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STUDY OF INELASTIC PROCESSES ON (d,p) REACTIONS IN DEFORMED NUCLEI[†]

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Abstract: The stripping reaction (d,p) in deformed nuclei is usually treated as a single-step process leading directly from the ground to the product state. This paper adds to the usual direct amplitude those others that proceed through intermediate rotational states in the target and final nucleus. These higher order processes introduce large connections for the weaker states that sometimes alter the angular distribution and change the magnitude of the cross section by up to a factor of ten or more. In using these reactions to determine the amplitudes in the Nilsson wave functions for the single particle states in odd nuclei, it is therefore essential that these processes be taken into account.

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1. Introduction

It was pointed out by Satchler[1] some years ago that the intensity of the (d,p) reaction, as then treated, could be used to measure directly the decomposition of the single particle wave functions in deformed nuclei. The Nilsson wave function is expressed as a superposition of components having various angular momenta

$$\chi_K = \sum_{\ell j} C_{\ell jK} \chi_{N\ell jK} \quad .$$

The cross section to a member of the rotational band with spin I, based on this single-particle state, is proportional to $C^2_{\ell IK}$, and thus the reaction can be to determine the components in the Nilsson functions. This technique has been extensively investigated[2], particularly by Sheline and collaborators and Elbek and Tjøm. However, the above proportionality holds strictly only if the reaction proceeds directly from the ground state of the even target nucleus to the state in question in the odd nucleus. If either the target or final nucleus is set into rotation by the free particle, this no longer is true, because the transferred neutron, in this case, is not obliged to carry all the angular momentum into the final state. Since the cross sections for inelastic scattering by deformed nuclei are large, we undertook to compute the effect of inelastic processes on the (d,p) reaction by use of the source term technique[3], to evaluate the validity of the usual type of analysis.

2. Theory for Including Inelastic Processes

2.1. THE SOURCE TERM AND IT'S EQUIVALENCE TO THE PENNY-SATCHLER T-MATRIX

It has been suggested by Penny and Satchler[4]) that the usual DWBA for calculating stripping cross sections could be modified to include inelastic processes in a conceptually straightforward manner. The idea is simply that the reaction amplitude is computed from

$$T_{p,d} = \langle \chi_p^{(-)} | V_{np} | \chi_d^{(+)} \rangle \tag{1}$$

in which, unlike the usual DWBA, χ_p and χ_d are solutions of coupled equations which involve those excited states which are strongly coupled to the ground states by the particle-nucleus interaction. They thus contain components involving excited nuclear states, whereas in the usual DWBA they contain only the ground state.

However the numerical complexity involved in following this program is such that only approximate (or partial) solutions have been achieved in the past.

A new approach, called the source term method, was suggested by Ascuitto and Glendenning[3]) who propose solving the coupled inhomogeneous equations for protons

$$(E-H)\, \Psi_p^{(+)} = V_{np}\, \chi_d^{(+)} \tag{2}$$

where the right side is a source of protons due to the stripping reaction. The deuteron wave function $\chi_d^{(+)}$ is the same function as would appear in (1) and satisfies

$$(E-H)\, \chi_d^{(+)} = 0 \tag{3}$$

The solutions of (2) and (3) with the desired boundary conditions may be formally written as

$$\chi_d^{(+)} = \phi_d + \frac{1}{E^{(+)}-H} V_d \phi_d \tag{4}$$

$$\Psi_p^{(+)} = \frac{1}{E^{(+)}-H} V_{np} \chi_d^{(+)} \tag{5}$$

where $V_d = V(d, A)$ is the deuteron-target interaction, and ϕ_d is a plane wave in the relative motion. Note that $\Psi_p^{(+)}$ contains only outgoing waves, as required by the physics of the problem, while $\chi_d^{(+)}$ contains a plane wave representing the incident deuteron beam.

Equations (2) and (3) correspond to a coupled channel solution of

$$(E-H) \Psi = 0 \tag{6}$$

in which Ψ is expanded in the redundant spaces referring both to the d-A and p-(A+1) partitions, and in which the stripping interaction, V_{np}, is retained to first order.

The two methods are equivalent[5]) in their results, as we now demonstrate by showing that the amplitude for protons at infinity in (5) is given by (1). To do this we introduce the following notation.

$$H = H_o + H_p, \quad H_o = H_{A+1} + T_p, \quad V_p + \sum_{i=1}^{A+1} V_{pi} \tag{7}$$

and use in (5) the identity

$$\frac{1}{E-H} = \frac{1}{E-H_o} \left[1 + V_p \frac{1}{E-H} \right] \tag{8}$$

so that

$$\psi_p^{(+)} = \frac{1}{E^{(+)}-H_o} \left[1 + V_p \frac{1}{E^{(+)}-H}\right] V_{np}\, \chi_d^{(+)} \tag{9}$$

Examining the limit $r \to \infty$ in the r-representation of (9) yields the amplitude

$$T_{p,d} = \langle \phi_p | \left[1 + V_p \frac{1}{E^{(+)}-H}\right] V_{np} | \chi_d^{(+)} \rangle \tag{10}$$

with ϕ_p being a plane wave in the relative motion of p-(A+1);

$$(E-H_o)\,\phi_p = 0 \tag{11}$$

Thus, since

$$\chi_p^{(-)} = \phi_p + \frac{1}{E^{(-)}-H^\dagger} V_p^\dagger \phi_p \tag{12}$$

one sees that (10) is identical to (1) as was to be proved. In fact, it is obvious that, depending on the approximations with which (2) and (3) are solved, the amplitude for outgoing protons will correspond to that of Penny-Satchler[4]) when the full coupling between channels is retained, of Iano and Austern[4]) when they are solved to first order in the inelastic transitions, or the usual DWBA when the inelastic transitions are neglected altogether.

Of course in practice the space in which the Schroedinger equations are solved must be truncated so as to yield a finite set of coupled equations. Thus V_p and V_d are treated as effective interactions as described in detail elsewhere[6]). Since, in this paper, we are treating deformed nuclei, they are in fact taken to be deformed optical potentials.

2.2 COUPLED CHANNEL EQUATIONS

The first stage of our stripping calculation is the determination of $\chi_d^{(+)}$. This we do by expanding in a limited space of nuclear states:

$$\chi_{d\pi I}^{(+)^M} = \frac{1}{R} \sum_{d'} v_{d'}^{d\pi I}(R)\, \phi_{d'\pi I}^{\;M}(\hat{R},A) \tag{13}$$

We use the same notation as Ref. [3], and in particular the channel quantum number d denotes a whole set of quantum numbers defining the nuclear state and the relative motion between nucleus and deuteron. When both d, and d' occur in an equation, d is understood to refer to a channel that contains the ground state. Substitution of (13) into (3) yields, for each d', the coupled equation

$$(T_{d'} + V_{d'd'}^{\pi I}(R) - E_{d'})u_{d'}^{\,d\pi I}(R) + \sum_{d''\neq d'} V_{d'd''}^{\pi I}(R)\, u_{d''}^{d\pi I}(R) = 0 \qquad (14)$$

where

$$V_{dd'}^{\pi I}(R) = \langle \phi_{d\pi I}^{M}(\hat{R},A) \,|\, V(d,A) \,|\, \phi_{d'\pi I}^{M}(\hat{R},A) \rangle \qquad (15)$$

A detailed expression for this matrix element is given later, for our case of scattering by a deformed optical potential and rotational nuclear states.

