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Using Cooperative Adaptive Cruise Control (CACC) to Form High-Performance Vehicle Streams. Microscopic Traffic Modeling

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Microscopic Traffic Modeling

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INTRODUCTION

This document summarizes the microscopic traffic simulation models used in the project entitled Using Cooperative Adaptive Cruise Control (CACC) to Form High-Performance Vehicle Streams. The major components of the microscopic traffic model include the vehicle dispatching model, human driver model and ACC/CACC model. The vehicle dispatching model determines how a modeled vehicle enters the simulation network and the distribution of different types of vehicles across the multi-lane highway. The human driver model and ACC/CACC model specify the car following and lane changing behaviors of the human drivers and ACC/CACC equipped drivers, respectively. The proposed models can capture drivers' specific behaviors as the traffic management strategies are activated.

In this project, we have considered the following strategies:

- High Occupancy Vehicle (HOV) lane.
- CACC vehicle managed lane, which serves CACC vehicles and human driven vehicles equipped with Vehicle Awareness Device (VAD). The managed lane can have either continuous access or restricted access.
- CACC string operation that allows consecutive CACC vehicles to follow closely.

The details of each model are described in the following sections.

VEHICLE DISPATCHING MODEL

The vehicle dispatching model is intended to generate very high volumes of vehicles at the source section, under steady state conditions. This is essential for simulating CACC strings that have much shorter time gaps between the consecutive vehicles than conventional manually driven vehicles.

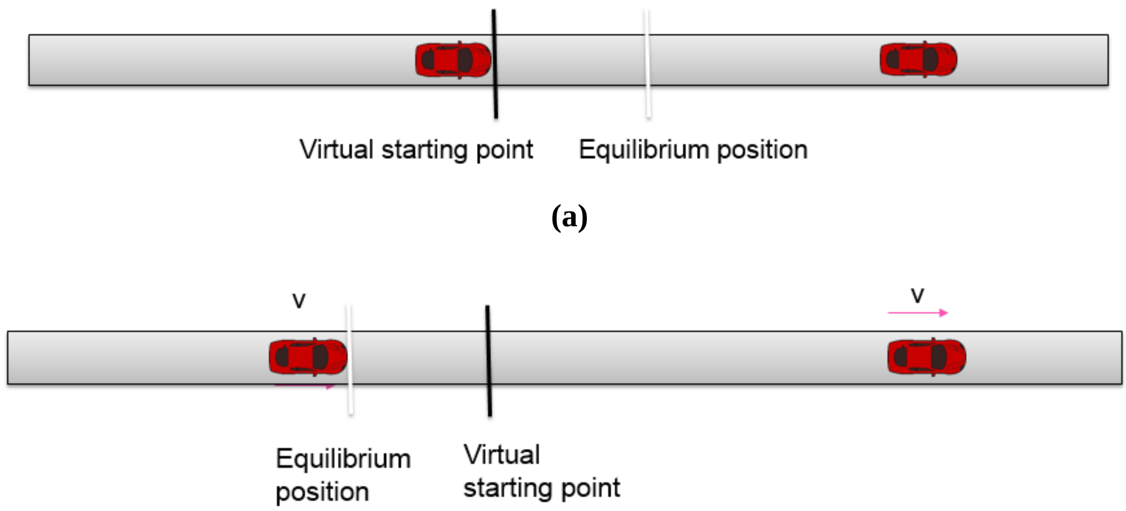
Once we specify the input volume and the minimum headway, the vehicle dispatching model can determine the arrival time of individual modeled vehicles. The time interval between two consecutive vehicles (t) is a random number drawn from the shifted negative-exponential distribution such as:

$$f(t) = \begin{cases} \lambda e^{-\lambda(t-t_{min})}, & t \geq t_{min} \\ 0, & t < t_{min} \end{cases} \quad (1)$$

where,

t_{min}: minimum headway [s]
λ: distribution parameter $\lambda = 1/(t_{avg} - t_{min})$
t_{avg}: average headway $t_{avg} = q/3600$ where q is the lane-based hourly volume [s]

Once a vehicle is generated in a lane, the vehicle type will be assigned based on the fleet composition (e.g., percentages of human driven vehicles, ACC vehicles and CACC vehicles) specified in the simulation inputs. The destination of each individual vehicle is determined based on the user specified O-D table or turning percentage at individual intersections and interchanges.



(b)

Figure 1 Illustration of vehicle dispatching.

An important part in the traffic generation model is the "holding" function that releases vehicles at the equilibrium position—a position where the subject vehicle has an acceleration of zero and speed equal to the desired speed. We also defined a virtual starting point (see Figure 1), which represents the most downstream position in the source link where a vehicle can be released. If the equilibrium position is downstream of the virtual starting point, the vehicle will be released at the virtual starting point with the same speed as the leader as shown in Figure 1(a). Otherwise, the vehicle will be released at the equilibrium position as shown in Figure 1(b).

If the CACC managed lane is activated, the CACC vehicles are first assigned to the managed lane at the source section. If the managed lane cannot accommodate all the CACC vehicles, the remaining CACC vehicles are released into the general-purpose lanes, with priority given to the lane immediately adjacent to the managed lane. For example, in a four-lane freeway, the leftmost lane is the CACC managed lane. If the CACC market penetration is 40%, the managed lane cannot serve all the CACC vehicles because each lane can only take 25% of the traffic load. In this case, the remaining CACC vehicles are assigned to the second left lane. If we define the ID of the leftmost lane as 1 and rightmost lane as 4, the percentages of CACC vehicles in individual lanes are as follows: lane 1—100% CACC vehicles and 0% human driven vehicles; lane 2—60% CACC vehicles and 40% human driven vehicles; lane 3—0% CACC vehicles and 100% human driven vehicles; and lane 4—0% CACC vehicles and 100% human driven vehicles.

The above vehicle generation scheme will lead to a 'stable' (i.e., steady state equilibrium) traffic condition at the beginning of the simulated network. It represents the traffic flow after the mixed fleet has traveled in a sufficiently long freeway segment without disturbance induced by the merging and departing traffic. On the other hand, if the CACC vehicles are evenly distributed across the lanes at the start, they will perform lane changing maneuvers towards the managed lane shortly after entering the network, causing unrealistic disturbances to the traffic flow.

HUMAN DRIVER MODEL

Microscopic driver car-following behavior and their interactions with the nearby vehicles determine the overall traffic pattern at the macroscopic level. The proposed human driver model that is intended to describe such driver interactions is built upon the basic framework of the NGSIM oversaturated flow model proposed by Yeo et al. (2008). Some important extensions and modifications were made to depict detailed car following and lane changing behaviors that were not represented in the original model. This section elaborates the human driver model formulations and model parameters.

