut Rates Histogram

Notes: Haircut rates are in percentages. Each bar width represents 5 percent increments. "Government agency repurchase agreement" are repos backed by US government agencies bonds. "Treasury repurchase agreements" are repos back by US Treasury securities. "Other repurchase agreements" are repos backed by the remaining securities types. "Total" denotes the frequency distribution among all three repurchase agreements classes.

Repo Rates Histogram Annual Interest Rates

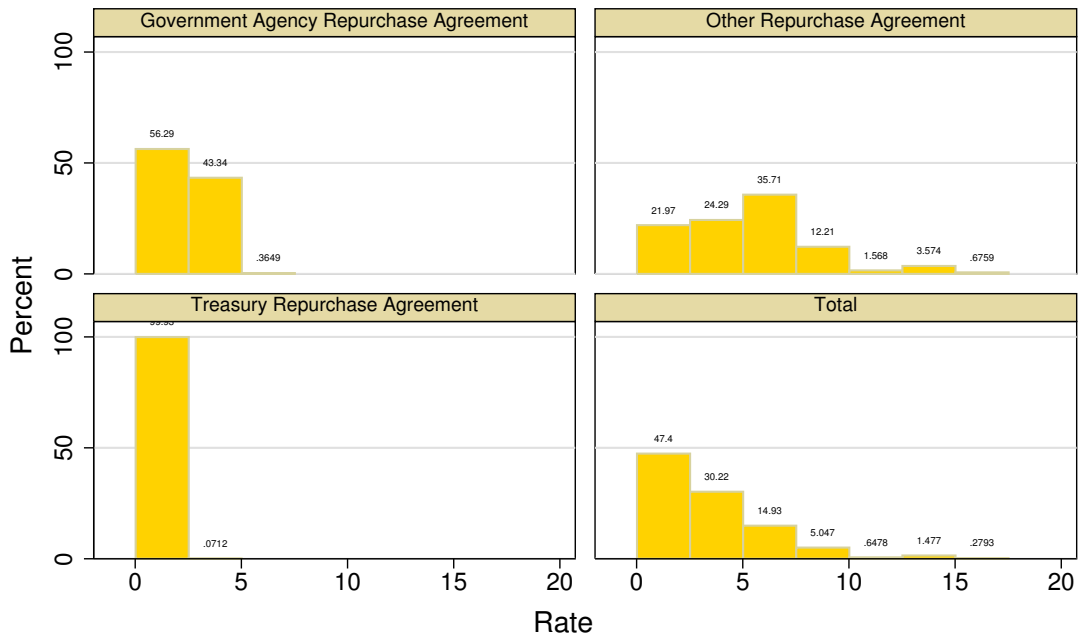


Graphs by Domain

Figure 2.7: Repo Interest Rates

Notes: Interest rates are in percentages. Each bar width represents .5 percent increments. "Government agency repurchase agreement" are repos backed by US government agencies bonds. "Treasury repurchase agreements" are repos back by US Treasury securities. "Other repurchase agreements" are repos backed by the remaining securities types. "Total" denotes the frequency distribution among all three repurchase agreements classes.

Transaction Cost Histogram Transaction Cost



Graphs by Domain

Figure 2.8: Implied Transaction Cost Histogram

Notes: Implied transaction costs are in percent of collateral pool price. Each bar width represents 2.5 percent increments. "Government agency repurchase agreement" are repos backed by US government agencies bonds. "Treasury repurchase agreements" are repos back by US Treasury securities. "Other repurchase agreements" are repos backed by the remaining securities types. "Total" denotes the frequency distribution among all three repurchase agreements classes.

percentile	Treasury Securities			Government Agency Securities			Other Securities		
	Haircut	Maturity	Int. Rate	Haircut (%)	Maturity	Int. Rate	Haircut	Maturity	Int. Rate
1%	.9901	1	.01	1.7305	1	.01	1.9608	1	.11
5%	1.8608	1	.02	1.9607	1	.05	1.9623	1	.18
10%	1.9607	1	.03	1.9607	1	.06	1.9732	1	.23
25%	1.9608	1	.06	1.9609	1	.1	4.1337	1	.31
50%	1.9609	1	.13	1.9803	1	.17	5.2614	4	.45
75%	1.9628	1	.17	2.9126	4	.21	7.4684	16	.65
90%	1.9998	4	.2	2.9647	5	.24	8.9197	49	.82
95%	2.1179	5	.22	3.8338	5	.26	11.4655	60	.92
99%	2.7818	14	.285	4.7619	32	.5	16.4619	93	1.01
mean	1.9573	1.7904	.1223	2.9176	2.9176	.1638	5.8656	13.41	.4882
std.dev.	.2296	3.4561	.0706	.6782	5.9149	.0961	3.0083	21.06	.2274

Table 2.1: Distribution of haircuts, maturities and interest rates of repos by collateral type

Notes: All distributions are weighted by the repurchase agreement principal amount. "Int. Rate" denotes interest rate. Haircuts and interest rates are in percentages. Maturities are in business days.

percentile	Treasuries	Gov. Agency	ABS	Corporate	Equity	MBS	Mixed
1%	1.6006	1.8144	1.9427	1.9382	1.9703	4.1662	1.7678
5%	1.9487	1.9492	1.9576	2.7827	4.6921	4.7490	1.8924
10%	1.9514	1.9513	2.0200	2.9049	4.7514	6.2049	1.9402
25%	1.9534	1.9542	4.7368	4.7430	6.4168	7.0833	1.9688
50%	1.9560	1.9920	6.6703	4.7565	7.3959	7.3139	3.0020
75%	1.9590	2.9032	7.3928	7.2851	7.4004	7.3853	5.9263
90%	1.9852	2.9911	7.5378	7.9803	7.4119	7.4104	8.8013
95%	2.0692	3.6866	7.6302	7.9841	8.1927	7.4957	12.8488
99%	2.6292	5.9833	17.6802	16.1997	9.3548	7.9211	16.6398
mean	1.9678	2.4200	6.0235	5.6402	6.7993	7.0357	4.5122
std.dev.	.1519	.7328	2.5393	2.2190	1.3915	.8288	3.3443

Table 2.2: Implied Transaction Distribution

Notes: The transaction cost distribution reported is weighted by principal amount of debt. Costs are in percentages. "Treasuries" denotes US Treasury securities, "government agency" denotes bonds issued by US Government Agencies, "ABS" denotes asset backed securities, "corporate" denotes US Corporate bonds (both investment grade and junk), "equity" denotes US stocks and exchanged traded funds, "MBS" denotes mortgage backed securities (both agency and private labels), and "mixed" denotes collateral pools that includes more than one of the aforementioned categories.

CHAPTER 3

Quality Disclosure and Competition: Evidence from the Los Angeles Restaurant Market

3.1 Introduction

Quality disclosure schemes such as mandatory quality ratings have become an increasingly popular approach to ensuring informed consumer choices and incentivizing firm quality investment. The context for these systems are varied, ranging from the performance of public schools to the mortality rate in dialysis centers. There is also a burgeoning academic literature evaluating the effectiveness of these programs in influencing consumer choice (Ippolito and Mathios [1990], Jin and Sorensen [2006]), incentivizing firm quality investment (Powers et al. [2011], Bennear and Olmstead [2008]), and as well as attempts to manipulate or game the system (Dranove et al. [2003]).

Standard models of oligopolistic competition suggest that the optimal design and the resulting efficacy of information disclosure schemes may depend crucially on the structure of competition present in the markets which the systems are implemented in. In this paper we present the first study, to our knowledge, of the interaction between market structure and mandatory quality disclosure schemes. To do this, we revisit the results of Jin and Leslie [2003] who demonstrate that a new mandatory quality disclosure law, which required restaurant to prominently display hygiene inspection grade cards to consumers, caused an increase in restaurants' hygiene

quality levels. They also find that the quality disclosure also caused consumer demand to become more sensitive to changes in hygiene score, and caused foodborne illness hospitalizations to decrease. Shih [2011] uses the same data in conjunction with locational data to demonstrate that restaurants under a voluntary hygiene disclosure policy improved hygiene more when located near a restaurant with a mandatory disclosure policy.