A practical complication arises in the solution of (14) because the deuteron has spin unity. When $\pi = (-)^{I+1}$, there are two possible entrance channels, one with angular momentum $\ell = I-1$ and one with $\ell = I+1$. (We treat even targets whose ground state spin is therfore zero.) In this case two solutions of the coupled system are needed, one with incoming waves in the ground state channel having $\ell = I-1$, and the other with $\ell = I+1$. In all other channels the desired solution has only outgoing waves.

Having found the solution $\chi_d^{(+)}$ which occurs in the source of (2), the next step is to solve this inhomogeneous equation subject to the condition that there are only outgoing proton waves, as indicated in (5).

Making an expansion of Ψ_p on a limited basis

$$\Psi_{p\pi I}^{M} = \frac{1}{r} \sum_{p'} \omega_{p'}^{d\pi I} \, \phi_{p'\pi I}^{M} (\hat{r}, A+1) \tag{16}$$

and substituting in (2) yields, for each p'

$$(T_{p'} + V_{p'p'}^{\pi I} (r'_p) - E_p) \, \omega_{p'}^{d\pi I}(r'_p) + \sum V_{p'p''}^{\pi I}(r'_p) \, \omega_{p''}^{d\pi I}(r'_p) = \rho_{p'}^{d\pi I}(r'_p) \tag{17}$$

where

$$\rho_{p'}^{d\pi I} = r_p \sum_{d'} \langle \phi_{p'\pi I}^{M} (\hat{r}_p, A+1) \,|\, V_{np}(r) \,|\, \phi_o(r) \, \phi_{d'\pi I}^{M} (R, A) \, u_{d'}^{d\pi I}(R)/R \rangle \tag{18}$$

The solutions $\omega_{p}^{d\pi I}$ of the proton system are also labelled by the deuteron (entrance channel) quantum numbers since the source term depends on these.

In the above equations the center of mass substitution

$$\underset{\sim}{r}'_p \rightarrow \underset{\sim}{r}_p - \frac{m_p}{M_{A+1}} \underset{\sim}{r}_n \tag{19}$$

is understood.

By employing the zero range approximation

$$V_{np}(r) \, \phi_o(r) = D_o \, \delta(\underset{\sim}{r}) \tag{20}$$

which causes $\underset{\sim}{r}'_p = \frac{A}{A+1} \underset{\sim}{R}$, where $\underset{\sim}{R}$ is the center-of-mass coordinate of the deuteron, the source term can be evaluated as

$$\rho_{p'}^{d\pi I} = \frac{A}{A+1} D_o \sum_{d'\ell_n j_n N} \beta_{N\ell_n j_n K}(J_{d'} J_{p'}) \, (-)^{\ell_{d'}+j_{p'}+J_{p'}+I} \left(\frac{\hat{j}_{p'} \hat{\ell}_{p'} \hat{\ell}_n \hat{j}_{d'}}{4\pi} \right)^{1/2}$$

$$\times \begin{Bmatrix} j_{p'} & \ell_n & j_{d'} \\ J_{d'} & I & J_{p'} \end{Bmatrix} \begin{pmatrix} \ell_{p'} & \ell_n & \ell_{d'} \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} \ell_{p'} & 1/2 & j_{p'} \\ \ell_n & 1/2 & j_n \\ \ell_{d'} & 1 & j_{d'} \end{bmatrix} \chi_{N\ell_n j_n K}(r) \, u_{d'}^{d\pi I}(r) \tag{21}$$

where β is a parentage factor

$$\beta_{N\ell jK}(J_d, J_p) = \langle \chi_{N\ell jK}(r_n) \, [\Phi_{J_d}(A) \, \mathcal{Y}_{(\ell \, 1/2)j}(\hat{r}_n, \sigma_n)]^M_{J_p} \, | \, \Phi^M_{J_p}(A+1) \rangle \tag{22}$$

and χ is a neutron bound state wave function (see next section) and Φ denotes a nuclear wave function.

A solution of the system (17) having the outgoing boundary condition is needed. From the amplitude of the outgoing wave, the cross section can be computed as detailed in the appendix of Ref. [3]).

For the problem of interest in this paper, namely stripping reactions on a deformed even target, the particular form that $V_{d'd''}$, $V_{p'p''}$, and $\beta_{\ell_n j_n}$ take will be given later.

3. Nuclear Model

3.1. SINGLE-PARTICLE WAVE FUNCTIONS

The even-even target nucleus is taken to have the simple adiabatic wavefunction of Bohr and Mottelson

$$| JM0 \rangle = \sqrt{\frac{2J+1}{8\pi^2}} \, | 0 \rangle \, D^J_{M0} \quad . \tag{23}$$

The odd (A+1) nucleus we take to have wave function

$$| JMK \rangle = \sqrt{\frac{2J+1}{16\pi^2}} \left\{ a^\dagger_K \, | \tilde{0} \rangle \, D^J_{MK} + (-)^{J+K} \, a^\dagger_{-K} \, | \tilde{0} \rangle \, D^J_{M,\,-K} \right\} \tag{24}$$

where we have employed the phase conventions stated by Nilsson[7]. The operator $a^\dagger_K$ creates a single-particle neutron of 3-axis angular momentum projection K on a core $| \tilde{0} \rangle$. This latter may, in principle, be perturbed from $| 0 \rangle$ appearing in eq. (23). Of course, other quantum numbers than K are required to specify the state. Although using wavefunctions (24) it is possible[8] to account for a wide range of phenomena in appropriate nuclei the true wavefunction, even in the strong coupling model, is itself the solution to a complicated coupled channel calculation. The importance of this in our case is the sensitivity of stripping to the nucleus tail, the asymptotic form of which is determined by the Q-value only. The most obvious simple solution is to use (24) while matching the radial wavefunction to a Hankel tail at some suitable radius. The complication of the problem is evident from the fact that <u>each</u> j-component must be matched for <u>each</u> state of the residual nucleus band considered for <u>each</u> state of the target nucleus considered (usually, 0^+, 2^+, and 4^+), for it is the <u>transition</u> Q-value which determines the asymptotic form. In the present computations* we simply used wavefunctions of an appropriately deformed Woods-Saxon potential. These single-particle wavefunctions are obtained by diagonalizing the one-particle Hamiltonian, containing a deformed Woods-Saxon potential, on a finite basis of harmonic oscillator functions $R_{N\ell}$, thus

$$\chi_K \equiv a_K^\dagger |\tilde{0}\rangle = \sum_{N\ell j} C_{N\ell jK} \; R_{N\ell}(\nu r^2) \; [Y_\ell(\hat{r}) \; \chi_{1/2}(\sigma)]_j^K \tag{25}$$

where N is the oscillator quantum number. These should be considerably better than Nilsson wavefunctions involving only one major shell and hopefully would give a reasonable representation of the wavefunction in the region of the deformed surface. The error in the asymptotic region which is incurred is likely to be greatest for weakly bound states and where the moment of inertia is low so that the Q-value varies more widely for a given problem. This problem is discussed further in our account[8]) of stripping on magnesium.

