Title
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Journal
Proceedings of the PowerUp Conference, 1(1)

Authors
Desai, Maitraya Avadhut
Escobar, Francisco
Hug, Gabriela

Publication Date
2026-09-10

DOI
None/powerup_proceedings.69119

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Maitraya Avadhut Desai, Francisco Escobar, Gabriela Hug

Power Systems Laboratory, ETH Zurich

Suggested Citation

M. A. Desai, F. Escobar, and G. Hug, "Generalized Vector Locus Transformation for Unbalanced Three-Phase Systems," in *Proceedings of the PowerUp Conference*, 2026.

BibTeX Entry

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@inproceedings{desai2026generalizedvector,
  author = {Desai, Maitraya Avadhut and Escobar, Francisco and Hug, Gabriela},
  title = {Generalized Vector Locus Transformation for Unbalanced Three-Phase
      Systems},
  booktitle = {Proceedings of the PowerUp Conference},
  year = {2026},
}
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Generalized Vector Locus Transformation for Unbalanced Three-Phase Systems

Maitraya Avadhut Desai, *Graduate Student Member, IEEE,* Francisco Escobar, *Member, IEEE,* Gabriela Hug, *Senior Member, IEEE*

***Abstract*—Coordinate transformations significantly simplify power systems computations. Most notably, the classical Clarke and $dq0$ transformations are widely used in three-phase systems, as together they transform balanced abc quantities into constant-valued signals. However, during unbalanced operation, the utility of these transformations diminishes, since a null 0-coordinate cannot be ensured and oscillating signals emerge. While recently proposed transformations ensure a null 0-coordinate, they either do not lead to constant-valued signals in the $dq0$ domain or fail under various unbalanced scenarios. In this paper, we propose a Generalized Vector Locus (GVL) transformation that ensures both a null 0-coordinate and constant-valued signals. Moreover, we show that, in the balanced case, the classical amplitude-invariant Clarke transformation is an instance of the proposed GVL transformation.**

***Index Terms*—Clarke, coordinate transformation, $dq0$, Park, reference frame, unbalanced three-phase systems, vector locus.**

I. Introduction

Coordinate transformations are crucial for the modeling, analysis, and control of power systems. Two notable examples are the Clarke transformation, which maps abc coordinates to a stationary $\alpha\beta0$ reference frame, and the $dq0$ transformation, which maps $\alpha\beta0$ coordinates to a rotating $dq0$ reference frame [1]. In the Clarke transformation, the basis vectors are chosen such that a balanced set of three-phase quantities produces α- and β-coordinates of equal amplitude and in quadrature with each other. In the $dq0$ transformation, the speed of rotation is set equal to the electrical frequency, in which case the α- and β-coordinates further simplify to constant-valued signals. Both transformations produce a null 0-coordinate in the balanced case. Among other advantages, these simplifications enable the control of balanced three-phase systems using elementary techniques, e.g., proportional-integral control.

In an *unbalanced* four-wire system, however, the same choice of basis vectors for the Clarke transformation fails to achieve the desired simplifications. This occurs because the locus traced out by the three-phase quantities in Cartesian space morphs from a circle in a canonical plane to an ellipse in a different plane, which is determined by the degree of unbalance.

Recently, transformations have been proposed to address the aforementioned drawbacks of classical coordinate transformations. In this paper, we restrict our focus to transformations that rely on the locus of the space vector. In [2], a stationary mno reference frame is proposed, whose orientation depends on the voltage unbalance. However, this frame is not defined for voltage conditions such as zero voltage amplitude in phase a. In [3], a non-orthogonal reference frame was proposed for three-phase four-wire systems; however, when any of the phase amplitudes is null or the phase difference between any two of them is 0 or π radians, this transformation fails. The Reduced Reference Frame (RRF) proposed in [4] takes advantage of the locus geometry by rotating and orienting the reference frame as unit vectors along the semi-major and semi-minor axes. As a result, dimensional reduction is obtained in the signals; nonetheless, constant-valued signals are not obtained by further application of the $dq0$ transformation. Finally, a generalization of the Park transformation has been proposed in [5] using the Frenet-Serret formulae, but, it also fails to obtain constant-valued quantities.

In this paper, we propose a Generalized Vector Locus (GVL) transformation that overcomes the shortcomings of the aforementioned transformations and is valid for any type of unbalance in a three-phase, four-wire system. The proposed transformation maps the three-phase quantities to two sinusoids of unit amplitude, in quadrature with each other. Thus, subsequent application of the $dq0$ transformation results in two constant-valued signals. We also show that, in the balanced case, the proposed transformation simplifies to the classical amplitude-invariant Clarke transformation up to a constant scalar factor.

