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Agenda Chasing and Contests Among News Providers*

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Abstract

This article studies competition in contests with a focus on the news industry that is increasingly influenced by social media. The model assumes publishers to pick a single topic from a large pool based on the topics' prior "success" probabilities, thereby "chasing" potentially successful topics. Firms that publish topics that become successful divide a "reward" which can change with the number of competing firms and the number of successful topics. The results show that share structures can be categorized into three types that, in turn, lead to qualitatively different outcomes for the contest. Furthermore, topic diversity increases with the number of participating firms only if the share they get from the reward dissipates rapidly.

Keywords: Media Competition, Contests, Agenda Setting

JEL codes: C72, D81, L82

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1 Introduction

Contests – competitive situations where winners share a reward – describe many economic interactions, including, for example, the rivalry among R&D teams, competition among forecasters or the race among job candidates. In this article, we develop a generalized contest model to describe competition between online news publishers. Largely due to the emergence of the Internet, news media have undergone qualitative change. There are many facets to this transformation but – we argue – an important aspect of it is that news outlets increasingly compete in a contest where (i) they try to publish stories that are likely to become "viral" due to consumers' online sharing activities and (ii), if successful, they divide online ad revenues closely linked to pageviews. The resulting contest has important implications for the type and diversity of news that end up being published.

Traditionally, mainstream media (the press or broadcast media) had a strong role in so-called "agenda setting". By 2010, the appearence of online media sites combined with the broad adoption of social networks lead to a more "diluted" public agenda where news outlets seem to grab considerable attention from the public by publishing seemingly marginal topics. A 2014 article in the *Financial Times*, tellingly entitled *"You won't believe what viral content does to news"* describes the trend as follows: "In 2014, the fastest-growing form of online 'content' is an epidemic of heartwarming videos [...], funny lists [...] and click-bait headlines from sites such as BuzzFeed, Upworthy and ViralNova. Rather than being found on news sites or through search engines, they flourish on social networks such as Facebook and Twitter" (Gapper 2014). Indeed, there is ample anecdotal evidence that social media may propel so-called trivia topics in the mainstream news (see, e.g. Deighton and Kornfeld (2010)).

Competition between online news sites is fierce. Barriers to entry in the news industry have declined substantially in recent years leading to the emergence of new players, such as the Huffington Post, BuzzFeed or The Business Insider, to only name the most prominent ones. Many of these new entrants came to prominence by developing proprietary forecasting technologies to rapidly identify popular themes that are likely to become mainstream online. The Huffington Post pioneered technology to monitor the public's search behavior to identify emerging topics. BuzzFeed succeeded by indentifying "trending" content based on technology that monitors sharing behavior on popular social media sites, such as Facebook or Twitter. In a world where social media has become the dominant source of traffic for news[1], such technologies have shown rapid results in terms of pageviews attracting large investments by established media conglomerates. Traditional publishers have had no choice but to follow the new entrants' tactics. In fact, the line between traditional and new media is increasingly blurred as even iconic examples of the respective categories copy each other's strategies. Traditional media have realized the need to link to external sources of content to remain relevant.[2] Even television news providers have taken notice of the emerging trend and have launched their own 'Web corners' not to miss out on the potential traffic generated by social media. In the meantime, new entrants have hired dozens of reporters, many of them previously employed by traditional news outlets, to improve their content quality.

It is important to realize that news sites cannot consider reporting on everything even though they do not face physical space limits. Consumers' attention is limited and they can quickly switch between pages. Similarly, if consumers use search engines, they only consider the first few items on the search engine's results list. Thus, to have a chance at becoming the most shared item on a particular topic requires considerable editorial 'investment', hence the hiring of traditional journalists at many of the news startups that discover the importance of "curation" beyond just focusing on forecasting algorithms for trending content.

In sum, in this new environment dominated by social media, where consumers' sharing behavior can quickly propel a news item to the top, competition among news providers is best

[1]See, for example, the data presented by Thompson (2014) in *The Atlantic* with the telling subtitle: "Social networks are the new front page and homepage for news. [..]". Facebook's growing influence on news is also reflected in that it increasingly requires publishers to host their content on its servers (Alpert 2015).

[2]An internal report on innovation by the *New York Times* that leaked to the public in March 2014 clearly reveals that the newspaper seeks to adopt the techniques that its newly minted competitors apply to generate audiences from social media.

described as a permanent contest where firms have to "bet" on a (few) topics and the "winners" divide the advertising revenues generated from pageviews (the "prize" of the contest).[3] As such, rather than "setting the agenda", as they did in the past, today's news providers are "chasing the agenda" mostly set by consumers' sharing behavior on the Internet. Indeed, the central question of the article is how this contest nature of competition between news providers is likely to influence the public agenda? That is, what type of news will emerge from the massive amount of diverse content available? In particular, will competition focus firms on a relatively few important topics or will such competition lead to the vast proliferation of published topics with a fragmented news scene? How will the distribution of content, the number of competitors and the division of ad revenues across winning firms influence the nature of published news and its diversity? Answers to these questions are important to understand our political/social environment. In the second part of the article, we focus our analysis on firms' competitive strategies. Specifically, we ask how asymmetries across news providers impact equilibrium outcomes? In particular, we explore firm assymmetries in forecasting capabilities and customer loyalty (e.g. brand equity).

To answer the above questions, we develop a generalized contest model in which firms have to choose one news item from a large set of items with varying prior probabilities of success. We allow for the simultaneous and/or correlated success of multiple items. Importantly, we model *all* relevant ways in which 'winning sites' (in the sense of reporting on eventually popular topics) may divide the reward for successful news. This ensures that we capture a broad range of competitive scenarios between publishers.

We generally find that the variety/diversity of topics as well as the weight given to 'marginal' (*a priori* unlikely) topics increases the more correlated the success of *a priori* likely topics are. In other words, whereas topics with high prior probabilities of success are obvious targets for publishers, their likelihood of being chosen is greatly reduced if their success is correlated. More

[3] The idea of a 'contest' is not entirely foreign to the news industry. It describes *some* aspect of competition among traditional news media who had to pick topics from a relatively large set generated by upstream information providers such as the Associated Press, Reuters or Agence France Press.

importantly, the number of competing firms has a non-trivial effect on the diversity of news. Interestingly, as long as the contest is "not too strong" among sites, their choice of published topics is concentrated on the ones with the highest prior success probabilities and this is even more so the more sites enter the market. In this case, increased entry actually reinforces the concentration of news (or tightens the public agenda). In contrast, when the intensity of the contest among sites is beyond a certain threshold, competition tends to rapidly increase the fragmentation of published news: as the number of competing publishers increases, more and more *a priori* unlikely topics are reported resulting in a large diversity of published topics (or a broader public agenda). This result is reminiscent of the emergence of "funny lists" and "heartwarming videos" in the news mentioned by the *Financial Times* (Gapper 2014). Our analysis extends to pure- and mixed-strategies and distinguishes between cases with small or large number of competing publishers. The connections between equilibrium outcomes in these various scenarios are established.

Next, in a model with firm asymmetries, we find that when some firms have better technology to forecast the popularity of topics, then, surprisingly, the overall diversity of news published by the remaining firms declines as these firms tend to take refuge in publishing 'safer' topics. When a subset of firms have extra revenue from a published 'hit' from loyal users then these 'branded' publishers tend to be conservative in their choice of topics as their loyal customer base represents 'insurance' against the contest. In contrast, the diversity of news published by unbranded outlets increases as unbranded publishers tend to avoid branded ones by putting more weight on *a priori* unlikely stories. These results are consistent with anecdotal evidence in the news industry where traditional (branded) news outlets are more conservative in their reporting whereas new entrants (that also tend to have an advantage in forecasting viral content) do not shy away from controversial stories. The findings are also conform to the broadly observed increase of diversity in the public agenda by communication theorists (see, e.g. McCombs and Zhu (1995)).

In a final analysis, we consider endogenous success probabilities. It is widely accepted that the media often 'makes the news' in the sense that a topic may become relevant simply because it got published. Interestingly, such a dynamic has an ambiguous effect on the diversity of published topics. If the contest is very strong then it results in a concentrated set of *a priori* likely topics. When the contest is moderate then the diversity of topics may be higher depending on the number of competing outlets.

The article is organized as follows. In the next section, we summarize the relevant literature. This is followed by the description of the basic model and its analysis where we first present a variety of results concerning symmetric competitors. Next, we extend the model to explore the impact of asymmetries across firms. Our last extension considers the case of endogenous success probabilities. The article ends with a discussion of the results and their applicability to other contexts. To facilitate reading, all proofs are relegated to the Appendix.

2 Relevant literature

The topic of this article is generally related to the literature on agenda setting (see McCombs (2004) for an excellent recent review) that studies the role of media in focusing the public on certain topics instead of others. It is broadly believed that agenda setting has a greater influence on the public than published opinion whose explicit purpose is to influence the readers' perspective. As the famous saying by Bernard Cohen (1963) goes: "The media may not be successful in telling people what to think but they are stunningly successful in telling their audiences what to think about". The literature examines the mechanisms that lead to the emergence of topics and the diversity of topics across media outlets. In particular, McCombs and Zhu (1995) show that the general diversity of topics as well as their volatility has been steadily increasing over time. The general focus of our article is similar: we show that the nature of competition is an important mechanism affecting the diversity of public agenda.

Agenda setting is also addressed in the literature studying the political economy of mass media (see Prat and Stromberg (2013) for an excellent review).[4] The standard theory states that media coverage is higher for topics that are of interest for (a) larger groups, (b) with larger advertising potential, and (c) when the topic is journalistically more "newsworthy" and (d) cheaper to distribute. Although there is little empirical evidence to support (b), the other hypotheses are generally supported (see Stromberg (2004) and Snyder and Stromberg (2010), among others). Hypotheses (c) is particularly interesting from our standpoint. Eisensee and Stromberg (2007) show that the demand for topics can vary substantially over time. For example, sensational topics of general interest (e.g. the Olympic Games) may crowd out other 'important' topics (e.g. natural disasters) that would be covered otherwise. This supports the general notion that media needs to constantly forecast the likely success of topics and select among them accordingly. Our main interest is different from this literature's as we primarily focus on media competition as opposed to what causes variations in demand. Taking the demand as given, our goal is to understand how the competitive forces between media firms influences the selection and diversity of topics, which then has a major impact on the public agenda.

As such, the article also relates to the literature on media competition where strategic behavior influences product variety. Early thoretical work by Steiner (1952) and Beebe (1977) on the "genre" selection of broadcasters explains cases of insufficient variety provision in an oligopoly. Interestingly, they show that although certain situations lead to the duplication of popular genres (neglecting niche segments), other scenarios may lead to a "lowest common denominator" outcome where no consumer's first choice of genre is ever served. A good discussion of these models and their extensions can be found in Anderson and Waldfogel (2016). Our work is different from this literature in two important ways: (i) we do not have consumer heterogeneity and, (ii) we do not rely on barriers to entry (fixed costs) to explain limited variety. In

[4]Although not our focus, this literature also addresses the related issue of media bias and how it is affected by media competition – see Mullainathan and Shleifer (2005) and Xiang and Sarvary (2007) for relevant analytical models and Gentzkow and Shapiro (2010) and Larcinese et al. (2011) for empirical evidence. The latter article is interesting because it shows how newspapers can achieve bias by overrepresenting favourable topics to politicians close to their voters.

fact, we study variety precisely when these factors' importance is greatly diminished.

On the empirical side, research on competition primarily focuses on how media concentration affects the diversity of news both in terms of the issues discussed in the media as well as the diversity of opinion on a particular issue. For example, George and Oberholzer-Gee (2011) show that in local broadcast news, "issue diversity" grows with increased competition (as measured by the number of local TV stations) even though political diversity tends to decrease. Franceschelli (2011) studies the impact of the Internet on news coverage, in particular the recent decrease in the lead-time for catching up with missed breaking news. He argues that missing the breaking news has less impact, as the news outlet can catch up with rivals in less time. This might lead to a free-riding effect among media outlets, where there is less incentive to identify the breaking news. Both of these articles have consistent empirical findings with our results/assumptions.

In terms of the analytical model, we rely on the literature studying competitive contests among forecasters. For example, Ottaviani and Sorensen (2006) use a similar framework to model competition among financial analysts. Our model is different in that we explore in more detail the structure of the state space, we generalize the contest model by considering all possible prize-sharing structures and extend it in a variety of ways, most notably by analyzing asymmetries across players.

3 Base model

As mentioned earlier, contests may be relevant in a variety of social/economic interactions. Yet, contest models need to be adapted to particular contexts, in this article, to the context of online news providers. Specifically, we assume that each outlet tries to identify and publish the topics that will ultimately become of central interest to the public. We assume that these topics attract an audience (e.g. pageviews) that is then monetized (e.g. translate to advertising revenues), which essentially constitutes the prize of the contest. This prize is divided across the 'winning' sites - those who report on ex post popular topics. The challenge for the media outlets is that, in addition to accurately forecasting the likely success of stories from a vast amount of content, they also need to try to identify unique stories that other sites did *not* publish. The contest nature of competition comes from the fact that successful sites need to share the audiences if they all identified the same story(ies). How this sharing is implemented is of particular interest because it essentially captures the nature of competition between the publishers.

To formalize this setup assume N competing news providers and K topics with p_k probabilities ($k = 1,\ 2, \dots,\ K$), where p_k measures the prior probability that topic k will become successful (i.e. capture the attention of the public). One could imagine, for example, that the K topics are pieces of content appearing on the Internet, some of which become viral. We assume that p_k and the entire joint distribution of the K events is exogenous and common knowledge across all outlets, that is, we assume that media firms have identical technology to forecast the likely success of available content.[5] To denote the joint distribution of topics, we use P_S for the probability that exactly events in set S become successful for $S \subseteq \{1, 2, \dots, K\}$.