We have estimated the single-particle energy of the intrinsic state, Σ_K, from the relation

$$E_J = \Sigma_K + \left(\frac{\hbar^2}{2\mathcal{J}}\right) [J(J+1) - 2K^2 + \delta_{K,1/2} \, a(-)^{J+1/2}(J + 1/2)] \tag{26}$$

and attempted to fit these energies using programs which solve the Schrödinger equation for a particle bound in a deformed Woods-Saxon potential. The intrinsic wavefunction is obtained as a sum over harmonic oscillator wavefunctions. The cross section is somewhat sensitive to the number of radial oscillator quanta included in the wavefunction and to the parameters of the binding potential. We found that seven or eight radial components were adequate

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*We were at the limit of the capability of the computing facilities.

since in most cases the eighth component made only a few percent difference at the most forward angles only. A complete specification of wavefunctions used may be found in Ref. [5]).

3.2. SPECTROSCOPIC FACTORS

The spectroscopic factors β which are required for the source term, eq. (20) have been calculated by Satchler[1]). They are, for the case of stripping onto states J_d of the K = 0 ground band of an even-even target nucleus:

$$\beta_{N\ell jK}(J_d,\ J_p) = \sqrt{2}\ \left(\frac{\hat{J}_d}{\hat{J}_p}\right)^{1/2}\ C_{N\ell jK} < \tilde{0}|0 > \quad C^{J_d j J_p}_{0\ K\ K} \ . \tag{27}$$

It is clear from this equation how entrance channel inelastic processes permit otherwise j-forbidden transfers to occur: if $J_d \neq 0$, then j need not equal J_p.

3.3. THE INELASTIC SCATTERING COUPLING MATRICES

The scattering within the rotational bands of the target and residual nuclei are calculated in a simple macroscopic model in which the free particle interacts with the nucleus through deformed coulomb and nuclear optical potentials. The optical potential can be written in the scalar product form

$$V(r,\ \omega,\ \Omega) = V(r) + \sum_{LK>0} V_{LK}(r)\ \ Y_L(\hat{r})\ .\ \left(\frac{D^{L}_{\cdot K}(\Omega) + D^{L}_{\cdot -K}(\Omega)}{1 + \delta_{K,\ 0}}\right) \tag{28}$$

where ω represents the angular coordinates of the point in the laboratory frame at which the potential is evaluated, and Ω represents the angular orientation of the deformed nucleus. The conditions under which this form is valid is discussed in Ref. [5]). The calculation of the factors $V_{LK}(r)$ in terms of the deformation parameters is given in Refs. [5,6]). The interaction matrices $V^{I\pi}_{c_1c_2}$ that occur in eq. (16) and eq. (17) above may be written, for particles of spin S

$$V^{I\pi}_{c_2c_1}(r) = V(r)\,\delta_{c_1c_2} + (-)^{I+J_{c_2}+2j_{c_1}+S}\left(\frac{\hat{j}_{c_1}\hat{j}_{c_2}\hat{\ell}_{c_1}\hat{\ell}_{c_2}}{4\pi}\right)^{1/2}$$

$$\times \sum_{LK \geqslant 0} (\hat{L})^{1/2}(-)^{L}\begin{pmatrix}\ell_{c_2} & L & \ell_{c_1}\\ 0 & 0 & 0\end{pmatrix} B^{LK}_{c_2c_1}\; V^{LK}_{c_2c_1}(r)\begin{Bmatrix}L & \ell_{c_1} & \ell_{c_2}\\ S & j_{c_2} & j_{c_1}\end{Bmatrix}\begin{Bmatrix}j_{c_1} & L & j_{c_2}\\ J_{c_2} & I & J_{c_1}\end{Bmatrix} \tag{29}$$

In this equation, $V(r)$ is the central complex potential, which includes the coulomb interaction as do the V^{LK}. The expressions $B^{LK}_{c_2c_1}$ are the reduced matrix elements of $(D^{L}_{,K} + D^{L}_{-K})/(1 + \delta_{K0})$. They are listed for even-even nuclei by Glendenning[6]). For odd nuclei we get

$$B^{L0} = \langle J_1K_1 \,\|\, D^{L}_{,0} \,\|\, J_2K_2 \rangle = (\hat{J}_1\hat{J}_2)^{1/2}(-)^{J_1-K_1}\begin{pmatrix}J_1 & L & J_2\\ -K_1 & 0 & K_2\end{pmatrix} \quad (L\ \text{even})$$

$$= 0 \quad L\ \text{odd} \tag{30}$$

In eq. (29) we have written D for the case of $K = 0$, i.e., axial symmetry, the only case we are concerned with. The use of eq. (30) involves the neglect of that part of the excitation interaction connected with the odd particle. These parts of the interaction correspond to the flipping of the intrinsic state and is expected to be most serious for bands of $K = 1/2$ which can be flipped by an $\ell = 2$ component in the interaction. The resulting amplitudes will be of single-particle strength and are probably in order of magnitude smaller than the collective part, although their coherent effect for $A \sim 25$ could be large.

4. Stripping Calculations For Strongly Deformed Rare Earth Nuclei

We present here the results of a series of stripping calculations on deformed rare earth nuclei. The target nuclei were even-even, and were treated as stiff, axially symmetric rotors. In each case we have ignored coriolis induced band mixing and have employed deuteron wavefunctions calculated according to the procedure outlined above. These wavefunctions are tabulated in Ref. [5]. This reference also discusses in full the procedure used for determining the optical potentials, which are tabulated there. The optical potentials are also tabulated and discussed in other publications[10,11]. A discussion of these latter, together with scattering fits will be published separately[10]. We should comment, however, that whereas the 'coupled channel' and 'distorted wave' proton optical potentials are defined to give identical elastic scattering in the appropriate calculations, this was not quite so for the deuteron potentials in most cases discussed below. Rather, the potential for the coupled channel calculation was adjusted so that it gave an equally good but not identical fit to the same (rather meagre) elastic data. We shall indicate for one of the cases discussed below how this might affect our results considered as model calculations.