II. Generalized Vector Locus Transformation

We consider the three-dimensional space $\mathbb{R}^3$ with orthogonal axes abc and canonical basis vectors $\hat{\mathbf{e}}_a = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^\intercal$, $\hat{\mathbf{e}}_b = \begin{bmatrix}0 & 1 & 0\end{bmatrix}^\intercal$, and $\hat{\mathbf{e}}_c = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^\intercal$. Inside this space, any set of three-phase quantities can be represented as a space vector of the form $\mathbf{v} = v_a\,\hat{\mathbf{e}}_a + v_b\,\hat{\mathbf{e}}_b + v_c\,\hat{\mathbf{e}}_c$. For illustration purposes, we consider $\mathbf{v}$ to be a vector of phase-to-neutral voltages, but it can represent other three-phase quantities, such as currents or magnetic fluxes. In the general case, the individual phase coordinates are

$$\begin{aligned} v_a &= V_a \cos\left(\omega t + \varphi_a\right), \\ v_b &= V_b \cos\left(\omega t + \varphi_b - 2\pi/3\right), \\ v_c &= V_c \cos\left(\omega t + \varphi_c + 2\pi/3\right), \end{aligned} \tag{1}$$

where ω denotes the fundamental angular frequency. Rearranging the coordinates, the voltage vector can be written as

$$\mathbf{v} = \underbrace{\cos\left(\omega t - \varphi_o\right)}_{v_1} \mathbf{e}_1 + \underbrace{\sin\left(\omega t - \varphi_o\right)}_{v_2} \mathbf{e}_2\,, \tag{2}$$

where $\varphi_o \in [0, 2\pi)$ is an arbitrary angle and the constant vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are given by

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{v}|_{\omega t = \varphi_o} = \begin{bmatrix} V_a \cos\left(\varphi_o + \varphi_a\right) \\ V_b \cos\left(\varphi_o + \varphi_b - 2\pi/3\right) \\ V_c \cos\left(\varphi_o + \varphi_c + 2\pi/3\right) \end{bmatrix}, \\ \mathbf{e}_2 &= \mathbf{v}|_{\omega t = \varphi_o + \pi/2} = - \begin{bmatrix} V_a \sin\left(\varphi_o + \varphi_a\right) \\ V_b \sin\left(\varphi_o + \varphi_b - 2\pi/3\right) \\ V_c \sin\left(\varphi_o + \varphi_c + 2\pi/3\right) \end{bmatrix}. \end{aligned} \tag{3}$$

This research was supported by the Swiss National Science Foundation under NCCR Automation, grant agreement 51NF40_180545.

M. A. Desai, F. Escobar and G. Hug are with the Power Systems Laboratory at ETH Zurich, 8092 Zurich, Switzerland. Emails: {desai, escobarprado, hug}@eeh.ee.ethz.ch. *(Corresponding Author: M. A. Desai)*

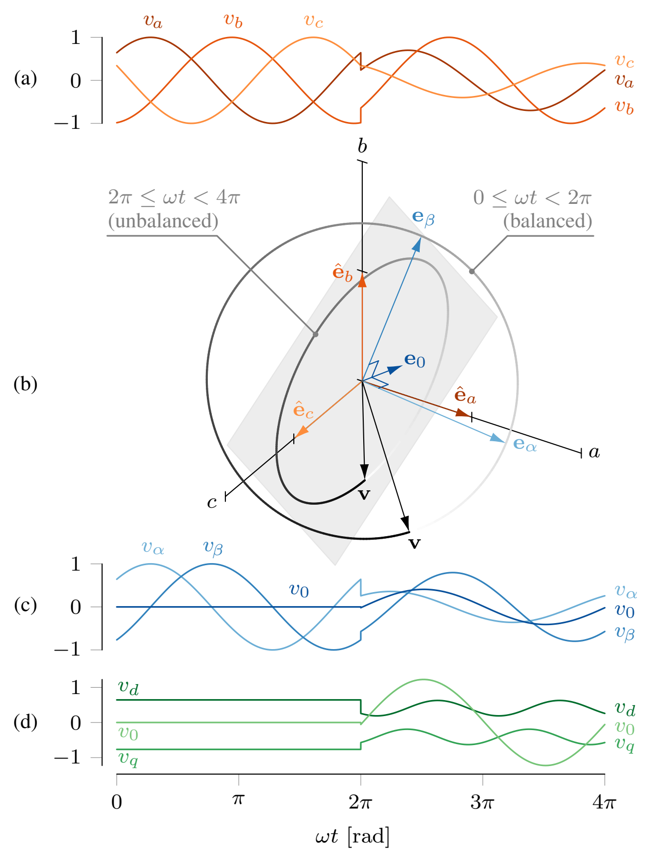

Fig. 1. Clarke and $dq0$ transformations applied to three-phase quantities. (a) Coordinates of $\mathbf{v}$ in abc during balanced and unbalanced operation. (b) Corresponding loci of $\mathbf{v}$ in $\mathbb{R}^3$ and basis vectors of both abc and $\alpha\beta 0$. (c) Coordinates of $\mathbf{v}$ in $\alpha\beta 0$. (d) Coordinates of $\mathbf{v}$ in $dq0$ for axes that rotate at the (synchronous) electrical frequency.