Without loss of generality, we rank events in decreasing order of prior probabilities ($p_{k_1} > p_{k_2}$ if $k_1 < k_2$).[6] Note that we allow for $\sum_k p_k \geq 1$, i.e. it is possible that several topics succeed and become mainstream news simultaneously. We assume that the media outlets can choose only one of the K topics for publication, which may be thought of as the editorial profile of the news outlet. This assumption reflects the idea that consumers have limited attention and only consider the first few topics. News providers can only select a small set from the large pool of topics for publication and need to put quite some effort in becoming a "potential" destination for these topics.[7]

[5] In Section 5, we relax this assumption and explore the potential for asymmetry in sites' forecasting capabilities. In Section 6 we also examine the case when the topics' success is endogenous.

[6] For technical convenience, we assume that all p_k probabilities are different, but this is not a crucial assumption.

[7] The assumption for selecting only one topic as opposed to a small number of topics is for technical reasons. In Appendix B, we explore an alternative model that allows publishers to distribute a finite resource across all topics. We find that, in equilibrium, the proportion of the resource allocated to a topic is the limit probability of our mixed-strategy distribution as N approaches infinity.

As an illustration, consider the simplified problem, where $K = 2$, i.e. the set of topics is $\{1,\ 2\}$ and, without loss of generality, $p_1 > p_2$. To complete the problem, assume that the probability that both events become mainstream is $P = P_{\{1,2\}}$. Then, the probability that only event 1 is successful is $p_1 - P$ and the probability that only event 2 is successful is $p_2 - P$. The probability that no event is successful in capturing the interest of the public is then $1-p_1-p_2+P$. One way to think about this simplification is that there are a few "important" topics that have a high probability of becoming mainstream (major wars, sport events, elections, etc.). Besides these however, there is a very large number of "marginal" topics (say, the long tail of Web content) that have a low probability to become mainstream. A central question is: how likely is it that a firm chooses from the low-probability topics? In this context, $p_1/p_2 > 1$ measures how skewed the prior distribution of content is, i.e. p_2 represents the mass corresponding to the long tail of this content. Although this simple example captures some basic properties of the success distribution, explicitly modeling the individual long tail topics in the general case of $K > 2$ provides important additional insights.

To model the reward of publishers who end up choosing a topic that becomes a success, let us first normalize the reward that a single publisher gets when picking an event that becomes the only successful topic to 1. This is the baseline value of the audience of a single successful topic. We introduce two types of parameters to capture competition among topics and among publishers. To capture the competition between topics, let γ_ℓ measure the potential value of the audience of one out of ℓ successful topics, where $\gamma_1 = 1$ and γ_ℓ is (weakly) decreasing in ℓ.[8] Similarly, let β_n measure the competition between publishers for this audience. When one out of n publishers picks a particular, successful topic, it receives β_n portion of the potential reward for that topic. We assume that $\beta_1 = 1$ and $\beta_n \geq 0$ is (weakly) decreasing in n, converging to $\beta_\infty = \inf\{\beta_n, n \geq 1\}$.

With these definitions, let y_i denote site i's choice and let n_k denote the number of sites

[8] We assume that γ_ℓ does not depend on the topic's prior probability. This seems to be a limitation of the model as one could imagine, for example, that an a priori more likely topic can sustain more firms reporting on it. However, it is easy to see that by rescaling the p_k parameters we obtain an equivalent model.

that choose k. Thus, n_{y_i} counts all firms who picked topic y_i. If S^* is the set of ℓ topics that became mainstream then site i's payoff is

$$\pi_i = \begin{cases} \beta_{n_{y_i}} \gamma_\ell & \text{if } y_i \in S^* \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The simplest example is a pure contest where $\beta_n = 1/n$, and a fixed reward (γ_ℓ) is divided equally between the sites who publish a single topic that is among the ℓ successful ones. However, the intensity of competition may be higher or lower than this baseline. A constant $\beta \equiv 1$ implies no competition between sites (e.g. when each site's customers are perfectly loyal), whereas a sharply decreasing β_n series describes a market that gets very competitive with more players.[9] An extreme example is $\beta_1 = 1$ and $\beta_n = 0$ for $n \geq 2$, which would correspond to Bertrand competition among publishers for the advertisers' business, e.g. when advertisers (or, in a more extreme case, a monopolist advertiser) have strong negotiation power facing publishers. In general, the 'nature' of the β_n sequence (formally characterized below) captures a broad variety of competitive scenarios among firms.

The number and set of topics that become successful also affect the payoffs. If multiple topics are successful, then the potential reward accrued to the sites that publish one of these is smaller and more so the more topics become successful, i.e. more successful news items reduce the demand for any individual one. It is possible that the potential reward accrued to all successful sites is larger than 1 (that is, when $\ell\gamma_\ell \geq 1$). In other words, a larger number of interesting news events may increase total media demand.

[9] A common scenario is when publishers need a critical audience size to attract advertisers to bid for so-called "premium" inventory. If they fall short in attracting a large audience, they miss out on lucrative contracts and are forced to sell advertising in more competitive settings leading to lower revenue. Another common reason for harsh competition for online advertising is, that in practice, an advertiser usually wants to pay for a given number of impressions (to manage decreasing returns to advertising). Such, self-imposed limits are often called "frequency capping". If there are multiple sites with sufficient audiences each, then the advertiser can shop around and negotiate a lower price compared to the case when there is only one (or a few) publisher(s).

4 Analysis of symmetric firms

For a single news provider ($N = 1$) the problem is trivial: choose $y = 1$, the topic with the highest prior probability. When there are multiple providers, however, there might be an incentive to choose from the topics with the lower priors because if providers choose different topics, the prize needs to be shared with fewer competitors. Moreover, the lower the prior, the fewer the sites willing to publish the corresponding event. As we show further down, there are multiple pure-strategy equilibria in which different sub-groups coordinate on different topics with group sizes being larger for higher probability topics. Given the multiplicity of pure-strategy equilibria and the difficulty of coordination across a large number of firms, it makes sense to consider equilibria in mixed-strategies (Cabral 1988). Indeed, a symmetric mixed-strategy equilibrium allows us to better characterize the expected distribution of topics resulting from firms' choices. Nevertheless, we will also explore equilibria in pure strategies and establish a connection between pure- and mixed-strategy equilibria further along the article. Let $q_k^{(N)}$ denote the equilibrium probability that a firm chooses topic k when there are N players. In order to present our main results, let us first define

$$v_k = p_k - \sum_{\{k \in S,\ |S| \geq 2\}} (1 - \gamma_{|S|}) P_S \tag{2}$$

and let $\omega(.)$ be a permutation defined by a decreasing order, $v_{\omega(1)} \geq v_{\omega(2)} \geq \ldots \geq v_{\omega(K)}$. As we show below, v_k represents the value of a topic and players choose between topics according to a decreasing order of value:

Proposition 1 *The game with $N > 1$ players has a unique symmetric equilibrium in mixed strategies with the following properties.*

1. *If $v_i > v_j$ holds for a pair of topics (i, j), then $q_i^{(N)} > q_j^{(N)}$ for any $N \geq 1$.*
2. *There exists an increasing $\overline{K_N}$ series such that topic $\omega(j)$ is chosen with positive probability, i.e. $q_{\omega(j)}^{(N)} > 0$, if and only if $j \leq \overline{K_N}$.*

3. If topic k is chosen with positive probability, then $v_k > v_{\omega(1)}\beta_N$.

The proposition has two key messages. First, it shows that the "value" of topics for firms, measured by v_i does not necessarily correspond to their prior probabilities of success. Rather, it is also a function of their correlation structure, i.e. the likelihood that some of them become successful *together*. We get $v_i = p_i$ only if $\gamma_\ell = 1 \ \forall \ell$ (i.e. multiple topics becoming successful simply multiplies the overall demand for news) or if $P_S = 0 \ \forall |S| \geq 2$, (i.e. topics are mutually exclusive). Although a reversal of value ($v_i > v_j$ when $p_i < p_j$) cannot happen for $K = 2$, it is possible for $K \geq 3$. As an example, assume three topics, i, j and k with $p_i > p_j > p_k$, where two of the events are highly correlated: say, if topic i becomes successful it is likely that topic j becomes successful too ($P_{\{i,j\}}$ is large). If event k is negatively correlated with the other two events and γ_2 is not too large, then topic k may attract disproportionate share of choice from firms even with a low prior. This happens because event k is likely to happen alone and, therefore, the demand will not be divided between two events. In this case, the probability that firms choose k in equilibrium may exceed k's prior probability, p_k. This simple example shows that under competition, the correlation structure between topics may also have a major role in determining what gets published from a large number of topics. In particular, in a contest, even quite unlikely topics may make it to the news if they are "unique" compared to others.

As an example, think of general themes like a presidential election or the Olympic games. Most 'relevant' news related to these themes (e.g. the state of the economy, political news, gold medals won), are likely to attract the attention of the public, i.e. their joint probability of "success" is high. Our model predicts that with increased competition between news outlets 'irrelevant' news (e.g. gossip about the presidential candidates or the missteps of athletes) may get over-represented in the news. Often, news consists of signals or forecasts about future events that are of general relevance to the public. For example, news may report poll results forecasting the outcome of elections, that of a referendum or an important vote in congress, etc. If the success of these news is related to the likelihood of them forecasting the truth then our model may explain why "controversial" or "surprising" forecasts about future events may be over-represented in the media: a strong prior about the future event would make most forecasts (those consistent with the prior) correlated (i.e. to be true *together*). In contrast, whereas the success of an inconsistent poll is low, its success is negatively correlated with the "non-surprising" forecasts. For instance, a poll predicting the failure of the front-runner in the presidential election is likely to be wrong and, therefore, less likely to become a successful story but it is also likely that, *if successful* it is a unique story. This latter aspect provides an extra incentive for competing media firms to report it.

The second insight from Proposition 1 is related to the level of competition, measured by the number of news outlets, N in the model. The higher the number of firms who compete in the contest for successful news the more there is a chance for "low value" (either unlikely or highly correlated) topics to make it to the news. The condition in the last part of the proposition sets a lower bound to the value of a topic for it to be considered: its value needs to be higher than the payoff from choosing the highest value topic if all other firms made the same choice. As $N \to \infty$, all K topics will be published with positive probability. In other words, competition tends to generally increase topic diversity. Yet, as we demonstrate below, the level of competition in the contest between the publishers has a major impact on topic diversity.

A particular example where we obtain a closed form solution is the one capturing Bertrand-type competition among news publishers:

Example 1 *If $\beta_1 = 1$ and $\beta_n = 0$ for $n \geq 2$, then for $N \geq 2$, we have*

$$q_k^{(N)} = 1 - \frac{K-1}{v_k^{1/(N-1)} \sum_{i=1}^{K} v_i^{-1/(N-1)}}. \tag{3}$$

The mixing probabilities demonstrate the fundamental result of Proposition 1 as higher value topics are more likely to selected by each player. However, obtaining a closed form solution allows us to examine how the probabilities change as the number of players increases. Interestingly, we find that all mixing probabilities converge to the same probability, $1/K$. Due to the intense competition, players have a strong incentive to avoid each other, especially when there are a high number of them, leading to an outcome with maximal diversity in topic coverage. Motivated by this extreme example, we examine in detail how the exact nature of the β_n series affects our results for various numbers of players throughout the rest of this section.

Large number of competing publishers

We first explore the equilibrium pattern for a large number of publisher. We examine the evolution of probabilities or the "attention" that topics get in the news as a function of competition, defined by the *nature* of the contest between publishers. To describe how competition increases with the number of players, we define $r(\beta) = -\log_2 \left(\lim_{n\to\infty} \frac{\beta_{2n}}{\beta_n} \right) \geq 0$, which measures how fast the β sequence converges to 0.[10] For example, if $\beta_n = 1/n^s$ then $r(\beta) = s$. When β_n does not converge to 0, then $r(\beta)$ is clearly 0, but even if it decreases slowly, such as when $\beta_n = 1/\log n$, we get that $r(\beta) = 0$. On the other extreme, when β_n decreases exponentially or even faster, $r(\beta) = \infty$. Depending on how fast β_n decreases, we get substantively different results as the number of players approaches infinity.

Proposition 2 *As $N \to \infty$, the equilibrium mixing probabilities converge as follows.*

1. *If $r(\beta) = 0$, then $q^{(N)}_{\omega(1)} \to 1$ and $q^{(N)}_{\omega(i)} \to 0$ for any $i \geq 2$.*
2. *If $r = r(\beta) \in (0, \infty)$, then $q^{(N)}_k \to (v_k)^{1/r} / \sum_{j=1}^{K} (v_j)^{1/r}$ for any $1 \leq k \leq K$ topic.*
3. *If $r(\beta) = \infty$, then $q^{(N)}_k \to 1/K$ for any $1 \leq k \leq K$ topic.*

In Proposition 1, we have seen that a higher number of players always leads to more topics covered with positive probability, but this probability strongly depends on how competitive the contest is. When there is absolutely no competition, i.e., $\beta_n \equiv 1$, players always chose the most promising topic regardless of the number of players. More interestingly, even if the contest is only mildly competitive (i.e. if competition does not increase very much with the number of players: $\lim_{n\to\infty} \beta_n > 0$ or even if $\lim_{n\to\infty} \beta_n = 0$, but slowly), in the limit, everyone will

[10] We assume that β is strictly positive and is such that the limit exists throughout the rest of this section.

choose the topic with the highest prior probability.[11] When competition is more intense, e.g. $\beta_n = 1/n^r$, the limit will be a diverse set of choices with probabilities proportional to $v_k^{(1/r)}$. To better illustrate this case, let us explore the simplest case with $\beta_n = 1/n$ and only two topics ($K = 2$) where $p_1 + p_2 = 1$, $P = P_{\{1,2\}} = 0$, and $p_1 > p_2$, i.e. only one of the two topics becomes mainstream for sure. Then, in the mixed-strategy equilibrium with only two players ($N = 2$), each firm will choose the less likely topic with probability $2 - 3p_1$, which is larger than 0 as long as $p_1 < 2/3$. However, as the number of firms increases, the probability of choosing the less likely topic increases until it reaches p_2 as $N \to \infty$. Similarly, the probability of choosing the more likely topic decreases. Figure 1 shows how increased competition reduces the probability of choosing the more likely topic as a function of its prior, p_1. Note that as $N \to \infty$ this probability converges exactly to p_1. According to our results, for the more general case with $K = 2$, the ratio of mixing probabilities converges to $\frac{q_2^{(\infty)}}{q_1^{(\infty)}} = \frac{p_2-(1-\gamma_2)P}{p_1-(1-\gamma_2)P}$. As we have seen before, for $P = 0$ or $\gamma_2 = 1$ this yields that, with many firms, the relative probability of publishing each topic corresponds to the relative proportion of prior probabilities.[12]

Insert Figure 1 around here

The final case in Proposition 2 is also interesting. It considers a very tough contest in which the reward dissipates very quickly as more publishers choose the same successful topic(s). The Proposition says that, in this case, news outlets randomize their choice of topics uniformly by putting equal weight on each of them leading to extreme topic diversity in the news. As the number of players goes to infinity, this pattern closely matches the outcome captured in

[11] This outcome does not contradict the result of Proposition 1. When $r(\beta) = 0$ the evolution of probabilities follows an intriguing pattern. As N increases additional topics with lower and lower v_i-s obtain a positive weight in firms' choice. However, after obtaining a positive weight, this weight decreases, converging to 0 with more firms entering the market. Therefore, from a practical perspective, when the contest is not too competitive, the concentration of topics increases with entry.