4.1. THE REACTION ^{166}Er(d,p) AT 12.1 MeV

This reaction was studied by Harlan and Sheline[12]. The relative cross section at one angle, 45°, was measured for many levels. We have fitted here data from a more recent study by Tjøm and Elbek[13] (not utilized in Ref. [5]), who give absolute cross sections at three angles.

The $[633]\frac{7}{2}^{+}$ Band

The $[633]\frac{7}{2}^{+}$ (ground) band is probably relatively free of coriolis admixtures. The amplitudes for the $j = \frac{7}{2}$ and $\frac{11}{2}$ components are very weak suggesting that the population of the corresponding levels would take place largely through indirect stripping amplitudes. Unfortunately the neutron is almost entirely in the $j = \frac{13}{2}$ state. Our calculation was limited by the capacity of the program ($\sum(2J + 1) \leq 30$) and we included only the $\frac{7}{2}^+$ $\frac{9}{2}^+$ and $\frac{11}{2}^+$ states together in our calculation. Our neglect of the $\frac{13}{2}^+$ state is probably not too serious because of the intrinsically low amplitude for $\ell = 6$ transitions. As far as entrance channel inelastic processes are concerned, the $j = \frac{13}{2}^+$ component is taken into account, being present in the neutron wavefunction employed. The weak $\frac{7}{2}^+$ state will be little affected by the absence of the $\frac{13}{2}^+$ exit state as it is not coupled to it by the lowest order Y_2 deformation.

The results of stripping calculations for this band are exhibited in fig. 1 and summarized in Table 1 where they are compared with the data of Ref. [13]. The overall fit to the data is greatly improved. However, the normalization is ambiguous in the sense that if it had been at 60° instead of 90°, the $\frac{11}{2}^+$ level would have been fitted instead of the $\frac{7}{2}^+$ the cross section of which would in this case have been to high. According to which scheme of normalization is used to compare our calculation with the data, this disagreement may be interpreted in two ways. According, then, to the first interpretation the disagreement might be due to our omission of the strong $\frac{13}{2}^+$ state and the other interpretation, it might be due to an incorrect radial form for the $\frac{7}{2}^+$ component of the neutron wavefunction. The DWBA-Nilsson calculations are at considerable variance with the data in this case. We have not attempted to compare normalizations between bands as we had no reliable means of calculating quasiparticle occupation factors.

Certain clear implications for stripping calculations with strongly deformed nuclei emerge from the following observations.

(a) The differential cross sections of the $\frac{7}{2}^{+}$ and $\frac{11}{2}^{+}$ are increased over a wide angular range by a factor of the order of four when the full inelastic processes are included in the calculation.

(b) The stronger $\frac{9}{2}^{+}$ level is little changed in overall cross section. A local reduction (40%) around 60°, a data angle for Ref. [13]). This level is certainly likely to be affected in lowest order by exit channel coupling to the $\frac{13}{2}^{+}$ state.

(c) For the $\frac{7}{2}^{+}$ level, at least, inelastic processes are about equally important in the exit and entrance channels. The exit channel scattering (when the $\frac{13}{2}^{+}$ state is excluded, at least) has a lesser effect on the $\frac{11}{2}^{+}$ state except at forward angles.

(d) In this particular case, the exit channel scattering appears to have imposed a diffraction pattern upon the angular distribution of the protons. In other cases described below this does not occur.

The $[512]\frac{5}{2}^{-}$ Band

According to Kanestrøm and Tjøm[14]) this band has considerable coriolis induced admixtures which will particularly affect the $\frac{9}{2}^{-}$ state.

The results of a series of calculations in which the three lowest levels of this band are included together (and one in which the $\frac{7}{2}^{-}$, $\frac{9}{2}^{-}$, and $\frac{11}{2}^{-}$ states are coupled) are illustrated in figs. 2 and 3. A comparison of the DWBA and the complete CCBA calculation with the data[13]) is given in Table 2. Numerical results at particular angles corresponding to all the curves in the figures is given in Table 3.

From all this, the following points can be made:

(a) The cross sections of the weaker levels ($\frac{5}{2}^{-}$ and $\frac{9}{2}^{-}$) have been considerably increased by the inelastic processes, exit and entrance channel effects being again of comparable importance.

(b) The effect of transitions through the target 4^+ state is notably significant for the $\frac{5}{2}^-$ state, but not for the other states. The Q-value for such stripping transitions in fact being larger than for those from the target ground state; these transitions might have been somewhat over-emphasized on account of our incorrect asymptotic neutron wavefunction. Judging from fig. 10 of Ref. [15]) this might be a small effect. (See Ref. [5]) for their discussion).

(c) The small amplitude $\ell = 1$ components in the intrinsic state single-particle wavefunctions were of some importance to the $\frac{9}{2}^-$ state, to the extent of a 10% reduction in cross section. The effect on the other states (through target 4^+ channels or exit scattering) is small.

(d) The replacement of the weak $\frac{5}{2}^-$ state by the weak $\frac{11}{2}^-$ state has a small effect on the other states; the exit channel scattering is dominated by the strong $\frac{7}{2}^-$ state.

(e) The exit channel scattering has increased the cross section to <u>all</u> the states. This does not happen in every case, and in principle makes the measurement of pairing occupation factors somewhat uncertain.

(f) The agreement with the experimental data of Ref. [12]) and with the more extensive data of Ref. [13]) is greatly improved. The under-population of the $\frac{9}{2}^-$ state is consistent with the findings of Kanestrøm and Tjøm[14]) that coriolis admixtures are important for increasing the cross section of this state.

We note here that in principle both the entrance and exit channel inelastic processes which take place may increase the effect of coriolis coupled admixtures. In particular, both processes might affect any state for which the

$C_{j\ell}$'s of strongly admixed single particle states are small (where j is the level spin) thus having little effect on DWBA calculations. This is most obvious where the level examined is strongly mixed with a single particle state for which there is a large $C_{j'}$, where $|j - j'| \leq 2$. These higher order effects will be most important for weaker states and the complete description of stripping will entail the coherent sum of a large number of such transitions.

In this particular case we repeated the DWBA calculations with deuteron optical potentials defined to give precisely* the same elastic scattering as coupled channel calculations including the 2^+ target state. The $\ell = 3$ cross sections were reduced 10-20% over the range of the experimental points, the $\ell = 5$ state somewhat less except at the most forward angles.

4.2. THE REACTION ^{156}Gd(d,p)

This reaction has been studied by Tjøm and Elbek[16]) at 12.1 MeV. We have carried out calculations of stripping to the $[521]\frac{3}{2}^-$ band at this energy and also purely model calculations at 16 MeV where there is no data. Kanestrøm and Tjøm[14]) have shown that this band is somewhat influenced by coriolis admixing in the neighboring nuclide ^{159}Gd. The effect should be rather smaller in ^{157}Gd where the $[521]\frac{3}{2}^-$ band is much more widely separated from the $[523]\frac{5}{2}^-$ band.