In this paper, we determine what factors induce firms to provide higher hygiene quality. We estimate a game of incomplete information, where the payoffs of restaurants are functions of competitors' quality choice, as well as covariates including the demographics of the market and restaurant characteristics. We model two static games: one before and one after the hygiene quality disclosure scheme is implemented. We proceed in this way because we do not have access to revenue or prices, therefore we choose not to directly model consumer choice and, instead, consider how the new scheme affects the payoff structure of firms. We use a two step estimation method as in Bajari et al. [2006]. The method breaks the estimation into a reduced form first step estimation and uses the fitted value probabilities to estimate a standard discrete choice model in the second step. At their core, our results decompose the average treatment effects found in Jin and Leslie [2003] into heterogeneous effects that depend on the characteristics of local competition and the market.

Our results show that initially the quality choice of competitors had a negligible effect on a restaurant's own quality choice, although on average having more competitors increased the probability of providing higher hygiene quality. The effect changes after the law, when the results of previous hygiene inspections are mandatorily disclosed to the consumer. Now competitors' choice have a positive effect with magnitude ten times greater than before, which means that if firm i believes that its competitors will provide high quality, it will be optimal for i to provide high quality

too. The intuition is that before the law, consumers did not have easy access to hygiene quality information, except possibly a noisy signal by word-of-mouth from other consumers. Since the probability of realizing the true hygiene quality is small for the consumer, firms do not have incentives to invest in it. After the law, the disclosure of information boosts competition, since the consumer can punish low hygiene practices by going to a competitor that offers higher hygiene standards.

The first step estimation reveals that, while the average payoff was specified in a way that does not include competitors' quality choices, the total number of firms in the zip code area had a positive effect in quality provision in both periods. The effect is nonlinear and becomes more intense when restaurants start displaying the grade cards. In markets with a small number of firms, an additional restaurant increases the probability that all other firms will provide high quality. The effect is positive until we reach some threshold, after which additional firms have negative effects on quality provision. Although the number of firms in the market is not a perfect proxy for competition intensity, the result is interesting since we do not know any theory paper, other than Bar-Isaac [2005], which explores the subject¹. We provide a possible source of estimation bias, but the intuition relies on the fact that more competition allows consumers to more easily switch from low quality firms to higher quality options, but as the number of firms becomes large, competition becomes too fierce and it is harder to keep customers with investments in quality. The investment is as costly as before, but the expected return on reputation investment is lower. In all, these results not only confirm the findings of Jin and Leslie [2003], but also suggest that the pathway through which information disclosure incentivizes restaurants to invest more in the quality is competition. With this consideration,

¹The author provides a specific example of a demand function from Sutton [1991] where the number of firms has a nonlinear effect on reputation incentives. However his example generates a different result: incentives are high when the number of firms are either small or big. At intermediate levels the incentives to build reputation for quality is small.

policymakers can design better quality disclosure schemes that seize upon potential social welfare gains from information to consumers.

Our paper is structured in the following way. In Section 2 we describe the model structure and define an equilibrium of the incomplete information game used. Next we describe the two step approach to estimate discrete games and describe identification issues. In Section 3, we describe the restaurant data used and specify functional forms in the first and second steps of estimation. In Section 4, we describe the estimation process and discuss results in Section 5. In Section 6, we conclude our paper. We also include in the appendix the derivation of the estimator’s asymptotic distribution.

3.2 Model

Although the data set has a panel structure from 1995 to 1998, we choose to model the game as a static one, because neither revenue nor price information were available. As a result, our best option was to treat consumer behavior as a primitive of the game. We therefore chose to focus our model on the firms’ choices, specifying a restaurant’s profit to be a function of its quality choice and several covariates from the data, which serves as a proxy for what determines consumer behavior. In addition, we note that there is a structural break in the data at the end of 1997, which was when the new hygiene information disclosure law was adopted in Los Angeles County. If consumer behavior changed as a result of this event, we should observe changes in the primitives of our model. To look for these changes, we decided to estimate two different static games—the first played before the law was passed and the second played afterwards—and compare parameter estimates.

3.2.1 Environment

In each market $m = 1, \dots, M$, there is a finite number of restaurants denoted $i = 1, 2, 3 \dots n_m$ that each have a unique manager. Throughout this paper, the words manager and restaurant will be used interchangeably. The managers simultaneously choose an action $a_i \in \{L, H\}$ to maximize the payoff to their respective restaurants, where L and H represent low and high hygiene quality levels, respectively. We define $A = \{L, H\}^{n_m}$ to be the set of all possible combination of actions involved in the game and $a = (a_1, a_2, a_3, \dots, a_{n_m})$ to be an element of this set. The variable $s_i \in S_i$ denotes the state for restaurant i and $s = (s_1, s_2, s_3, \dots, s_{n_m}) \in S$ denotes the vector of all restaurants states, where $S = \times S_i$.

The state s is common knowledge to the players and observable to the econometrician. However, associated with each restaurant action is another state variable $\varepsilon_i(a_i)$, which is private information only to the manager of restaurant i . We assume the private states to be identically and independently distributed across restaurants with density $f(\varepsilon_i(a_i))$ with support E . The payoff to the manager i has the following additive form:

$$U_i(a, s, \varepsilon_i) = \pi_i(a_i, a_{-i}, s) + \varepsilon_i(a_i) \quad (3.1)$$

The manager's payoff consists of an observable part which is common knowledge to all players and a privately observed state. The value of the observable part depends on the actions of all players in the market and the realized state $s \in S$ of the game. The optimal policy of the manager for restaurant i is a mapping $g : S \times E \mapsto \{H, L\}$, so that his decision does not depend on ε_{-i} since it consists of privately known information of the other players. Given this function and knowledge about the structure of game, the other players are able to infer the probability that manager i chooses H as the hygiene quality conditional on the observable state s in

the following way:

$$\sigma_i(a_i = H|s) = \int_{\{\varepsilon_i \in E: g(s, \varepsilon_i) = H\}} f(d\varepsilon_i) \quad (3.2)$$

Using Equation 3.2 we can calculate the conditional probability of observing action $a \in A$ given the state s . Let $\sigma(a|s) = \times_{i=1,2,3..n_m} \sigma_i(a_i|s)$. Using the conditional distribution of the actions of the $-i$ players, we can compute the expected utility of manager i 's choice $a_i \in \{H, L\}$ conditional on the state s .

$$E_{\sigma_{-i}} [U_i(a, s, \varepsilon_i)|s] = \sum_{-i} \sigma_{-i}(a_{-i}|s) \pi_i(a_i, a_{-i}, s) + \varepsilon_i(a_i) \quad (3.3)$$

Then, the optimal decision for agent i given s is to choose an action in $\{H, L\}$ such that the conditional expected utility given by Equation 3.3 is maximized. The maximization problem is the following:

$$g_i(s, \varepsilon_i) = \operatorname{argmax}_{a_i \in \{H, L\}} \{E_{\sigma_{-i}} [U_i(a, s, \varepsilon_i)|s]\} \quad (3.4)$$

The probability that manager i chooses high quality is calculated in the following way.

$$\begin{aligned} \sigma_i(a_i = H|s) &= P[\{\varepsilon_i \in E : \sum_{-i} \sigma_{-i}(a_{-i}|s) \pi_i(H, a_{-i}, s) + \varepsilon_i(H) \\ &\geq \sum_{-i} \sigma_{-i}(a_{-i}|s) \pi_i(L, a_{-i}, s) + \varepsilon_i(L)\}] \end{aligned} \quad (3.5)$$

Finally, we define an equilibrium of the hygiene quality choice game below.