[12] See Laster et al. (1999) for a similar result in the context of economic forecasting.

Example 1 where a firm's reward falls to 0 as soon as another firm also reports on the same topic, resembling a Bertrand-like competition.

In summary, our model predicts that, with a competitive contest, the more firms enter the market the less players choose topics that are likely to succeed *a priori* and topic diversity tends to increase in the published news. Note, however, that even with a large number of firms, the average representation of topics may not correspond to the marginal distribution of priors, as correlations distort the representation of topics, with the exception of mutually exclusive topics, when $v_i = p_i$. In the final case of Proposition 2, i.e. when the contest is extremely competitive, it drives players to differentiate as much as possible and, in the limit, they choose each topic with the same likelihood, irrespective of the topics' priors. Figure 2 compares our results for different r values, showing the different outcomes depending on how competitive the contest is.

Insert Figure 2 around here

Having determined the limits as N goes to infinity, we turn our attention to describing how the probabilities change as N increases. We know from Proposition 2 that the probability of choosing the most valuable topic eventually has to decrease for $r > 0$ as the limit probability is less than one. Similarly, other topics which are not as valuable and may be not selected at all when there are a small number of players, will eventually increase towards a positive limit. Because $r = 0$ is a special case, we will focus on $r > 0$ here.

To determine how the mixing probabilities converge to their limits, we first introduce some additional notation. For a given $r(\beta) = r$, the β series decreases approximately as $\beta_n \approx \frac{C}{n^r}$, where $C = C(\beta) = \lim_{n\to\infty} n^r \beta_n$. As we show in the Appendix, the best representative β series for a given r and C is $C\beta_n^{*r}$ where $\beta_n^{*r} = \frac{1}{n(n+1)\cdots(n+r-1)}$. We call β_n^{*r} the "focal" series for r. To determine the convergence of the equilibrium probabilities for a particular β_n series with $r(\beta) = r > 0$, we define $s(\beta_n) = \inf\left\{s : \lim_{n\to\infty} \left|n^{s+r}\left(\beta_n - C\beta_n^{*r}\right)\right| > 0\right\}$. That is, s measures the magnitude of difference between β_n and the focal β_n^* series. A higher s means a smaller, magnitude $\frac{1}{n^{s+r}}$ difference and, by definition, we always have $s(\beta) > 0$.

To gain insight into how mixing probabilities change with N, we determine the differential of the probabilities as N increases. Let $D = \lim_{n\to\infty} n^{r+s}(\beta_n - \beta_n^*)$.

Proposition 3 *If $0 < s < \infty$, then there exists a δ_N series such that $\lim_{N\to\infty} N^{s+1}\delta_N = 0$ and*

$$q_k^{(N)} - q_k^{(N-1)} = \frac{D(r+s)}{CrN^{s+1}} \cdot \left(\frac{v_k^{1/r}}{\sum_{j=1}^K v_j^{1/r}}\right)^{2-s} \cdot \left(\frac{\sum_{j=1}^K v_j^{1/r}}{v_k^{1/r}} - \frac{\sum_{j=1}^K \left(v_j^{1/r}\right)^{1-s}}{\left(v_k^{1/r}\right)^{1-s}}\right) + \delta_N.$$

The result shows a clear pattern. First the magnitude of the differential is $1/N^{s+1}$ implying that the speed of convergence to the limit only depends on s: the difference from the limit probabilities is of the magnitude $1/N^s$. For example, when $\beta_n = 1/n^r$, then for any $r > 1$ integer[13], we obtain $s = 1$, therefore the equilibrium probabilities converge relatively slowly, with magnitude $1/N$.

More interestingly, the change in probabilities is negative for high value topics and positive for low value topics. The differential gradually increases as the topic value decreases. In particular, the probability of choosing the highest value topic drops the most as the number of players increases, followed by the second highest topic, and so on. A further implication is that above a certain value threshold all probabilities will decrease, whereas below the threshold all probabilities will increase in N. We call this a *shrinking* pattern as the range of topic choice probabilities shrinks. When $s = 1$ (a good example is $\beta_n = 1/n^r$ for $r > 1$) the results are particularly clean. The differential of q_k is negative if and only if $v_k^{1/r}$ is higher than the average. For $s > 1$, fewer and fewer topics will have equilibrium probabilities converging increasingly as s increases. For a high enough s, all but the lowest value topic will have a decreasingly converging probability function. These patterns are readily discernible on Figure 2.

The case of $r = 0$ does not fit the shrinking pattern described above but it is easy to analyze. For $r = 0$, the probability of the highest value topic converges (increasingly) to 1 whereas the

[13]The case of $r = 1$ is an exception, because $\beta_n^{*1} = 1/n$ for $r = 1$, therefore $s = \infty$. As we show in the Appendix (Corollary 4), the convergence is exponential when $\beta_n = \beta_n^{*r}$, for all r including $r = 1$.

other probabilities converge (decreasingly) to 0.

In sum, we find that all β_n reward sharing structures of the contest can be characterized by a positive real number r that completely determines the limit of the equilibrium mixing probabilities as the number of firms approaches infinity. Moreover, the speed of convergence to this limit only depends on how "different" the β_n series are from a characteristic/focal β_n^{*r} series. The pattern of convergence is a shrinking pattern where the range of topic probabilities gets gradually narrowed (see Figure 2). These results greatly reduce the set of qualitatively different outcomes of the contest.

Small number of competing publishers

We have now seen how different properties of the β_n series affect the equilibrium mixing probabilities for a large number of players. The results for a high N are mostly determined by the series' behavior as it converges to infinity. In this section, we examine the topic choice probabilities for small values of N. By definition, the results for a given N are only affected by β_n values which have a lower index $n \leq N$. In combination with the previous section, we can conclude that the equilibrium probabilities for small and large number of players are independent from each other and only depend on the small or large indexed β_n values.[14]

To examine the case of small N's, we start with solving for the topic choice probabilities for $N = 1$, $N = 2$, and $N = 3$, showing a shrinking pattern regardless of the parameter values. We then show that the same pattern holds under a specific family of β_n series as we derive a closed form solution. Finally, we demonstrate that starting from $N = 4$, no clear pattern holds and a variety of configurations are possible in the general case.

Proposition 4 *As N increases from 1 to 3, the equilibrium probabilities exhibit a shrinking pattern: there exists critical indices $\widehat{K}_2 = 1$ and $\widehat{K}_3$ such that $q_{\omega(h)}^{(N)} \leq q_{\omega(h)}^{(N-1)}$ if $h \leq \widehat{K}_N$ and $q_{\omega(h)}^{(N)} \geq q_{\omega(h)}^{(N-1)}$ otherwise.*

[14] For large N's β_n values with a small n do have a small effect on the equilibrium probabilities, but these effects disappear as $N \to \infty$.

The shrinking pattern is clear as N increases from 1 to 2. In the former case, the single player chooses the highest value topic with probability one. But, as another player enters, the probability of choosing other topics may become positive. The same pattern continues as a third player enters. The probability of choosing the most valuable topic never increases and the probability of choosing lower valued topics starts to increase below a threshold.

We can show that the same pattern continues in the specific case of a linearly decreasing β_n series by deriving a closed-form solution for the probabilities.

Proposition 5 *Let $\beta_n = 1 - \delta(n-1)$ and $N < 1/\delta$. Define $\overline{K}$ as the largest index such that $\sum_{j=1}^{\overline{K}} \frac{v_{\omega(\overline{K})}}{v_{\omega(j)}} > \overline{K} - (N-1)\delta$. Then*

$$q_{\omega(h)}^{(N)} = \frac{1}{(N-1)\delta}\left(1 - \frac{K-(N-1)\delta}{\sum_{j=1}^{\overline{K}} \frac{v_{\omega(h)}}{v_{\omega(j)}}}\right) \text{ if } h > \overline{K} \text{ and } q_{\omega(h)}^{(N)} = 0 \text{ otherwise.}$$

These probabilities also exhibit a shrinking pattern.

When $N = 4$, the general pattern breaks down: depending on the parameters it is possible that the highest valued topic's probability decreases further as shown in the previous proposition, but it is also possible that it increases.

Example 2 *Let $\beta_1 = 1$, and $\beta_2 = \beta_3 = \beta_4 = 1/2$. Then, for $K = 2$ topics with values $v_1 = 3$ and $v_2 = 2$ the equilibrium probabilities are*

$$q_1^{(1)} = 1,\ \ q_1^{(2)} = 4/5 = 0.8,\ \ q_1^{(3)} = 3 - \sqrt{5} \approx 0.764,\ \ q_1^{(4)} = 4/5 = 0.8,$$

$$q_2^{(1)} = 0,\ \ q_2^{(2)} = 1/5 = 0.2,\ \ q_2^{(3)} = \sqrt{5} - 2 \approx 0.236,\ \ q_2^{(4)} = 1/5 = 0.2.$$

Clearly, the probabilities are shrinking for $N = 1, 2, 3$, but the trend reverses for $N = 4$.

Summarizing the case with a small number of competitors, we find that, for $N = 1, 2, 3$, the equilibrium probabilities are shrinking as they do for the case with large N as long as $r > 0$. However, from $N = 4$, there is no clear discernible pattern until N becomes large again.

Split topics

We have seen how the nature of the contest and the number of players affect the choice of topics between different players, generally leading to more diversification and differentiation in case of more players. It is also important to consider how a change in the available topics changes players' choices. Let us assume that the set of available topics is modified in a way that topics $1, 2, 3, \ldots, K-1$ remain the same, but topic K is split into two separate topics such that the value of the split topic is equal to the sum of the values of the two individual topics. A good example is the case of mutually exclusive topics, where the value of a topic is simply the probability of it becoming successful. In this case, a topic can be refined into two versions such that players can pick one version or the other. The following proposition shows when splitting topics in such a way decreases the likelihood that each player picks them.

Proposition 6 *Assume $\beta_n = 1/n^r$ and let $q_i^{(N)}$ denote the symmetric equilibrium probabilities of choosing between the K topics with values $v_1, v_2, ..., v_K + v_{K+1}$ and let $s_i^{(N)}$ denote the equilibrium probabilities of choosing between the $K+1$ topics with values $v_1, v_2, ..., v_K, v_{K+1}$. Then $s_K^{(N)} + s_{K+1}^{(N)} \leq q_K^{(N)}$ for any p_k, γ_l and N parameter values if and only if $r \leq 1$.*

The proposition says that, if multiple topics get integrated into a "theme", the latter is more likely to be represented in the news than the cumulative representation of individual topics only if $r \leq 1$. When N approaches infinity, this result follows from Proposition 2, because the mixing probability for each topic is proportional to $v_k^{(1/r)}$ and the $(1/r)$th power of a sum is greater than the sum of the components' powers if and only if $(1/r)$ is higher than 1. Our additional results here show that this relationship holds for any number of players as long as $r \leq 1$. The intuition is that when competition is less intense and multiple outlets reporting on the same topic are not hurt too much, the tradeoff between avoiding each other and reporting on high value topics sways incentives in the latter direction. Interestingly, when competition is intense there is no clear relationship and the comparison can go either way depending on the number of players.

The topic integration examined above may happen when the public categorizes various news items together. Examples may include events with multiple aspects that represent different phases of a bigger event, such as, for example, the Financial Crisis. It is possible that people were originally interested only in certain aspects of this broad phenomenon but later, realizing the importance of the event they might be interested in anything related to it. It is relevant to ask whether such integration is beneficial to publishers.

Corollary 1 *In the case of split topics publisher profits are lower if and only if the joint probability of choosing the split topics is smaller than the probability of choosing the joint topic.*

Therefore, profits always go down with topics being split when $r \leq 1$. On the other hand, when $r > 1$, profits can either decrease or increase depending on the parameter values, but always increase for a large number of players. In summary, publishers tend to favor a more fragmented presentation of news as competition increases.

Pure and mixed strategies

So far we have focused on equlbria in mixed strategies. We have done so for multiple reasons. First, we believe that mixed strategies are a more realistic representation of the prediction game where the difficulty of coordination between players would make it implausible for players to sustain equilibria where players choose distinct topics. Second, there is a unique symmetric equilibrium in mixed strategies whereas there is typically no symmetric equilibrium in pure strategies. Indeed, in this section, we will show that the pattern exhibited by the pure-strategy equilibria is rather irregular with many cases of multiple equilibria. Nevertheless, we will also show that as the number of players converges to infinity the pure- and mixed-strategy equilibria become essentially identical further supporting the validity of using mixed-strategies.