4.3. 12.1 MeV DEUTERONS

The results of the various calculations performed in our study are shown in fig. 4 and listed numerically with data[5]) in Table 4. The calculations comprise (i) a DWBA calculation; (ii) a CCBA calculation including the $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$ states of the residual nucleus and the 0^+, 2^+, and 4^+ target states; (iii) a similar calculation in which the deformation of the residual nucleus is increased

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*within the meaning discussed in Ref. [5]).

from $\beta_2 = .25$ to $\beta_2 = .3$ (without, however, changing the proton optical potential); (iv) a calculation similar to (ii) but with amplitudes of the $\ell = 3$, $j = \frac{7}{2}$ reduced by a factor of .85. In the DWBA calculation corresponding to (iv) the cross section of the $J = \frac{7}{2}$ state is scaled by $(.85)^2$ but is otherwise identical to (i). These various cases are shown in fig. 4 with the data uniformly normalized to the theoretical result for case (iv) at 60°. It is clear from the figure and from the table that the overall fit to the data has been considerably improved by the inclusion of inelastic processes. The scaling of the $C_{\frac{7}{2}-}$ by .85 was necessary to get the fit shown. It is interesting to note that the cross section of the $\frac{5}{2}-$ level was scaled by close to $(.85)^2$ whereas the $\frac{7}{2}-$ level cross section was reduced by a greater factor, suggesting that the $\frac{3}{2}-$ component has contributed destructively to the cross section for this state. This is consistent with the reduction in cross section for this level that occurs in the CCBA calculation, although this latter is probably due in part to the optical model ambiguity in the DWBA case as explained for the $\frac{5}{2}-$ band in ^{167}Er above.

As was also found[5,9]) for the case of Mg, there is approximate proportionality to C^2 over a small range of variation of this factor. This could be exploited in some cases to measure C^2 without the necessity of extensively repeated runnings of coupled channel programs.

The distinctive features of this case are (i) the destructive nature of the transition amplitudes involving the $\frac{3}{2}-$ component in the $\frac{7}{2}-$ state; (ii) the possibility of fitting the $\frac{9}{2}-$ state; (iii) the very large effect on the $\frac{5}{2}-$ level. This level is not listed in the tabulation of Ref. [16]) but appears in their fig. 11 as a bump of about the right size. They attribute this to band mixing. (This level has been seen[17]) in ^{155}Sm and was reasonably well fitted in calculations mentioned below). The j-component amplitudes corresponding to our final fit are $C_{\frac{3}{2}} = -.366$, $C_{\frac{7}{2}} = .651$, $C_{\frac{9}{2}} = -.515$, etc. compared with Vergnes and Sheline's[2]) -.32, +.73, -.5 (respectively). Although mixing with the $[523]\frac{3}{2}^-$ band greatly affects the $\frac{9}{2}^-$ state in ^{159}Gd the much greater band separation here should make the effect much smaller.

We mention here calculations that were also carried out for the same band in ^{155}Sm (for details, see Ref. [5])). The general results were similar to the Gd case, but there were additional qualitative findings. These calculations suggested in the first place the importance of good neutron wavefunctions; and it was established that the target 4^+ state is only of significant importance for the $\frac{9}{2}^-$ state, this state also being more affected by exit channel scattering than entrance channel scattering at the experimental angles. The general point is, of course, that it is in general difficult to know *a priori* what the effect in a given band will be (Cf. the $\frac{5}{2}^-$ band in Er).

4.4. 16.0 MeV DEUTERONS

The DWBA and CCBA calculations of the $[521]\frac{3}{2}^-$ band in ^{157}Gd were performed at 16.0 MeV with extrapolated optical potentials (see Refs. [5,11])) which gave 'identical' elastic scattering in appropriate calculations. The result is shown in fig. 5. Apart from the obscuring effect of the diffraction pattern in the $\frac{7}{2}^-$ state, the effects are qualitatively and quantitatively similar to those at 12.1 MeV. In particular, the importance of inelastic processes in describing stripping to the $\frac{9}{2}^-$ level is emphasized.

4.5. COMPARISON WITH OTHER DATA

There is insufficient data to judge directly whether our predicted modifications of DWBA angular distribution are consistently in accord with what would be measured. Confidence in spectroscopic factors is surely undermined by poor fits to angular distributions, and we have seen that these are certainly changed by inelastic processes. We can make one qualitative comparison with existing data, as follows; for (d,t) reactions of large ℓ-transfer, the angular distribution is like that for (d,p). For this, see Jaskola, Nybø, Tjøm, and Elbek[18]). The $\ell = 6$ transfer case that we have illustrated in fig. 1 suggests that indirect transitions may provide a remedy for the poor angular distribution which Jaskola et al. found (see their fig. 8), and suggests, perhaps, an overall renormalization of the level involved.

5. Conclusions

In general, the inclusion of inelastic processes in stripping calculations for deformed nuclei is important for both the angular distribution and relative strengths of particular levels. The effect on the latter is of the same order as that resulting[14]) from coriolis admixed components. Strongly excited levels within a band are generally not greatly affected in angular distribution but their overall strength may be changed. For weaker levels, the angular distribution can be considerably altered and the overall cross section can be changed by as much as a factor of ten or more. A comparison of the 16 MeV results in one case with those at 12 MeV does not indicate a rapid fall off in effect with energy, in this energy region.

We have not succeeded in fitting data exactly, (although we have achieved considerable improvement), but we have established that the inclusion of intraband inelastic processes (and, possibly, inter-band transitions) is a prerequisite for doing this. The complete calculation is thus very elaborate, but this work has demonstrated the utility of the source term technique for such a calculation. We have shown, however, that the disagreement often apparent[14]) between DWBA calculations (including coriolis admixing) and data can probably be understood without abandoning the overall Mottelson-Nilsson picture.

Unfortunately, it has not been possible to check the details of our angular distribution results owing to the dearth of experimental data. Few detailed angular distributions have been determined for (d,p) reactions in the lower half of the deformed region (where computing time requirements for calculations such as ours are somewhat less stringent) as it has not seemed necessary for the determination of ℓ-transfer. However, we have shown that in some cases there may be considerable alteration in angular distribution as a result of indirect transition and indeed there seem* to be systematic differences between experiment and DWBA calculations.

The present experimental data is not sufficient to assure us that these reactions are well understood. The current procedure for measuring spectroscopic factors in deformed nuclei is model dependant and suspect for two reasons. Firstly where more detailed angular distributions are measured (see Refs. [18,19]) the agreement with DWBA is not good, showing apparently systematic departures; and, as we have seen, our calculations predict considerable departures from DWBA form accompanied by changes in overall normalization. It thus seems that more detailed angular distributions are one prerequisite before confidence in either the nuclear model or the stripping model is possible. Furthermore, in view of the fact that, as we found, stripping is more sensitive to the optical potential than scattering, the presently existing elastic and inelastic scattering data are probably insufficient to define optical and collective parameters[10]) as closely as may be desired. In particular, owing to the uncertain way that the deuteron optical potential extrapolates in the Coulomb barrier region, any realistic theoretical study of the energy dependence of inelastic processes in stripping in this region would have to be preceded by deuteron scattering measurements over the appropriate energy range.