Definition 3.2.1. An equilibrium is a set of policy functions $\{g_i\}_i$ and choice probabilities $\{\sigma_i\}_i$ such that for any player i , given the policy function of the other players g_{-i} , the inferred choice probabilities σ_{-i} are given by Equation 3.2, g_i is the solution to the maximization problem in Equation 3.4 and σ_i is given by Equation 3.5.

The existence of such an equilibrium has been proven in previous papers including McKelvey and Palfrey [1996], but the equilibrium may not be unique. We address this important question in the next subsection.

3.3 Data

We use panel data on restaurants in Los Angeles County from Jin and Leslie [2003] to estimate our model. Each observation in the data set is the result of an official hygiene inspection of a restaurant. Along with the date of and the resulting score from the inspection, the data includes census tract demographic information and a significant number of restaurant details. The census data includes information about average income, ethnic composition, and socioeconomic makeup in each census tract. The restaurant information includes whether or not it has a license to sell alcoholic beverages and dummies to indicate whether the restaurant size is small, medium, or large. Other characteristics are whether or not the restaurant is part of a chain and the age of the restaurant. The data set covers hygiene inspections conducted from June 1995 to December 1998, and includes the type of inspection (e.g. if it was a routine inspection or initiated by a case of food poisoning). The inspections done from 1995 to 1997 were mostly initiated by routine inspections and the average number of inspections was 1.9 from 1995 to 1997 and 2.1 in 1998.

The hygiene scores are on a scale from 0 to 100, but scoring criteria were changed during the sample period. Before July 1997, the inspections consisted of objective and subjective assessments. Under this design, the DHS inspector first deducts pre-specified points for each violation. For instance, a food temperature violation results in a deduction of 5 points, an improperly sanitized eating utensils violation is worth 5 points, and an indication of the presence of cockroaches is worth 3 points. Before July 1997, inspectors were also supposed to give a subjective assessment of the status of the establishment, which could be excellent (no deduction), average (20 points), fair (30 points) or poor (40 points). In the whole sample period, restaurants could be closed if they receive a score below 60 in two consecutive inspections or when there is a “severe” hygiene problem. In the period after the inspection

procedure was redesigned to be more objective, we observe an immediate increase of scores by an average of 8.9 points. In view of this inspection standards discrepancy, we normalized the hygiene inspection scores after the redesign occurred in our estimation.

In December 1997 the Los Angeles County Board of Supervisors voted in favor of the grade card ordinance, which would come into effect in the next month to make the display of hygiene inspection results mandatory. Despite this legislation, it was the cities themselves that had the authority to determine when the law would take effect. It took several months until all cities in the county had implemented the law. Before then, restaurants were free to display the grades voluntarily if their city had not implemented the law yet. The grades are displayed on a letter scale, where “A” represents a hygiene score between 90 and 100, “B” represents one between 80 and 89, “C” represents one between 70 and 79 and if the score is less than 70 the restaurant is issued a card that reports the actual number score.

The data initially contain hygiene inspection results for 24304 restaurants, but after dropping the ones with missing census data and dropping restaurants that were listed as being a bar or a cafe (e.g. Starbucks), we were left with a total of 16673 restaurants. The rationale behind dropping the bars and cafes is that those types of establishments are not in direct competition with traditional restaurants. All the demographic data comes from the U.S. Census, where the units of observation are census tracts, regions in general smaller than zip code areas. The total number of census tracts and zip codes are 1373 and 304, respectively. We have raised demographic data from the census tracts to the zip code level, because our study analyzes the strategic interaction between restaurants in providing hygiene quality. The census tract cannot be used as a unit level of analysis because due to its small size, restaurants in some census tract are likely to be competing with restaurants from another census tract.

A good proxy for the level of competition between two restaurants would be the pair's distance or a variable to designate food style similarities. Unfortunately, the locations of restaurants are not available to us and most of the dataset is missing the restaurant's style or food type. We instead make the assumption that a zip code is a market and that every restaurant is a direct competitor of the others in the same zip code. A consequence of this assumption is that the number of players in one instance of the game can exceed 100. This fact reinforces our need for a specific parameter choice to identify the model as discussed in the last section.

We merged the information of all census tracts contained under a zip code to be the demographics of that particular zip code. The zip code per capita income is computed as the average of per capita income of the census tracts in the area. We do so because neither the population of each census tract nor an identifier of the census tract is available, so we assume that the census tracts' coverage is divided evenly within a zip code.

We also calculated correlations between the scores, number of restaurants and average income in the zip codes. Before the implementation of the law, the correlation between average hygiene score and per capita income in a market was 0.322. A scatterplot of the average scores as a function of per capita income is given in Figure 3.7.1.1.

The correlation between average hygiene scores and the number of restaurants in a market in the sample before the law is -0.145. A scatterplot of the average scores as function of the number of restaurants is given in Figure 3.2. After the law was implemented, correlation between average income and average hygiene score in a market is reduced to 0.03, while the correlation between average scores and the number of restaurants is more negative than before, -0.236. We repeat the graphs for the sample after the law in Figures 3.3 and 3.4.

3.4 Estimation

We estimate two sets of primitives for the two different games observed, where one is played before and the other after the law is implemented. In the first game, the consumers do not observe the scores, and have to rely on learning from past experiences to choose which restaurant to draw services from. We consider hygiene inspections from the beginning of the dataset to June 30, 1997. We exclude inspections from July 1997 to when the law was passed, because the inspection standards changed in this period, and because we believe that the news investigations (which inspired the law) may have temporarily changed the game dynamics, creating noise in our estimation.

In the second game, the local government provides direct information to consumers about recent hygiene practices, so consumers can condition their choices on the information available. If consumers start making their decisions on that information, the restaurant payoff function should change. We consider all hygiene inspections from January 1998 to the end of our dataset, including restaurants that did not post their hygiene score because their cities had not started enforcing the law. These restaurants were included, because they account for only 1,002 out of 32,273 hygiene inspections in this period of time. Our main questions are what variables determine quality, what is the importance of competition to quality provision, and how the law affected the game equilibrium.

Throughout this paper, we assumed that hygiene quality is a discrete choice variable with only two values, high and low. This assumption simplifies enormously the estimation process. Although we could have a greater number of choices, we kept the choice to be binary for simplicity, even though we believe that the results would not be quite different.

Our estimation depends on actions $a_i \in \{H, L\}$ which we do not observe. Our

solution was to use the hygiene score as a proxy for quality. We choose a threshold \bar{L} and establish a rule such that if the restaurant's recent average score is below \bar{L} , the restaurant was producing a low quality product. Since our threshold choice is ad hoc, we use three different thresholds for each game, resulting in a total of 6 estimations. Because after July 1997 the grades were inflated by an average of 8.9 points, we adjusted the threshold values for the estimation after the law. The smallest threshold in the game before the law is 70, the second 75 and the third 80 which lead to respective thresholds after the law of 78.9, 83.9, and 88.9, as displayed in the 3.1 below.

3.4.1 Identification

Before describing the estimation method, we discuss identification issues. Our main objective is to estimate $\{\pi_i\}_{i=1,2,3..n_m}$ which consists of the primitives of the game. We will say the model is identified if different values for the primitives generate different choice probabilities. This allows us to uniquely recover the functions $\{\pi_i\}_{i=1,2,3..n_m}$ from the choice probabilities, as long as the distribution of the private state variable is known. We normalize $\pi_i(L, a_{-i}, s) = 0$ and assume the difference between the two error terms in Equation 3.5 is generated from a type I extreme value distribution. The normalization is necessary for identification and the extreme value distribution assumption is convenient for approaching this problem using maximum likelihood estimation. Inverting the distribution, we form a system of n_m equations for each s as below.

$$F^{-1}(\sigma_i(a_i = H|s)) = \sum_{-i} \sigma_{-i}(a_{-i}|s)\pi_i(H, a_{-i}, s) \quad (3.6)$$

If we assume the choice probabilities to be known, we can solve for the sequence $\{\pi_i\}_{i=1,2,3..n_m}$, but not uniquely since there are $n_m 2^{n_m-1}$ unknowns for each state. In

order to make the model identified, we need exclusion restrictions so that we have at least the same number of equations as unknowns. A common way to achieve this is to restrict the payoff function π_i to depend only on s_i and not s_{-i} . If the data has enough variation in the states, such that s_{-i} conditional on s_i has at least 2^{n_m-1} points, the model is non-parametrically identified. Unfortunately our model has many players and the limited extent of the data does not allow us to non-parametrically identify the model. Our approach is instead to specify a parametric form and make exclusion restrictions, the details of which we describe in the next subsection.