In the balanced case, where $V_a = V_b = V_c = V$ and $\varphi_a = \varphi_b = \varphi_c$, we have $\|\mathbf{e}_1\| = \|\mathbf{e}_2\| = \sqrt{3/2}\,V$. Thus, (2) reduces to the parametric equation of a circle centered at the origin with a radius of $\sqrt{3/2}\,V$. The amplitude-invariant Clarke transformation maps the canonical basis vectors $\hat{\mathbf{e}}_a$, $\hat{\mathbf{e}}_b$, and $\hat{\mathbf{e}}_c$ to $\mathbf{e}_\alpha = \begin{bmatrix}1 & -1/2 & -1/2\end{bmatrix}^\mathsf{T}$, $\mathbf{e}_\beta = \begin{bmatrix}0 & \sqrt{3}/2 & -\sqrt{3}/2\end{bmatrix}^\mathsf{T}$, and $\mathbf{e}_0 = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^\mathsf{T}$, respectively. This results in two perpendicular basis vectors coplanar with the balanced voltage vector locus and one basis vector perpendicular to this plane. Therefore, during balanced operation, the Clarke transformation reduces the three original signals to two displaced by a quarter of the period, and further application of the $dq0$ transformation yields constant-valued signals. However, in the general case, (2) represents the parametric equation of an ellipse centered at the origin. Thus, under unbalance, the Clarke transformation does not achieve the objectives mentioned above, as shown in Fig. 1.

The reformulated voltage vector as expressed in (2) facilitates the realization of the intended GVL transformation. We observe that, for any choice of φ_o, the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are the conjugate semi-diameters of the elliptical locus of the voltage vector during unbalanced operation. Thus, we define a third vector $\mathbf{e}_3$ that is perpendicular to this plane as

$$\mathbf{e}_3 = (\mathbf{e}_1 \times \mathbf{e}_2)/\lambda\,, \tag{4}$$

where the scaling factor $\lambda = \|\mathbf{e}_1 \times \mathbf{e}_2\|/\sqrt{3}$ allows backward compatibility with the classical Clarke transformation in the balanced case. Note that the vector $\mathbf{e}_3$ ensures that the 3-coordinate v_3 is always null. Consequently, we propose a transformation such that the canonical basis vectors $\hat{\mathbf{e}}_a$, $\hat{\mathbf{e}}_b$, and $\hat{\mathbf{e}}_c$ are transformed into $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. The inverse of the proposed transformation can therefore be represented as

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \underbrace{\begin{bmatrix} | & | & | \\ \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ | & | & | \end{bmatrix}}_{\mathbf{T}^{-1}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}. \tag{5}$$

The proposed GVL transformation $\mathbf{T}$ reduces the three original unbalanced signals to two sinusoidal signals displaced by a quarter of the period. Moreover, the two signals have the same (unit) amplitude. This is enabled by the form of $\mathbf{e}_1$ and $\mathbf{e}_2$ in (3), which retains the essential information about the unbalance (in contrast to normalized vectors). The further application of the $dq0$ transformation can also be visualized as rotating the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ such that, for every time instant, the corresponding conjugate semi-diameters of the ellipse are obtained. Thus, this application leads to two constant-valued signals.

At first glance, we note that $\mathbf{T}$ is written in terms of the unbalanced parameters. According to [6], these can be estimated. However, the key observation is that $\mathbf{e}_1$ and $\mathbf{e}_2$ are samples of $\mathbf{v}$ separated by a quarter of the period and $\mathbf{e}_3$ is obtained using (4). This observation can be leveraged in closed-loop implementations by using a frequency estimator to enforce the quarter period separation without using an explicit delayed measurement.