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*See, for example, figs. 8 and 10 of Rickey and Sheline[19]) where angular distributions are given for ^{177}Hf. The $\ell = 3$ cases have some qualitative similarities to those in our figures 2 and 3.

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Table 1. Comparison of DWBA and CCBA for the ground band of ^{167}Er. The data is also given normalized to the respective theoretical results for the $\frac{9}{2}+$ state at 90° CM. The improvement is good for the $\frac{7}{2}+$ state but less so for the $\frac{11}{2}+$ state, possibly reflecting the neglect of the $\frac{13}{2}+$ state.

	60° CM		90° CM		125° CM		Arrangement of Table	
	1				~ 0.3		DATA	
$\frac{7}{2}+$	.000609	.00218[a]	.000536		.000298	~.000654[a]	DWBA	data[a]
	.00206	.00211[b]	.00143		.000667	~.000634	CCBA	data[b]
	19		9		8		DATA	
$\frac{9}{2}+$	.0241	.0414[a]	.0196	.0196[a]	.0121	.0174	DWBA	DATA[a]
	.0177	.0401[b]	.0190	.0190[b]	.00856	.0169	CCBA	DATA[b]
	~ 3				~ 1		DATA	
$\frac{11}{2}+$	.00112	.00654[a]	.00150		.000888	~.00218[a]	DWBA	DATA[a]
	.00336	.00634[b]	.00231		.000874	.00211[b]	CCBA	DATA[b]

[a]Obtained by multiplying data by $\frac{.0196}{9} = .00218$

[b]Obtained by multiplying data by $\frac{.0190}{9} = .00211$

Table 2. Comparison of DWBA and CCBA for the $\frac{5}{2}-$ band in ^{167}Er. The table is arranged in the same way as Table 1, and with the same normalization procedure.

	60° CM		90° CM		125° CM		
	13				3		
$\frac{5}{2}-$	.00802	.0203[a]	.00582		.00245	.00467[a]	DWBA
	.0385	.02295[b]	.0119		.00468	.00530[b]	CCBA
	304		260		112		
$\frac{7}{2}-$	.555	.474[a]	.405	.405[a]	.1667	.1745[a]	
	.661	.536[b]	.459	.459[b]	.2077	.1978[b]	
	15		11		8		
$\frac{9}{2}-$	.00525	.0234[a]	.00281	.01713[a]	.0210	.01246[a]	
	.0133	.0264[b]	.0101	.0194[b]	.00800	.0141[b]	

[a]Obtained by multiplying data by $\frac{.405}{260} = .001558$

[b]Obtained by multiplying data by $\frac{.459}{260} = .001765$

Table 3. Stripping of 12.1 MeV deuterons leading to the [512]5/2- band in ^{167}Er. The experimental cross section exists only for 45°.

State	Angle	Expt.[40]	N^{+} / N^{++}	Y / N	Y / Y	Y^{b} / Y	$Y^{b,c}$ / Y^{c}
	45°	.054	.00769	.0159	.0187	.0345	.0339
5/2-	60°		.00802	.0157	.0215	.0385	.0386
	90°		.00582	.0090	.00746	.0118	.0116
	45°	.567	.549	.456	.475	.567[a]	.571
7/2-	60°		.555	.558	.641	.661	.675
	90°		.405	.381	.414	.459	.456
	45°	.0587	.00382	.00581	.0112	.0120	.0132
9/2-	60°		.00525	.00761	.0112	.0133	.0142
	90°		.00281	.00638	.0105	.0101	.0113

N^{+} signifies no coupling in entrance channels, Y signifies coupling.

N^{++} signifies no coupling in exit channels, Y signifies coupling.

[a] We normalize the data to this number.

[b] Includes the 4+ state of the target nucleus. The other calculations include 2+ only.

[c] Calculation excludes the l = 7 components of the single-particle wavefunction.

Table 4. Stripping of 12.1 MeV deuterons leading to the [521]3/2- band in ^{157}Gd. The upper numbers of each pair are the theoretical numbers as calculated; the lower are the values of (Experimental/Theory) for that angle. These numbers should ideally all be equal; the great reduction in the range of values when inelastic processes are included is evident -- especially if the 125° results are ignored.

State		Expt.[51]	N+ / N++	Y / Y
3/2-	60	146	.0309 4725	.0416 3510
	90	55	.0152 3618	.0141 3901
	125	23	.00639 3333	.00422 5450
5/2-	60	0	.000189	.00442
	90	0	.000129	.00152
	125	0	.000063	.000529
7/2-	60	236	.1213 1946	.0981 2343
	90	132	.0660 1999	.0539 2449
	125	69	.0229 3013	.0199 3469
9/2-	60	23	.003263 6345	.0055 4182
	90	9	.0199 4520	.0034 2639
	125	11	.0134 8209	.0024 4490

N^{+} In top row, N or Y signifies absence of presence of coupling in entrance channels.

N^{++} Same for exit.

(continued)

Table 4 (continued)

State		Y	N	Y^b
		Y^a	N	Y
		.0406	a, b	.0415
	60	3596		3518
		.0139		.0145
3/2-	90	3957		3846
		.0042		.00456
	125	5470		5044
	60	.00492		.00333
5/2-	90	.00157		.00126
	125	.000515		.000447
		.0981	.08764	.0725
	60	2405	2639	3255
		.0505	.0447	.0393
7/2-	90	2614	2766	3358
		.0182	.0166	.0145
	125	3791	4170	4759
		.00587		.00507
	60	3918		4536
		.00355		.00312
9/2-	90	2535		2885
		.0026		.00222
	125	4255		4955

[a]In this case β_2 = .3 in proton channels.

[b]In these cases, $c_{7/2-}$ is multiplied by .85.

[c]Where not given, as for the other N, N column.

