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Full range of predictions for $B$ physics from isosinglet down quark mixing

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We extend the range of predictions of the isosinglet (or vector) down quark model to the fully allowed physical ranges, and also update this with the effect of new physics constraints. We constrain the present allowed ranges of $\sin(2\beta)$ and $\sin(2\alpha)$, $\gamma$, $x_s$, and $\Delta B$. In models allowing mixing to a new isosinglet down quark (as in $E_6$) flavor changing neutral currents are induced that allow a $Z^0$ mediated contribution to $B \to \bar{B}$ mixing and which bring in new phases. In $(p, \eta)$, $(x_s, \sin(\gamma))$, and $(x_s, \Delta B)$ plots for the extra isosinglet down quark model which are herein extended to the full physical range, we find new allowed regions that will require experiments on $\sin(\gamma)$ and/or $x_s$ to verify or to rule out an extra down quark contribution.

I. INTRODUCTION

The ‘‘new physics’’ class of models we use are those with extra iso-singlet down quarks, where we take only one new down quark as mixing significantly. An example is $E_6$, where there are two down quarks for each generation with only one up quark, and of which we assume only one new iso-singlet down quark mix strongly. This model has shown large possible effects in $B \to \bar{B}$ mixing phases. The approaching $B$ factory experiments will also set limits on the phases of the mixing angles to the new iso-singlet down quark in this model. In previous analyses [1,2], we focused on ranges of variables in which the standard model (SM) results occurred, in the sense of looking for small deviations in setting limits. As emphasized by Wolfenstein [3], we now explore the full range of output in variables $\eta_s$, $\sin(\gamma)$, and the $B_s$ asymmetry to indicate the full possible range of outcomes for these experiments due to new physics models.

A significant number of improved constraints have appeared in the last two years, and most importantly, some of the $R_b$ experiments now give results in agreement with the standard model. Since the mixing to a new down quark can only decrease the diagonal neutral current, these results now give useful limits on the parameters. The other improved experiments are $K^+ \to \pi^+\nu\bar{\nu}$, the new D0 limit on $B \to \mu\mu X$, improved $V_{ub}$ limits, and the CERN $e^+ e^-$ collider LEP lower bounds on $|\Delta m_s|$ or $x_s$. We also now have an exact method of combining the one event Poisson result on $K^+ \to \pi^+\nu\bar{\nu}$ with the Gaussian probability experiments which results in a chi-squared distribution [4].

We also project to a range of results from the $B$ factory experiments. For different $\sin(2\alpha)$ cases, we find extended multiple regions in $(p, \eta)$ that will require experiments on $\sin(\gamma)$ or $x_s$ to decide between, and experiments on both could be required to bound out or to verify the model. We also find a sizeable range for the $B_s \to \bar{B}_s$ mixing asymmetry in the extra down quark model, while in the SM this asymmetry is very small. In setting limits we use the method of a joint $\chi^2$ fit to all constraining experiments.

II. ISO-SINGLET DOWN QUARK MIXING MODEL

Groups such as $E_6$ with extra SU(2)$_L$ singlet down quarks give rise to flavor changing neutral currents (FCNC) through the mixing of four or more down quarks [2.5–8]. We use the $4 \times 4$ down quark mixing matrix $V$ which diagonalizes the initial down quarks $(d_{ijL})$ to the mass eigenstates $(d_{ijL})$ by $d_{ijL} = V_{ijL} d_{ijL}$. The flavor changing neutral currents we have are [7,8] $-U_{us} = V^u_d V_{us}$, $-U_{sb} = V^u_d V_{sb}$, and $-U_{ub} = V^u_d V_{ub}$. These FCNC with tree level $Z^0$ mediated exchange may contribute part of $B_{u+d} \to \bar{B}_{u+d}$ mixing and of $B^0_s \to \bar{B}^0_s$ mixing, and the constraints leave a range of values for the fourth quark’s mixing parameters. $B_{u+d} \to \bar{B}_{u+d}$ mixing may occur by the $b \to d$ quarks in a $\bar{B}_d$ annihilating to a virtual $Z$ through a FCNC with amplitude $U_{db}$, and the virtual $Z$ then creating $\bar{b} \to d$ quarks through another FCNC, again with amplitude $U_{db}$, which then becomes a $B_d$ meson. If these are a large contributor to the $B_d \to \bar{B}_d$ mixing, they introduce three new mixing angles and two new phases over the standard model (SM) into the CP violating $B$ decay asymmetries.