3.4.2 Estimation Model

In our model, restaurant i enjoys a utility level which depends on whether it chooses the $a_i = H$ or $a_i = L$ and is linear in the observable state and the aggregate decisions of competing restaurants. This is captured in the following specification:

$$\pi_i(a_i, a_{-i}, s) = \begin{cases} s'_i \alpha^1 + \delta^1 \sum_{j \neq i} I(a_j = H) & \text{when } a_i = H \\ s'_i \alpha^2 + \delta^2 \sum_{j \neq i} I(a_j = H) & \text{when } a_i = L \end{cases} \quad (3.7)$$

With the utility specification as shown in Equation 3.7, we cannot identify the parameters α^1 , α^2 , δ^1 , or δ^2 separately. The best we can do is to estimate a model with a linear form commonly used in firm entry games as in Berry [1992] given by:

$$\pi_i(a_i, a_{-i}, s) = \begin{cases} s'_i \alpha + \delta \sum_{j \neq i} I(a_j = H) & \text{when } a_i = H \\ 0 & \text{when } a_i = L \end{cases} \quad (3.8)$$

where $\delta = \delta^1 - \delta^2$ and $\alpha = \alpha^1 - \alpha^2$.

Plugging in Equation 3.8 into Equation 3.3 along with the extreme value distribution assumption on the unobservable terms yields the probability of agent i choosing high hygiene quality given the state s , which is given by:

$$\sigma_i(a_i = H|s) = \frac{\exp(s'_i\alpha + \delta \sum_{j \neq i} \sigma_j(a_j = H|s))}{1 + \exp(s'_i\alpha + \delta \sum_{j \neq i} \sigma_j(a_j = H|s))} \quad (3.9)$$

We estimate the model above using a two step method approach as in Bajari et al. [2004] and Bajari et al. [2006]. This method is as efficient as a one step method but is computationally faster. The two step method consists of forming a reduced form model in the first step, yielding consistent estimators for the choice probabilities $\{\sigma_i\}$. We decided to use probit estimation for this step. In the second step we use the fitted values $\hat{\sigma}_i(a_i = H|s)$ to estimate the primitives of the model, using a binary choice estimation. To do this, we find the parameters that maximize the following likelihood function:

$$L(\beta) = \prod_{m=1}^M \prod_{i=1}^{n_m} \left(\frac{\exp(x_{i,m})}{1 + \exp(x_{i,m})} \right)^{I(a_{i,m}=H)} \left(1 - \frac{\exp(x_{i,m})}{1 + \exp(x_{i,m})} \right)^{I(a_{i,m}=L)} \quad (3.10)$$

where $x_{i,m} = s'_i\alpha + \delta \sum_{j \neq i} \sigma_j(a_j = H|s)$.

The estimation is consistent and gives efficient estimators unless agents play different equilibria across the markets. Since multiple equilibria is a possibility, we make the strong assumption that players coordinate on the same equilibrium in the sample. In this first step we use the following two specifications for the probit model.

Specification 1:

$$\sigma(a_i = H|s) = \Phi(\gamma_{INC}INC_i + \gamma_{AL}AL_i + \gamma_{NIC}NIC_i + \gamma_N N_i + \gamma_{PAL}PAL_i + \gamma_{PNIC}PNIC_i) \quad (3.11)$$

Specification 2:

$$\sigma(a_i = H|s) = \Phi(\gamma_{INC}INC_i + \gamma_{AL}AL_i + \gamma_{NIC}NIC_i + \gamma_N N_i + \gamma_{N^2} N_i^2 + \gamma_{PAL}PAL_i + \gamma_6 PAL_i^2 + \gamma_7 PNIC_i) \quad (3.12)$$

where the symbol Φ denotes the standard normal cumulative distribution function.

The variable INC_i denotes average income per capita in the zip code area of restaurant i , AL_i is a indicator variable which is equal to 1 if the restaurant has a license to sell alcoholic beverages, and NIC_i is a binary variable which is equal to 1 if the restaurant is not part of chain. The variable N_i is the number of restaurants in the zip code area and lastly PAL_i and $PNIC_i$ are the percentages of restaurants that have alcohol license and are not part of a chain in the zip code area, respectively. The reduced form estimation above uses the each zip code as a repetition of the (same) game with differing numbers of players. We view each market as a repeated game because the cost structure of maintaining hygiene levels in a restaurant should be the same across markets. Since the number of players is not constant throughout the zip code areas, the number of restaurants is included as one of the explanatory variables.

After estimating the first step, we use the fitted values of the choice probabilities $\hat{\sigma}_i(a_i = H|s)$ as inputs in the likelihood function in Equation 3.10. The linear specification with the actual variables used in our regressions is given by:

$$\pi_i(a_i, a_{-i}, s) = \begin{cases} \alpha_1 INC_i + \alpha_2 AL_i + \alpha_3 NIC_i + \delta \sum_{j \neq i} I(a_j = H) & \text{if } a_i = H \\ 0 & \text{otherwise} \end{cases} \quad (3.13)$$

We note that only income per capita, whether the restaurant has a liquor license, and whether the restaurant is part of a chain are explanatory variables. The other regressors used in the first step, such as number of competitors in the neighborhood, percentage of restaurants with license, and percentage of chain restaurants are not included in the second step, which is a part of our identification strategy. If the first stage estimates $\hat{\sigma}_i(a_i = H|s)$ and the term $s'\alpha$ in Equation 3.8 depend on $s \in S$, our regression would suffer from severe collinearity, which would make identifying the separate effects of the vector $(\alpha, \delta)'$ on the choices probabilities difficult. The prob-

lem is similar to a standard simultaneous equation model, meaning that exclusion restrictions are necessary to identify parameters.

It seems intuitive that neither the number of competitors, the proportion of competitors with alcoholic license nor the proportion of independent restaurants should enter directly into restaurant i 's profit function. On the contrary, these variables indirectly affect the profits through competitive behavior, which is captured by the term δ . With these restrictions, our model is identified.

3.5 Results

The results from specification (1) are found in Tables 3.2 and 3.3. The parameter associated with competitors' hygiene quality choice δ is either negative and close to zero, or insignificant under any specification before the law. The intuition behind this result is that poor hygiene practices are mostly observable when a customer suffers from food poisoning, which should only happen with sufficiently small probability since the DHS shuts down restaurants when the chance of contamination is high enough. If the probability of observing the true quality is small, the incentives to invest in high quality are smaller. Restaurant i should not care much about the actions of other restaurants, because choosing to produce high hygiene quality would have a small effect on its reputation. If more competitors are choosing high quality, the reputation gain to firm i choosing high quality is even lower, and may not be large enough to compensate for the cost of that choice. In this case, it is possible to have competing restaurants' high quality choices negatively affecting manager i 's probability of choosing high quality, as is observed in the estimates.

On the other hand, the new law dramatically changes the primitives of the payoff function. This can be explained by noting that the government now directly informs the consumer about the past quality choice of managers. If consumers

believe that the grades are good proxies for the current hygiene practices, they will avoid restaurants with lower grades. Then the hygiene quality choices of the competitors directly affects restaurant i 's choice, because i can be punished with the loss of customers if it chooses quality lower than its competitors.