The instant used to compute the first basis vector is set by the angle φ_o introduced earlier. There are infinitely many valid choices of φ_o, each yielding a different pair of conjugate semi-diameters $\mathbf{e}_1$ and $\mathbf{e}_2$; we discuss two. The first choice is the value $\varphi_o^\diamond = -\varphi_a$, which corresponds to measuring $\mathbf{v}$ when v_a reaches its maximum. In the balanced case, this leads to the simplification of $\mathbf{T}$ to the amplitude-invariant Clarke transformation up to a constant factor that ensures coordinates with unit amplitude. The second choice is the value $\varphi_o^\star = \arg\max\|\mathbf{v}\| = \arg\max\|\mathbf{v}\|^2$. Since $\|\mathbf{v}\|^2 = v_a^2 + v_b^2 + v_c^2$, using trigonometric identities, it can be succinctly represented in the form $\|\mathbf{v}\|^2 = C - A\cos(2\omega t + \phi)$ for constants C, A, and ϕ. Thus, $\varphi_o^\star = -\pi/4 - \phi/2$, where

$$\tan\phi = -\frac{\begin{array}{r} V_a^2\cos 2\varphi_a + V_b^2\cos(2\varphi_b - 4\pi/3) + \\ V_c^2\cos(2\varphi_c + 4\pi/3) \end{array}}{\begin{array}{r} V_a^2\sin 2\varphi_a + V_b^2\sin(2\varphi_b - 4\pi/3) + \\ V_c^2\sin(2\varphi_c + 4\pi/3) \end{array}}\,. \tag{6}$$

In this case, we obtain an orthogonal set of basis vectors such that $\mathbf{e}_1$ and $\mathbf{e}_2$ are aligned with the major and minor axes of the ellipse, respectively, which simplifies the instantaneous power calculation (see Section III). Normalizing $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ for this $\varphi_o^\star$ recovers the RRF of [4]. Fig. 2 gives the geometric interpretation of the special choices.

We note that (2) describes a linear locus, and the matrix $\mathbf{T}^{-1}$ becomes non-invertible, *only* if all three abc coordinates are linearly dependent. Specifically, this linear dependence can appear if (i) all abc coordinates are in phase with each other, (ii) at least two of their amplitudes are 0, or (iii) the amplitude of

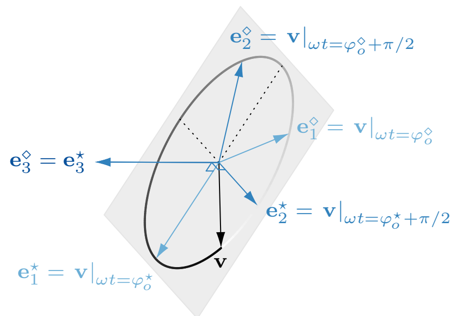

Fig. 2. Special choices of the basis vectors, found by measuring $\mathbf{v}$ when v_a reaches its peak ($\diamond$) or $\|\mathbf{v}\|$ is maximized ($\star$). The vector $\mathbf{e}_3$, being orthogonal to the ellipse and having length $\sqrt{3}$, is independent of these choices.

one coordinate is 0 *and* the phase difference between the other two coordinates is 0 or π radians. These particular unbalanced scenarios, where only one effective signal exists instead of three, renders further dimensionality reduction from one to two signals illogical. Such an attempt does not decrease the dimensionality, but rather introduce potential inaccuracies and redundancy by trying to add an unsupported dimension.

A comparison with the other transformations discussed in Section I is presented in Table I. In summary, the proposed GVL is the only transformation that ensures equal-amplitude, in-quadrature outputs and constant $dq0$ components under unbalance. Moreover, its sole failure mode is the genuinely degenerate case in which the voltage locus collapses to a line, a limitation shared by all such transformations; the mno and Tan-Sun frames, in contrast, also fail under conditions that leave the locus non-degenerate, namely a single 0 phase amplitude or any two phase angles differing by 0 or π.

III. Power Calculations

We consider voltage and current space vectors $\mathbf{v}, \mathbf{i} \in \mathbb{R}^3$. Without loss of generality, we note that the proposed transformation maps the canonical basis $\{\hat{\mathbf{e}}_a, \hat{\mathbf{e}}_b, \hat{\mathbf{e}}_c\}$ to a basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ span the voltage locus plane and $\mathbf{e}_3$ is normal to it. Under this construction, we have $\mathbf{v} = v_1\,\mathbf{e}_1 + v_2\,\mathbf{e}_2 + 0\,\mathbf{e}_3$ and $\mathbf{i} = i_1\,\mathbf{e}_1 + i_2\,\mathbf{e}_2 + i_3\,\mathbf{e}_3$.

The instantaneous active power is given by the dot product of the voltage and the current space vectors [7, §3.3.6], i.e.,

$$\begin{aligned} p &= \mathbf{v}\cdot\mathbf{i} \\ &= v_1 i_1 \|\mathbf{e}_1\|^2 + v_2 i_2 \|\mathbf{e}_2\|^2 + (v_1 i_2 + v_2 i_1)\,\mathbf{e}_1\cdot\mathbf{e}_2 \,. \end{aligned} \tag{7}$$

If $\varphi_o = \varphi_o^\star$ is used to form the basis vectors, the active power simplifies to $p = v_1 i_1 \|\mathbf{e}_1\|^2 + v_2 i_2 \|\mathbf{e}_2\|^2$, since $\mathbf{e}_1\cdot\mathbf{e}_2 = 0$. This generalizes the usual $\alpha\beta$ dot product, with scaling factors equal to the squared lengths of the semi-major and semi-minor axes.