For $B_d \to \bar{B}_d$ mixing with the four down quark induced $b \to d$ coupling, $U_{db}$, we have [9]

$$x_d = \frac{2G_F/3v^2 B_B f^2 m_B \eta_B \tau_B}{U^2_{std-db} + U_{db}^2}$$

where, with $y_i = m_i^2/m_W^2$,

$$U^2_{std-db} = \frac{\alpha i (4 \pi \sin^2 \theta_W) y_s f^2 (V_{id}^s V_{tb})^2}{\Gamma_B \Delta m_{B_d} \Delta m_{B_d}},$$

and $x_d = \Delta m_{B_d} / \Gamma_B \Delta m_{B_d}$.

The $CP$ violating decay asymmetries depend on the combined phases of the $B^0_d \to \bar{B}^0_d$ mixing and the $b$ quark decay
amplitudes into final states of definite CP. Since we have found that Z mediated FCNC processes may contribute significantly to $B_d^-\bar{B}_d^0$ mixing, the phases of $U_{db}$ would be important. Calling the singlet down quark $D_0$, to leading order the mixing matrix elements to $D$ are $V_{td} \approx s_{13} e^{-i\delta_{13}}$, and $V_{ud} \approx s_{14} e^{-i\delta_{14}}$. The complete $4 \times 4$ mixing matrix was given previously [9,12]. The FCNC amplitude $U_{db}$ to leading order in the new angles is

$$U_{db} = (s_{34} - s_{24} s_{23} e^{i\delta_{23}})(s_{34} V_{td}^* + s_{14} e^{-i\delta_{14}} s_{24} e^{-i\delta_{24}} s_{12}),$$

where $V_{id} \approx (s_{12} s_{23} - s_{13} e^{i\delta_{13}})$ and $V_{ub} = s_{13} e^{-i\delta_{13}}$.

III. JOINT CHI-SQUARED ANALYSIS FOR CKM AND FCNC EXPERIMENTS

FCNC experiments put limits on the new mixing angles and constrain the possibility of new physics contributing to $B^0_d - \bar{B}^0_d$ and $B^0_d - \bar{B}^0_d$ mixing. Here we jointly analyze all constraints on the $4 \times 4$ mixing matrix obtained by assuming only one of the SU(2)$_L$ singlet down quarks mixes appreciably [7]. We use the nine experiments for the $3 \times 3$ CKM sub-matrix elements [1], which include: those on the five matrix elements $V_{td}, V_{cd}, V_{us}, V_{ub}, V_{cb}$ of the $u$ and $c$ quark rows; $|\epsilon|$ and $K_L - \mu\mu$ in the neutral $K$ system [13]; $B_d - \bar{B}_d$ mixing ($\lambda_3$); and the new limits on $\Delta m_s$, or $\lambda_3$. For studying FCNC, we have four experiments which include the bound on $B - \mu\mu X$ (which constrains $b \rightarrow d$ and $b \rightarrow s$) for which we have the UA1 and the new D0 [14] results, the new first event in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [4,11,15–17] and new results on $R_b$ in $Z^0 \rightarrow b \bar{b}$ [11,18] (which directly constrains the $V_{cb}$ mixing element). FCNC experiments will bound the three amplitudes $U_{ds}, U_{ub},$ and $U_{bd}$ which contain three new mixing angles and three phases. We use the mass of the top quark as $m_t = 174$ GeV. We use a method for combining the Bayesian Poisson distribution for the average for the one observed event in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [17] with the chi-squared distribution from the other experiments. We take $(n) = 2.7 \times 10^9 |U_{td}|^2$, ignoring the SM contribution, since the observed event is at a rate four times the SM result.

In maximum likelihood correlation plots, we use for axes two output quantities which are dependent on the mixing matrix elements and phases, such as $(\rho, \eta)$, and for each possible bin with given values for these, we search through the nine dimensional angular data set of the $4 \times 4$ down quark mixing angles and phases, finding all sets which give results in the bin, and then put into that bin the minimum $\chi^2$ among them. To present the results, we then draw contours at several $\chi^2$ in this two dimensional plot corresponding to given confidence levels.

IV. CONSTRAINTS ON THE STANDARD MODEL CKM MATRIX AT PRESENT

We first analyze the standard model using the present constraints on the eight Cabibbo-Kobayashi-Maskawa (CKM) related experiments. We use the results for $\sin(2\beta)$.
these asymmetries are not directly related to angles in a triangle in this model. The asymmetries with FCNC contributions included are

$$\sin(2\beta) = A_{B_d}^{(0)} \Phi \Psi K^*_y = \text{Im} \left[ \frac{(U^2_{\text{std}} - \Phi ) + U_{db}^2}{U^2_{\text{std}} + U_{db}^2} \right] \frac{(V^*_c V_5)}{(V^*_c V_5)^*}$$

(5.1)

$$\sin(2\alpha) = -A_{B_d}^{(0)} \pi + \pi^* = -\text{Im} \left[ \frac{(U^2_{\text{std}} - \Phi ) + U_{db}^2}{U^2_{\text{std}} + U_{db}^2} \right] \frac{(V^*_c V_5)}{(V^*_c V_5)^*}$$

(5.2)

with $U_{\text{std}}$ defined in Eq. (2.2).