We next state the main result regarding pure strategy equilibria. As players are symmetric, there are always multiple equilibria because players can switch roles and it does not matter which particular player picks which topic. The relevant metric to describe an equilibrium is the number of players choosing a certain topic k. We denote this number by $M_k^{(N)}$ when there are a total of N players.

Proposition 7 *At least one pure strategy equilibrium always exists. Let the series $M_k^{(N)}$ denote the number of players choosing topic k in an arbitrary series of pure-strategy equilibria. Then*

$$\frac{M_k^{(N)}}{N} \underset{N\to\infty}{\longrightarrow} q_k^{(\infty)}.$$

The proposition establishes a connection between the proportion of players choosing a certain topic in a pure-strategy equilibrium and the probability of choosing a topic in a mixed-strategy equilibrium. As the number of players converges to infinity, this proportion converges to the topic choice probability. In other words, the pure- and mixed-strategy equilibria capture essentially the same outcome when there are a large number of players. When the number of players is small, the connection between the two types of equilibria is not as simple. Pure-strategy equilibria can exhibit a fairly irregular pattern as illustrated by the following example. We use the notation $\lfloor x \rfloor$ for the largest integer not larger than x and $\lceil x \rceil$ for the smallest integer not smaller than x.

Example 3 *Let $K = 2$ and $v_1 = 2v_2$. If $N = 2, 5, 8, ...$ then we have either*

$$M_1^{(N)} = \left\lfloor \frac{2N}{3} \right\rfloor, M_2^{(N)} = \left\lceil \frac{N}{3} \right\rceil \quad \text{or} \quad M_1^{(N)} = \left\lceil \frac{2N}{3} \right\rceil, M_2^{(N)} = \left\lfloor \frac{N}{3} \right\rfloor.$$

Otherwise, we always have

$$M_1^{(N)} = \left\lfloor \frac{2N}{3} \right\rfloor, M_2^{(N)} = \left\lceil \frac{N}{3} \right\rceil.$$

In summary, with a small number of players, the contest's outcome is quite irregular with multiple equilibria possible in some cases but not others. However, as the number of players becomes large, pure-strategy equilibria essentially converges to the mixed-strategy equilibrium, further supporting the use of mixed-strategies to our context that is typically characterized by many players.

5 The role of firm asymmetries

In the basic model, we studied a general setup with symmetric publishers. In this section, we explore the role of asymmetries. First, we consider how "branded" publishers with a loyal consumer segment behave and how this impacts the rest of the players. Second, we model the potential differences in players' abilities in determining which topic is going to be successful.[15]

Branded publishers

To account for branded news providers, we assume that brand value manifests itself in a loyal segment that only consumes a given provider's stories. Each branded provider has a loyal segment of size $\psi > 0$, and each of these consumers provides a revenue of 1 if the branded provider's story is successful and 0 otherwise. We modify the payoff function in (1) for branded publishers to

$$\pi_i^B = \begin{cases} \beta_{n_{y_i}} \gamma_\ell + \psi & \text{if } y_i \in S^* \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Recall that y_i above denotes the choice of player i, where S^* is the set of ℓ successful topics. Therefore, the payoff includes the same competitive component as before from non-loyal consumers, but also includes a unit profit from all loyal customers. The payoff of all non-branded sites is identical to π_i as before in (1). For simplicity we assume that all branded sites have the same amount of loyal consumers and we use α to denote the proportion of branded providers. It is not hard to map this setup to today's situation in the news media: branded news providers are represented by traditional firms (e.g. the *New York Times*, *Washington Post*, etc. for newspapers or CNN, NBC or Fox News for television broadcasters) whereas entering, mostly online news outlets (BuzzFeed or Huffington Post) represent unbranded providers.

To compare this setting to our main findings, we search for equilibria in mixed strategies where the strategies only depend on the type of provider and do not differ within the set of

[15] Importantly, although we restrict the presentation of the results to the most interesting/relevant cases, the technical results of Section 4 pertaining to the equilibrium analysis apply to the case of asymmetric firms with the appropriate modifications.

branded players or within the set of non-branded players. We call this a symmetric equilibrium. For a first look, let us consider the case with only one branded player, say player 1. It is clear that if non-branded players are indifferent between two topics k_1 and k_2 then the branded player will choose the topic with the higher prior probability, that is k_1 iff $p_{k_1} > p_{k_2}$. As such, the branded player will chose topic 1, the topic with the highest prior probability. This, in turn will incentivize non-branded players to avoid topic 1. We state the result in a general fashion for settings with at least one, but not too many branded players below.

Proposition 8 *There exists $\overline{\alpha} > 0$, such that if $0 < \alpha < \overline{\alpha}$, then the game has a unique symmetric equilibrium, where all the branded players always choose topic 1, that is $q_1^B = 1$. The non-branded players' q_k^{NB} mixing probabilities satisfy $q_1^{NB} < q_1$ and $q_k^{NB} > q_k$, for any $k \geq 2$, where q_k are the equilibrium probabilities obtained by applying Proposition 1 to only the $(1-\alpha)N$ non-branded players.*

The result shows that when the number of branded players is not too high, they all bet on the topic with the highest prior probability so that they do not disappoint their loyal customers. The concentration of branded players leads non-branded providers to avoid this topic and pick others.

We can obtain a more precise picture of the magnitudes by examining the case of $N \to \infty$.

Corollary 2 *Assume $r = r(\beta) > 0$ and $\alpha < (v_1)^{1/r} / \sum_{j=1}^{K} (v_j)^{1/r}$. Then, as $N \to \infty$*

$$q_1^{NB(N)} \to \frac{(v_k)^{1/r} / \sum_{j=1}^{K} (v_j)^{1/r} - \alpha}{1-\alpha},$$

$$q_k^{NB(N)} \to \frac{(v_k)^{1/r} / \sum_{j=1}^{K} (v_j)^{1/r}}{1-\alpha},$$

for any $k \geq 2$.

The corollary reveals that the α proportion of branded providers dominate topic 1, thereby increasing the likelihood of other topics chosen by the non-branded providers. The more branded providers there are the more non-branded ones turn away from topic 1 as shown by the mixing probabilities that are increasing in α for topics $k \geq 2$. Figure 3 illustrates the results for $\alpha = 0.15$ and $\alpha = 0.3$.

Insert Figure 3 around here

Note that although topic 1 has the highest prior probability it is not necessarily the highest value topic in the absence of loyal customers. Depending on the correlation structure, these two topics can differ. When they do, we observe more of a horizontal differentiation with branded providers choosing the most likely topics, whereas non-branded providers choosing the most likely *but also* unique topic. If the two coincide then the differentiation is more vertical because non-branded providers choose topics that are less likely to be successful. The varying pattern is a result of the asymmetries in risk preferences created by the loyal segment. Branded providers do not have to worry about other topics, thus they can focus on the most likely outcome. Thus, non-branded providers want to avoid the most likely success topic, but they are not necessarily forced to settle with a lower value topic leading to a more horizontal than vertical type of differentiation.

The result described by Proposition 8 and Corollary 2 is consistent with the casual observation that traditional (branded) media is relatively "conservative" in their choices of top stories. Although recently even established media firms ventured into publishing less traditional news on their websites, these are often relegated to special sections that are clearly suggested to be taken "more lightly" (see, e.g. CNN's "Distraction videos"). In contrast, new media sites are much more venturesome in their editorial process often reporting stories that could be easily qualified as 'rumor' or 'gossip'.

Publishers with better predictive ability

So far, we assumed symmetric information across publishers. However, some publishers may have an advantage in determining which topic would become successful in the future. To study information asymmetry, we start from our basic model using the payoffs given in (1) but we assume that a μ proportion of players can perfectly predict which topic(s) will be successful. The remaining $1-\mu$ proportion has the same prior information as before. For tractability, we assume that the topics are mutually exclusive,[16] that is, $P_S = 0$ for any $|S| \geq 2$. Again, this setup can be easily mapped to today's changing news media landscape. Here, media firms/sites with better forecasting ability often correspond to new online entrants (e.g. the Huffington Post or Buzzfeed), who claim to have superior technology in terms of predicting the success of emerging stories using social media or search engines.

Naturally, in this setting, high-ability publishers will choose a topic that will eventually become successful as they can perfectly predict success. The question is how this behavior affects the topic choice of the remaining $(1-\mu)N$ players.

Proposition 9 *The game has a unique symmetric equilibrium in mixed strategies for less-informed players. For any μ, there exists a $\overline{K_N}(\mu)$ series, increasing in N, such that topic $\omega(j)$ is chosen with positive probability by less-informed players, i.e. $q_{\omega(j)}^{L(N)} > 0$, if and only if $j \leq \overline{K_N}(\mu)$. If topic k is chosen with positive probability, $v_k > v_{\omega(1)}\beta_N/\beta_{\mu N+1}$.*

The special case of $\beta_n = 1/n$ illustrates our results best. When $\mu = 0$, that is, all players have the same forecasting ability, the necessary condition for a topic to be chosen is that topic k's value, $v_k > v_{\omega(1)}/N$. However, as a positive proportion of μ players have high abilities, the condition becomes $v_k > v_{\omega(1)}(\mu N+1)/N$, essentially ruling out topics with a value less than μ times the highest value topics. In other words, the higher the proportion of high-ability players, the less uninformed players chose unlikely topics. As $N \to \infty$, we can derive the mixing probabilities for less-informed players.

[16] This implies $v_k = p_k$, but for notational consistency, we still use the v_k values.

Corollary 3 *Assume $r = r(\beta) > 0$. Then, $\overline{K_N}(\mu) \to \overline{K_\infty}(\mu)$, where $\overline{K_\infty}(\mu)$ is decreasing in μ and is defined as the largest integer satisfying*

$$\frac{(v_{\omega(\overline{K_\infty}(\mu))})^{1/r}}{\sum_{j=1}^{\overline{K_\infty}(\mu)}(v_\omega(j))^{1/r}} \geq \frac{\mu}{(\overline{K_\infty}(\mu)-1)\mu+1}.$$

Furthermore, as $N \to \infty$, for any $j \leq \overline{K_\infty}(\mu)$

$$q_{\omega(j)}^{L(N)} \to \frac{\left((v_{\omega(j)})^{1/r} / \sum_{j=1}^{\overline{K_\infty}(\mu)}(v_j)^{1/r}\right)\left((\overline{K_\infty}(\mu)-1)\mu+1\right)-\mu}{1-\mu}.$$

For $j > \overline{K_\infty}(\mu)$, we have $q_{\omega(j)}^{L(N)} \equiv 0$.

Using the $\beta_n = 1/n$ case again, we can see that with a positive μ, not all topics are chosen by low-ability news providers even if there are a large number of them. When $N \to \infty$, topics that have a relatively low value (in proportion to the other topics) are not in the mix. The threshold is decreasing with μ, that is, the higher the proportion of high-ability players, the fewer, and higher value topics will be chosen by low-ability players, as illustrated in Figure 4.

Insert Figure 4 around here

This outcome is, again, consistent with what we observe in practice. News outlets with better forecasting ability are often the ones that report on an unexpected topic (e.g. "click-bait" headlines) as they are more likely to foresee general interest for such topics by the public. In contrast, and maybe as a response, traditional media is much more conservative in its editorial choice of topics.

It is also interesting to ask: what happens if branded providers also have better predictive ability? Which effect dominates in this case? We find that in such a model, brand is dominated by predictive ability. If every branded news provider also has superior predictive ability then brand doesn't matter anymore: every such provider will choose the news that will become successful and all other firms become more conservative in their reporting. There is a caveat however. In our present model, superior predictive ability means that the corresponding firms know for sure which topics will become successful. Clearly, the interplay of these two effects should depend on their relative effectiveness.

6 Endogenous success

So far, we have considered topic success to be exogenous. This is a reasonable assumption in a world where readers/viewers behavior on social media (sharing, in particular) is the primary driver of "success". Yet, the success of a news story may also depend on how much it is reported by providers. In this section, we explore the endogenous component of success probabilities that can depend on who chooses to report which story. Recall that in our basic setup P_S denoted the probability that exactly stories in set S become successful. We call these purely exogenous success probabilities. At the other extreme, we define purely endogenous success probabilities, where exactly one topic can become successful and the probability is proportional to the reporting. Let $\tilde{p_k} = n_k / \sum_{j=1}^{K} n_j$ denote the probability that topic k becomes successful in the purely endogenous setting.[17] Then the probability that exactly topics in set S become successful is

$$\tilde{P_S} = \begin{cases} \tilde{p_k} = n_k / \sum_{j=1}^{K} n_j & \text{when} \quad S = \{k\} \\ 0 & \text{when} \quad |S| \geq 2. \end{cases} \tag{5}$$

Most of the time success probabilities are not completely exogenous or endogenous, hence we use a λ parameter to measure the strength of the endogenous component. That is,

$$P'_S := \Pr(\text{exactly topics in set } S \text{ become a success}) = \lambda \tilde{P_S} + (1 - \lambda) P_S.$$

Clearly, the case of $\lambda = 0$ corresponds to our basic model, where success probabilities are not affected by players' choices. On the other extreme, when $\lambda = 1$, success is purely determined by which topics players choose. If all of them choose the same topic, that topic becomes a

[17] The proportional specification is somewhat arbitrary, but we study the effects in relation to the β_n series which is very general. An alternative would be to pick a specific β_n series and a general function for the success probabilities.

success for certain. Indeed, symmetric pure-strategy equilibria may exist in this case. When all players choose topic k, each of them has a payoff of β_N, because N players have to share the benefits. By deviating to another topic one player could get all the benefits, but only if that topic becomes a success, which happens with probability $1/N$. Therefore, there exists a symmetric equilibrium with all players choosing the same (any) topic if and only if $\beta_N \geq 1/N$. We call this a self-fulfilling equilibrium, as a topic becomes successful only because all players choose it. In a purely endogenous setting these equilibria exist if competition is not very intense relative to how much the players' actions determine success. For tractability, we assume that $\beta_n = 1/n^r$ and look at pure-strategy symmetric equilibria throughout this section. When $\lambda = 1$, self-fulfilling equilibria exist for any topic if and only if $r \leq 1$. In the general case, we first show that such an equilibrium can only exist for the highest value topics.

