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RADIATIVE AND NONRADIATIVE BOSON DECAY INTO LEPTONS

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The decay $K_{\mu 2} \rightarrow \mu + \nu$, which is the commonest K-meson decay, is strikingly similar to $\pi \rightarrow \mu + \nu$. A simple phase-space estimate gives a K- μ lifetime less than one-tenth the π - μ lifetime, while the observed lifetimes are more nearly equal. In this note, we wish to point out that the long $K_{\mu 2}$ lifetime and the absence of $K_{e 2}$ are both understandable in terms of the same interaction (axial vector) as has been invoked¹ to explain the absence of π -e decay. Since this interaction is one that suppresses the emission of fast electrons, it had been expected² that radiative decays like $\pi \rightarrow e + \nu + \gamma$ might be relatively important. That this is not so, however, can be shown in a simple way by a generalized equivalence theorem.

Since γ_5 merely inverts neutrino spins, and in the final state neutrino spins are summed over, the decay of a scalar meson by scalar (vector) coupling is identical with the decay of a pseudoscalar meson by pseudoscalar (pseudovector) coupling. The essential feature of derivative coupling is that the matrix element squared is proportional to $1 - v/c = (m/M + m)^2$, where v and m are the electron or muon velocity and mass, and M is the pion or K-meson mass. The transition rate is

$$\gamma^{-1} = g^2(M^2 - m^2)^2 / 2M^3 \cdot (m/M)^2, \quad (1)$$

where g is the effective boson-lepton coupling constant. The pseudovector interaction for the pion decay was motivated by the small ratio 1.3×10^{-4} that this equation gives for the pion decaying into $e + \nu$ rather than $\mu + \nu$

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From Eq. (1) the ratio of K-e to K- μ decay is

$$\tau_{Ke2}^{-1} / \tau_{K\mu2}^{-1} = 2.5 \times 10^{-5}. \quad (2)$$

When the coupling constant in Eq. (1) is adjusted to the observed lifetime,

$$\tau_{K\mu} = (0.70) \tau_{\pi\mu} = 1.8 \times 10^{-8} \text{ sec}. \quad (3)$$

No K_{e2} has in fact been reported, and the lifetime in Eq. (3) is in good agreement with the observed value.³

The derivative coupling suppresses the transition rate so long as $1 - v/c = 1 + (\vec{p}_\nu/E_\nu) \cdot (\vec{p}_e/E_e) \approx 0$, a relation which is altered by photon emission. If radiative transitions were to be somewhat enhanced relative to nonradiative decays, K_{e3} and $K_{\mu3}$, which appear to be a few percent of $K_{\mu2}$, might be interpretable (at least in part) as radiative decays. That this is not the case, but rather that the radiative decay is of order $\alpha/\pi = 0.2\%$ of the nonradiative decay follows immediately from an equivalence theorem relating (pseudo)scalar and (pseudo)vector interactions. From $(\not{p}_e - e\mathcal{K}) \psi_e = m_e \psi_e$ and $\not{p}_\nu \psi_\nu = m_\nu \psi_\nu$, it follows, for every order in e , that

$$g \langle \psi_{e,\mu} | (1 - e\mathcal{K}) \not{\phi} | \psi_\nu \rangle = g \langle \psi_{e,\mu} | (\not{p}_e - \not{p}_\nu - e\mathcal{K}) \not{\phi} | \psi_\nu \rangle \\ (m_{e,\mu} \mp m_\nu) g \langle \psi_{e,\mu} | \not{\phi} | \psi_\nu \rangle. \quad (4)$$

The minus sign holds when ϕ is a scalar meson wave function and the plus sign when ϕ is γ_5 times a pseudoscalar wave function. For $e = 0$, the matrix element of Eq. (4) is that for the nonradiative decay; if terms are kept linear in e , the matrix elements of Eq. (4) are those of the

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radiative decay. For both the radiative and the nonradiative decay, on derivative coupling, the matrix element for electron emission is m_e/m_μ times the matrix element for muon emission. The ratio of radiative and nonradiative decay rates of spin-zero mesons is the same on derivative and on direct coupling.⁴

In all four cases (S(S),⁵ P(P), S(V), P(A)), the probability of radiative decay with the emission of muons or electrons of momentum p in the energy interval dE is

$$P(E)dE = (\alpha/\pi)(M^2/M^2 - m^2) \left\{ \frac{4}{M^2 + m^2 - 2ME} \left[E \ln \left(\frac{E+p}{E-p} \right) - 2p \right] \right.$$

$$\left. \frac{M^2 + m^2 - 2ME}{M(M^2 - m^2)} \ln \frac{M(E+p) - m^2}{M(E-p) - m^2} \right\} dE .$$

It is interesting to observe that for all four couplings the spectrum obtained is essentially that expected from classical radiation damping.

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2. M. Ruderman, Phys. Rev. 85, 157L (1952).
3. We are grateful to S. Yamaguchi for pointing out that a similar calculation has been done by Ogawa, Okonagi, and Oneda, Prog. Theor. Phys. 11, 330L (1954). They calculate, however, a K_{M2} lifetime shorter by an order of magnitude, apparently by a numerical error.
4. In Reference 2 the emission of photons by virtual nucleon pairs was unjustifiably discounted. Their inclusion gives agreement with the above result. Simultaneously with the preparation of this letter, one of us received a preprint from Treiman and Wyld who, as part of a more extensive calculation, find this same result for the unimportance of radiative decay with pseudovector coupling. We are grateful to Drs. Treiman and Wyld for informing us of their results.
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