Our choice to have three different thresholds for high and low quality choices helps to clarify the last result. The estimate for δ after the law reduces from 0.0108 to 0.0075 and 0.005 as we increase the threshold from 78.9 to 83.9 and 88.9. The intuition is that having a grade below "C", when others have grades above, is much worse than having a grade lower than "B", when other restaurants have grades of "B" or "A".

Another variable of interest is per capita income in the market. In our estimation, the probability of high hygiene quality is increasing in income, although the effect of income diminishes after the law is implemented. As can be seen in Table 3.3, the estimates for $\hat{\alpha}_{INC}$ are always positive, but they decrease from 0.0836 to 0.0717 in the first, from 0.0593 to 0.0426 in the second, and from 0.0477 to 0.0285 in third, when the law implemented. Although we do not have data on other product dimensions, such as food quality, it seems reasonable to believe that wealthier zip code areas in the sample contain higher level restaurants. These tend to have higher prices and consequently a smaller customer base. With a smaller clientele, the restaurant have more to lose when one client is not satisfied. This happens because first, the per client revenue is larger and second, negative information flows faster in wealthier neighborhoods since high level restaurants are frequently rated by professional surveys such as Zagat.

The main question we attempt to answer in this paper is how the degree of competition and incentives to produce quality are related. An ideal proxy for competition would be number of restaurants divided by some market size index, which would be some function of population and income. Unfortunately we do not have

census data about population size in the zip codes areas, so our next best choice was to use the number of restaurants as our proxy for competition intensity. Our estimates for γ_N are negative in all specifications, both before and after the law. This means that an additional firm in the game reduces the probability that the current restaurants produce high hygiene. That does not seem to be compatible with the fact the δ estimate becomes strictly positive after the law. For this reason, we also analyzed specification (2), which also includes in the payoff function the number of restaurants squared in the zip code.

The results of the specification (2) are found in Tables 3.4 and 3.5. The additional term in the first stage does not change the second stage estimates much, except for $\hat{\delta}$ which became slightly positive in the first two benchmarks before the law. Our interpretation does not change since the parameter is very close to zero in both of these benchmarks, and not significant in the second benchmark. This indicates that competition might have nonlinear effects on quality choice. Our results show that one additional firm has a positive effect on quality choice when the number of restaurants is below 92, in benchmark (1) and (3), and when below 69 in benchmark (2) before the law. After some threshold is achieved, an increase in the number of restaurants reduces the probability to produce high hygiene. We do not know of any theoretical papers that could help explain this empirical fact.

After the law was implemented, the negative effects of competition are reduced, since the effect of an additional firm in the zip code area is positive until the total number reaches 152, 146 and 105 in benchmarks (1), (2) and (3), respectively. The regressions show that more competition provides incentives to invest in quality, but eventually the incentives are reduced after some threshold number of restaurants is reached. One possible explanation is that if a market is saturated by restaurants, that is, many firms serve a relatively small customer pool, it may become harder for restaurants to keep clients loyal to their businesses, what could reduce the returns

of building a reputation for being clean.

The other parameters in the payoff function were indicators for if the restaurant has a license to sell alcohol, and if it is not part of a chain. On average, the alcohol license reduces the probability of restaurant choosing high quality (in all specifications), both before and after the law. However, before the law, chain restaurants were more prone to provide quality, which is reasonable since reputation of the whole brand is affected, when one of its affiliates starts shirking on quality. Surprisingly, after the law, the parameter changes sign in benchmark (1) and (2).

One possible source of estimation bias is that in some of our markets, there may be no reputational incentives for restaurants to invest in hygiene quality. For example, in areas where tourists are commonly found such as Hollywood or Universal City, we might expect consumers to be one-shot (i.e. tourists who eat at the restaurant once during their vacation and never return). In our estimation, we ignore this possibility because in every zip code, regardless of whether it's popular among tourists or not, there are residents who can be viewed as repeat customers of these restaurants.

3.6 Conclusion

In this paper, we have analyzed the effect of an oligopolistic market structure on the incentives to produce quality, in the context of quality disclosure. Our approach was to search for empirical evidence in favor of our view that a lack of competition reduces quality. We then estimated a discrete quality choice game using Los Angeles County restaurant hygiene inspection data, from 1995 to 1998. As a simplification, we analyzed the restaurants' hygiene choice, despite the fact that the good they supply has many other dimensions, such as food quality, location, and service. This choice worked with our estimation strategy, because hygiene procedures can be

treated as an homogeneous good, and the supply cost should be very similar for the restaurants in the sample. That permitted us to use each zip code as a realization of the game and to estimate the primitives of the restaurants payoff function.

Our results suggest that competition improves average hygiene quality and that the increase in the information flow to consumers strengthens this effect. It also shows that too much competition may reduce incentives for investment quality. We do not know any theory that explains this fact, but our intuition is that too much competition can make harder for firms keep clients, so that the incentives to invest in quality are reduced, or that consumers learn too slowly about individual firms in a saturated producer market. With these results, policymakers can better understand the mechanism through which information improves quality in markets and design optimal information disclosure schemes to maximize welfare gains.

3.7 Appendix

3.7.1 Asymptotic Variance of Two Step Estimator

The moments associated with the two-step estimation are derived from the first order conditions of the log-likelihood functions in both the first and the second steps. They are given as follows:

Step 1:

$$\begin{aligned} & \frac{\partial}{\partial \gamma} \sum_{m=1}^M \sum_{i=1}^{n_m} \log(\Phi(x'_i \gamma)^{y_i} (1 - \Phi(x'_i \gamma))^{1-y_i}) = 0 \\ \rightarrow & \frac{1}{\sum_{m=1}^M n_m} \sum_{m=1}^M \sum_{i=1}^{n_m} \frac{\partial}{\partial \gamma} \log(\Phi(x'_i \gamma)^{y_i} (1 - \Phi(x'_i \gamma))^{1-y_i}) = 0 \end{aligned}$$

Step 2:

$$\begin{aligned} & \frac{\partial}{\partial \theta} \sum_{m=1}^M \sum_{i=1}^{n_m} \log\left(\frac{\exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}\right)^{y_i} \\ & \quad \left(\frac{1}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}\right)^{1-y_i} = 0 \\ \rightarrow & \frac{1}{\sum_{m=1}^M n_m} \sum_{m=1}^M \sum_{i=1}^{n_m} \frac{\partial}{\partial \theta} \log\left(\frac{\exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}\right)^{y_i} \\ & \quad \left(\frac{1}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1 | s_m))}\right)^{1-y_i} = 0 \end{aligned}$$

where $\theta = (\beta, \delta)'$, β and γ are column vectors of parameters, δ is a scalar, s_i is the individual state variable, $s_m = (s_1, \dots, s_{n_m})$ is the market state variable, and x_i is a subvector of s_i which includes additional market state variables.