Lemma 1 *Let $\overline{j}(\lambda, N)$ denote the largest integer such that a pure-strategy symmetric equilibrium where all players choose topic $\omega(\overline{j}(\lambda, N))$ exists. Then, there also exists an equilibrium where all players choose topic $\omega(j)$ for any $j < \overline{j}(\lambda, N)$.*

The lemma shows that a topic can only be the pure-strategy equilibrium if all topics that are of higher value are also pure-strategy equilibria. We have established above that $\overline{j}(1, N) = K$ if $r \leq 1$ and $\overline{j}(1, N) = 0$ if $r > 1$. For intermediate values of λ we get the following.

Proposition 10 *Assume $0 \leq \lambda < 1$. The threshold $\overline{j}(\lambda, N)$ is increasing in λ.*

1. *If $r \geq 1$, then $\overline{j}(\lambda, N) \leq 1$.*

2. *If $r < 1$, then $\overline{j}(\lambda, N) > 1$ if λ is high enough.*

3. *For each j and high enough λ, there exist $1 < \underline{N}(j) \leq \overline{N}(j)$ such that $j \leq \overline{j}(\lambda, N)$ iff $\underline{N}(j) \leq N \leq \overline{N}(j)$.*

The results describe how and when the coordination on an *a priori* not necessarily high value topic is an equilibrium. First, when competition is intense and $r \geq 1$, only the highest value topic can be chosen by all players and true self-fulfilling equilibria do not exist. Note that when $r = 1$ this is in contrast to the benchmark case of $\lambda = 1$. When competition is less intense and $r < 1$, self-fulfilling equilibria exist when the endogenous component of the success probability is strong enough. Finally, for any given topic (that is not the highest value), an equilibrium with all players chosing that topic exists as long as λ is high enough and N is in an intermediate range.

The non-monotonicity in N is a result of the combination of two forces. On one hand, more players make it more worthwhile to go with the mainstream as deviating to an alternative topic is hard when success is strongly determined by the amount of reporting. On the other hand, when there are too many players, even though the probability of reporting the right topic is high for everyone who coordinates, the contest makes it less appealing to participate in this equilibrium. Overall, this extension shows that although the general dynamics identified in our base model remain valid, endogeneity somewhat moderates our results in terms of the resulting agenda diversity.

7 Discussion and concluding remarks

This article studies competition among news providers who compete in a contest to publish on a relatively small number of topics from a large set when these topics' prior success probabilities differ and when their success may be correlated. We show that the competitive dynamic generated by a strong enough contest (i.e. one that results in a low share of revenue for winners) causes firms to publish 'isolated' topics with relatively small prior success probabilities. The stronger the competition (either because of a larger number of competitors or – more markedly – because firms are forced to give up a larger proportion of the winners' reward), the more diverse the published news is likely to be.

Applied to the context of today's news markets characterized by increased competition between firms, new entrants and reduced customer loyalty, we expect a more diverse set of topics covered by the news industry. Although direct evidence is scarce, there seems to be strong empirical support for the general notion that the public agenda has become more diverse over time while also exhibiting more volatility McCombs (2004). This general finding is consistent with our results. Although diversity of news may generally be considered a good thing, agenda setting, i.e. focusing the public on a few, worthy topics (arguably a core function of the news industry) maybe impaired by increased competition.

In a next step, we explore differences across news providers and find that branded outlets with a loyal customer base are likely to be conservative with their choice of reporting in the sense that they report news that is *a priori* agreed to be important. Facing new competitors with better forecasting ability also makes traditional media more conservative. In sum, if the public considers traditional media and not the new entrants as the key players in agenda setting, then increased competition may actually make for a more concentrated set of *a priori* important topics on the agenda. It is not clear however, that traditional news outlets can maintain forever their privileged status in this regard. Some new entrants (e.g. the Huffington Post) have managed to build a relatively strong 'voice' over the last few years.

We also explore what happens when the success of news is endogenous, i.e. if the act of publishing a topic ends up increasing its likely success. Interestingly, we find that an excessively strong contest tends to concentrate reporting on topics with the highest *a priori* success probabilities. We also find that the number of competitors has a somewhat ambiguous effect on the outcome. If there are too few or too many competing firms then, again agenda setting tends to remain conservative in the sense of focusing on the *a priori* likely topics. These results also resonate to anecdotal evidence concerning today's industry dynamics.

Our analysis did not consider social welfare. This is hard to do as it is not clear how one measures consumer surplus in the context of news. Indeed, the model is silent as to what is consumers' (i.e. readers') utility when it comes to the diversity of news. Although policy makers generally consider the diversity of news as a desirable outcome, a view that often guides policy and regulatory choices, it is not entirely clear that, beyond a certain threshold, more diversity is always good for consumers. As mentioned in the introduction, the media does have an agenda setting role and it is hard to argue that every topic equally represented in the news is a useful agenda to coordinate collective social decisions (e.g. votes at elections). Nevertheless, our goal was to identify the competitive forces that may play a role in determining the diversity of news in today's environment increasingly dominated by social media. Our analysis indicates that these forces do not necessarily have a straightforward impact on diversity.

The generalized contest model presented has implications for other economic situations that may be well-described by contests. In this sense, our most relevant results are those that describe the outcome as a function of the reward-sharing patterns across winners. Indeed, we characterize all such patterns with a simple parameter, r, and show that depending on r there are only three qualitatively different outcomes leading to vastly different firm behaviors. Different r's may characterise different contexts. For our case a finite, albeit varying r seemed appropriate and $r = \infty$ is less likely. In the case of a contest describing R&D competition, $r = \infty$ is quite plausible. Conversely, the case of $r = 0$ may well apply to contests among forecasters whose reward might be linked more closely to actually forecasting the event and less to how many other forecasters managed to do so. Our analysis of the case with a small number of firms may also be useful in particular situations (again, R&D contests may well be characterized by a few competitors); we show that this case is tractable and shares many characteristics with the case involving many players. An important insight from our analysis is that contest models need to be carefully adjusted to the particular situations studied.

Our framework can be extended in a number of directions. So far, we assumed a static model, one where repeated contests are entirely independent. One could also study the industry with repeated contests between media firms where an assumption is made on how success in a period may influence the reward or the predictive power of a medium in the next period. A similar, setup is studied with a Markovian model by Ofek and Sarvary (2003) to describe industry evolution for the hi-tech product categories. Finally, our article generated a number of hypotheses that would be interesting to verify in future empirical research.

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Figures

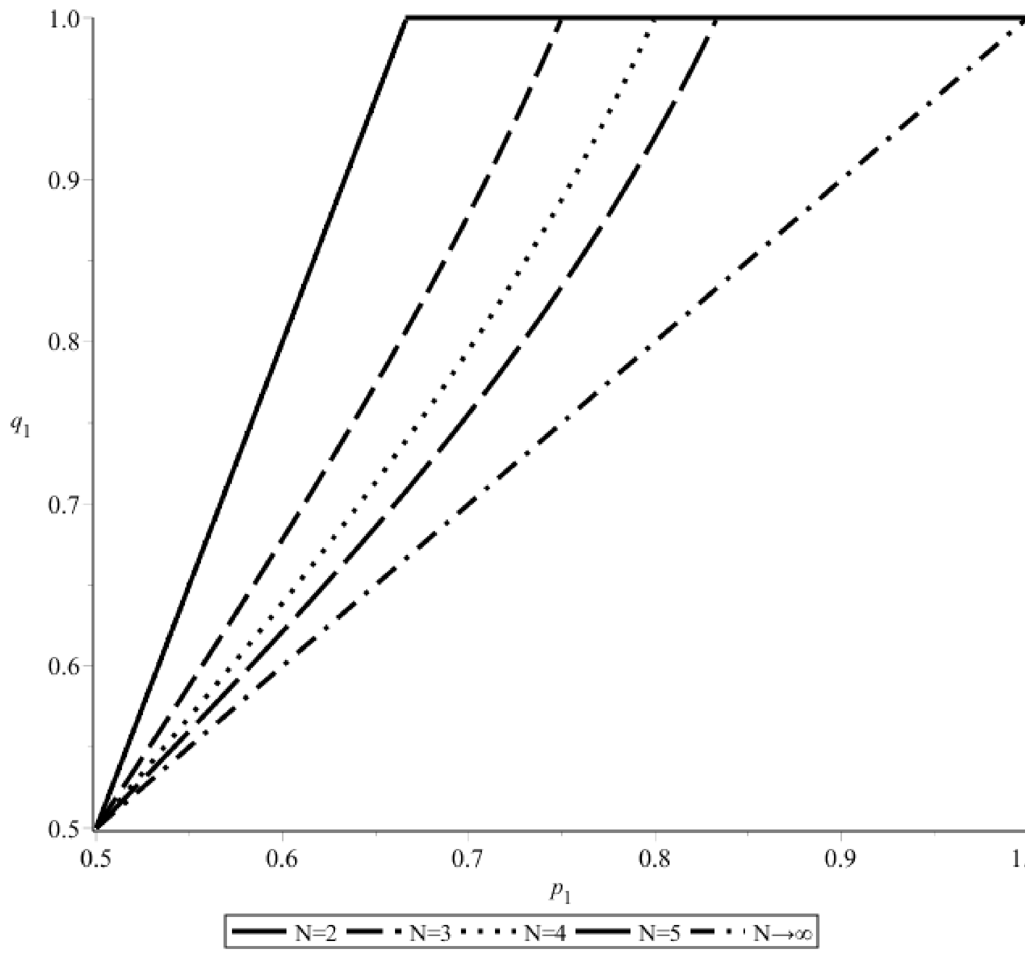


Figure 1: Equilibrium probability of choosing topic 1 for $N = 1, 2, 3, 4, 5$ and $N \to \infty$.

