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A Time-Marching Scheme for Analyzing Transient Scattering from Nonplanar Doubly Periodic Structures

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Introduction

This paper presents a novel marching-on-in-time (MOT) based scheme for solving a time domain integral equation (TDIE) pertinent to the analysis of plane wave scattering from (potentially nonlinearly loaded) nonplanar doubly periodic structures with unit cells comprising perfect electrically conducting (PEC) elements and inhomogeneous dielectric volumes (Fig. 1). Historically, the analysis of transient scattering from doubly periodic structures has been carried out predominately using finite difference time domain methods, see e.g. [1]. Unfortunately, when the periodic structure is excited by obliquely incident fields, noncausal terms enter the FDTD update equations (unless specific measures are taken). Within a TDIE framework, the same problem arises, though it is easily resolved through the use of time-shifted temporal basis function in combination with bandlimited extrapolation methods [2]. Unfortunately, at present, these TDIE solvers remain computationally expensive, which precludes their application to the analysis of real-world, complicated structures. A recently developed Floquet wave-based TDIE solver [3] relieves this burden by expanding fields generated by quiescent and bandlimited periodic currents into time domain Floquet waves (TDFWs) [4]. By evolving this expansion using so-called blocked FFT methods [5], the solver detailed in [3] permits the efficient analysis of discretely planar, free-standing PEC structures. This paper presents an improvement of the scheme in [3] aimed at rendering the solver applicable to nonplanar structures. Specifically, it introduces a scheme for evolving TDFW mode amplitudes along one-dimensional domains that permits their efficient evaluation not only on the source plane, but also removed from it. The proposed scheme evolves individual TDFWs using a spectral scheme supplemented with Huygen's based boundary conditions. The resulting scheme applies without difficulty to nonplanar doubly periodic structures comprising PEC and sculptured dielectric substrates.

The time-marching scheme

Consider a doubly periodic combined PEC-dielectric structure that is excited by an oblique incident plane wave of maximum temporal frequency \( \omega_{\text{max}} \). To analyze scattering from this structure, a TDIE can be constructed in terms of the induced PEC surface and dielectric polarization currents residing in one of the structure's cells (typically referred to as the mother cell). This equation can be solved by MOT methods. Key to this scheme is a capability to evaluate the so-called "scattered field", produced by present and past currents residing on the array. The reader is referred to [2] for details regarding the construction of the MOT system. In the remainder of this paper, the focus is on an efficient scheme for computing the fields produced by doubly periodic transient...
though bandlimited sources using TDFW concepts. Only a scalar version of the scheme is presented to minimize notational overhead.

Consider a source at \( r' \) (on the mother cell), and with a temporal signature \( g(t) \) (bandlimited by \( \omega_{\text{max}} \)), which produces a scalar field \( F(r, t) = G(r, r', t) \ast g(t) \), where "\( \ast \)" denotes temporal convolution, the Green's function is

\[
G(r, r', t) = \frac{\sin \theta_{\text{inc}} \cos \phi_{\text{inc}} - \sin \theta_{\text{inc}} \sin \phi_{\text{inc}} \cos \theta_{\text{inc}} - \cos \theta_{\text{inc}}}{|r' + r_\text{in} - r|}
\]

where \( \theta_{\text{inc}} \) is the direction of the incident wave, \( c \) is the wave speed in the free space, \( r_\text{in} = m_1 D_x + n_1 D_y \), and \( D_x \) and \( D_y \) denote unit cell dimensions along \( x \) and \( y \), respectively. To efficiently evaluate \( F(r, t) \), the source signal \( g(t) \) is decomposed in terms of approximate prolate spheroidal wave functions (APSWFs) as

\[
g(t) = \sum_{n} \sqrt{\omega_n} P(t - iAt, T, \omega_n, n)
\]

where

\[
\omega_n = \frac{\omega_n - \omega_{\text{max}}}{2}, \quad \Omega = (\omega_n - \omega_{\text{max}}) / 2 \quad \text{and} \quad T = T_{\text{pt}} \quad \text{is the temporal support of} \quad P(t, T_{\text{pt}}, \omega_n, n). \quad \text{Correspondingly, the field is also decomposed into sub-fields, viz.} \quad F(r, t) = \sum_{n} F_n(r, t) \quad \text{and each sub-field is due to the sample} \quad g(iAt) P(t - iAt, T, \omega_n, n) \quad \text{of the source signal. It follows that each sub-field} \quad F_n(r, t) \quad \text{if observed after the corresponding APSWF vanishes, can be efficiently calculated using a TDFW-based scheme; specifically,}
\]

\[
F_n(r, t) = \left[ g(iAt) P(t - iAt, T, \omega_n, n) \right] \ast \tilde{G}(r, r', t) \quad \text{when} \quad t > iAt + T_{\text{pt}}
\]

