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July 3, 1952

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ABSTRACT

Nuclear forces yielded by the symmetrical pseudoscalar theory are discussed in terms of a perturbation expansion. It is shown that up to terms in the square of the coupling constant that the pseudoscalar coupling is equivalent to a scalar pair coupling of the pseudoscalar field plus a pseudovector coupling of that field. The dominant contribution to the fourth order nucleon potential is then obtained in a simple way by using this result.

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Introduction

The nuclear forces given by the symmetrical pseudoscalar meson theory with pseudoscalar coupling has been re-examined in recent years by a number of authors¹ treating the meson-nucleon interaction as weak. They have shown that the contribution to these forces due to processes involving transport of momentum between nucleons by a pair of mesons are larger than those due to a single meson. The pseudoscalar character of the meson implies that simultaneous emission or absorption of S-state meson pairs by a single nucleon cannot involve nucleon spin change. Similarly the symmetrical theory implies that there can be no isotopic spin change for these processes. As a consequence the forces due to them are spin and charge independent.

The importance of these effects suggests that one should be able to exhibit them explicitly in the meson-nucleon interaction hamiltonian. In what follows the nuclear forces are obtained in a simple manner by taking advantage of this possibility. Besides having the aforementioned properties it is shown that these forces are highly singular and have a range of half the meson compton wavelength.

-4-

1. Transformation of the Hamiltonian

The equation of motion, in the interaction representation², of the state vector, $\psi[\sigma]$, of the coupled meson and nucleon fields is determined by the coupling hamiltonian

$$H(x) = i \bar{\psi}(x) \gamma_5 \tau_\alpha \psi(x) \phi_\alpha(x). \quad (1)$$

Here and in the following ψ , $\bar{\psi}$, ϕ represent the nucleon and meson field variables, τ_α is the nucleon isotopic spin and

$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4$ is the Dirac pseudoscalar. Units are chosen so $\hbar = c = 1$ and the meson and nucleon masses are μ and K_0 , respectively.

If one now applies the transformation $e^{iS[\sigma]}$

$$S[\sigma] = - \frac{if}{2K_0} \int d\sigma_\mu \bar{\psi}(x) \gamma_\mu \gamma_5 \tau_\alpha \phi_\alpha(x) \psi(x) \quad (2)$$

used by Dyson³ to the state vector $\psi[\sigma]$

$$\psi[\sigma] = e^{iS[\sigma]} \psi'[\sigma] \quad (3)$$

one finds the new hamiltonian is, up to terms in f^3 ,

$$H'(x) = H(x) - i \left[S[\sigma], H(x) \right] + \frac{\delta S[\sigma]}{\delta \sigma(x)} - \frac{i}{2} \left[S[\sigma], \frac{\delta S[\sigma]}{\delta \sigma(x)} \right]. \quad (4)$$

The choice of $S[\sigma]$ insures that the new hamiltonian contains no term in $H(x)$ since

-5-

$$\frac{\delta S[\sigma]}{\delta \sigma(x)} = -H(x) - \frac{if}{2K_0} \bar{\psi}(x) \gamma_\mu \gamma_5 \gamma_\alpha \partial_\mu \phi_\alpha(x) \psi(x) \quad (5)$$

If this expression for $\delta S[\sigma]/\delta \sigma(x)$ is inserted in Eq. (4) one finds that the commutators can be readily evaluated to give⁴

$$\begin{aligned} H'(x) = & \frac{f^2}{2K_0} \bar{\psi}(x) \psi(x) \phi_\alpha^2(x) - \frac{if}{2K_0} \bar{\psi} \gamma_\mu \gamma_5 \gamma_\alpha \partial_\mu \phi_\alpha(x) \psi(x) \\ & + \frac{1}{2} \left(\frac{f}{2K_0} \right)^2 \left[\gamma_\mu \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) \right]^2 - \left(\frac{f}{2K_0} \right)^2 \bar{\psi}(x) \gamma_\mu \epsilon_{\alpha\beta\gamma} \gamma_\alpha \phi_\beta(x) \partial_\mu \phi_\gamma(x) \psi(x). \end{aligned} \quad (6)$$

The main terms in $H'(x)$ correspond to a scalar pair coupling of a pseudoscalar field plus two terms corresponding to the usual pseudovector coupling. The important thing to notice is that only a single power of the nucleon rest mass is contained in the denominator of the pair coupling. A similar result has been obtained by Foldy⁵ using a different canonical transformation. It can also be shown that the scalar pair coupling yields the dominant contribution to the nuclear forces in a strong coupling approximation.⁶

2. Determination of the Nuclear Forces

The interaction hamiltonian, $H'(x)$, may now be treated by standard Feynman-Dyson techniques to yield the nuclear forces.

The second order nuclear force due to the pseudovector coupling has been obtained by many authors⁷ so only the force yielded by the pair coupling will be discussed here.

-6-

The term in the scattering matrix corresponding to the lowest order effect involving two nucleons of the scalar pair coupling is

$$S_4 = \frac{-3 f^4}{16 K_0^2} \int dx_1 \int dx_2 \bar{\psi}(x_1) \psi(x_1) \bar{\psi}(x_2) \psi(x_2) \Delta_F^2(x_1 - x_2). \quad (7)$$

$\Delta_F(x)$ is the Feynman Δ -function as defined by Dyson⁸.

The nuclear force may be inferred from this expression by replacing all quantities in Eq. (7) by their appropriate non-relativistic approximation. One finds, in the centre of momentum system, for initial and final nucleon momenta $P_\mu, Q_\mu; P'_\mu, Q'_\mu$,

$$S_4 = -2\pi i \delta(P'_0 + Q'_0 - P_0 - Q_0) (P' Q' | V_4(r) | PQ) \quad (8)$$

where

$$V_4(r) = -\frac{3 f^4}{16 K_0^2} \int_{-\infty}^{\infty} dt \Delta_F^2(t^2 - r^2) \quad (9)$$

and $(r, t) = (x_1 - x_2, t_1 - t_2)$, the relative coordinates of the two nucleons.

The integral in Eq. (9) may be evaluated by transforming to momentum space or by using one of the integral representations⁸ for $\Delta_F(x)$. One finds

$$V_4(r) = -\left(\frac{f^2}{4\pi}\right)^2 \left(\frac{\mu}{2x_0}\right)^2 \frac{3\mu}{\pi(\mu r)^2} \int_{\mu r}^{\infty} K_0(2\lambda) d\lambda + \text{contact term} \quad (10)$$

for the pair-exchange nuclear potential. There are, of course,

-7-

other terms in the nuclear potential proportional to f^4 ; these are, however, much smaller and contribute less than ten percent to the nucleon-nucleon interaction.

At small distances $V_4(r)$ represents a highly singular potential

$$V_4(r) \sim (\mu r)^{-1} \ln(\mu r) . \quad (11)$$

Whereas for large distances it has half the range of a Yukawa potential.

$$V_4(r) \sim (\mu r)^{-5/2} e^{-2\mu r} . \quad (12)$$

This potential has been studied by Levy⁹ in connection with a hard core model for the nuclear forces.

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