For notational brevity, we let $n = \sum_{m=1}^M n_m$ and renumber the observations to write these conditions as:

$$\frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \gamma} \log(\Phi(x'_i \gamma)^{y_i} (1 - \Phi(x'_i \gamma))^{1-y_i}) = 0$$

$$\frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \theta} \log\left(\frac{\exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s))}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s))}\right)^{y_i} \left(\frac{1}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s))}\right)^{1-y_i} = 0$$

Assuming that all $z_i = (x_i, s_i, y_i)$ are i.i.d., the law of large numbers yields:

$$\mathbb{E}(m(z_i; \gamma_0)) = 0 \quad (3.14)$$

$$\mathbb{E}(g(z_i; \theta_0, \gamma_0)) = 0 \quad (3.15)$$

where

$$m(z_i; \gamma_0) = \frac{\partial}{\partial \gamma} \log(\Phi(x'_i \gamma)^{y_i} (1 - \Phi(x'_i \gamma))^{1-y_i}) \quad (3.16)$$

$$m(z_i; \gamma_0) = \frac{\partial}{\partial \gamma} \log(f_1(z_i, \gamma_0)) \quad (3.17)$$

$$g(z_i; \theta_0, \gamma_0) = \frac{\partial}{\partial \theta} \log\left(\frac{\exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s_m))}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s_m))}\right)^{y_i} \left(\frac{1}{1 + \exp(s'_i \beta + \delta \sum_{j \neq i} \hat{\sigma}(a_j = 1|s_m))}\right)^{1-y_i} \quad (3.18)$$

$$g(z_i; \theta_0, \gamma_0) = \frac{\partial}{\partial \theta} \log(f_2(z_i, \theta_0, \gamma_0)) \quad (3.19)$$

A feasible estimator for the moment condition given in Equation 3.15 is given by:

$$\frac{1}{n} \sum_{i=1}^n g(z_i; \hat{\theta}, \hat{\gamma}) = 0.$$

Invoking the mean value theorem, an expansion around the true value θ_0 yields the condition:

$$\frac{1}{n} \sum_{i=1}^n g(z_i; \theta_0, \hat{\gamma}) + \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial g(z_i; \theta^*, \hat{\gamma})}{\partial \theta'}\right) (\hat{\theta} - \theta_0) = 0.$$

where θ^* lies between (or on the line connecting) θ_0 and $\hat{\theta}$. Rearranging this expression yields:

$$\sqrt{n}(\hat{\theta} - \theta_0) = -\left(\frac{1}{n} \sum_{i=1}^n \frac{\partial g(z_i; \theta^*, \hat{\gamma})}{\partial \theta'}\right)^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \hat{\gamma})\right) \quad (3.20)$$

The second term on the right hand side of Equation 3.20 can be expanded around the true value γ_0 in a similar way:

$$\begin{aligned} \frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \hat{\gamma}) &= \frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) + \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \frac{\partial g(z_i; \theta_0, \gamma^*)}{\partial \gamma'} \right) (\hat{\gamma} - \gamma_0) \\ \rightarrow \frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \hat{\gamma}) &= \frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) + \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial g(z_i; \theta_0, \gamma^*)}{\partial \gamma'} \right) \sqrt{n} (\hat{\gamma} - \gamma_0) \end{aligned}$$

where γ^* lies between γ_0 and $\hat{\gamma}$.

We also have as an estimator for the moment condition given in Equation 3.14, which can be expanded by again appealing to the Mean Value Theorem:

$$\begin{aligned} \frac{1}{n} \sum_{i=1}^n m(z_i; \hat{\gamma}) &= 0 \\ \rightarrow 0 &= \frac{1}{n} \sum_{i=1}^n m(z_i; \gamma_0) + \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial m(z_i; \gamma^*)}{\partial \gamma'} \right) (\hat{\gamma} - \gamma_0) \end{aligned} \quad (3.21)$$

$$\rightarrow \sqrt{n} (\hat{\gamma} - \gamma_0) = - \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial m(z_i; \gamma^*)}{\partial \gamma'} \right)^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^n m(z_i; \gamma_0) \quad (3.22)$$

Then, combining Equations 3.20, 3.21, and 3.22 yields:

$$\begin{aligned} \sqrt{n} (\hat{\theta} - \theta_0) &= - \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial g(z_i; \theta^*, \hat{\gamma})}{\partial \theta'} \right)^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) \right. \\ &\quad \left. + \frac{1}{n} \sum_{i=1}^n \frac{\partial g(z_i; \theta_0, \gamma^*)}{\partial \gamma'} \left(- \left(\frac{1}{n} \sum_{i=1}^n \frac{\partial m(z_i; \gamma^*)}{\partial \gamma'} \right)^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^n m(z_i; \gamma_0) \right) \right) \end{aligned} \quad (3.23)$$

Assuming that $\hat{\theta}$ and $\hat{\gamma}$ are consistent and under some regularity conditions, we can rewrite this result as:

$$\begin{aligned} \sqrt{n} (\hat{\theta} - \theta_0) &= - \left(\frac{1}{n} \sum_{i=1}^n \left(\frac{\partial g(z_i; \theta_0, \gamma_0)}{\partial \theta'} + o_p(1) \right) \right)^{-1} \\ &\quad \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) + \frac{1}{n} \sum_{i=1}^n \left(\frac{\partial g(z_i; \theta_0, \gamma_0)}{\partial \gamma'} + o_p(1) \right) \right. \\ &\quad \left. \left(- \left(\frac{1}{n} \sum_{i=1}^n \left(\frac{\partial m(z_i; \gamma_0)}{\partial \gamma'} + o_p(1) \right) \right)^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^n m(z_i; \gamma_0) \right) \right). \end{aligned} \quad (3.24)$$

Denote:

$$G_\theta = \mathbb{E}\left(\frac{\partial g(z_i; \theta_0, \gamma_0)}{\partial \theta'}\right), G_\gamma = \mathbb{E}\left(\frac{\partial g(z_i; \theta_0, \gamma_0)}{\partial \gamma'}\right)$$

. With z_i being i.i.d., the law of large numbers and Slutsky's theorem together yield:

$$\begin{aligned} \sqrt{n}(\hat{\theta} - \theta_0) &= -G_\theta^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) \right) \\ &\quad - G_\theta^{-1} G_\gamma \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \left(-\mathbb{E}\left(\frac{\partial m(z_i; \gamma_0)}{\partial \gamma'}\right)^{-1} m(z_i; \gamma_0) \right) \right) + o_p(1). \end{aligned}$$

Call $\alpha(z_i) = G_\gamma \psi(z_i)$ where $\psi(z_i) = \mathbb{E}\left(\frac{\partial m(z_i; \gamma_0)}{\partial \gamma'}\right)^{-1} m(z_i; \gamma_0)$.

$$\sqrt{n}(\hat{\theta} - \theta_0) = -G_\theta^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n g(z_i; \theta_0, \gamma_0) + \alpha(z_i) \right) + o_p(1)$$

The central limit theorem yields:

$$\begin{aligned} \sqrt{n}(\hat{\theta} - \theta_0) &\rightarrow_d N(0, G_\theta^{-1} \text{Var}(g(z_i; \theta_0, \gamma_0) + \alpha(z_i)) G_\theta^{-1})' \\ &= {}_d N(0, G_\theta^{-1} \mathbb{E}(g(z_i; \theta_0, \gamma_0) g(z_i; \theta_0, \gamma_0)' + g(z_i; \theta_0, \gamma_0) \alpha(z_i)' \\ &\quad + \alpha(z_i) g(z_i; \theta_0, \gamma_0)' + \alpha(z_i) \alpha(z_i)') G_\theta^{-1}) \end{aligned} \tag{3.25}$$

3.7.1.1 Figures and Tables

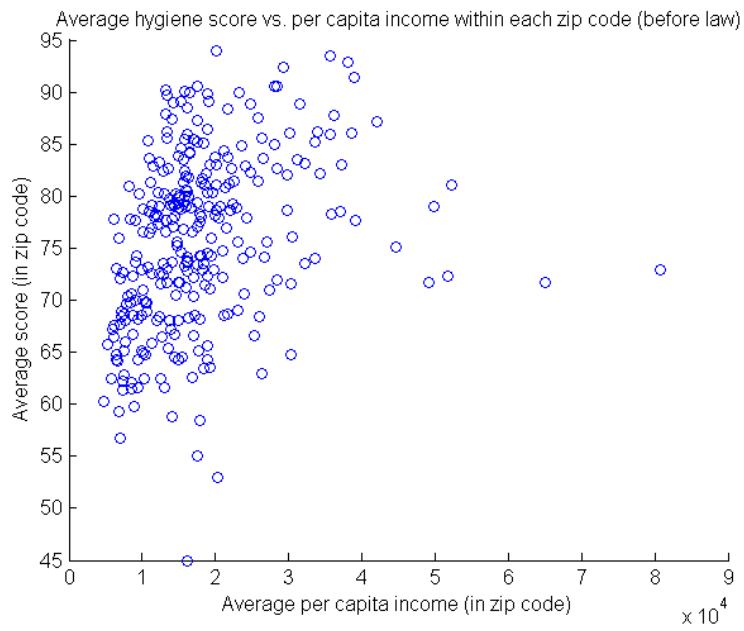


Figure 3.1: Average hygiene score plotted against average per capita income in each zip code before the law (June 1995-Dec 1997)