According to [8], the instantaneous reactive power is defined as the cross product of the voltage and the current space vectors, i.e., $\mathbf{q} = \mathbf{v}\times\mathbf{i}$. Thus,

$$\begin{aligned} \mathbf{q} &= (1/\lambda)\,(v_2 i_3 \|\mathbf{e}_2\|^2 + v_1 i_3\,\mathbf{e}_1\cdot\mathbf{e}_2)\,\mathbf{e}_1 \\ &\quad - (1/\lambda)\,(v_1 i_3 \|\mathbf{e}_1\|^2 + v_2 i_3\,\mathbf{e}_1\cdot\mathbf{e}_2)\,\mathbf{e}_2 \\ &\quad + \lambda\,(v_1 i_2 - v_2 i_1)\,\mathbf{e}_3 \,. \end{aligned} \tag{8}$$

Using $\varphi_o^\star$ in the basis vectors, this simplifies to

$$\begin{aligned} \mathbf{q} &= \sqrt{3}\,v_2 i_3 \|\mathbf{e}_2\|\,\hat{\mathbf{e}}_1 - \sqrt{3}\,v_1 i_3 \|\mathbf{e}_1\|\,\hat{\mathbf{e}}_2 \\ &\quad + (v_1 i_2 - v_2 i_1)\,\|\mathbf{e}_1\|\,\|\mathbf{e}_2\|\,\hat{\mathbf{e}}_3 \,, \end{aligned} \tag{9}$$

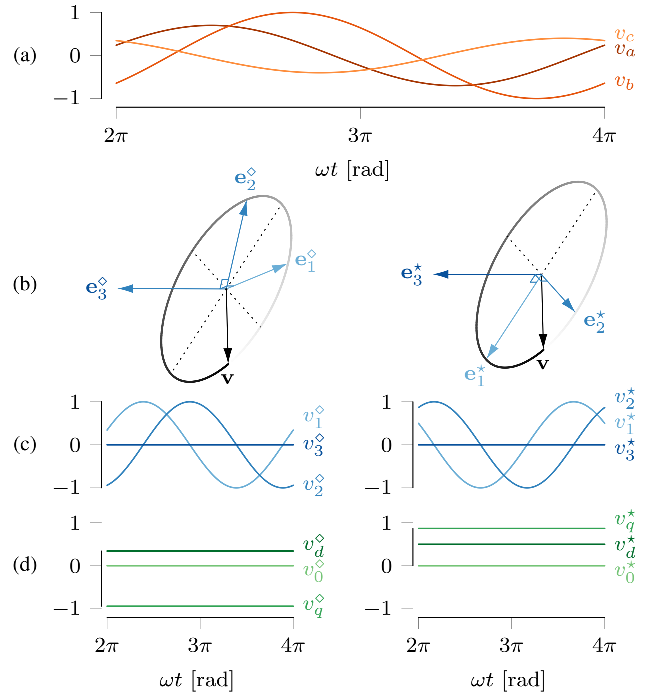

Fig. 3. Proposed transformation applied to an unbalanced three-phase system. (a) Coordinates of $\mathbf{v}$ in abc. (b) Choices of the basis vectors. (c) Corresponding coordinates of $\mathbf{v}$ under the new transformation. (d) Coordinates of $\mathbf{v}$ in $dq0$ for axes that rotate at the (synchronous) electrical frequency.

where $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$, $\hat{\mathbf{e}}_3$ are the respective unit vectors. It should be noted here that the third term in (8) and (9) is the component of the reactive power perpendicular to the voltage plane, analogous to the $\alpha\beta$ reactive power term. In contrast, the first two terms are components in the voltage plane and arise only when the out-of-plane current component is nonzero, which occurs due to the presence of the zero-sequence current.