In the proposed model, a driver's car following and lane changing behaviors are partitioned into fundamental driving modes (or movement phases):

CF: Regular car following mode
LC: Lane change mode, which includes discretionary lane change (DLC), active lane change (ALC) and mandatory lane change (MLC)
ACF: After lane changing car following mode (a driver temporarily adopts a short gap after a lane change maneuver)
BCF: Before lane changing car following mode (a driver speeds up or slows down to align with an acceptable gap in the target lane)
RCF: Receiving car following mode (a driver temporarily adopts a short gap after a vehicle from the adjacent lane merges in front)
YCF: Yielding (cooperative) car following mode

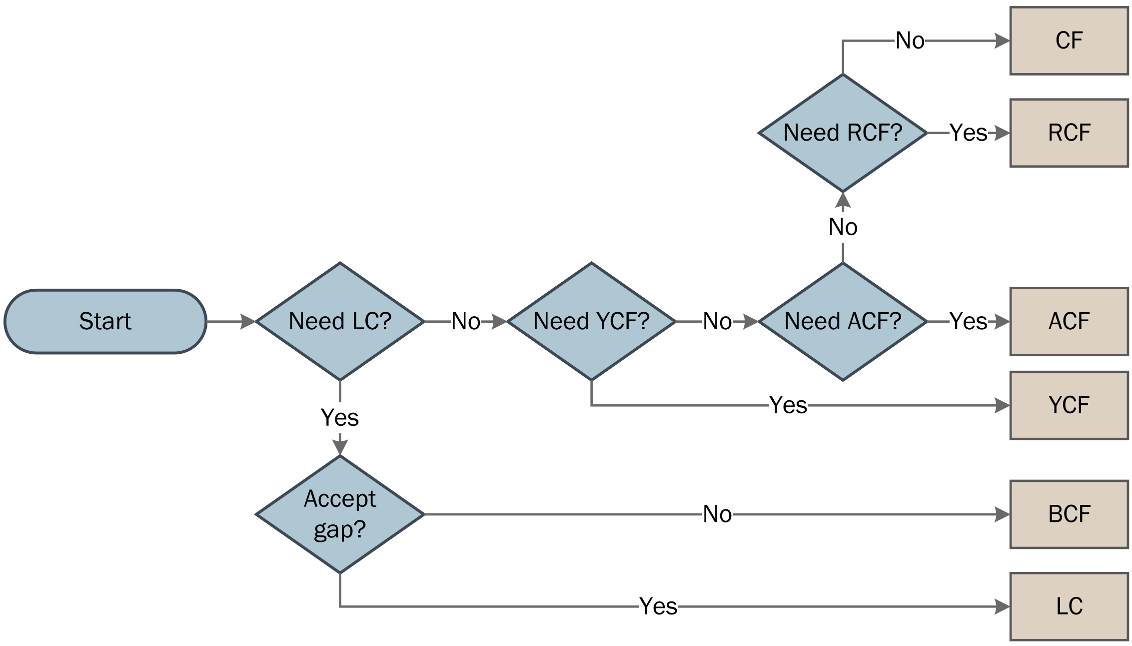


Figure 2 Human driver model structure.

As depicted by Figure 2, at the beginning of each simulation update interval, a subject driver's driving mode will be determined based on a set of car following and lane changing rules that are described in the following subsections. Each driving mode is associated with specific car following and lane changing algorithms, which are used to determine the driver's speed and position at the end of the update interval. Such an update process is executed iteratively for every modeled vehicle in the simulation environment, resulting in a trajectory for each vehicle.

Discrete Kinematic Model

To determine the trajectory of a vehicle at a microscopic level, it is necessary and sufficient to iteratively determine its location at each time step, which can be realized through a discrete kinematic model if the desired acceleration and current speed are known. The latter is known from the last step calculation. The former is determined by the dynamic interactions with the adjacent vehicles, geometric constraints, and the overall traffic conditions. The vehicle behaviors in the dynamic interactions are determined by time/clearance gaps for safety and mobility, and possible scenarios associated with lane changes.

The discretized kinematic model is used at the microscopic level to determine vehicle position x for the next simulation time step $t + \Delta t$ based on all the information at the current time step t. The following are the first and second order Taylor series expansion approximations of $x_{des}(t + \Delta t)$, the desired location of the subject vehicle (SV) for the next time step:

$$x_{des}(t + \Delta t) \approx x(t) + \Delta t \cdot v_{des}(t + \Delta t) \approx x(t) + \Delta t \cdot v(t) + \frac{\Delta t^2}{2} \cdot a_{des}(t) \qquad (2)$$

Equation 1 states that the expected (or desired) location of the subject vehicle can be determined as follows:

(a) First order approximation: if one knows the desired speed v_{des} and current location $x(t)$
(b) Second order approximation: if one knows the desired acceleration a_{des} and the current location $x(t)$ and speed $v(t)$

We use the term *desired* (or expected) because the actual speed is subject to constraints imposed by adjacent traffic conditions, and the maneuvers of the subject vehicle (SV), which will be discussed in the following. Similarly, one could easily deduce the expression for the relative distance with respect to the leading vehicle.

Car Following Model

The car following model used in this study is Newell's simplified car following model (Newell, 2002) with constraints for safety and free-flow accelerations. The safety acceleration is derived from the safe distance term in Gipps' car-following model (Ciuffo et al., 2012). It specifies a subject vehicle's maximum allowable acceleration under collision avoidance constraint. The free-flow acceleration is derived from the free-flow component of the Intelligent Driver Model (Treiber et al., 2000), which provides the upper limit of the acceleration when a vehicle accelerates in light traffic. The free flow acceleration a_F is described by the following equation:

$$a_F = a_M \left[1 - \left(\frac{v(t)}{V_0}\right)^{\alpha}\right] \quad (3)$$

where,

V_0: free flow speed [m/s]
a_M: maximum acceleration [m/s^2]
α: acceleration exponent

The acceleration (a_G) given by the safe distance term of the Gipps model reads:

$$a_G(t) = \frac{v(t+\tau_r) - v(t)}{\tau_r} \quad (4)$$

$$v(t+\tau_r) = A(t) + \sqrt{A(t)^2 - C(t)} \quad (5)$$

$$A(t) = b_f \tau_r \quad (6)$$

$$C(t) = b_f\left[2\left(d(t) - d_{jam}\right) - v(t)\tau_r - v_l(t)^2/(\hat{b})\right] \quad (7)$$

where,

τ_r: reaction time [s]
$v(t+\tau_r)$: speed of the subject vehicle after reaction time [m/s]
$v_l(t)$: speed of the preceding vehicle [m/s]
b_f: most severe braking that the subject driver wishes to undertake (<0) [m/s^2]
$\hat{b}$: the subject driver's estimate of preceding vehicle's most severe braking capabilities (<0) [m/s^2]
$d(t)$: clearance gap regarding the leader at time t [m]
d_{jam}: jam gap [m]

The acceleration of the Newell model (a_N) is given by the following equation:

$$a_N(t) = \frac{\left(d(t) - d_{jam}\right)/\tau - v(t)}{\tau/2} \quad (8)$$

where τ is the headway parameter [s].

The final desired acceleration a_{des} equation is:

$$a_{des} = \min(a_F, a_N, a_G) \quad (9)$$

For smooth transition between different car following modes, the following transition treatment is adopted:

$$a(t) = a(t - \Delta t) + \frac{a_{des}(t) - a(t - \Delta t)}{\omega} \quad (10)$$

where $\omega(t)$ is a smoothing factor (≥ 1).

