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Hadronic Instabilities in Very Intense Magnetic Fields

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Composite hadronic states exhibit interesting properties in the presence of very intense magnetic fields, such as those conjectured to exist in the vicinity of certain astrophysical objects. We discuss three scenarios. (i) The presence of vector particles with anomalous magnetic moment couplings to scalar particles, induces an instability of the vacuum. (ii) A delicate interplay between the anomalous magnetic moments of the proton and neutron makes, in magnetic fields $B \geq 2 \times 10^{14}$ T, the neutron stable and for fields $B \geq 5 \times 10^{14}$ T the proton becomes unstable to a decay into a neutron via β emission. (iii) In the unbroken chiral σ model magnetic fields would be screened out as in a superconductor. It is the explicit breaking of chiral invariance that restores standard electrodynamics. Astrophysical consequences of all these phenomena are discussed.

1. Introduction

Very intense magnetic fields have been conjectured to exist in connection with several astrophysical phenomena. Supernova may contain fields up to 10^{10} T [1]; a recent model for extrgalactic gamma ray bursts [2] involves fields of 10^{13} T and fields of strengths greater than 10^{14} T are associated with cosmic strings [3]. The latter are due to [4] currents $I = 10^{20}$ A in strings whose thickness is $1/M_W$. The presence of fields with electromagnetic couplings of non-electromagnetic origin induces instabilities in such large fields. The electro-weak theory itself, due to anomalous magnetic moments of the charged gauge fields, produces vacuum instabilities in the presence of fields of 10^{20} T [5]. In this report we wish to summarize effects due to ordinary hadronic physics which will manifest themselves in much lower fields, $B = 10^{14}$ T. We will discuss three such phenomena. (i) The presence of magnetic dipole transitions between hadronic resonances induces a vacuum instability [6]; (ii) in fields of such magnitude, due to a subtle interplay of the anomalous magnetic moments of the proton and neutron, the former becomes heavier than the latter and decays into it by positron emission [7]; (iii) chiral symmetry breaking screens magnetic fields larger than 10^{14} T [8]. We shall discuss each of these topics in turn and some possible consequences at the end.

2. Instabilities due to Magnetic Dipole Transitions

Among the low lying hadronic states are scalar and vector mesons with opposite parities and charge conjugations coupled to each other by magnetic dipole transitions. We will point out that this will lead to an instability in a large magnetic field. The simplest model to exhibit this effect contains a neutral scalar field $s(x)$ with mass μ and a vector field $v^\mu(x)$ with mass M and with parity and charge conjugation opposite to that of s. An magnetic dipole transition couples the electromagnetic field to s and v. The Lagrangian for such a model is

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu s \partial^\mu s - \frac{1}{2}\mu^2 s^2 - \frac{1}{4}(\partial_\mu v_\nu - \partial_\nu v_\mu) \\ &\quad (\partial^\mu v^\nu - \partial^\nu v^\mu) + \frac{1}{2}M^2 v_\mu v^\mu \\ &\quad -\frac{1}{2}\frac{e}{\Lambda} s(\partial_\mu v_\nu - \partial_\nu v_\mu)F^{\mu\nu}\,; \qquad (1)\end{aligned}$$

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$F^{\mu\nu}$ is the electromagnetic field strength tensor and the last term is the aforementioned magnetic dipole coupling. Λ has the dimensions of a mass and its magnitude provides the strength of this coupling. For $F^{\mu\nu}$ constant we can easily find the eigenmodes of this Lagrangian. The two transverse states of the vector meson remain uncoupled while the longitudinal state mixes with the scalar particle yielding two modes whose dispersion relation is obtained from a solution of

$$(p^2-\mu^2)(p^2-M^2)+\frac{e^2}{\Lambda^2}p_\lambda p_\eta F^{\lambda\nu}F^{\eta}{}_{\nu}=0\,. \quad (2)$$

For a constant magnetic field pointing in the z direction Eq. (2) takes the form

$$(p^2-\mu^2)(p^2-M^2)-\frac{e^2}{\Lambda^2}\mathbf{p}_\perp{}^2H^2=0\,; \quad (3)$$

$\mathbf{p}_\perp{}^2=p_x{}^2+p_y{}^2$. The energies satisfy

$$p_0^2=\mathbf{p}^2\pm\left[\left(\frac{M^2-\mu^2}{2}\right)^2+\frac{e^2}{\Lambda^2}\mathbf{p}_\perp{}^2H^2\right]^{\frac{1}{2}}\,. \quad (4)$$

For sufficiently large magnetic fields the lower solution becomes negative indicating an instability. This will occur whenever

$$H\geq H_c=\frac{(M+\mu)\Lambda}{|e|}\,. \quad (5)$$

The instability will manifest itself in that fields $H\geq H_c$ will create pairs of the s and v particles. Γ, the decay rate per unit time per unit volume of a "vacuum" with such a large field may be calculated by standard methods [9]

$$\Gamma = Re\int\frac{d^4p}{(2\pi)^4}\ln\left[(p^2-\mu^2)(p^2-M^2)-\frac{e^2}{\Lambda^2}\mathbf{p}_\perp{}^2H^2\right]\,. \quad (6)$$