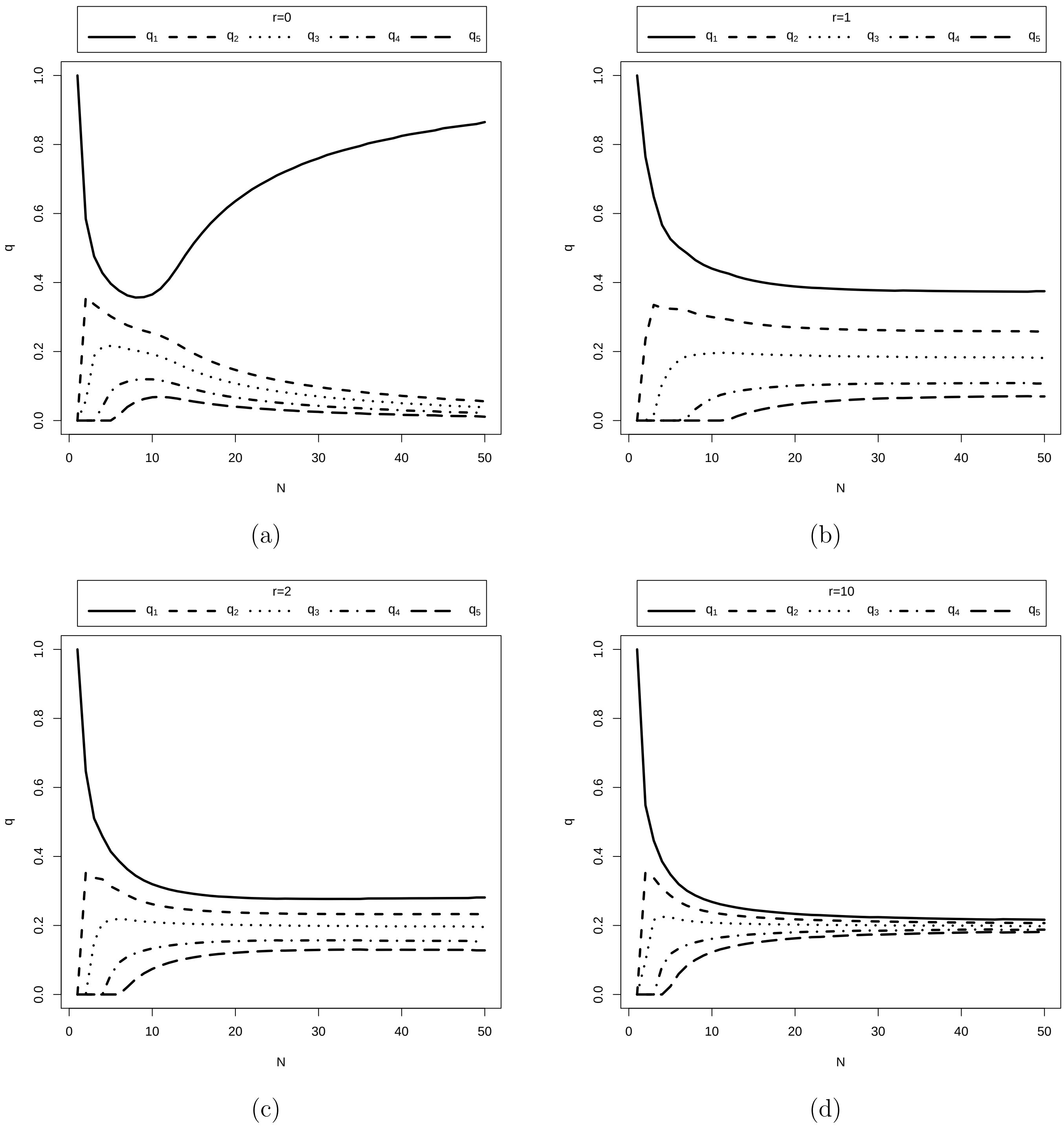
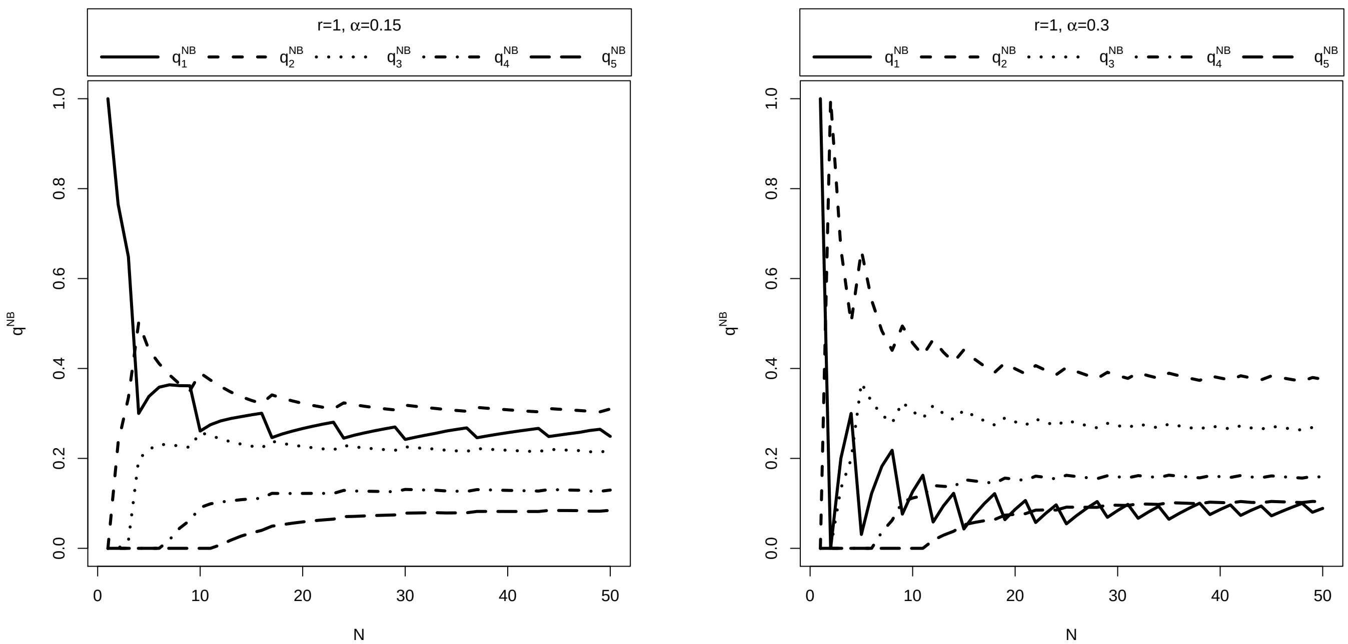
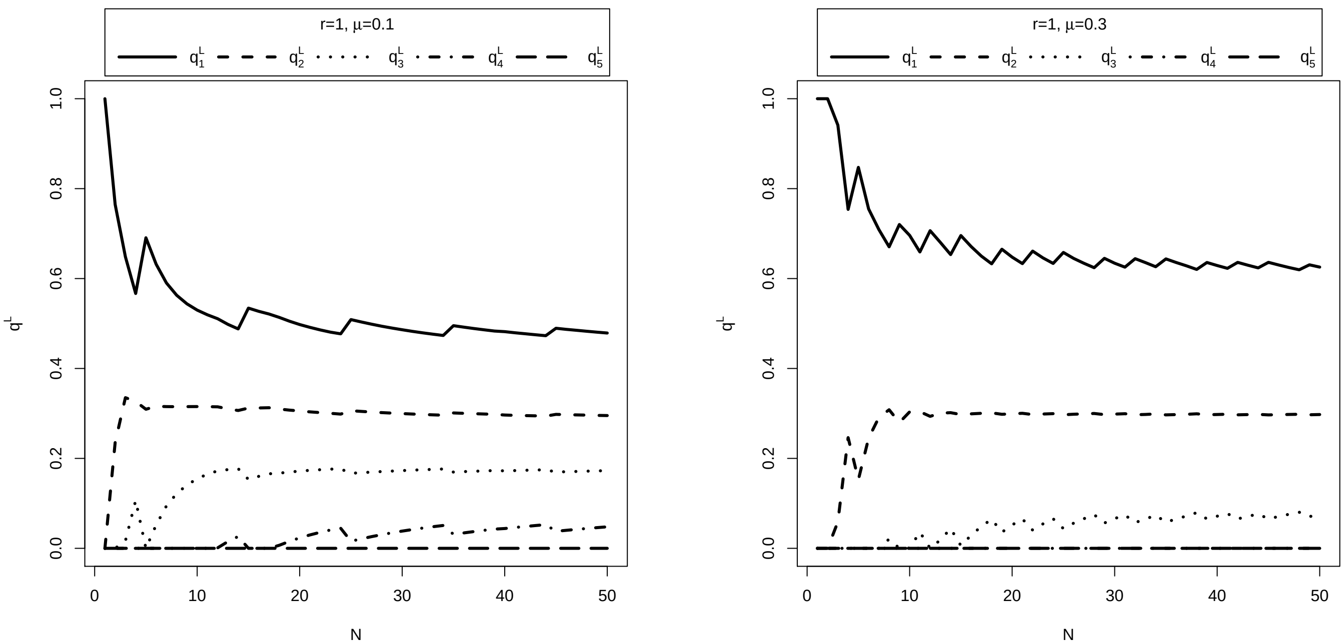


Figure 2: Mixing probabilities for $K = 5$ with $v_1 = 0.5, v_2 = 0.35, v_3 = 0.25, v_4 = 0.15, v_5 = 0.1$. As N increase, more topics are choosen with positive probability. However, when $r = 0$ all probabilities except q_1 converge to 0. When $r > 0$, the mixing probability for large N's is proportional to $v_k^{(1/r)}$, eventually leading to equal probabilities as $r \to \infty$.



(a) (b)

Figure 3: Non-branded provider's mixing probabilities for $K = 5$ with $v_1 = 0.5, v_2 = 0.35, v_3 = 0.25, v_4 = 0.15, v_5 = 0.1$ for different values of α (0.15, 0.3). A high proportion of branded providers makes non-branded providers turn away from high value topics.



(a) (b)

Figure 4: Low-ability provider's mixing probabilities for $K = 5$ with $v_1 = 0.5, v_2 = 0.35, v_3 = 0.25, v_4 = 0.15, v_5 = 0.1$ for different values of $\mu(0.15, 0.3)$. A high proportion of high-ability providers makes low-ability providers turn away from low value topics.

Appendix A: Proofs

Proof of Proposition 1: Throughout the proof, we drop the superscript (N) and q_k denotes the probability that any given site out of N players chooses topic k in the symmetric mixed strategy equilibrium. Let $\{k_1, k_2, k_{K+}\}$ denote the set of topics that players pick with a positive probability in a potential (symmetric) equilibrium in such an order that $v_{k_1} \geq v_{k_2} \geq \ldots \geq v_{k_{K+}}$. We calculate the expected payoff of firm i when choosing topic k_1 as

$$\mathbf{E}\,\pi_i^{(k_1)} = \sum_{l=1}^{K} \gamma_\ell \Pr(\text{topic } k_1 \text{ is a success among } \ell \text{ topics}) \sum_{m=1}^{N} \beta_m \Pr(m{-}1 \text{ other players choose topic } k_1) \tag{6}$$

The first probability is simply $\sum_{k_1 \in S, |S|=l} P_S$, therefore

$$\sum_{l=1}^{K} \gamma_\ell \Pr(\text{topic } k_1 \text{ is a success among } \ell \text{ topics}) = p_{k_1} - \sum_{\{k_1 \in S,\ |S| \geq 2\}} (1-\gamma_{|S|}) P_S = v_{k_1}$$

The second probability in (6) can be written as

$$\sum_{n_{k_2}+\ldots+n_{k_{K+}}=N-m} \frac{(N-1)!}{(m-1)!n_{k_2}!n_{k_3}!\ldots n_{k_{K+}}!} q_{k_1}^{m-1} q_{k_2}^{n_{k_2}} q_{k_3}^{n_{k_3}} \ldots q_{k_{K+}}^{n_{k_{K+}}}.$$

Because $q_{k_2} + \ldots + q_{k_{K+}} = 1 - q_{k_1}$, we can write the above as

$$\frac{(N-1)!}{(m-1)!(N-m)!} q_{k_1}^{m-1} \sum_{n_{k_2}+\ldots+n_{k_{K+}}=N-m} \frac{(N-m)!}{n_{k_2}!n_{k_3}!\ldots n_{k_{K+}}!} q_{k_2}^{n_{k_2}} q_{k_3}^{n_{k_3}} \ldots q_{k_{K+}}^{n_{k_{K+}}} =$$

$$= \frac{(N-1)!}{(m-1)!(N-m)!} q_{k_1}^{m-1} (1-q_{k_1})^{N-m}. \tag{7}$$

Therefore,

$$\mathbf{E}\,\pi_i^{(k_1)} = v_{k_1} \sum_{m=1}^{N} \beta_m \frac{(N-1)!}{(m-1)!(N-m)!} q_{k_1}^{m-1} (1-q_{k_1})^{N-m}. \tag{8}$$

Notice that the above sum is an expectation of β_m, where $m-1$ is distributed Binomially. Let $X(n,q) \sim \text{Binom}(n,q)$ be a random variable and let $G_N(q) = \mathbf{E}\,\beta_{1+X(N-1,q)}$. It is clear that $\mathbf{E}\,\pi_i^{(k)} = v_k G_N(q)$ for any other k topic. Following from its definition $G_N(q)$ is a decreasing, continuous function on $[0,1]$ with $G_N(0) = 1$ and $G_N(1) = \beta_N$ for any $N \geq 1$. Furhtermore $G_N(q)$ is decreasing in N for any fixed q value. An equilibrium has to satisfy

$$v_{k_1} G_N(q_{k_1}) = v_{k_2} G_N(q_{k_2}) = \ldots = v_{k_{K^+}} G_N(q_{k_{K^+}}) \tag{9}$$

and $v_{k_1} G_N(q_{k_1}) > v_k G_N(q_k)$ for any other k topic. Because $G_N(q)$ is decreasing, it follows that if players put a positive probability on a topic then they have to put a positive probability on all other topics with higher values. Otherwise deviating to the higher value topic would be profitable. That is, the set $\{k_1, k_2, \ldots, k_{K^+}\}$ has to be of the form $\{\omega(1), \omega(2), \ldots, \omega(K^+)\}$, a set of the K^+ highest value topics. Together with (9), this implies *Part 1* as $G_N(q)$ is decreasing.

For *Part 2*, let $\overline{K_N}$ denote the highest integer for which (9) has a solution with positive q_k iff $v_{k_{\overline{K_N}}} N > v_{k_1}$. The solution with a given set of $\{\omega(1), \omega(2), \ldots, \omega(\overline{K_N})\}$ has to be unique due to the decreasing $G_N(q)$ function. Furthermore, there is no solution, where less than $\overline{K_N}$ topics receive a positive probability.

To see that $\overline{K_N}$ is increasing in N, assume on the contrary that, for some $N_1 > N_2$, we have $\overline{K_{N_1}} \leq \overline{K_{N_2}}$. Because $G_N(q)$ is decreasing in both q and N, it follows that $q_{\omega(j)}^{(N_1)} \leq q_{\omega(j)}^{(N_2)}$ for any $j \leq \overline{K_{N_2}}$. However, then $1 = \sum_{j=1}^{\overline{K_{N_1}}} q_{\omega(j)}^{(N_1)} < \sum_{j=1}^{\overline{K_{N_2}}} q_{\omega(j)}^{(N_2)} = 1$, which is a contradiction.

Finally, for *Part 3*, note that for any k topic that is chosen in equilibrium with positive probability $v_{\omega(1)} G_N(q_{\omega(1)}) = v_k G_N(q_k)$. Because $G_N(q)$ falls between β_N and 1, this implies $v_k > v_{\omega(1)} \beta_N$. $\square$

Proof of Example 1: As $G_N(q) = (1-q)^{N-1}$ in this case, we get $v_i(1-q_i)^{N-1} = v_k(1-q_k)^{N-1}$ for any pair of topics i, k by plugging into (9). Therefore, $1 - q_i = (v_k/v_i)^{1/(N-1)}(1-q_k)$. Summing for all i, we then obtain

$$K - 1 = \sum_{i=1}^{K} (1 - q_i) = (1 - q_k) v_k^{1/(N-1)} \sum_{i=1}^{K} v_i^{-1/(N-1)} \tag{10}$$

yielding the formula presented in the example. $\square$

Proof of Proposition 2: Let us define $f_\beta(x) = \lim_{n\to\infty} \frac{\beta_{nx}}{\beta_n}$ for any rational $x \geq 1$. The definition implies that $f_\beta(x_1 x_2) = f_\beta(x_1) f_\beta(x_2)$ for any $x_1 \geq 1, x_2 \geq 1$. Also it is clear that $f_\beta(1) = 1$ and that $f_\beta(.)$ is decreasing. Let $x' = 2^{(a/b)}$, where a, b are integers so that $\log_2(x')$ is rational. Then $f_\beta(x') = f_\beta(2^{(a/b)})$. If $f_\beta(2) = 0$, this implies $f_\beta(x') = 0$. Because $f_\beta()$ is decreasing, $f_\beta(x) = 0$ for any $x > 0$. If $f_\beta(2) > 0$, then $f_\beta(x') = (2^{-\log_2(f_\beta(2))})^{(a/b)} = (2^{(a/b)})^{-\log_2(f_\beta(2))} = x'^{-r}$. Therefore, for a monotone decreasing β_n series the $f_\beta(x)$ function can be either $f_\beta(x) = 1/x^r$ with a non-negative r, or a discontinuous function with $f_\beta(1) = 1$ and $f_\beta(x) = 0$ for any $x > 1$, corresponding to $r(\beta) = \infty$.