Speed Friction across Lanes

It is generally recognized that most drivers do not drive significantly faster than those in the adjacent lane due to safety concerns, and significantly reduce their speeds if planning a lane change into the slower adjacent lane. A model for the *friction effect* is proposed to account for this real-world scenario:

$$V_0 = \begin{cases} min\{v_r, v_l\} + \frac{V - min\{v_r, v_l\}}{c_f} + \varphi & V > min\{v_r, v_l\} \\ V + \varphi & Otherwise \end{cases} \quad (11)$$

where,

V_0: desired speed adjusted for lane friction [m/s]
V: free flow speed [m/s]
v_l, v_r: speeds ahead on the left/right lane, respectively [m/s]
φ: a random number with the mean of 0 m/s and the standard deviation equal to the standard deviation of the desired speed in driver population
c_f: coefficient of lane friction, a constant tunable parameter adjusted in model calibration

Lane Change Models

The terminology used in the LC algorithms is defined in the following figure:

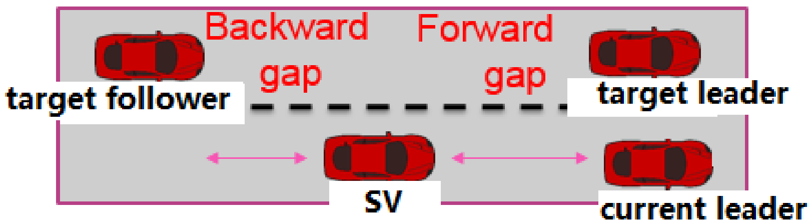


Figure 3 Terminology used in the LC algorithm.

In the LC algorithm, there are three different types of LC maneuvers: mandatory lane change (MLC), active lane change (ALC), and discretionary lane change (DLC). MLC describes drivers' behavior when they merge into the freeway from the acceleration lane and exit the freeway from the mainline. ALC is used in four cases: 1) when drivers in the rightmost lane change to the left adjacent lane to avoid conflict with the on-ramp merging traffic; 2) when drivers who intend to use the HOV lane make lane changes towards the HOV lane; 3) when drivers who intend to use the CACC managed lane make lane changes towards the CACC managed lane; and 4) when drivers that intend to exit the freeway—we will call them departing drivers afterwards—change toward the right lane at a long distance (e.g., 1 mile) upstream from the freeway exit. A driver will take a DLC if she or he wants to travel faster in the target lane. As we will show in the following sections, these LC types differ in terms of lane changing motivation generation, gap acceptance, and car following patterns in the BCF mode.

Lane Changing Motivation Generation

We use different functions to generate the lane changing motivation for MLC, ALC and DLC maneuvers. The output of these functions is a desire index γ between zero and one. Zero means the driver has no intention to change lane, while one indicates the driver has the highest intention. The driver will start the gap searching and lane change process if her or his γ is larger than a lane changing desire threshold Γ.

Mandatory Lane Change (MLC) Motivation

If the driver must merge onto the freeway from the on-ramp, a mandatory lane change desire $\gamma_{m,\{l,r\}}$ is generated by using the following equation:

$$\gamma_{m,\{l,r\}} = \begin{cases} 0 & d \geq E_{max}, t \geq T_{max} \\ 1 - \min(\frac{d - E_{min}}{E_{max} - E_{min}}, \frac{t - T_{min}}{T_{max} - T_{min}}) & otherwise \\ 1 & d \leq E_{min}, t \leq T_{min} \end{cases} \quad (12)$$

where,

l, r: subscripts indicating left and right, respectively
d: distance to the end of the acceleration lane [m]
t: d/v, time to the end of the acceleration lane [s]
E_{max}, E_{min}: maximum and minimum distances to the end of the acceleration lane [m]
T_{max}, T_{min}: maximum and minimum times to the end of the acceleration lane [s]

When a driver wants to exit the freeway, she or he usually starts the departing behavior upstream from the off-ramp lane to avoid making difficult MLC maneuvers. The driver only makes MLC maneuvers as she or he approaches the end of the off-ramp. In the proposed LC algorithm, such a departing behavior is modeled by using both ALC and MLC modes. A departing driver will first try to make ALC maneuvers if the current distance to the end of the off-ramp is larger than a threshold. If the driver cannot find a gap with the ALC mode, she or he will start making MLC maneuvers. As Figure 4 shows, a freeway departing area is divided into three zones. In Zone 1, a subject driver will try to make ALC maneuvers. The driver will perform MLCs in Zone 2 and Zone 3 (the BCF behavior is different in Zone 2 and Zone 3). The MLC lane changing motivation in these zones is defined as:

$$\gamma_{m,\{l,r\}} = \begin{cases} 1 - \min(\frac{d_e - E_{min}}{E_{max} - E_{min}}) & E_{min} < d_e \leq E_{max} \\ 1 & d_e \leq E_{min} \end{cases} \quad (13)$$

$$d_e = d - N_{lc} \cdot v \cdot (T_{LC} + T_{search}) \quad (14)$$

where,

d_e: equivalent distance to the end of the off-ramp
N_{lc}: number of lane changes required
T_{LC}: time required to complete the lane changing maneuver
T_{search}: time required to search for an acceptable gap

In the above equation, an equivalent distance to the end of the off-ramp is used instead of the actual distance. The equivalent distance accounts for the number of lane changes required for a subject driver to exit the freeway. The more lane changes are required, the smaller the equivalent distance is. As the equivalent distance reduces, the room left for the driver to make lane changes becomes smaller and subsequently the driver's lane changing motivation gets stronger.

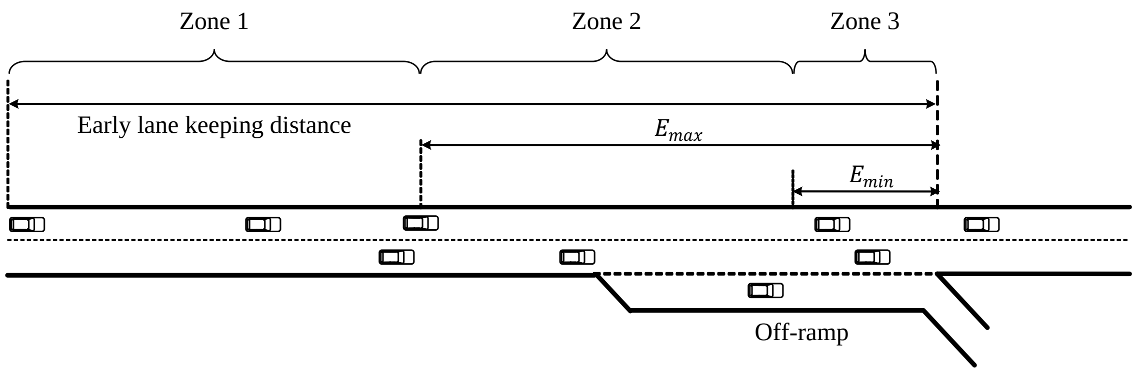


Figure 4 Definition of zones for the departing drivers.

Active Lane Change (ALC) Motivation

If a departing driver enters Zone 1 defined in Figure 4, she or he will start considering making lane changes towards the rightmost lane. The drivers will initiate the departing behavior at various locations. While a conservative driver may begin the departing behavior at an upstream location, an aggressive driver may want to stay in the left (and fast) lane longer and start the lane change at a downstream location. In this case, drivers will randomly pick locations in Zone 1 as the start point of the lane change behavior. We adopted a random lane change motivation generation function to model such a behavior:

$$\gamma_{a,\{l,r\}} = U_{[0,b]} \quad E_{early} \leq d_e \leq E_{max} \quad (15)$$

$$\left(\frac{\Gamma}{b}\right)^{N_{\Delta t}} = P \tag{16}$$

where,

$U_{[0,b]}$: uniformly distributed random variable between 0 and b, which is computed by using Equation 16
Γ: lane change desire threshold
$N_{\Delta t}$: number of update intervals considered in the ALC process
P: percentage of vehicles that choose not to make ALC maneuvers in Zone 1 within $N_{\Delta t}$ update intervals

$N_{\Delta t}$ and P are model parameters to be determined in the model calibration step. For example, if we find that 60% of drivers will consider making an ALC in Zone 1 in a 30-second period and the update interval is 0.1 second, $N_{\Delta t} = 30/0.1 = 300$ and $P = 1 - 60\% = 40\%$.