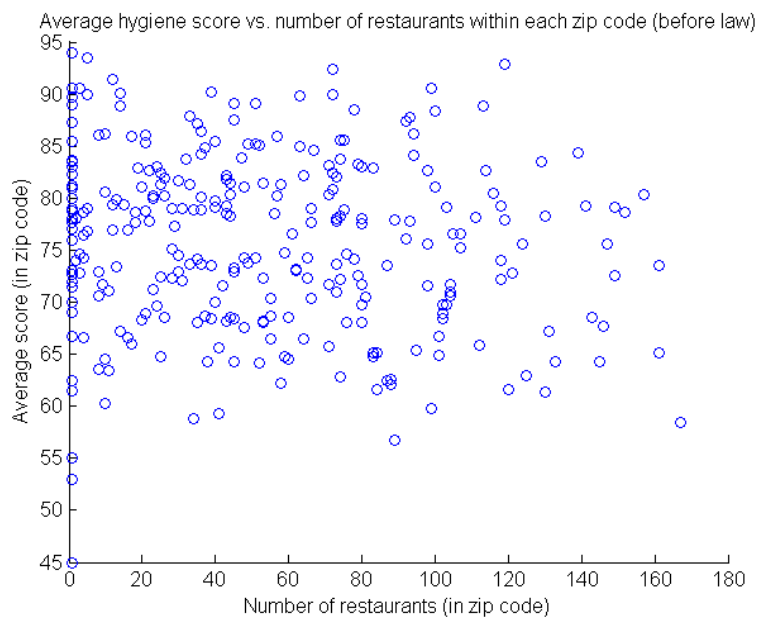


Figure 3.2: Average hygiene score plotted against the number of restaurants in each zip code before the law (June 1995-Dec 1997)

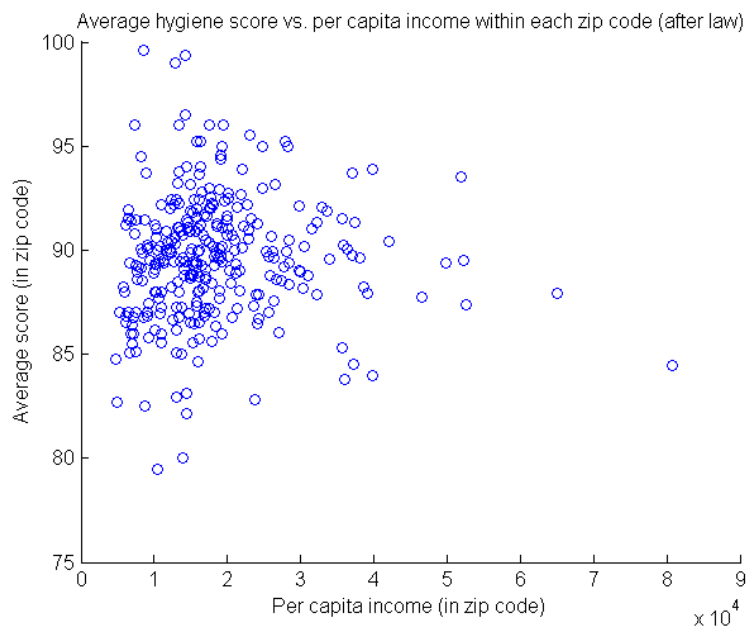


Figure 3.3: Average hygiene score versus average per capita income in each zip code after the law (Jan 1998-Dec 1998)

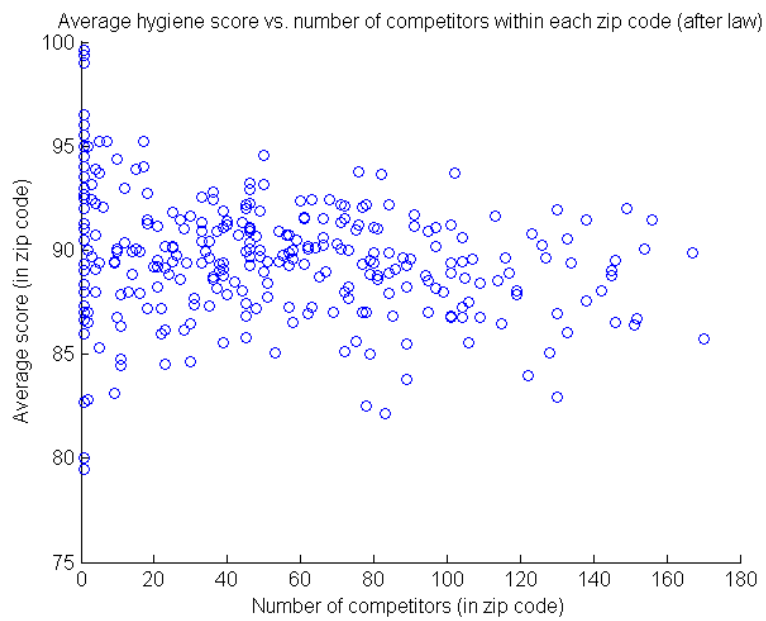


Figure 3.4: Average hygiene score versus the number of restaurants in each zip code after the law (Jan 1998-Dec 1998)

	L1	L2	L3
Before Law	70	75	80
After Law	78.9	83.9	88.9

Table 3.1: Thresholds Used in Estimation

Notes: L1, L2 and L3 denotes the inspection score beyond which the restaurant is considered as providing high quality.

Period	Grade Threshold	<i>INC</i>	<i>AL</i>	<i>NIC</i>	<i>N</i>	<i>PAL</i>	<i>PNIC</i>	Log-likelihood
	70	0.0254*** (0.0011)	-0.0894*** (0.0238)	-0.6314*** (0.0388)	-0.0028*** (0.0003)	1.9559*** (0.1239)	0.2715*** (0.0629)	-8673.6
Before	75	0.0224*** (0.0011)	-0.0919*** (0.0228)	-0.5644*** (0.0331)	-0.0025*** (0.0003)	1.6280*** (0.1174)	-0.1216** (0.0572)	-9726.2
	80	0.0262*** (0.0012)	-0.0962*** (0.0234)	-0.5693*** (0.0310)	-0.0022*** (0.0003)	1.2550*** (0.1229)	-0.6267*** (0.0581)	-9198.7
	78.9	0.0006 (0.0017)	-0.0686** (0.0333)	-0.5287*** (0.0777)	-0.0021*** (0.0004)	0.7573*** (0.1813)	2.2057*** (0.1091)	-3817.9
After	83.9	-0.0002 (0.0014)	-0.0402 (0.0256)	-0.6399*** (0.0485)	-0.0018*** (0.0003)	0.8166*** (0.1399)	1.5719*** (0.0740)	-7258.8
	88.9	0.0015 (0.0013)	-0.0216 (0.0225)	-0.7186*** (0.0350)	-0.0016*** (0.0003)	0.8910*** (0.1229)	0.8422*** (0.0599)	-9915.5

Table 3.2: 1st Stage Estimates, Specification (1)

Notes: The period "Before" and "After" denotes the time period before and after the law, respectively. The grade threshold denotes the quality level beyond which the restaurant is considered as providing high quality. For each period, different grade thresholds are used. Thresholds are either 70, 75 and 80 before the law, and 78.9, 83.9 and 88.9 after law. The variable *INC* denotes average income per capita in the zip code area of restaurant, *AL* is a indicator variable which is equal to 1 if the restaurant has a license to sell alcoholic beverages, and *NIC* is a binary variable which is equal to 1 if the restaurant is not part of chain. The variable *N* is the number of restaurants in the zip code area and lastly *PAL* and *PNIC* are the percentages of restaurants that have alcohol license and are not part of a chain in the zip code area, respectively. Standard errors are in parentheses. * Significant at the 10 percent level. ** Significant at the 5 percent level. *** Significant at the 1 percent level.