It is only for $H\geq H_c$ that the above integral develops a real part

$$\Gamma = \frac{1}{96\pi}\frac{eH}{\Lambda}\left\{\left(\frac{eH}{\Lambda}\right)^2-2\left(M^2+\mu^2\right)+\left[\frac{\left(M^2-\mu^2\right)\Lambda}{eH}\right]^2\right\}^{\frac{3}{2}}\,. \quad (7)$$

The masses involve are of the order of a few hundred MeV; we likewise expect Λ to be of the same order and thus the critical fields will be in the range of 10^{15} T. Zeeman splittings in such fields will be of the order of tens of MeV's and we expect the effective Lagrangian of Eq. (1) to be valid.

3. Proton β Decay in Large Fields

In this section we shall show how a delicate interplay between the anomalous magnetic moments of the proton and neutron makes, in magnetic fields $B\geq 2\times10^{14}$ T, the neutron stable and for fields $B\geq 5\times10^{14}$ T the proton becomes unstable to a decay into a neutron via β emission.

The spectrum of Dirac particles with anomalous magnetic moments, placed in uniform external magnetic fields can be obtained in a straightforward manner; the previous statement is true as long as the energy shifts due to the anomalous part of the moments are smaller than the masses of the particles. For the field strengths we shall study, this will be true for the proton and for the neutron. This treatment has to be modified for the electron, as even in these fields, the shift due to the QED correction to the moment of the electron would be larger than the mass itself.

Although a fully relativistic treatment may be given [7], we shall, for the proton and neutron quote the non-relativistic results. For the field **B** along the z direction the energy of a proton in the lowest Landau level and with spin along the magnetic field is

$$E_p(p_z)=\tilde{M}_p+\frac{p_z{}^2}{2\tilde{M}_p}\,, \quad (8)$$

where the effective mass $\tilde{M}$ is

$$\tilde{M}_p=M_p-\frac{e}{2M_p}\left(\frac{g_p}{2}-1\right)B\,; \quad (9)$$

M_p is the proton's mass and $g_p=5.58$ is the proton's Landé g factor.

For the neutron with momentum **p** and spin opposing the magnetic field, the non-relativistic energy is

$$E(\mathbf{p})=M_n+\frac{e}{2M_n}\left(\frac{g_n}{2}\right)B+\frac{\mathbf{p}^2}{2M_n}\,, \quad (10)$$

where this time M_n is the neutron's mass and $g_n=-3.82$.

The calculation of the energy of an electron in a strong magnetic field is more subtle. For fields $B \geq M_e^2/e$ the point formalism breaks down and we have to solve to one loop QED in the external field. This problem was treated by Schwinger [10]. The energy of an electron with $p_z = 0$, spin up and in the lowest Landau level is

$$E_{m,p_z=0} = M_e \left[1 + \frac{\alpha}{2\pi} \ln\left(\frac{2eB}{M_e{}^2}\right)\right] ; \tag{11}$$

for field strengths of subsequent interest this correction is negligible.

From Eq. (9) and Eq. (10) we note that the neutron becomes stable against β-decay when the following inequality is satisfied

$$-\frac{e}{2M_n}\left(\frac{g_n}{2}\right)B - \frac{e}{2M_p}\left(\frac{g_p}{2}-1\right)B \geq M_n - M_p - M_e , \tag{12}$$

or $B \geq 2 \times 10^{14}$ T. On the other hand the proton becomes unstable for decay into a neutron and a positron whenever

$$\frac{e}{2M_n}\left(\frac{g_n}{2}\right)B - \frac{e}{2M_p}\left(\frac{g_p}{2}-1\right)B \sim 0.12\mu_N B \geq M_n + M_e - M_p , \tag{13}$$

or for $B \geq 5 \times 10^{14}$ T.

The positron spectrum for the decay of the proton in such a field is

$$\frac{d\Gamma}{dp_{z,e}} = \frac{4}{3}\frac{G_F^2 M_p}{(2\pi)^6}\frac{E_e + p_{z,e}}{E_e}(\Delta - E_e)^3 ; \tag{14}$$

where $\Delta = 0.12\mu_N B - M_n + M_p$. For $\Delta \gg M_e$ the total rate is easily obtained

$$\Gamma = \frac{2}{3}\frac{G_F^2 M_p}{(2\pi)^6}\Delta^4 . \tag{15}$$

The lifetime is $\tau \sim 1.5 \times 10^2 (10^{15}\mathrm{T}/B)^4$ s.