Equation 15 and 16 are also used to generate lane change motivation for the other types of ALC cases: 1) when drivers in the rightmost lane change to the left adjacent lane to avoid conflict with the on-ramp merging traffic; 2) when drivers make lane changes towards the HOV lane; and 3) when drivers make lane changes towards the CACC managed lane. Different $N_{\Delta t}$ and P may be used to determine the upper bound b for those ALC types. A subject driver will not have the MLC and ALC motivations at the same time. For example, if an HOV driver wants to exit the freeway, she or he only has MLC motivation towards the off-ramp. The ALC motivation towards the HOV lane is set to zero.

Discretionary Lane Change (DLC) Motivation

The discretionary lane change motivation is generated based on the anticipated speed ahead on the current lane and the adjacent lanes. The anticipated speed is determined as:

$$v_{ant} = \min\left(\tilde{v}_{\{l,r\}}, v_{l'}\right) \tag{17}$$

where,

v_{ant}: anticipated speed on the target lane [m/s]
$\tilde{v}_{\{l,r\}}$: average speed of the target lane (left or right) ahead [m/s]
$v_{l'}$: speed of the leader in the target lane [m/s]

The DLC motivation represents the percentage of speed increase a subject driver can achieve in the target lane. The lane change incentive $\gamma_{d\{l,r\}}$ is determined by the following equation.

$$\gamma_{d\{l,r\}} = \min\left(\max\left(0, \frac{v_{ant} - \tilde{v}_0}{\max(\tilde{v}_0, V_{dlc})}\eta_{\{l,r\}}\right), 1\right) \tag{18}$$

where,

$\tilde{v}_0$: average speed of the current lane ahead [m/s]
$\eta_{\{l,r\}}$: parameter (=1 for left lane change and <1 for right lane change)
V_{dlc}: minimum speed parameter to avoid division by zero [m/s]

Combining MLC, ALC and DLC Motivations

At each time step, the following lane change motivations are generated for each driver:

- left-lane mandatory lane change motivation $\gamma_{m,l}$,
- right-lane mandatory lane change motivation $\gamma_{m,r}$,
- left-lane active lane change motivation $\gamma_{a,l}$,
- right-lane active lane change motivation $\gamma_{a,r}$,
- left-lane discretionary lane-change motivation $\gamma_{d,l}$, and
- right-lane discretionary lane-change motivation $\gamma_{d,r}$.

All six are combined, with highest priority given to mandatory lane changes, medium priority to active lane changes and lowest priority to discretionary lane changes.

Case 1: Mandatory lane change motivation is larger than zero ($\gamma_{m,r} > 0$ or $\gamma_{m,l} > 0$). If the driver has a mandatory lane change motivation for a target lane (left/right), then the active and discretionary lane change motivation for the opposite target lane (right/left) is set to zero. That is, if $\gamma_{m,r} > 0$ $(\gamma_{m,l} > 0)$, then $\gamma_{d,l} = 0$ and $\gamma_{a,l} = 0$ ($\gamma_{d,r} = 0$ and $\gamma_{a,r} = 0$). The final desire is then determined by:

$$\gamma = \gamma_m + \gamma_a + \gamma_d$$

Case 2: Mandatory lane change motivation is zero ($\gamma_{m,r} = 0$ and $\gamma_{m,l} = 0$), but active lane change motivation is not. If the driver has an active lane change motivation for a target lane (left/right), then the discretionary lane change motivation for the opposite target lane (right/left) is set to zero. That is, if $\gamma_{a,r} > 0$ $(\gamma_{a,l} > 0)$, then $\gamma_{d,l} = 0$ ($\gamma_{d,r} = 0$). The final desire is then determined by:

$$\gamma = \gamma_a + \gamma_d$$

Case 3: Mandatory and active lane change motivation is zero ($\gamma_{m,r} = 0$, $\gamma_{m,l} = 0$, $\gamma_{a,r} = 0$, and $\gamma_{a,l} = 0$). In this case, the desire is determined by:

$$\gamma = \max(\gamma_{d,l}, \gamma_{d,r})$$

For MLC and ALC, the direction of the lane change (i.e., left or right) is determined based on the location of the target lane (e.g., HOV lane, CACC managed lane or off-ramp). In DLC, a subject driver will target the left lane if the left lane change motivation is larger than the right one, and vice versa. A random variable Γ, that follows a normal distribution with the driver's average lane changing desire threshold ($\bar{\Gamma}$) as the mean, is generated at the beginning of the simulation. If $\gamma > \Gamma$, the driver decides to change lane at the current time step and starts scanning gaps in the target lane, otherwise, the driver remains in the current lane.

Gap Acceptance Model

Before making a LC maneuver, a subject driver will check the forward and backward gaps in the target lane (see Figure 2). If both gaps are accepted, the vehicle will make a LC maneuver immediately. The lane change gap acceptance model is defined separately for mandatory, active and discretionary lane changes. For mandatory and active lane changes, safety is the primary concern. For discretionary lane changes, both comfort and safety are taken into account. The forward and backward gaps are considered separately since, in practice, the distance to the target leader can be shorter than that with respect to the target follower for the driver's safety and comfort.

MLC and ALC Gap Acceptance

The gap acceptance of MLC and ALC is based on the minimum gap that the driver anticipates to be available after the lane change maneuver. A target gap is accepted if the anticipated minimum forward and backward gaps meet the following conditions:

$$g_{f,min} \geq d_{jam} \text{ and } g_{b,min} \geq d_{jam} \tag{19}$$

To estimate the minimum forward gap, a subject vehicle driver assumes that the target leader will adopt an acceleration a_l, and the subject driver her/himself will apply an acceleration a_f during the lane change maneuver. The anticipated minimum gap appears at various times depending on whether the subject driver or the target leader stops and who stops first. In this regard, we compare the following three time periods: the time it takes for the target leader to stop $t_l = -v_l/a_l$; the time it takes for the subject driver to stop $t_f = -v_f/a_f$; and the time it takes for the target leader and the subject driver to reach the same speed $t_e = -\Delta v/\Delta a$. If both the target leader and the subject vehicle stop, and the target leader stops prior to the subject driver (i.e., $t_f \geq t_l \geq 0$), the anticipated minimum gap is:

$$g_{min} = g_0 + v_l \cdot t_l + 0.5 \cdot a_l \cdot t_l^2 - v_f \cdot t_f - 0.5 \cdot a_f \cdot t_f^2 \tag{20}$$

where, g_0 is the gap at the beginning of the lane change maneuver [m].

If both the target leader and the subject vehicle stop, and the subject driver stops prior to the target leader (i.e., $t_l \geq t_f \geq 0$), the anticipated minimum gap is:

$$g_{min} = \begin{cases} g_0 & t_e \leq 0 \\ g_0 + \Delta v \cdot t_e + 0.5 \cdot \Delta a \cdot t_e^2 & 0 \leq t_e \leq t_f \\ g_0 + \Delta v \cdot t_f + 0.5 \cdot \Delta a \cdot t_f^2 & t_f \leq t_e \end{cases} \tag{21}$$

where, $\Delta v = v_l - v_f$ and $\Delta a = a_l - a_f$.