Period	Grade Threshold	$\hat{\alpha}_{INC}$	$\hat{\alpha}_{AL}$	$\hat{\alpha}_{NIC}$	$\hat{\delta}$	log-likelihood
Before	70	0.0836*** (0.0019)	-0.0431 (0.0392)	-0.6824*** (0.0444)	-0.0001 (0.0007)	-8788.9
	75	0.0593*** (0.0017)	-0.0634* (0.0363)	-0.8968*** (0.0387)	-0.0014* (0.0008)	-9808.4
	80	0.0477*** (0.0019)	-0.1077*** (0.0374)	-1.3402*** (0.0375)	-0.0055*** (0.0012)	-9337.7
After	78.9	0.0717*** (0.0029)	-0.0696 (0.0660)	0.6145*** 0.0830	0.0108*** (0.0008)	-4192.4
	83.9	0.0426*** (0.0020)	-0.0069 (0.0526)	0.1429*** (0.0475)	0.0075*** (0.0006)	-7679.6
	88.9	0.0285*** (0.0016)	0.0269 (0.0354)	-0.4547*** (0.0414)	0.0050*** (0.0007)	-10151.3

Table 3.3: 2nd Stage Estimates, Specification (1)

Notes: The period "Before" and "After" denotes the time period before and after the law, respectively. The grade threshold denotes the quality level beyond which the restaurant is considered as providing high quality. For each period, different grade thresholds are used. Thresholds are either 70, 75 and 80 before the law, and 78.9, 83.9 and 88.9 after law. The variable *INC* denotes average income per capita in the zip code area of restaurant, *AL* is a indicator variable which is equal to 1 if the restaurant has a license to sell alcoholic beverages, and *NIC* is a binary variable which is equal to 1 if the restaurant is not part of chain. The variable *N* is the number of restaurants in the zip code area and lastly *PAL* and *PNIC* are the percentages of restaurants that have alcohol license and are not part of a chain in the zip code area, respectively. Standard errors are in parentheses. * Significant at the 10 percent level. ** Significant at the 5 percent level. *** Significant at the 1 percent level.

Period	Grade Threshold	$\hat{\gamma}_{INC}$	$\hat{\gamma}_{AL}$	$\hat{\gamma}_{NIC}$	$\hat{\gamma}_N$	$\hat{\gamma}_{N^2}$	$\hat{\gamma}_{PAL}$	$\hat{\gamma}_{PAL^2}$	$\hat{\gamma}_{PNIC}$	log-likelihood
	70	0.0264*** (0.0011)	-0.0825*** (0.0239)	-0.6425*** (0.0388)	0.0092*** (0.0013)	-0.0001*** (0.0000)	1.3521*** (0.3555)	0.6616 (0.4602)	-0.0916 (0.0857)	-8636.6
Before	75	0.0221*** (0.0011)	-0.0913*** (0.0229)	-0.5985*** (0.0330)	0.0069*** (0.0013)	-0.0001*** (0.0000)	2.0098*** (0.3457)	-0.5145 (0.4527)	-0.5136 (0.0792)	-9671.4
	80	0.0286*** (0.0012)	-0.0901*** (0.0234)	-0.5725*** (0.0310)	0.0092*** (0.0013)	-0.0001*** (0.0000)	1.1180*** (0.3725)	-0.2114 (0.4837)	-1.0529*** (0.0841)	-9158.2
	78.9	0.0191*** (0.0016)	-0.0892*** (0.0325)	-0.0380 (0.0788)	0.0303*** (0.0019)	-0.0002*** (0.0000)	0.0560 (0.5468)	0.2407 (0.6946)	-0.0102 (0.1318)	-4020.0
After	83.9	0.0110*** (0.0014)	-0.1091*** (0.0256)	-0.6517*** (0.0481)	0.0146*** (0.0014)	-0.0001*** (0.0000)	-0.3303 (0.4143)	0.2114 (0.5415)	1.0161*** (0.0973)	-7305.4
	88.9	0.0053*** (0.0013)	-0.0187 (0.0225)	-0.7084*** (0.0350)	0.0105*** (0.0013)	-0.0001*** (0.0000)	0.7058* (0.3637)	-0.0135 (0.4778)	0.2900*** (0.0827)	-9903.8

Table 3.4: 1st Stage Estimates, Specification (2)

Notes: The period "Before" and "After" denotes the time period before and after the law, respectively. The grade threshold denotes the quality level beyond which the restaurant is considered as providing high quality. For each period, different grade thresholds are used. Thresholds are either 70, 75 and 80 before the law, and 78.9, 83.9 and 88.9 after law. The variable INC denotes average income per capita in the zip code area of restaurant, AL is a indicator variable which is equal to 1 if the restaurant has a license to sell alcoholic beverages, and NIC is a binary variable which is equal to 1 if the restaurant is not part of chain. The variable N is the number of restaurants in the zip code area and lastly PAL and $PNIC$ are the percentages of restaurants that have alcohol license and are not part of a chain in the zip code area, respectively. Standard errors are in parentheses. * Significant at the 10 percent level. ** Significant at the 5 percent level. *** Significant at the 1 percent level.

Period	Grade Threshold	$\hat{\alpha}_{INC}$	$\hat{\alpha}_{AL}$	$\hat{\alpha}_{NIC}$	$\hat{\delta}$	Log-likelihood
Before	70	0.0808*** (0.0019)	-0.0474 (0.0393)	-0.7189*** (0.0453)	0.0014* (0.0007)	-8787.2
	75	0.0560*** (0.0017)	-0.0696* (0.0364)	-0.9374*** (0.0391)	0.0009 (0.0009)	-9809.3
	80	0.0439*** (0.0020)	-0.1152*** (0.0374)	-1.3712*** (0.0376)	-0.0019 (0.0013)	-9345.1
After	78.9	0.0711*** (0.0029)	-0.0685 (0.0660)	0.6301*** (0.0825)	0.0108*** (0.0009)	-4193.5
	83.9	0.0419*** (0.0021)	-0.0037 (0.0810)	0.1388*** (0.0457)	0.0077*** (0.0006)	-7676.7
	88.9	0.0276*** (0.0016)	0.0261 (0.0356)	-0.4715*** (0.0417)	0.0057*** (0.0007)	-10145.1

Table 3.5: 2nd Stage Estimates, Specification (2)

Notes: The period "Before" and "After" denotes the time period before and after the law, respectively. The grade threshold denotes the quality level beyond which the restaurant is considered as providing high quality. For each period, different grade thresholds are used. Thresholds are either 70, 75 and 80 before the law, and 78.9, 83.9 and 88.9 after law. The variable *INC* denotes average income per capita in the zip code area of restaurant, *AL* is a indicator variable which is equal to 1 if the restaurant has a license to sell alcoholic beverages, and *NIC* is a binary variable which is equal to 1 if the restaurant is not part of chain. The variable *N* is the number of restaurants in the zip code area and lastly *PAL* and *PNIC* are the percentages of restaurants that have alcohol license and are not part of a chain in the zip code area, respectively. Standard errors are in parentheses. * Significant at the 10 percent level. ** Significant at the 5 percent level. *** Significant at the 1 percent level.

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