Recall that in equilibrium (9) holds for every topic with high enough value. As $N \to \infty$, we get $G_N(q)/(\beta_{Nq}) \to 1$ due to the central limit theorem. Also, when $\beta_N \to 0$, we also have $G_N(1) \to 0$, that is, for a large enough N, each topic is chosen with positive probability. Therefore, for any two topics $\beta_{Nq_{k_1}^{(N)}}/\beta_{Nq_{k_2}^{(N)}} \to G_N(q_{k_1}^{(N)})/G_N(q_{k_1}^{(N)}) = v_{k_2}/v_{k_1}$. Because the $f_\beta(x)$ limit exists for any x and is decreasing in x, $q_{k_1}^{(N)}/q_{k_2}^{(N)}$ converges for any pair of topics. The sum of all q values is 1, hence each $q_k^{(N)}$ probability converges to a limit denoted by q_k^∞. When $r(\beta) > 0$, the definition of the $f_\beta()$ function implies $v_{k_2}/v_{k_1} = f_\beta(q_{k_1}^\infty/q_{k_2}^\infty) = (q_{k_2}^\infty)^{r(\beta)}/(q_{k_1}^\infty)^{r(\beta)}$. Because the sum of probabilities is 1, we get the stated results for any $r(\beta) > 0$, including $r(\beta) = \infty$. When $r(\beta) = 0$, all probabilities converge to 0 except one and that has to belong to the highest value topic and converge to 1. $\square$

Proof of Proposition 3: For notational convenience, but without losing generality we can assume in this proof that $\omega(i) = i$. Throughout the proof, we also use the notation $\xi_n = o(\zeta_n)$ for any $\xi_n \underset{n\to\infty}{\longrightarrow} 0$ series such that $\lim_{n\to\infty} \xi_n/\zeta_n = 0$. In other words $o(\zeta_n)$ denotes a series that converges to 0 faster than ζ_n. Recall from (9) that $v_i G_N(q_i) = v_j G_N(q_j)$ for any i, j topic pair. Although $G_N(.)$ is originally defined for integer N's, it is straightforward to extend the definition to any positive real N. We can differentiate with respect to N, yielding

$$v_i \left(\frac{\partial G_N(q_i)}{\partial N} + \frac{\partial G_N(q_i)}{\partial q} \cdot \frac{\partial q_i}{\partial N} \right) = v_j \left(\frac{\partial G_N(q_j)}{\partial N} + \frac{\partial G_N(q_j)}{\partial q} \cdot \frac{\partial q_j}{\partial N} \right) \tag{11}$$

To differentiate $G_N(.)$, we first determine $G_N^*(.)$ defined by the β_n^* series.

$$G_N^*(q) = C \frac{1 - \sum_{h=1}^{r} \binom{N+r-1}{r-h} q^{r-h} (1-q)^{N+h-1}}{q^r N(N+1)\dots(N+r-1)} = \frac{C}{q^r N(N+1)\dots(N+r-1)} + o(N^{r-1}/e^N)$$

Because $\lim_{n\to\infty} n^{r+s}(\beta_n - \beta_n^*) = D$, we have

$$G_N(q) = \frac{C}{q^r N(N+1)\dots(N+r-1)} + \frac{D}{q^{r+s}N^{r+s}} + o(1/N^{r+s})$$

Differentiating thus yields

$$\frac{\partial G_N(q)}{\partial N} = -C\frac{\frac{1}{N} + \ldots + \frac{1}{N+r-1}}{q^r N(N+1)\dots(N+r-1)} - \frac{(r+s)D}{q^{r+s}N^{r+s+1}}(1+o(1))$$

$$\frac{\partial G_N(q)}{\partial q} = -C\frac{r}{q^{r+1} N(N+1)\dots(N+r-1)} - \frac{(r+s)D}{q^{r+s+1}N^{r+s}}(1+o(1))$$

Our goal is to determine $\frac{\partial q_i}{\partial N}$ as $N \to \infty$, therefore we can substitute the limit probability of $q_i = v_i^{1/r}/(\sum_{j=1}^K v_j^{1/r})$, which we denote by w_i, yielding

$$w_i^r \left(\sum_{j=1}^K v_j^{1/r}\right)^r \left(\frac{\partial G_N(w_i)}{\partial N} + \frac{\partial G_N(w_i)}{\partial q}\cdot\frac{\partial q_i(w_i)}{\partial N}\right) = -\frac{\frac{1}{N} + \ldots + \frac{1}{N+r-1}}{N(N+1)\dots(N+r-1)} + A_i + B_i\frac{\partial q_i}{\partial N}$$

where $A_i = -\frac{(r+s)D}{w_i^s N^{r+s+1}}(1+o(1))$ and $B_i = -\frac{Cr}{w_i N(N+1)\dots(N+r-1)}(1+o(1))$. As the sum of probabilities is 1, the sum of their derivatives is 0, hence after differentiation, (9) translates to the set of equations $A_i + B_i\frac{\partial q_i}{\partial N} = A_K - B_K\sum_{j=1}^{K-1}\frac{\partial q_j}{\partial N}$ for every $i < K$. Solving these completes the proof:

$$q_i^{(N)} - q_i^{(N-1)} = \frac{\partial q_i}{\partial N}(1+o(1)) = \frac{-A_i\sum_{j\neq i}\prod_{h\neq i,j} B_h + \sum_{j\neq i} A_j \prod_{h\neq j,i} B_h}{\sum_{j=1}^K \prod_{h\neq j} B_h} =$$

$$\frac{-A_i\sum_{j\neq i}(1/B_j) + \sum_{j\neq i}(A_j/B_j)}{B_i\sum_{j=1}^K (1/B_j)} = \frac{D(r+s)}{Cr}\cdot\frac{w_i^{1-s} - w_i\sum_{j=1}^K w_j^{1-s}}{N^{s+1}}.$$

□

Corollary 4 *If* $\beta_n = \frac{C}{n(n+1)\cdots(n+r-1)}$, *then there exist* $\varepsilon_k^{(N)}$ *series such that* $\lim_{N\to\infty}\frac{e^N}{N^{r-1}}\varepsilon_k^{(N)} = 0$ *and*

$$q_k^{(N)} - q_k^{(N-1)} = E_k\left(1 - \frac{v_{\omega(K)}^{1/r}}{\sum_{j=1}^K v_j^{1/r}}\right)^N N^{r-1} + \varepsilon_N,$$

where the E_k constants do not depend on N. Furthermore, $E_k < 0$ for all $k \neq \omega(K)$ and $E_{\omega(K)} > 0$.

Proof: Corollary 4 When $\beta_n = \beta_n^* = \frac{1}{n(n+1)\dots n+r-1)}$, we have

$$G_N(q) = G_N^*(q) = \frac{1 - \sum_{h=1}^{r} \binom{N+r-1}{r-h} q^{r-h}(1-q)^{N+h-1}}{q^r N(N+1)\dots(N+r-1)} =$$

$$= \frac{1}{q^r N(N+1)\dots(N+r-1)} - \frac{(1-q)^N}{(r-1)!qN} + o(1/Ne^N)$$

Differentiating yields

$$\frac{\partial G_N(q)}{\partial N} = -\frac{\frac{1}{N} + \dots + \frac{1}{N+r-1}}{q^r N(N+1)\dots(N+r-1)} - \frac{(1-q)^N \log(1-q)}{(r-1)!qN}(1+o(1))$$

$$\frac{\partial G_N(q)}{\partial q} = -\frac{r}{q^{r+1} N(N+1)\dots(N+r-1)} + \frac{(1-q)^{N-1}}{(r-1)!q}(1+o(1))$$

As before, substituting $q_i = w_i$ yields

$$w_i^r \left(\sum_{j=1}^{K} v_j^{1/r}\right)^r \left(\frac{\partial G_N(w_i)}{\partial N} + \frac{\partial G_N(w_i)}{\partial q} \cdot \frac{\partial q_i(w_i)}{\partial N}\right) = -\frac{\frac{1}{N} + \dots + \frac{1}{N+r-1}}{N(N+1)\dots(N+r-1)} + A_i + B_i \frac{\partial q_i}{\partial N}$$

where $A_i = -w_i^{r-1} \frac{(1-w_i)^N \log(1-w_1)}{(r-1)!N}(1+o(1))$ and $B_i = -\frac{r}{w_i N(N+1)\dots(N+r-1)}(1+o(1))$. Solving the set of equations $A_i + B_i \frac{\partial q_i}{\partial N} = A_K - B_K \sum_{j=1}^{K-1} \frac{\partial q_j}{\partial N}$ for every $i < K$ now gives a simpler result, because $A_i = o(A_K)$ for any $i < K$ as $1 - w_K > 1 - w_i$. Hence, for $i < K$:

$$\frac{\partial q_i}{\partial N} = \frac{A_K}{B_i \sum_{j=1}^{K} (B_K/B_j)} = w_i \frac{w_K^r \log(1-w_K)(1-w_K)^N (N+1)\dots(N+r-1)}{(r-1)!} + o\left(\frac{N^{r-1}}{e^N}\right)$$

□

Proof of Proposition 4: As a single player picks the highest value topic with probability 1, the shrinking pattern is clear as N changes from 1 to 2 with $\widehat{K}_2 = 1$. To prove shrinking from $N = 2$ to 3, we first calculate $G_2(q) = 1 - (1-\beta_2)q$ and $G_3(q) = 1 - 2(1-\beta_2)q + (1 - 2\beta_2 + \beta_3)q^2$, yielding $G_2(q) - G_3(q) = (1-\beta_2)q - (1 - 2\beta_2 + \beta_3)q^2$. Note that $G_2(q) - G_3(q)$ is non-negative, is 0 at $q = 0$, is $\beta_1 - \beta_2$ at $q = 1$. Furthermore, it is increasing between 0 and 1 when $\beta_2 - \beta 3 > \beta_1 - \beta_2$. Otherwise, it is increasing from 0 to $\frac{1-\beta_2}{2-4\beta_2+\beta_3} \geq \frac{1}{2}$ and decreasing above. Hence $G_2(q) - G_3(q) \geq G_2(1-q) - G_3(1-q)$ for any $q \geq 1/2$. We know examine how the optimal probabilities for $N = 2$ behave when we substitute them into the equilibrium equations (9) for $N = 3$. Because only one probability can be above 1/2, it follows from the properties of $G_2(q) - G_3(q)$ described above, that $v_{\omega(g)} G_3\left(q^{(2)}_{\omega(g)}\right) \leq v_{\omega(h)} G_3\left(q^{(2)}_{\omega(h)}\right)$ when $\omega(g) < \omega(h)$. To prove shrinking, assume on the contrary that there is no threshold $\widehat{K}_3$ as defined in the proposition. This would imply that there are topics $\omega(g) < \omega(h)$ such that $q^{(3)}_{\omega(g)} - q^{(2)}_{\omega(g)} > 0$ and $q^{(3)}_{\omega(h)} - q^{(2)}_{\omega(h)} < 0$, yielding a contradiction:

$$v_{\omega(g)} G_3\left(q^{(2)}_{\omega(g)}\right) > v_{\omega(g)} G_3\left(q^{(3)}_{\omega(g)}\right) = v_{\omega(h)} G_3\left(q^{(3)}_{\omega(h)}\right) > v_{\omega(h)} G_3\left(q^{(2)}_{\omega(h)}\right).$$

□

Proof of Proposition 5: For notational convenience, we assume that $\omega(j) = j$w.l.o.g. When $\beta_n = 1 - \delta(n-1)$, we can derive that $G_N(q) = 1 - (N-1)\delta q$ is linear in q. For any k topic, we have $v_k G_N(q_k) = v_k - v_k(N-1)\delta q_k = C$, where C is constant taking the same value for all k topics. Therefore, $q_k = \frac{v_k - C}{v_k(N-1)\delta}$ and

$$1 = \sum_{j=1}^{K} q_j = \sum_{j=1}^{K} \frac{v_j - C}{v_j(N-1)\delta} = \frac{K}{(N-1)\delta} - \frac{C}{(N-1)\delta} \sum_{j=1}^{K} \frac{1}{v_j},$$

which yields $C = \frac{K-(N-1)\delta}{\sum_{j=1}^{K} \frac{1}{v_j}}$. Hence

$$q_k = \frac{v_k - \frac{K-(N-1)\delta}{\sum_{j=1}^{K} \frac{1}{v_j}}}{v_k(N-1)\delta} = \frac{1}{(N-1)\delta}\left(1 - \frac{K-(N-1)\delta}{\sum_{j=1}^{K} \frac{v_k}{v_j}}\right)$$

□

Proof of Proposition 6: We first examine the case of $r \leq 1$. Assume w.l.o.g that $\frac{v_K}{s_K} \geq \frac{v_{K+1}}{s_{K+1}}$. Then, we have $\frac{v_K + v_{K+1}}{(s_K + s_{K+1})^r} \geq \frac{v_{K+1}}{s^r_{K+1}}$. When $\beta_n = 1/n^r$, differentiating shows that we have $q^r G_N(q)$ is increasing in q. Hence for any k topic,

$$\frac{v_k}{s^r_k} s^r_k G(s_k) = \frac{v_{K+1}}{s^r_{K+1}} s^r_{K+1} G(s_{K+1}) \leq \frac{v_K + v_{K+1}}{(s_K + s_{K+1})^r} (s_K + s_{K+1})^r G(s_K + s_{K+1}).$$

If we had $q_K < s_K + s_{K+1}$, then this would yield

$$v_k G(s_k) < v_K G(q_K) = v_k G(q_k)$$

which would lead to $q_k < s_k$, which is a contradiction, Because the sum of the q probabilities is 1, just as the sum of the s probabilities. We have thus completed the proof of the "if" part of the proposition.