If the target leader stops but the subject vehicle does not (i.e., $t_l \geq 0 \geq t_f$), the two vehicles will collide. In this case, the anticipated minimum gap is set to a negative value. If the subject vehicle stops but the target leader does not (i.e., $t_f \geq 0 \geq t_l$), the anticipated minimum gap can also be estimated via Equation 21. If both the target leader and the subject vehicle do not stop (i.e., $t_l \leq 0$ and $t_f \leq 0$), the anticipated minimum gap is:

$$g_{min} = \begin{cases} g_0 & t_e \leq 0 \\ g_0 + \Delta v \cdot t_e + 0.5 \cdot \Delta a \cdot t_e^2 & t_e \geq 0 \end{cases} \quad (22)$$

When estimating the minimum backward gap, the above equations are used as well, taking the subject vehicle as the leader and the target follower as the follower. The forward gap accepted by a driver is usually shorter than the backward gap in a LC maneuver. To model this behavior, we assume that the subject driver will have different anticipated acceleration for the target leader and her/himself. In other words, the anticipated accelerations (i.e., a_l) used in the forward and backward minimum gap estimation have the following relationship:

$$a_{f,l} \geq a_{b,l} \quad (23)$$

where,

$a_{f,l}$: anticipated acceleration of the target leader in the minimum forward gap estimation [m/s^2].
$a_{b,l}$: anticipated acceleration of the subject driver in the minimum backward gap estimation [m/s^2].

DLC Gap Acceptance

Besides safety, gap acceptance for discretionary lane changing also considers the anticipated acceleration the subject driver or the target follower must apply due to the lane change maneuver. The anticipated acceleration is computed by the car-following model of Equation 9, assuming that the subject vehicle follows the target leader and the target follower follows the subject vehicle. The acceptable forward and backward gap should meet the following criteria:

- Forward gap:

$$g_{f,min} \geq d_{jam} \text{ and } a_{sv} \geq A_1 \quad (24)$$

- Backward gap:

$$g_{b,min} \geq d_{jam} \text{ and } a_f \geq A_2 \quad (25)$$

where,

a_{sv}: anticipatory acceleration of the subject driver [m/s^2]
a_f: anticipatory acceleration of the target follower [m/s^2]
A_1: minimum acceleration the subject driver will accept in a DLC
A_2: minimum acceleration the target follower will accept in a DLC

Gap Acceptance for Special Cases

When the subject driver's lane changing motivation is very large (i.e., $\gamma = 1$) or the relative speed between the current lane and the target lane is high, the above gap acceptance models might lead to overly conservative lane change behaviors. The former condition occurs as a driver in the MLC mode approaches the end of the on-ramp acceleration lane or the off-ramp lane. The latter is usually observed in the upstream segment of a merging area where the traffic in the rightmost lane moves slowly due to the merging disturbance but the traffic in the left lanes moves relatively faster. In these conditions, a driver is usually willing to take smaller gaps that do not satisfy the above gap acceptance models. To model the gap acceptance of these special cases, we adopt a headway based gap acceptance model. Particularly, a forward or backward gap is accepted if it meets the following condition:

$$\begin{aligned} g_{\{f,b\}}(t+\Delta t) &> v \cdot \tau_{min} + l \\ g_f(t+\Delta t) &= g_f(t) - v \cdot \Delta t + v_l \cdot \Delta t \\ g_l(t+\Delta t) &= g_l(t) + v \cdot \Delta t - v_f \cdot \Delta t \end{aligned} \tag{26}$$

where, τ_{min} is the minimum headway a driver would accept at the next update interval.

After Lane Change Car Following (ACF)

As a subject driver identifies an acceptable gap and completes the LC maneuver, she or he will switch to the ACF mode. In this mode, the driver will temporarily adopt reduced headway, jam gap and reaction time parameters and gradually return to the regular CF mode. The reduced headway, jam gap and reaction time are given by the following equation:

$$\begin{aligned} \tau_h' &= \varphi_h(n)\tau_h, \qquad 0 < \varphi_h < 1 \\ d_j' &= \varphi_j(n)d_j, \qquad 0 < \varphi_j < 1 \\ \tau_r' &= \varphi_r(n)\tau_r, \qquad 0 < \varphi_r < 1 \end{aligned} \tag{27}$$

where,

τ_h, τ_h': headway and reduced headway, respectively
d_j, d_j': jam gap and reduced jam gap, respectively
τ_r, τ_r': reaction time and reduced reaction time, respectively
$\varphi_h, \varphi_j, \varphi_r$: reduction factor used to adjust the headway, jam gap and reaction time, respectively. The reduction factors are time dependent variables. Assuming the number of transition time steps is denoted by I_s, the reduction parameters at the *n*th step ($n < I_s$), are determined as:

$$\varphi_h(n) = \frac{1 - \varphi_h(0)}{I_s} \cdot n + \varphi_h(0) \tag{28}$$

$$\varphi_j(n) = \frac{1 - \varphi_j(0)}{I_s} \cdot n + \varphi_j(0)$$
$$\varphi_r(n) = \frac{1 - \varphi_r(0)}{I_s} \cdot n + \varphi_r(0)$$

where the values of $\varphi(0)$ need to be determined in model calibration.

Receiving Car Following (RCF)

Once a subject vehicle finishes a lane change maneuver, the new follower in the destination lane will apply RCF mode. The RCF applies the same rules as the ACF and Equations 27 and 28 are used to determine the behavior parameters.

Before Lane Change Car Following (BCF)

If a subject driver cannot merge into the current gap, she or he will perform the BCF to either align the vehicle beside a different gap or wait for the current gap to grow larger. BCF is defined separately for on-ramp MLC, off-ramp MLC, ALC, and DLC.

BCF for On-ramp MLC

For on-ramp mandatory lane changes, if either the forward or the backward gap is insufficient due to the safety constraints (i.e., the gap does not satisfy Equation 19), the vehicle will adopt one of the two BCF modes: synchronizing or gap skipping.

On-ramp BCF synchronizing: If the forward gap is rejected and the backward gap is accepted, the driver will start to synchronize speed with the leader in the target lane. In this case, the driver will consider both the leaders in the target lane and in the current lane in car following. The synchronizing acceleration is determined as:

$$a_{sync} = \min(a_c, \max(a_s, a_{sm})) \tag{29}$$

where,

a_c: acceleration with respect to the current leader to keep a comfortable distance
a_s: acceleration with respect to the leader in the target lane
a_{sm}: acceleration required to maintain a minimum coasting speed while synchronizing with the gap in the target lane

Both a_c and a_s are calculated based on the basic car following model with shorter jam gap and reaction time and headway (see Equation 27); and a_{sm} is determined based on a minimum synchronizing coasting speed V_{sc}. If $v \leq V_{sc}$, $a_{sm} = 0$; otherwise, $a_{sm} = \min\left(b_f, (V_{sc} - v)/\Delta t\right)$.