Figure Captions

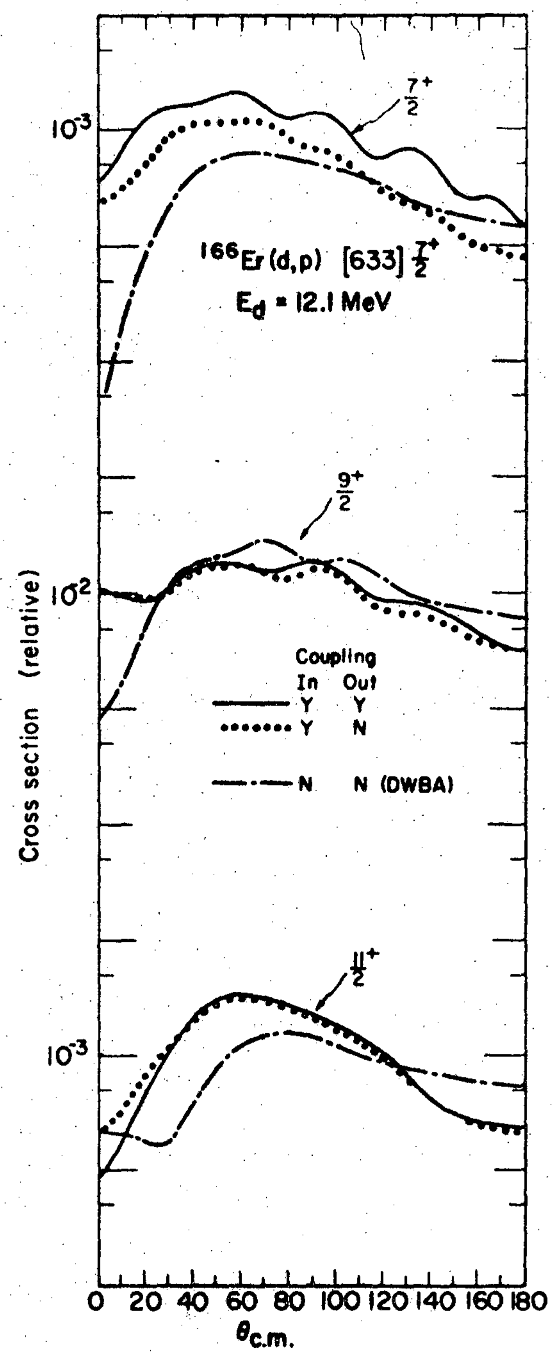
Fig. 1. Illustrating the influence of inelastic processes on the stripping of 12.1 MeV deuterons leading to states of the ground ($[633]\frac{7}{2}^+$) band of ^{167}Er. We have also illustrated the partial case where there are indirect transitions through the target 2^+ state but no ejectile scattering.

Fig. 2. The effect of indirect transitions on the stripping of 12.1 MeV deuterons leading to the $[512]\frac{5}{2}^-$ band in ^{167}Er. We compare calculations in which transitions through the target are included with those with the 2^+ state alone. We also give the result of a calculation in which the $\frac{7}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ levels of this band are coupled together instead of the $\frac{5}{2}^-$, $\frac{7}{2}^-$, and $\frac{9}{2}^-$.

Fig. 3. Further study of the reaction of fig. 2. Here we illustrate the effect of omitting exit channel coupling and of omitting the $\ell = 7$ components from the wavefunction of the bound neutron.

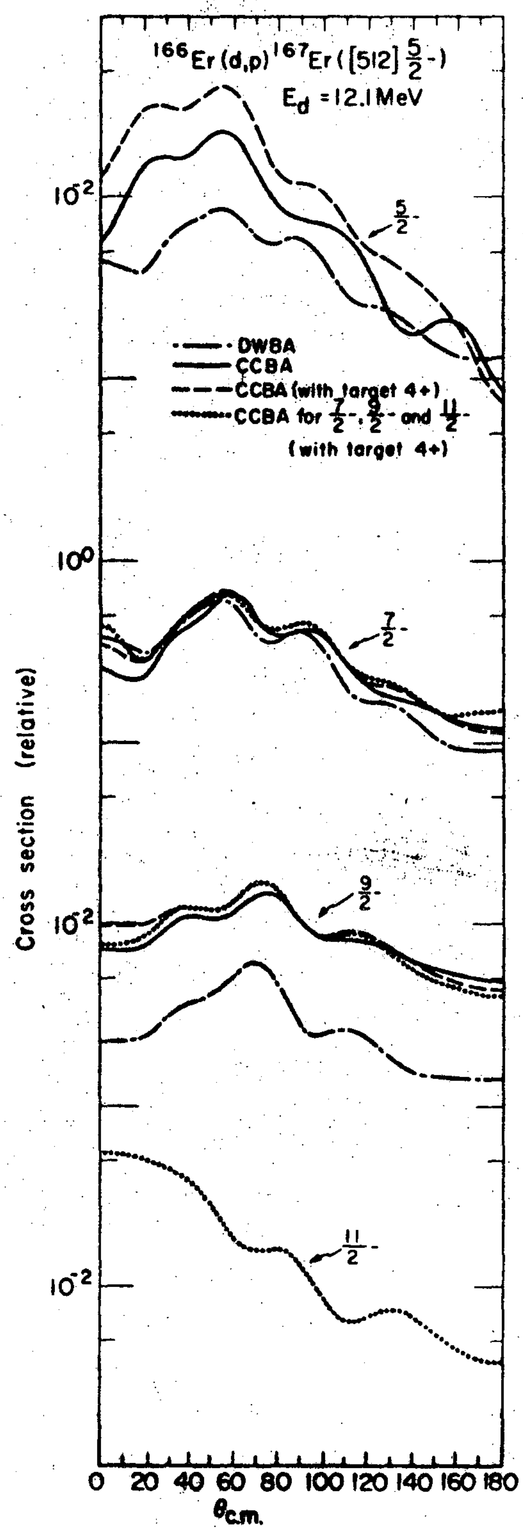
Fig. 4. Stripping to the ground band, $[521]\frac{3}{2}^-$, in ^{157}Gd at 12.1 MeV DWBA and CCBA are compared. Also shown are CCBA calculations in one of which the exit channel deformation is increased to .3, and in the other where the $\frac{7}{2}^-$ component was scaled in amplitude by .85. The data is normalized to the $\frac{7}{2}^-$ level calculated using CCBA with $C_{\frac{7}{2}-}$ scaled by .85. For the $\frac{7}{2}^-$ level, we also give the DWBA curve scaled by $(.85)^2$.

Fig. 5. The same reaction as fig. 4 at 16 MeV. CCBA and DWBA compared without scaling (there is no data).



XBL701-2226

Fig. 1



$^{166}Er(d,p)^{167}Er([512]\frac{5}{2}-)$
E_d = 12.1 MeV
DWBA
CCBA
CCBA (with target 4+)
CCBA for $\frac{7}{2}$-, $\frac{9}{2}$- and $\frac{11}{2}$- (with target 4+)
$\frac{5}{2}$-
$\frac{7}{2}$-
$\frac{9}{2}$-
$\frac{11}{2}$-
Cross section (relative)
$\theta_{c.m.}$
0 20 40 60 80 100 120 140 160 180
XBL701-2088