where the "pseudo-Green function" \( \tilde{G}(r, r', t) \) is composed of TDFWs [3]:

\[
\tilde{G}(r, r', t) = A_{\text{pt}}^\text{FW}(r, r', t) U(t - \tau_0)
\]

where

\[
\begin{align*}
A_{\text{pt}}^\text{FW}(r, r', t) &= \frac{e^{-j \eta_{\text{pt}} r}}{2D_xD_y \sqrt{1 - \eta^2}} e^{j \eta_{\text{pt}} \sqrt{1 - \eta^2} \tau_0} U(t - \tau_0) \\
\eta_{\text{pt}} &= \frac{\omega_n + \omega_{\text{max}}}{2} \quad \text{and} \quad T_{\text{pt}} = T_{\text{pt}} (\omega_n, n). \quad \text{To make} \quad G(r, r', t) \quad \text{converge, only a finite number} \quad N_{\text{pt}} \quad \text{of TDFWs suffices, determined by} \quad \omega_n \quad \text{and the unit cell size. By exploiting the plane wave nature of TDFWs, the} \quad F_n(r, t) \quad \text{can be expressed as}
\end{align*}
\]

\[
F_n(r, t) = \sum_{\lambda, \mu} A_{\lambda \mu}^\text{pt}(r - z_{\text{ref}}, \lambda - z_{\text{ref}}, \mu) e^{-j \omega_n (\lambda - \mu) \tau_0} U(t - \tau_0) \quad \text{when} \quad t > iAt + T_{\text{pt}}
\]

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It appears that \( u_{n}(x,t) \) solves the partial differential equation (PDE):
\[
\frac{1}{c^2} \frac{\partial^2 u_{n}(x,t)}{\partial t^2} - \frac{2\mu_0}{c^2} \frac{\partial u_{n}(x,t)}{\partial t} - \frac{1}{1 - \eta^2} \frac{\partial^2 u_{n}(x,t)}{\partial x^2} + \frac{\partial^2 u_{n}(x,t)}{\partial y^2} = \delta_{n}(x,t)
\]
with
\[
S_{n}(x,t) = e^{-j\omega_{n}(\eta - \rho_{n})}[t - \rho_{n} \cdot (\rho_{n} - \rho_{s})] / c.
\]
Equation (8) can be transformed to the spectral domain:
\[
\frac{1}{c^2} \frac{\partial^2 \hat{u}_{n}(k,x,t)}{\partial t^2} - \frac{2\mu_0}{c^2} \frac{\partial \hat{u}_{n}(k,x,t)}{\partial t} + \frac{\partial^2 \hat{u}_{n}(k,x,t)}{\partial x^2} + \frac{\partial^2 \hat{u}_{n}(k,x,t)}{\partial y^2} = \hat{S}_{n}(k,x,t)
\]
Here \( \hat{S}_{n}(k,x,t) = (k^2 + \tau_{n}^2)/(1 - \eta^2) \)
\[
\hat{u}_{n}(k,x,t) = \int_{-\infty}^{\infty} u_{n}(x,t)e^{j\omega t} \, dx
\]
and
\[
\hat{S}_{n}(k,x,t) = e^{-j\omega_{n}(\eta - \rho_{n})}[t - \rho_{n} \cdot (\rho_{n} - \rho_{s})] / c.
\]
Numerical results

The above-described time-marching scheme is applied to analyze transient scattering from doubly periodic square-loop elements imprinted on a grooved dielectric slab. The grooved dielectric slab is of a relative permittivity \( \varepsilon_r = 3.0 \) and thickness 7 mm. The side length of the square mother cell is 7 em. The dimension of the square-loop is shown in the inset of Fig. 2. The structure is illuminated by a \( \hat{\rangle} \) polarized normally incident field with time signature
\[
f(t) = \cos\left[2\pi f_{c}(t - t_{p})\right] \exp\left[-\left(t - t_{p}\right)^{2}/(2\sigma^{2})\right],
\]
where \( f_{c} = 1.2 \text{ GHz} \) is the center frequency of the incident wave, \( \sigma = 6/(2\pi f_{c}) \) and \( t_{p} = 6\sigma \) with \( f_{c} = 1.2 \text{ GHz} \) termed the "bandwidth" of the signal. The geometry is discretized in terms of 1908 spatial unknowns. The time step is \( \Delta t = 20.83 \text{ ps} \) and 1024 time steps are used in the analysis. Forty-nine TDFWs, viz. \( N_{mode} = 49 \), are used in the time-marching scheme. The result obtained from the time-marching scheme is Fourier transformed to the frequency domain and compared against data from a frequency
domain periodic method of moments (P-MoM) code. The power transmission coefficients of the structure are shown in Fig. 2. A good agreement between the two data sets is observed.

References