On the other hand, the subject driver will synchronize the speed with the target follower instead of the target leader if the following conditions are met: 1) the backward gap is rejected; 2) the target follower is yielding; and 3) the distance to the end of the acceleration lane is larger than a certain threshold (i.e., $d > d_{sync}$). The first two conditions mean that the target follower is slowing down to create a gap for the subject driver. The last condition implies that the driver can continue driving in the acceleration lane until the backward gap becomes sufficiently large. In this case, the synchronizing acceleration is given by the following equation:

$$a_{sync} = \min\left(a_{safe}, a_{follow}\right) \tag{30}$$

where,

a_{safe}: minimum acceleration required to avoid collision with the preceding vehicle
a_{follow}: acceleration to synchronize the speed with the target follower, $a_{follow} = (v_f + \mu - v)/\Delta t$, where μ is a small speed increment that allows the subject driver to travel slightly faster than the target follower

On-ramp BCF gap skipping: If the backward gap is rejected and the conditions for speed synchronizing are not met, the driver will slow down with a comfortable acceleration, $a_c = \phi_c b_f$, to skip the current gap and seek the next gap behind. Here ϕ_c is the comfortability factor ($0 < \phi_c \leq 1$). Note that the comfortable acceleration has a negative value. The minimum speed during this process cannot be lower than V_{sk}. The gap skipping acceleration is defined as:

$$a_{gs} = \max(a_c, a_{sm}) \tag{31}$$

BCF for Off-ramp ALC and MLC

If a subject driver is in Zone 1 (see Figure 4), she or he will make an ALC maneuver towards the right lane. In this case, the driver first examines the current gap in the target lane. If the current gap is too small to merge, the driver will accelerate to align the vehicle with the next gap in front. The acceleration is given by the following equation:

$$a_{ALC} = \min\left(a_{safe}, a_{comfort}, a_{target}\right) \tag{32}$$

where,

$a_{comfort}$: comfortable acceleration the driver will use to speed up
a_{target}: acceleration required to reach the average speed in the target lane, $a_{target} = (v_{target} - v)/\Delta t$, where v_{target} is the average speed of five downstream vehicles in the target lane

With the constraint of a_{target}, the subject driver will not drive much faster than the traffic in the target lane. Otherwise, the driver cannot safely merge into the target lane.

When the subject driver enters Zone 2, she or he begins to have MLC motivation. Because the driver still has room to maneuver in this zone, she will only consider downstream gaps if the current gap is insufficient. In this regard, the BCF behavior of the driver is the same as the behavior depicted by Equation 32. The only exception occurs when the target follower yields to the subject driver. In this case, the driver will synchronize the speed with the target follower. The acceleration is then determined by Equation 30.

When the subject driver is in Zone 3, she/he must force into the target lane to avoid missing the exit. In this case, the driver will synchronize the speed with the target follower (i.e., Equation 30) and force the target follower to yield (the yielding behavior is described in the next subsection). In the weaving segment, however, such a BCF behavior may cause a "deadlock" phenomenon if the subject driver wants to merge into the right lane but the target leader in the right lane is making an on-ramp MLC maneuver at the same time. Under such circumstance, neither of them can create a gap that satisfies the gap acceptance criterion. The target leader is synchronizing the speed with the subject vehicle, hoping that the subject driver would yield to increase the backward gap. The subject driver, instead of yielding to the target leader, is synchronizing the speed with the target follower. In the meantime, the target follower is slowing down to avoid collision with the target leader. In this case, the three vehicles will decelerate to a very close and slow speed. The gaps among them do not grow larger. To avoid this unrealistic behavior, we use gap skipping mode (Equation 31) to model the subject driver's acceleration. It will allow the subject driver to first overtake the target leader and then resume to the speed synchronizing behavior. Because of the gap skipping, the target leader in the right lane can merge into the freeway as well.

BCF for Other ALC Types

In our LC algorithm, the HOV drivers will take ALC maneuvers toward the HOV lane. The drivers in the rightmost freeway lane also perform ALC to the left lane to avoid potential conflict with the on-ramp merging vehicles. In some simulation analyses, the HOV rule might be activated in the middle of the simulation period. Before the activation of the HOV rule, some non-HOV vehicles may have entered the HOV lane. These vehicles will also make ALC maneuvers to exit the HOV lane. The acceleration of these vehicles is determined by Equation 31, which was adopted previously to model off-ramp drivers' ALC behaviors.

BCF for DLC

In the DLC maneuvers, a subject driver will not actively search for gaps in the upstream or downstream direction. If the current gap is not acceptable, the driver will resume to the normal car following mode.

Yielding Car Following (YCF)

At each simulation step, the simulation algorithm will check the driving mode of a subject driver's target leader in both the left and right lane. If the target leader has an intent for making a MLC lane change to the current lane, the algorithm will decide if YCF should be applied to the subject driver based on a cooperative factor ζ that ranges from 0 and 1. The factor is a random value generated for each driver at the beginning of the simulation, from a normal distribution with user specified mean $\bar{\zeta}$ and variance $\bar{\sigma}$. Before making the driver switch to YCF, the system generates a random number evenly distributed between 0 and 1. Any value less than the cooperative factor ζ will require the driver to apply YCF. Note that drivers only yield to MLC. In addition, the subject driver will abort the yielding behavior if the following conditions are met:

1) The current speed is less than a minimum threshold;
2) The yielding car following mode has been active longer than a certain period for the same lane changer;
3) The current spacing is negative (the front bumper of the subject vehicle has already passed the rear bumper of the potential lane changer).

In YCF mode, the subject driver will adopt a smaller jam gap, reaction time, and headway as indicated in Equation 27. The driver will take the lane changer as the leading vehicle. The car following model depicted by Equation 9 is used to update the driver's speed and position.

PROPOSED ACC AND CACC VEHICLE BEHAVIOR MODEL

We adopted the ACC and CACC car following models developed in (Milanes and Shladover, 2014). CACC equipped vehicles exhibit significantly different car following behavior from manual drivers and can form strings that allow them to follow the preceding vehicles with short gaps. The modelling framework of CACC vehicles is highlighted in Figure 5. Drivers of CACC equipped vehicles can also exit their closely coupled string and switch off CACC to make lane changes or exit the freeway. The LC behaviors of the C/ACC drivers are depicted by the C/ACC vehicle behavior model. Details of driving behavior for CACC equipped vehicles are discussed in the remainder of this section.

Although the CACC system implementation relies on information received from the leading vehicle in the CACC string as well as from the immediately preceding vehicle, the empirical models used in the simulation provide a simplified description of the closed-loop vehicle-following dynamics that are achieved relative to the immediately preceding vehicle.