IV. Numerical Validation

To validate the proposed GVL transformation, we first apply it to the unbalanced system of Figs. 1 and 2 (interval $2\pi \le \omega t < 4\pi$). The specific abc coordinates are given by

$$\begin{aligned} v_a &= 0.70\cos(\omega t - 7\pi/18)\,, \\ v_b &= 1.00\cos(\omega t - \pi/18 - 2\pi/3)\,, \\ v_c &= 0.40\cos(\omega t - \pi/2 + 2\pi/3)\,, \end{aligned}$$

and are plotted again in Fig. 3(a). We consider both choices $\varphi_o = \varphi_o^\diamond$ and $\varphi_o = \varphi_o^\star$, with the resulting coordinates shown in Fig. 3(c). The phase shift between, e.g., $v_1^\diamond$ and $v_1^\star$, reflects the time for $\mathbf{v}$ to traverse the arc between $\mathbf{e}_1^\diamond$ and $\mathbf{e}_1^\star$ in Fig. 3(b). As expected, the 1- and 2-coordinates have unit amplitude and are in quadrature, while the 3-coordinate is null throughout. Applying the standard $dq0$ transformation then yields the constant coordinates of Fig. 3(d), which could readily follow setpoints, e.g., via proportional-integral controllers. Moreover, since these signals are constant, zero-mean measurement noise can be attenuated by low-pass filtering without distorting them, unlike for transformations whose $dq0$ components remain oscillatory.

We next test the proposed transformation under the more severe unbalances of Fig. 4(a). Here, we test the cases where

TABLE I
COMPARISON OF THE PROPOSED GVL TRANSFORMATION AGAINST EXISTING TRANSFORMATIONS

Transformation	Generalizes	Output coords. under unbalance: Equal amplitude	Output coords. under unbalance: In quadrature	Resulting $dq0$ coords. under unbalance	Failure scenarios (non-invertible matrix)
mno [2]	$abc \mapsto \alpha\beta 0$	No	No	Oscillatory	Locus of $\mathbf{v}$ is linear; any amplitude is 0.
Tan-Sun [3], [6]	$abc \mapsto \alpha\beta 0$	Yes	Yes	Constant	Locus of $\mathbf{v}$ is linear; any amplitude is 0; any two phase angles differ by 0 or π.
RRF [4]	$abc \mapsto \alpha\beta 0$	No	Yes	Oscillatory	Locus of $\mathbf{v}$ is linear. [a]
Frenet frame [5]	$abc \mapsto dq0$	—	—	Oscillatory [b]	Locus of $\mathbf{v}$ is linear.
GVL (proposed)	$abc \mapsto \alpha\beta 0$	Yes [c]	Yes	Constant	Locus of $\mathbf{v}$ is linear.

[a] An additional check is implemented for the linear locus case and a new matrix is constructed as the original RRF transformation matrix is non-invertible.
[b] While the so-called [5] N- and B-coordinates (equivalent to q and 0, respectively) are both null, the T-coordinate (equivalent to d) is oscillatory.
[c] The common amplitude is 1.

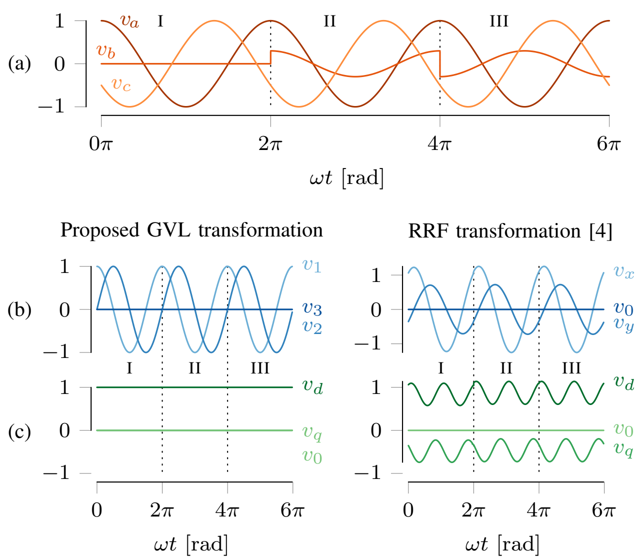

Fig. 4. Comparison of the proposed transformation and the RRF transformation [4] for different, severe degrees of unbalance, namely when $v_b = 0$ (I), when v_a and v_b are in phase (II), or when they are in counterphase (III). (a) Coordinates of $\mathbf{v}$ in abc. (b) Corresponding coordinates of $\mathbf{v}$ under the compared transformations. (c) Coordinates of $\mathbf{v}$ in $dq0$ for axes that rotate at the (synchronous) electrical frequency.

(I) any amplitude is 0, (II) the phase difference between any two phases is 0, and (III) the phase difference between any two phases is π. The proposed transformation handles all three without any special-case detection. Specifically, we fix $v_a = \cos\omega t$ and $v_c = \cos(\omega t + 2\pi/3)$ for $0 \le \omega t < 6\pi$, and then employ

$$\begin{aligned} v_b &= 0 && \text{and } \varphi_o = 0 \quad \text{for } 0 \le \omega t < 2\pi\,, \\ v_b &= 0.3\cos\omega t && \text{and } \varphi_o = 2\pi \text{ for } 2\pi \le \omega t < 4\pi\,, \\ v_b &= 0.3\cos(\omega t + \pi) && \text{and } \varphi_o = 4\pi \text{ for } 4\pi \le \omega t < 6\pi\,. \end{aligned}$$

The left plots of Fig. 4(b) and (c) show the 123 and $dq0$ coordinates from the proposed GVL transformation; the right plots show the RRF [4], which by Table I is the only other abc-to-$\alpha\beta 0$ generalization admitting these scenarios. The RRF yields quadrature sinusoids of unequal amplitude, so further $dq0$ application does not give constant quantities.