4. Screening of Fields by Chiral Symmetry Breaking

The breaking of the strong interaction chiral symmetry is well described by the σ model. This theory accounts for the interactions of the Goldstone (more precisely, the pseudo-Goldstone) modes. We shall show that in very intense magnetic fields, $B > 1.5 \times 10^{14}$ T, the breaking of the $SU(2) \times SU(2)$ symmetry arranges itself so that instead of the neutral σ field acquiring a vacuum expectation value it is the charged π field that does and at the same time the magnetic field is screened.

The Hamiltonian density for this problem, including the coupling of the electromagnetic potential to an external current $\mathbf{j}$ is

$$\begin{aligned} H &= \frac{1}{2}\nabla\sigma\cdot\nabla\sigma + \frac{1}{2}\nabla\pi_0\cdot\nabla\pi_0 \\ &+ (\nabla + e\mathbf{A})\pi^\dagger\cdot(\nabla - e\mathbf{A})\pi \\ &+ g(\sigma^2 + \pi\cdot\pi - f_\pi^2)^2 \\ &+ m_\pi^2(f_\pi - \sigma) + \frac{1}{2}(\nabla\times A)^2 - \mathbf{j}\cdot\mathbf{A} ; \end{aligned} \tag{16}$$

we have used cylindrical coordinates with ρ the two dimensional vector normal to the z direction. In the limit of large g the radial degree of freedom is frozen out and the chiral fields may be parameterized by angular variables.

$$\begin{aligned} \sigma &= f_\pi \cos\chi , \\ \pi_0 &= f_\pi \sin\chi\cos\theta , \\ \pi_x &= f_\pi \sin\chi\sin\theta\cos\phi , \\ \pi_y &= f_\pi \sin\chi\sin\theta\sin\phi . \end{aligned} \tag{17}$$

In terms of which the Hamiltonian density becomes

$$\begin{aligned} H &= \frac{f_\pi^2}{2}(\nabla\chi)^2 + \frac{f_\pi^2}{2}\sin^2\chi(\nabla\theta)^2 \\ &\quad \frac{f_\pi^2}{2}\sin^2\chi\sin^2\theta(\nabla\phi - e\mathbf{A})^2 \\ &+ m_\pi^2 f_\pi^2(1-\cos\chi) \\ &+ \frac{1}{2}(\nabla\times A)^2 - \mathbf{j}\cdot\mathbf{A} . \end{aligned} \tag{18}$$

The angular field ϕ can be eliminated by a gauge transformation.

We first look at he case where the external current is due to a thin wire along the z direction,

$$\mathbf{j} = I\delta(\rho)\mathbf{z} ; \tag{19}$$

An approximate solution, valid for $I/m_\pi >> 1$, is

$$\chi = \begin{cases} \frac{\pi}{2} & \text{for } \rho < \rho_0 \\ 0 & \text{for } \rho > \rho_0 , \end{cases}$$

$$\theta = \frac{\pi}{2},$$

$$A = \begin{cases} -\frac{I}{2\pi}\left[K_0(ef_\pi\rho) - I_0(ef_\pi\rho) \right. \\ \left. K_0(ef_\pi\rho_0)/I_0(ef_\pi\rho_0)\right] & \text{for } \rho < \rho_0 \quad (20) \\ \frac{I}{2\pi}\ln\frac{\rho}{\rho_0} & \text{for } \rho > \rho_0 ; \end{cases}$$

The parameter ρ_0 is determined by minimizing the energy. Doing this we obtain

$$\rho_0 = \frac{I}{2\sqrt{2}m_\pi f_\pi} . \tag{21}$$

We find that the magnetic fields are screened out to a distance ρ_0 from the wire; beyond this distance ordinary electrodynamics resumes.

Eq. (21) has a very straightforward explanation. It results in a competition of the magnetic energy density $\frac{1}{2}B^2$ and the energy density of the pion mass term $m_\pi^2 f_\pi^2(1-\cos\chi)$. The magnetic field due to the current I is $B = I/2\pi\rho$ and the transition occurs when $B = B_c$, with $B_c = \sqrt{2}m_\pi f_\pi \sim 1.5 \times 10^{14}$ T. Thus, the chiral fields will adjust themselves to screen out fields larger than B_c for any current configuration.

5. Conclusions

We have presented several mechanisms that may inhibit the existence of fields beyond 10^{14} T and that may induce unusual hadronic phenomena as proton β decay into a neutron. Which of these processes will dominate is, for the moment, an open question. As discussed in the Introduction, fields of such magnitude can only be realized in connection with conjectured astrophysical phenomena. Superconducting cosmic strings are supposed to carry currents of 10^{20} A; from Eq. (21) we note that the magnetic field will be screened out to distances of 40 cm. Should this screening be incomplete, as perhaps due to penetration by magnetic vortices, then proton β decay could occur. There are suggestions of anomalous astrophysical positron emission [11].

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