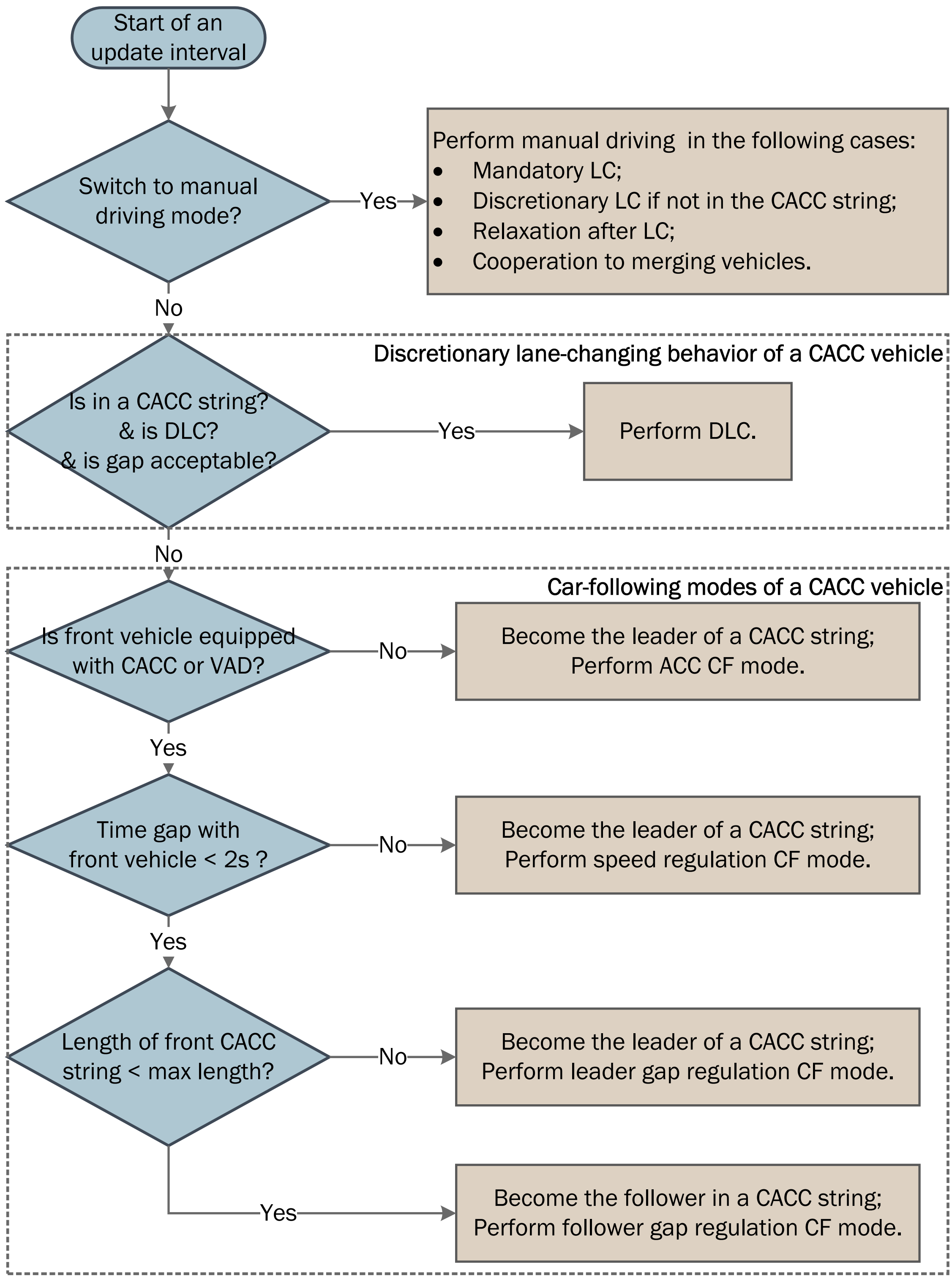


Figure 5 Car following and lane changing logic for CACC vehicles.

Car Following Behaviors of CACC Vehicles

As shown in Figure 5, CACC equipped vehicles can apply both ACC and CACC car following modes. We adopted the CACC car following models developed in (Milanes and Shladover, 2014). These simplified models were implemented because of the need for computational efficiency when simulating many CACC equipped vehicles in a complex environment.

The controllers used for automating a CACC vehicle's car following behavior are represented as follows. If a CACC equipped vehicle is following a vehicle not equipped with CACC or VAD, the CACC controller can only use the ACC car following mode. The ACC controller determines the car following rule according to the clearance distance between the subject vehicle and the preceding vehicle. If the distance is larger than a maximum threshold (e.g., 120 meters), then the preceding vehicle is beyond the on-board sensors' detection range so the ACC controller will apply the speed regulation mode:

$$a_{sv} = k_1(v_f - v_{sv}) \tag{33}$$

where,

a_{sv}: acceleration recommended by the ACC controller to the subject vehicle (m/s^2)
k_1: gain in the speed difference between the free flow speed and the subject vehicle's current speed ($k_1 = 0.4\ s^{-1}$ in this study)
v_f: free flow speed (m/s)
v_{sv}: current speed of the subject vehicle (m/s).

If the clearance distance is smaller than the minimum threshold (e.g., 100 meters), the ACC controller will turn on gap regulation mode and help the subject vehicle follow the motions of the preceding vehicle. The gap regulation mode is described by the following:

$$a_{sv} = k_2(d - t_{hw}v_{sv} - L) + k_3(v_l - v_{sv}) \tag{34}$$

where,
k_2: gain on position difference between the preceding vehicle and the subject vehicle ($k_2 = 0.23\ \ s^{-2}$ in this study)
k_3: gain on speed difference between the preceding vehicle and the subject vehicle ($k_3 = 0.07\ \ s^{-1}$ in this study)
d: distance between the subject vehicle's front bumper and the preceding vehicle's front bumper (m)
t_{hw}: desired time gap of the ACC controller (s), which is drawn from the distribution of time gaps selected by the ACC drivers in the field test described in Nowakowski et al. (2010): 31.1% of their vehicle following time at 2.2 s, 18.5% at 1.6 s; and 50.4% at 1.1s
L: length of the preceding vehicle (m)
v_l: current speed of the preceding vehicle (m/s).

If the clearance distance is between the maximum and minimum thresholds, the ACC controller will use the control rule implemented in the previous time step. This introduces a hysteresis in the control loop such that the ACC controller can perform a smooth transfer between the speed regulation mode and gap regulation mode.

If a subject vehicle is a CACC string leader and the preceding vehicle is a CACC vehicle in another CACC string, the subject vehicle may implement either of the following two modes. When the time gap between the subject vehicle and the preceding vehicle is more than 2 seconds, the subject vehicle will switch to the speed regulation mode, which is represented by Equation 33. If the time gap is less than 2 seconds and the preceding CACC string is operating at the maximum allowable string length, the subject vehicle will use the string leader gap regulation mode:

$$v_{sv}(t) = v_{sv}(t - \Delta t) + k_p e_k(t) + k_d \dot{e}_k(t) \quad (35)$$

$$a_{sv}(t) = (v_{sv}(t) - v_{sv}(t - \Delta t))/\Delta t \quad (36)$$

where
Δt: time step for each update (s)
k_p and k_d: gains for adjusting the time gap between the subject vehicle and preceding vehicle ($k_p = 0.45\ s^{-1}$ and $k_d = 0.0125$)
e_k: time gap error, which is described by the following:

$$e_k(t) = d(t - \Delta t) - t_1 v_{sv}(t - \Delta t) - L \quad (37)$$

$$\dot{e}_k(t) = v_l(t - \Delta t) - v_{sv}(t - \Delta t) - t_1 a_{sv}(t - \Delta t) \quad (38)$$

where t_1 is the constant time gap between the last vehicle of the preceding CACC string and the subject vehicle ($t_1 = 1.5\ s$ is the time gap that has been chosen for use in this study after evaluations of the effects of alternative values).

If the subject vehicle is a CACC string follower, there are two possible car-following modes. It will implement the in-string follower gap regulation mode if the time gap from the preceding vehicle is less than 1.5 seconds. The in-string follower gap regulation mode uses the algorithm represented by Equations 35 through 38, with a minor adjustment of replacing t_1 with t_2, which is the intra-string constant time gap. The values of t_2 are selected randomly in the simulation, drawn from the distribution of time gaps that were chosen by the CACC drivers in the field test described in Nowakowski et al (2010). In particular, the drivers in that test chose a time gap of 0.6 s for 57% of the time they were car following, 0.7 s for 24% of the time, 0.9 s for 7% and 1.1 s for 12%. For time gaps larger than 2 seconds, the subject vehicle will turn on the speed regulation mode (i.e., Equation 33). When the time gap is between 1.5 seconds and 2 seconds, the subject vehicle will use the hysteresis control rule, which applies the car-following mode implemented in the previous time step.