To prove the "only if" part, we show that $s_K + s_{K+1}$ can be either smaller or larger than q_K when $r > 1$. When N is large, Proposition 2 directly implies that $s_K + s_{K+1} > q_K$ for $r > 1$, as in the limiting case $(v_K)^{1/r} + (v_{K+1})^{1/r} > (v_K + v_{K+1})^{1/r}$. However, for $N = 2$, we always have v values such that $s_K + s_{K+1} \leq q_K$ as demonstrated by the formula derived in Proposition 5. $\square$

Proof of Corollary 1: If $s_K + s_{K+1} < q_K$, then $s_k > q_k$ for any $k < K$. The expected profit of a player is $v_1 G(s_1)$, when topics are split and $v_1 G(q_1)$ otherwise. As $G(.)$ is decreasing, $v_1 G(s_1) < v_1 G(q_1)$ follows, completing the proof. $\square$

Proof of Proposition 7: We prove the existence of at least one equilibrium by induction. When $N = 2$, both players will choose the highest value topic iff $v_1\beta_2 \geq v_2$, where $v_1 > v_2$ are the two highest topic values. Otherwise, one player will choose the highest value topic and the other will chose the second highest value topic. Now suppose that an equilibrium exists for $N = j$ and we add another player to the game. Let topic k be the topic that maximizes $M_k^{(j)}\beta_{M_k^{(j)}+1}$. That is, topic k is the best response of the extra player given that all other players do not change their behavior compared to the equilibrium with only j players. If the extra players choosing topic k and the other players do not change their actions, this will constitute and equilibrium of the game with $j+1$ players. No player choosing a topic other than k will want to deviate because all possible devation are at least as bad as with j players. Furthermore, no player choosing topic k will deviate, because they face the same maximization problem that player $j+1$ did. Therefore, we constructed an equilibrium for $j+1$ players.

To show the convergence, let us assume w.l.o.g. that topic 1 has the highest value. Clearly, the highest value topic will have the highest number of players choosing it, hence $M_1^{(N)} \underset{N\to\infty}{\to} \infty$. In a pure strategy equilibrium, the following must hold for any k topic: $v_1\beta_{M_1^{(N)}} \geq v_k\beta_{M_k^{(N)}+1}$ and $v_k\beta_{M_k^{(N)}} \geq v_1\beta_{M_1^{(N)}+1}$. Therefore, $\frac{\beta_{M_k^{(N)}+1}}{\beta_{M_1^{(N)}}} \leq \frac{v_1}{v_k} \leq \frac{\beta_{M_k^{(N)}}}{\beta_{M_1^{(N)}+1}}$. The difference between the LHS and RHS bounds converges to 0, because $M_1^{(N)} \underset{N\to\infty}{\to} \infty$. Hence $\frac{\beta_{M_k^{(N)}}}{\beta_{M_1^{(N)}}} \underset{N\to\infty}{\to} \frac{v_1}{v_k}$. Therefore, $\frac{M_1^{(N)}}{M_k^{(N)}} \underset{N\to\infty}{\to} \frac{v_1^{1/r}}{v_k^{1/r}}$ if β is such that $r > 0$. If $r = 0$, then $\frac{M_1^{(N)}}{M_k^{(N)}} \underset{N\to\infty}{\to} \infty$. As per the definition of $q_k^{(\infty)}$ this completes the proof. $\square$

Proof of Proposition 8: Let us first examine the decision of branded providers, assuming that non-branded providers mix between topics $k_1, k_2, ..., k_{K^+}$. If their proportion is small enough, a branded provider cannot be indifferent between any two topics in the above set, as their profit function is that same as non-branded providers' except for a ψp_i term. This additional term leads them to choose topic 1, the topic with the highest prior probability. Given the strategies of the branded providers, we can derive the mixing strategies of the non-branded providers. Without loss of generality, assume that $k_1 = 1$. Following the same lines as in the proof of Proposition 1, similarly to (9) we get that

$$v_1\tilde{G}_N(q_1^{NB}) = v_{k_2}G_{(1-\alpha)N}(q_{k_2}^{NB}) = \ldots = v_{k_{K^+}}G_{(1-\alpha)N}(q_{k_{K^+}}^{NB}) \tag{12}$$

and $v_{k_1}G_N(q_{k_1}) > v_kG_N(q_k)$ for any other k topic. $\tilde{G}_N()$ above is defined similarly to $G_N()$, as $\tilde{G}_N(q) = \mathbf{E}\,\beta_{\alpha N+1+X((1-\alpha)N-1,q)}$. Therefore, $\tilde{G}_N(q) < G_{(1-\alpha)N}(q)$, which implies that the solutions q_k^{NB} have to satisfy $q_1^{NB} < q_1$ and $q_k^{NB} > q_k$ for any other k, where q_k's are the solution of

$$v_1G_{(1-\alpha)N}(q_1) = v_{k_2}G_{(1-\alpha)N}(q_{k_2}) = \ldots = v_{k_{K^+}}G_{(1-\alpha)N}(q_{k_{K^+}}) \tag{13}$$

$\square$

Proof of Corollary 2: Following the same steps as in the proof of Proposition 2, we obtain for any two topics other than topic 1 that $\beta_{(1-\alpha)Nq_{k_1}^{(N)}}/\beta_{(1-\alpha)Nq_{k_2}^{(N)}} \to G_{(1-\alpha)N}(q_{k_1}^{(N)})/G_{(1-\alpha)N}(q_{k_1}^{(N)}) =$

v_{k_2}/v_{k_1}. For topic 1 and an arbitrary topic k, we obtain $\beta_{\alpha N+(1-\alpha)Nq_1^{(N)}}/\beta_{(1-\alpha)Nq_k^{(N)}} \to v_k/v_1$. These yield $v_{k_2}/v_{k_1} = (q_{k_2}^\infty)^{r(\beta)}/(q_{k_1}^\infty)^{r(\beta)}$ and $v_k/v_1 = (q_k^\infty)^{r(\beta)}/(q_1^\infty + \alpha/(1-\alpha))^{r(\beta)}$.

□

Proof of Proposition 9: High-type players clearly always chose the topic that becomes successful as they are able to predict perfectly which one it will be. The equilibrium strategies for low-type players can be determined along the the same lines as in the proof of Proposition 1. Equation (9) in this case is transformed to

$$v_{k_1}\hat{G}_{(N}(q_{k_1}^L) = v_{k_2}\hat{G}_N(q_{k_2}^L) = \ldots = v_{k_{K+}}\hat{G}_{(N}(q_{k_{K+}}^L) \tag{14}$$

where $\hat{G}_N(q) = \mathbf{E}\,\beta_{\mu N+1+X((1-\mu)N-1,q)}$. Because $\hat{G}_N(0) = \beta_{\mu N+1}$ and $\hat{G}_N(0) = \beta_N$, any k topic chosen with a positive probability has to satisfy $v_k > v_{\omega(1)}\beta_N/\beta_{\mu N+1}$. □

Proof of Corollary 3: We can show that all mixing probabilities converge as in the proof of Proposition 2. Furthermore, we obtain for any two topics that $\beta_{\mu N+(1-\mu)Nq_{k_1}^{(N)}}/\beta_{\mu N+(1-\mu)Nq_{k_2}^{(N)}} \to v_{k_2}/v_{k_1}$. This yields $v_{k_2}/v_{k_1} = \left(\frac{\mu+(1-\mu)q_{k_2}^\infty}{\mu+(1-\mu)q_{k_2}^\infty}\right)^{r(\beta)}$ if both $q_{k_1}^\infty$ and $q_{k_2}^\infty$ are positive. Clearly, this equation cannot hold for too small v_k values, giving the threshold stated in the proposition. Topics with values below the threshold cannot receive a mix probability converging to a positive number. For topics with a positive mix probability in infinity, the previous equation implies the stated formula by summing all positive probabilities to 1. □

Proof of Lemma 1: Player i's profit when all players choose topic k is $\pi_i^{(k)} = (1/N^r)(\lambda + (1-\lambda)v_k)$. When player i deviates to topic k', the profit becomes $\pi_i^{(k)} = \lambda/N + (1-\lambda)v_1$. Hence the most profitable deviation is to topic $\omega(1)$ if $k \neq \omega(1)$ and to topic $\omega(2)$ if $k = \omega(1)$. Therefore $k \neq \omega(1)$ can be an equilibrium topic choice for all players iff

$$D(k,r,N,\lambda) \stackrel{def}{=} \frac{1}{N^r}(\lambda + (1-\lambda)v_k) - \frac{\lambda}{N} - (1-\lambda)v_1 \geq 0 \tag{15}$$

Clearly, $D(k,r,N,\lambda)$ is decreasing in k, proving the lemma. □

Proof of Proposition 10: Checking that $D(k,r,N,\lambda)$ from (15) increases in λ proves that $\bar{j}(\lambda, N)$ is also increasing in λ.

For *Part 1*, let $k \neq \omega(1)$ be an arbitrary, but not the highest value topic. As $D(k,r,1,\lambda) = (1-\lambda)v_k < 0$ and $D(k,r,N,\lambda) \underset{N\to\infty}{\to} -(1-\lambda)v_1 < 0$, topic k cannot be a pure equilibrium for small or large N's. Furthermore,

$$\frac{\partial D(k,r,N,\lambda)}{\partial N} = \frac{1}{N^2}(\lambda - r(\lambda + (1-\lambda)v_k)N^{1-r}),$$

which is negative for small N's and positive a certain N^* value when $r > 1$ (and all negative when $r = 1$). Therefore, $D(k,r,1,\lambda) < 0$ when $r \geq 1$, proving *Part 1.*

When $r < 1$, the derivative is positive for small N's and negative for large N's with a single root at $N^* = \left(\frac{\lambda}{r(\lambda+(1-\lambda)v_k)}\right)^{1/(1-r)}$. Plugging this value in shows that $D(k,r,N^*,\lambda) > 0$ for high enough values of λ, proving *Part 2.*

Finally, as $D(k,r,N,\lambda)$ is inverse-U shaped in N, being negative at $N=1$ and $N \to \infty$, with a positive maximum for high λ's, there must be a $1 < \underline{N} \leq \overline{N}$ interval where it is positive, proving *Part 3.* $\square$

Appendix B: Weighted topic choice

We consider a modification of our main model in which, instead of picking and publishing a single topic, publishers can divide their effort between multiple topics and pick more than one. We keep the important constraint of a space limit on the publisher's site, therefore we assume that they divide a unit of weight between the different topics they choose.

If one were to micro-model the behavior of consumers, there are different possibilities with respect to what they read from a given publisher when facing multiple topics. If they choose randomly, one possibility is that consumers make their choice in a perfectly correlated fashion. That is, the first consumer to see the multiple topics makes a random choice and then all other consumers follow. In this case, an equilibrium in our modified model would be equivalent to the original model's mixed-strategy equilibrium, because the random unified consumer choice corresponds to the mixed strategy. On the other hand, if consumers make random choices independently, the reward may be distributed more proportionally. For example, if half the consumers pick one topic, with the remaining picking the other topic, the publisher can reap some benefits if either topic becomes successful. Although in our opinion, the former setting is more plausible, especially with the increased role of social media in news consumption driving herding, we formally analyze the latter situation below.

Let w_j^i denote the weight assigned to topic j by player i, where $\sum_{j=1}^K w_j^i = 1$ for any player i. Let $\beta(n)$ denote a differentiable function defined for any $n > 0$. Using the same notation of S^* and ℓ, we define the payoff function generalizing (1) as follows:

$$\pi_i = \sum_{j \in S^*} \gamma_\ell w_j^i \beta \left(\sum_{i=1}^N w_j^i \right).$$

This is a direct generalization of the main model as restricting all w_j^i to 0 or 1 gives back the payoff function in (1). We can show for a restricted family of $\beta(n)$ functions that the results are similar.

Example 4 *If $\beta(n) = 1/n^r$, then for $N > 1$, we obtain a unique symmetric equilibrium where the weight assigned to topic j by each player is $w_j^* = q_j^{(\infty)}$, where $q_j^{(\infty)}$ are the limits given in Proposition 2.*

PROOF: Focusing on topics 1 and 2, we use the notation $w_{-12}^* = \sum_{j=3}^K w_j^*$ for the sum of equilibrium weights put on the remaining topics. In any equilibrium, the choice of w_1^i has to maximize player i's profits given w_{-12}^* and $w_2^i = 1 - w_{-12}^* - w_1^i$. Differentiating the profits with respect to w_1^i and equating to 0 yields

$$\frac{v_1}{(Nw_1^*)^r} - w_1^* \frac{rv_1}{(Nw_1^*)^{r+1}} = \frac{v_2}{(Nw_2^*)^r} - w_2^* \frac{rv_2}{(Nw_2^*)^{r+1}}$$

for the equilibrium weights w_1^* and w_2^*. The above equation further translates to $\frac{v_1}{(w_1^*)^r} = \frac{v_2}{(w_2^*)^r}$. As this equation holds for any pair of topics, not only 1 and 2, and because the sum of weights is 1, we obtain $w_j^* = (v_k)^{1/r} / \sum_{j=1}^K (v_j)^{1/r} = q_j^{(\infty)}$. □

We have thus shown that the modified game has a symmetric equilibrium where the weights assigned to each topic are the same as the topic choice probabilities in the main model's limit case with the number of players going to infinity. However, these results are not as general as the ones we obtain in the main model as they are sensitive to the exact definition of the $\beta()$ function. This is due to the fact that the formulation of our generalized model is somewhat ad hoc with regards to the payoff function. There are alternative definitions that would all generalize the main model in meaningful ways although providing slightly different results.