Fig. 2

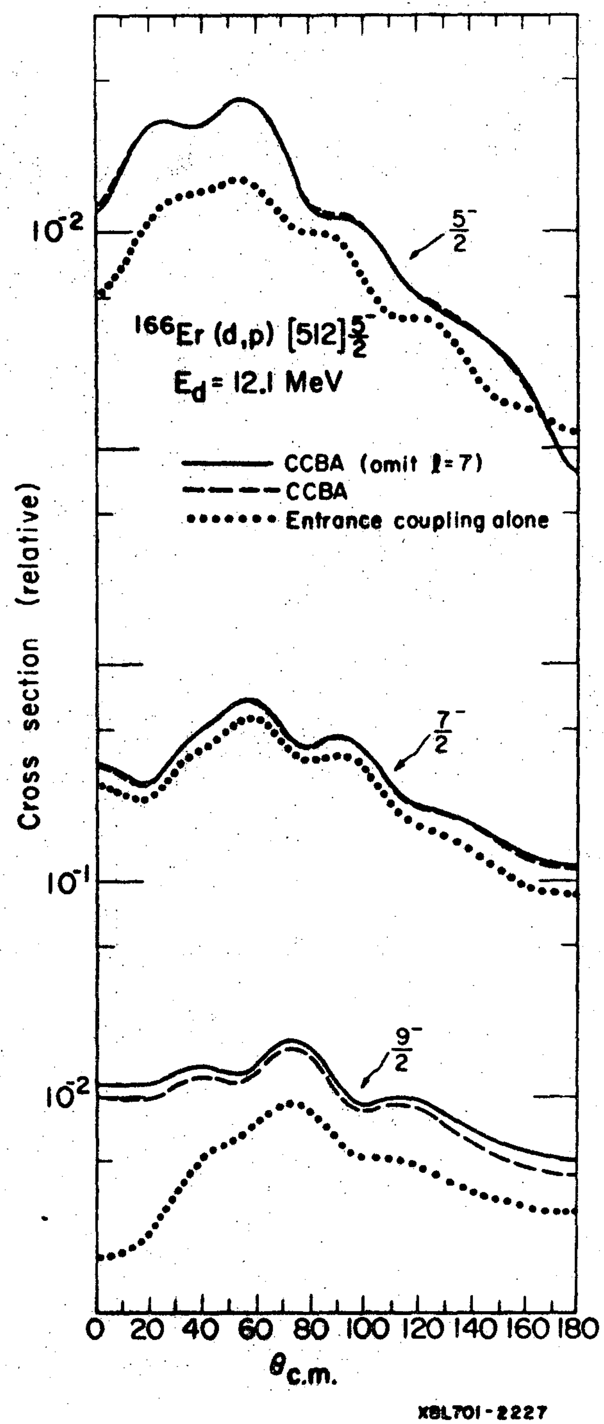

^{166}Er (d,p) [512]$\frac{5}{2}^-$
E_d = 12.1 MeV
CCBA (omit ℓ=7)
CCBA
Entrance coupling alone
$\frac{5}{2}^-$
$\frac{7}{2}^-$
$\frac{9}{2}^-$
10^{-2}
10^{-1}
10^{-2}
Cross section (relative)
0 20 40 60 80 100 120 140 160 180
$\theta_{c.m.}$
XBL701-2227

Fig. 3

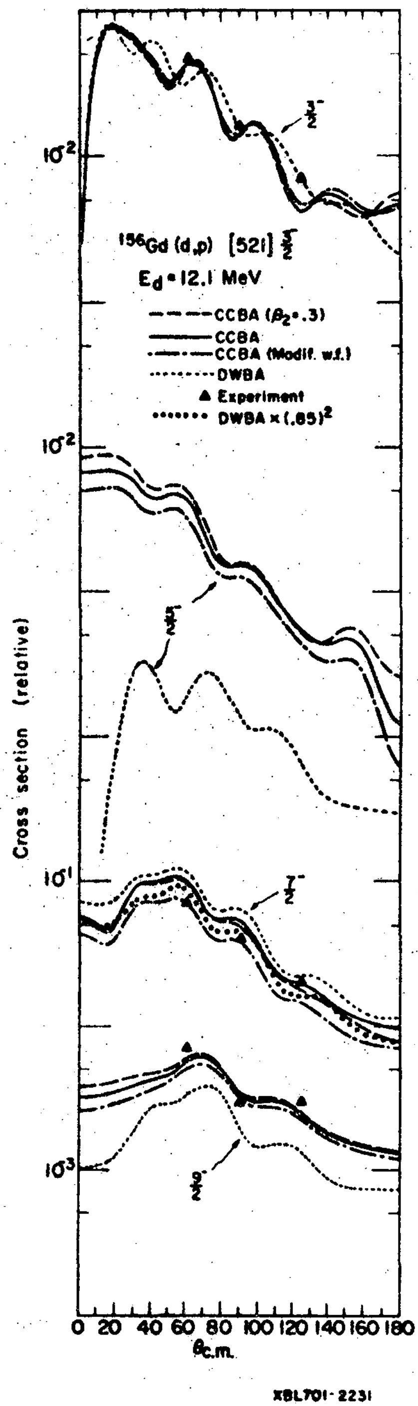

^{156}Gd (d,p) [521] 3/2
E_d = 12.1 MeV
CCBA (β_2=.3)
CCBA
CCBA (Modif. w.f.)
DWBA
Experiment
DWBA × (.85)^2
3/2
5/2
7/2
9/2
Cross section (relative)
10^2
10^2
10^1
10^3
0 20 40 60 80 100 120 140 160 180
$\theta_{c.m.}$
XBL701-2231

Fig. 4

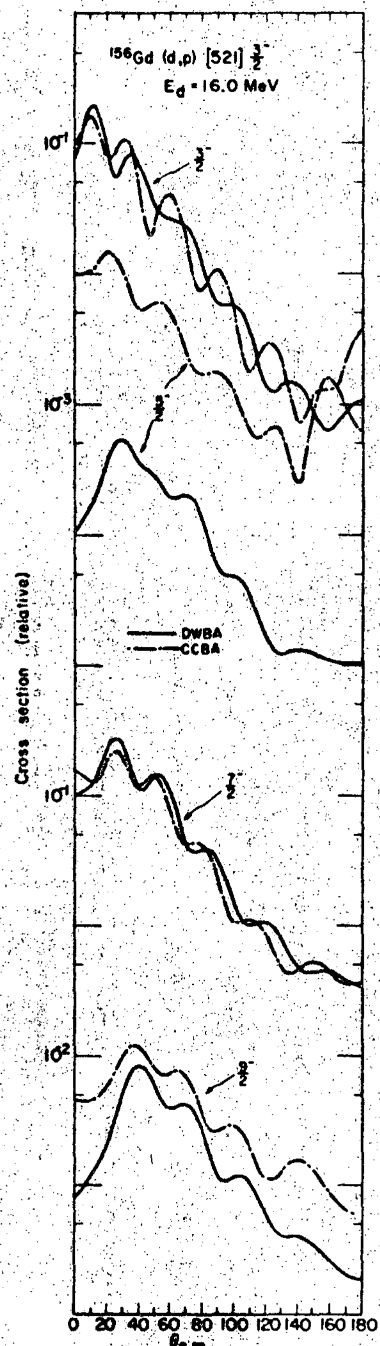


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Fig. 5

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