The CAMP forward collision warning algorithm (Kiefer et al., 2003) is included in the C/ACC car following modes to check if the gap between the subject vehicle and the preceding vehicle is sufficient for safe car following. The CAMP algorithm first determines a required deceleration for the subject vehicle:

$$d_{REQ} = -0.165 + 0.685 \cdot d_l + 0.080 \cdot \zeta - 0.00889 \cdot (v_{sv} - v_l) \tag{39}$$

$$\zeta = \begin{cases} 1 & v_l > 0 \\ 0 & otherwise \end{cases} \tag{40}$$

where,

d_{REQ}: deceleration required to avoid a rear-end collision (in g)
d_l: deceleration of the preceding vehicle (in g)

The required deceleration from the above empirical function represents the comfortable deceleration the subject driver may take to avoid a collision with the preceding vehicle given the current relative speed and deceleration of the preceding vehicle. If d_{REQ} is larger than zero, it means that the subject vehicle does not need to brake and the current gap is sufficient. If d_{REQ} is less than zero and the preceding vehicle stops prior to the subject vehicle, the required gap is:

$$g_{REQ} = max(0, \frac{v_{sv}^2}{-2 \cdot d_{REQ}} - \frac{v_l^2}{-2 \cdot d_l}) \tag{41}$$

If the preceding vehicle does not stop prior to the subject vehicle, the required gap is:

$$g_{REQ} = max(0, \frac{(v_{sv} - v_l)^2}{-2 \cdot (d_{REQ} - d_l)}) \tag{42}$$

When the current gap is smaller than the g_{REQ}, it implies that a crash will happen if both the subject vehicle and the preceding vehicle keep their current acceleration (i.e., d_{REQ} and d_l, respectively) for the next few seconds. In this case, the subject vehicle will switch to the manual driving mode to avoid the crash.

Lane Changing Behaviors of CACC Vehicles

The driver of a CACC equipped vehicle is expected to behave slightly differently from other drivers when making decisions about lane changing maneuvers. The specific lane changing behaviors of CACC drivers are detailed in the following:

- When in the CACC managed lane, a CACC driver will have small motivation to make DLC maneuvers to exit the managed lane: $\gamma_{exit\ ML} < \gamma_{d\{l,r\}}$.
- When a CACC driver is not in the CACC managed lane, she or he will have ALC motivation towards the managed lane if the speed in the managed lane is faster than that of the current lane. The ALC motivation is computed by

Equation 18, in which the speed difference between the target lane and the current lane is the only incentive for a lane change.

- Drivers of CACC equipped vehicles in the rightmost freeway lane would not have strong incentives to move toward the left lane when incoming on-ramp vehicles merge into the freeway because they are aware that the gap between them and other vehicles in the CACC string is too small to facilitate on-ramp traffic merging. Instead, they expect on-ramp traffic to merge into the larger gaps between two adjacent CACC strings. In this case, drivers of CACC equipped vehicles will stay in the CACC string and not perform ALC maneuvers to the left lane as the regular human drivers do.
- When a CACC driver merges into the managed lane, she or he is only allowed to merge into the gap between two CACC strings. Such a merging behavior will create small disturbances to the CACC strings in the managed lane.
- Drivers of CACC equipped vehicles will not switch to the BCF mode when making discretionary lane changes, they will instead remain in the CACC string until an acceptable gap is found in the target lane.
- CACC equipped vehicles do not perform receiving car following (RCF) mode when the CACC or ACC controller is on. The RCF describes the behavior that a human driver temporarily accepts a short time gap after another driver merges in front and then gradually increases the time gap to the desired level. Instead, the CACC controller will treat such a maneuver as a “cut-in” (to be described in the next section) and execute the gap regulation rule.

CACC String Operations

In this study, we chose a maximum string length of 10 vehicles based on our preliminary analysis of multiple candidate string lengths. Shorter string lengths would result in more CACC strings, which can lead to lower freeway capacity because inter-string gaps are larger than the gaps between consecutive vehicles within the string. On the other hand, long CACC strings would lead to less versatility since they impede lane changes and make merging more difficult for other vehicles.

Two consecutive CACC vehicle strings should keep a consistent time gap, in order to maintain stable traffic flow and facilitate lane change maneuvers. As Fernandes and Nunes (2015) suggested, there are two ways of implementing a constant time gap between two strings. One option is to maintain the constant time gap between the two leaders of the strings, which always leaves behind enough space for a full-size string (i.e., a string with maximum length) behind a string leader regardless of whether there are vehicles following the leader. However, this method reduces the freeway capacity when the strings do not operate at the maximum length. Another option is to regulate the time gap between the last vehicle of the preceding string and the first vehicle of the following string. This allows the dynamic assignment of string leaders to accommodate strings that are shorter than the maximum string length, and can prevent the negative impact on freeway capacity. Therefore, the second option is adopted with a 1.5 second constant time gap between strings based on our preliminary analysis of several alternative time gap settings.

The joining of two strings is modeled by the following process:

- Step 1: select a subject vehicle as the leader of a CACC string.
- Step 2: the subject vehicle is registered as the last follower of the preceding string if the time gap from the last vehicle of the preceding string is less than 2 seconds and the length of the preceding string is less than the maximum permitted string length.
- Step 3: vehicles in the following string update their position IDs (i.e., ID that reflects the location of a vehicle in the CACC string). If the new string would be longer than the maximum permitted length, the string split process will be executed (the split process is described later) by the vehicles behind the one that reaches the maximum allowable ID.
- Step 4: the subject vehicle switches to the speed regulation mode until the time gap to the preceding vehicle is less than 1.5 seconds. Afterwards, it switches to the CACC string follower gap regulation mode.
- Step 5: other vehicles in the following string continue to update their speeds using the CACC follower gap regulation mode.

A string needs to be split up if it is longer than the maximum allowed string length. The string split process is modeled as follows:

- Step 1: each vehicle in a string updates its position ID.
- Step 2: if a vehicle is the first vehicle in the string with a position ID larger than the maximum string length, that vehicle becomes the subject vehicle of the string split process.
- Step 3: the subject vehicle becomes the leader of a new string and switches from the CACC follower gap regulation mode to the CACC leader gap regulation mode.
- Step 4: each vehicle in the new string updates the position ID and starts following the new leader using the CACC follower gap regulation mode.

This study also modelled cut-in and cut-out maneuvers by using the combination of the lane changing and the string join and split algorithms. In the cut-out process, a subject vehicle first applies the lane changing mode to exit the CACC string. Afterwards, the string joining process is implemented for the remaining vehicles in the string. The cut-in process involves two cases. If the cut-in vehicle is not a CACC equipped vehicle, the string split process will be executed when the vehicle arrives in the lane of the CACC string. The vehicles in the first half of the original string will continue moving downstream without changing their driving modes. The first vehicle in the second half of the original string will become the string leader of a new string. The vehicle will execute string leader gap regulation mode to follow the cut-in vehicle. If the cut-in vehicle is a CACC equipped vehicle, the cut-in process executes the following steps:

- Step 1: after the cut-in vehicle completes the lane changing maneuver, it becomes the subject vehicle of the process.
- Step 2: the subject vehicle implements the after lane changing car following (ACF) mode, which is a manual driving mode. In the meantime, the string split process is executed. The vehicles in front of the subject vehicle form the preceding string, and the rest of the vehicles form the following string, with the subject vehicle as the leader of the following string.
- Step 3: After the ACF mode concludes, the subject vehicle turns on the CACC controller and initiates the string joining process. The subject vehicle could use CACC speed regulation mode or CACC string follower gap regulation mode, depending on the time gap between the subject vehicle and its preceding vehicle.
- Step 4: After the string joining process completes, the string length may become longer than the maximum string length. In such instance, the string split process is performed.

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