V. CONCLUSION

In this paper, we examined the shortcomings of the classical Clarke and $dq0$ transformations for the unbalanced operation of a three-phase system. We also listed the shortcomings of recently proposed transformations for unbalanced operation. Addressing these, we propose a GVL transformation, whose effectiveness is substantiated through numerical examples. The proposed transformation converts any unbalanced three-phase quantities into two unit-amplitude sinusoids in quadrature, leading to two constant-valued signals after applying the $dq0$ transformation. In the balanced scenario, it reduces to the classical Clarke transformation, scaled by a constant. Furthermore, the straightforward implementation of this GVL transformation, coupled with its backward compatibility with established classical transformations, advocates its adoption in the control of three-phase systems. Future work will explore the closed-loop implementation of the proposed transformation.

AI USAGE DISCLOSURE

The authors did *not* use Artificial Intelligence (AI) either in the research or in the writing of this paper.

REFERENCES

[1] C. J. O'Rourke, M. M. Qasim, M. R. Overlin, and J. L. Kirtley, "A geometric interpretation of reference frames and transformations: dq0, Clarke, and Park," *IEEE Trans. Energy Convers.*, vol. 34, no. 4, pp. 2070–2083, Dec. 2019.

[2] A. A. Montanari and A. M. Gole, "Enhanced instantaneous power theory for control of grid connected voltage sourced converters under unbalanced conditions," *IEEE Trans. Power Electron.*, vol. 32, no. 8, pp. 6652–6660, Aug. 2017.

[3] G. Tan, Z. Chen, Y. Zhang, J. Wei, X. Zeng, and X. Sun, "Three-dimensional coordinate transformation theory for three-phase four-wire unbalanced power electronic system," in *2021 IEEE 4th International Electrical and Energy Conference (CIEEC)*. Wuhan, China: IEEE, May 2021, pp. 1–6.

[4] F. Casado-Machado, J. L. Martinez-Ramos, M. Barragán-Villarejo, J. M. Maza-Ortega, and J. A. Rosendo-Macías, "Reduced reference frame transform: Deconstructing three-phase four-wire systems," *IEEE Access*, vol. 8, pp. 143 021–143 032, Jul. 2020.

[5] F. Milano, "The Frenet frame as a generalization of the Park transform," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 70, no. 2, pp. 966–976, Feb. 2023.

[6] G. Tan, J. Wei, W. Zhao, L. Qi, and X. Sun, "Application of three-dimensional unbalanced coordinate transformation to stand-alone four-leg voltage-source inverter," *IEEE Trans. Power Electron.*, vol. 37, no. 10, pp. 11 686–11 703, Oct. 2022.

[7] H. Akagi, E. Hirozaku Watanabe, and M. Aredes, *Instantaneous Power Theory and Applications to Power Conditioning*, 2nd ed., ser. IEEE Press Series on Power Engineering. Hoboken, NJ, USA: IEEE Press, 2017.

[8] F. Z. Peng and J.-S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," *IEEE Trans. Instrum. Meas.*, vol. 45, no. 1, pp. 293–297, Feb. 1996.

I. Review #68A

A. Paper summary

The paper constructs a new coordinate transformation that maps unbalanced three phase signals to two constant-valued signals. One use of this is designing control schemes.

B. Comments for authors

The paper is well-written. The discussion of properties, limitations, and comparison with existing work is appropriate. The numerical example serves to establish the properties of the coordinate transform.

C. Summarize your recommendation in one to two sentences

Not provided by the reviewer.

— Anonymous reviewer

II. Review #68B

A. Paper summary

The paper proposes a coordinate transformation method for time-varying signals, that overcomes weakness in unbalanced networks of established methods such as Park-Clarke and others, which may result in oscillatory signals.

B. Comments for authors

The paper is well-written, and I particularly appreciate the quality of the diagrams.

Edge cases where the inverse transformation due to any of the voltage magnitudes being 0 can be flagged ahead of time, as they arguably represent missingness of something. In that case fallback alternatives can be provided triggered, for any of the methods that suffer from this, not just the proposed one.

Are there differences in robustness in the presence of noise between your methods and established ones?

C. Summarize your recommendation in one to two sentences

A proof-of-concept study for an actual control system application would add value to the paper, currently it is not clear if the observed benefits will translate to better performance of things like power electronic converters.

— Frederik Geth

III. Review #68C

A. Paper summary

This paper proposes a new coordinate transformation that generalizes standard transformations to unbalanced three phase systems. It allows to transform a general three-phase unbalanced signal to two signals displaces by a quarter of a period and a zero component. This allows the application of dq0 transformation, to produce constant-value signals for all coordinates.

B. Comments for authors

The paper is very well written, easy to follow, and technically sound. My main recommendation would be to illustrate the usefulness of the GVL transformation on a more realistic application, rather than a simple toy example used.

C. Summarize your recommendation in one to two sentences

I think this note is a very good contribution to the PowerUp conference.

— Andrey Bernstein

IV. Brief response to reviewers (optional)

We thank the reviewers for their careful reading and positive assessment. We are glad that the clarity of the presentation, the treatment of properties and limitations, and the quality of the figures were appreciated (Comments 1.1, 2.1, and 3.1). We address the other specific comments below.

Comment 2.2 (edge cases and fallback). We agree that configurations in which the inverse transformation is undefined should be identifiable in advance. We emphasize, however, that the proposed GVL transformation does not fail when a single phase amplitude is zero: a missing phase still leaves a non-degenerate elliptical locus, so the transformation remains well defined and needs no special-case handling. This is in contrast to the mno and Tan-Sun frames, which are undefined in this situation. The only configuration in which the GVL matrix becomes singular is the genuinely degenerate one, where the locus collapses to a line and a single effective signal remains. This case is detectable a priori, since the basis vectors become linearly dependent, and a fallback can then be triggered, as suggested, for any affected method. We have edited the discussion following Table I and the second numerical example to make this explicit.

Comment 2.3 (robustness to noise). Two noise sources enter: the instantaneous measurement $\mathbf{v}_{\mathrm{abc}}$, and the transformation $\mathbf{T}$ of (5), which is itself built from measurements. Because $\mathbf{T}$ depends only on the quasi-static unbalance, it could be estimated and need not be read from two raw samples. An estimator in the closed-loop implementation would average many samples, which would suppress its noise and decouple it from the fast measurement noise. For such a $\mathbf{T}$, the map of $\mathbf{v}_{\mathrm{abc}}$ is, to first order, linear, and the equal-amplitude normalization distinguishing the GVL from the RRF is a per-coordinate rescaling that leaves the per-coordinate signal-to-noise ratio unchanged. As the GVL maps the unbalanced quantities to unit-amplitude quadrature sinusoids, its $dq0$ signal lies at DC, separated from the broadband noise, and can be recovered by averaging or low-pass filtering without distortion. Equivalently, a proportional-integral loop tracks the constant setpoint at low bandwidth, so the out-of-band noise is rejected. The RRF, mno and Frenet frames retain a double-frequency ripple, placing the signal away from DC. Thus, the same filtering would distort it, and faithful tracking instead requires bandwidth, or a resonant term, at the ripple frequency, which admits more noise. A quantitative study, including the noise transferred through the estimation of $\mathbf{v}_{\mathrm{abc}}$, is left as future work.

Comments 2.4 and 3.1 (realistic application). We agree that a converter-level study would strengthen the practical case. As this version is not re-reviewed and is bounded by the page limit, we have kept the scope unchanged. The closed-loop implementation in a power-electronic application is identified as future work.

V. Brief summary of changes made to the final paper

This is the camera-ready version. In line with the organizers' guidance, the content is essentially unchanged from the reviewed version; the edits are limited to deanonymization and small clarifications prompted by the reviews.

Deanonymization and required additions. We added the author names and affiliations, the corresponding-author designation, the funding acknowledgment (Swiss National Science Foundation, NCCR Automation), and the AI usage disclosure.

Clarifications from the reviews. Following Comment 2.2, we edited the summary after Table I to state that the proposed GVL transformation's sole failure mode is the genuinely degenerate (linear-locus) case shared by all such transformations, whereas the mno and Tan-Sun frames additionally fail under non-degenerate conditions, namely a 0 amplitude or two phases differing by 0 or π. We added a short note to the second numerical example emphasizing that the transformation handles the tested severe-unbalance cases, including the vanishing amplitude, without any special-case detection. Following Comment 2.3, we added one sentence to the numerical validation noting that the constant $dq0$ components allow zero-mean measurement noise to be attenuated by low-pass filtering, since the signal then lies at DC while the noise remains broadband, unlike for the oscillatory components of the other methods.

No changes were made to the theoretical development, the figures, or the numerical results. Some paragraphs were shortened for length so that the paper meets the 4-page limit, however, no technical content was removed.