In the four down quark model, what we mean by 
"sin($\gamma$)" is the result of the experiments which would give this variable in the SM [23]. Here, the four down quark model involves more complicated amplitudes, and sin($\gamma$) is not simply sin($\delta_{33}$):

$$\sin(\gamma) = \text{Im} \left[ \frac{(U^2_{\text{std}} - \Phi ) + U_{bs}^2}{U^2_{\text{std}} + U_{bs}^2} \right] \frac{(V^*_c V_5)}{(V^*_c V_5)^*}.$$  

(5.3)

We note that since sin($\gamma$) is an imaginary part of a complex amplitude, it can have values ranging from $-1$ to $+1$. We now extend the range of the previous analyses to cover the complete range.

In the four down quark model, $\gamma$ is no longer the simple ratio of two CKM matrix elements, but now involves the $Z$-mediated annihilations and exchange amplitudes as well

$$\gamma = 1.35 x_d \frac{U^2_{\text{std}} - \Phi + U_{bs}^2}{U^2_{\text{std}} - \Phi + U_{bs}^2},$$

(5.4)

where

$$U^2_{\text{std}} - \Phi = (a/(4 \pi \sin \theta_{W}^2)) y_4 f_2(y_3) (V^*_c V_5)^2.$$  

(5.5)

The asymmetry $A_{B_s}$ in $B_s$ mixing in the standard model with the leading decay process of $b \rightarrow c \bar{c} s$ has no significant phase from the decay or from the mixing which is proportional to $V_{ts}^2$. The near vanishing of this asymmetry is a test of the standard model [6], and a non-zero value can result from a "new physics" model. With the FCNC, the result is

$$A_{B_s} = \text{Im} \left[ \frac{(U^2_{\text{std}} - \Phi + U_{bs}^2)}{U^2_{\text{std}} - \Phi + U_{bs}^2} \right] \frac{(V^*_c V_5)}{(V^*_c V_5)^*}. $$

(5.6)

Again, since this is an imaginary part of a complex amplitude, we extend our studies to the full range including negative values for this. Since it concerns the $B_s$ mixing, we plot it against $\gamma$, which involves the magnitude of the amplitude used in $A_{B_s}$.

In the four-down-quark model with the unitarity quadrangle, what we plot for the ($\rho, \eta$) plot is the scaled vertex of the matrix element $V^*_c V_5$:

$$\rho + i \eta = V^*_c V_5 / |V_{cb} V_5|.$$  

(5.7)
Since $\eta$ is an imaginary part, it can have negative as well as positive values. While the negative values were not included before in comparing to the standard model, they are now included to show the full range of predictions of the four-down-quark model.

We then make maximum likelihood plots which include $(\sin(2\alpha), \sin(2\beta))$, $(\rho, \eta)$, $(x, \sin \gamma)$, and $(x, A_B)$.

The corresponding plots for the four down quark model are shown for present data and for projected $B$ factory data in the following figures. In the figures, we show $\chi^2$ contour plots with confidence levels (C.L.) at values equivalent to $1-\sigma$ and at 90% C.L. (1.64$\sigma$) for present data, and for projected $B$ factory results. Again, for results with the $B$ factories, we use the example of the most likely $\sin(2\beta) = 0.65$ with $B$ factory errors of $\pm 0.06$, and errors of $\pm 0.08$ on $\sin(2\alpha)$.

In Fig. 4 we have plotted the $\chi^2$ contours for the location of the vertex of $(\rho, \eta)$. We note that in contrast to the standard model, in Fig. 4(a) the presently allowed 90% C.L. contour in the four down quark model is an annular ring representing no constraint on $\delta = \delta_{13}$ which can result from the FCNC with its new phases $e^{i\delta_{13}}$ or $e^{i\delta_{23}}$ in $U_{db}$ causing the known $CP$ violation. In Figs. 4(b), 4(c) and 4(d) we show the $B$ factory cases of $\sin(2\alpha) = -1.0$ and 1, respectively, with contours at $1-\sigma$ and at 90% C.L. The existence of several regions, even now for negative $\eta$, requires that extra experiments in $\sin(\gamma)$ or $x$, will also be needed to verify or to bound out the extra down quark mixing model. Use of the slightly more conservative bound for $|V_{ub}/V_{cb}|$ of 0.08 $\pm 0.02$, which is used by some authors, still results in multiple regions.

![Four Down Quark Model, 1σ, 90%CL](image1)

**Four Down Quarks – Present, 1, 2, 3σ**

**Fraction of New in $\varepsilon$ vs. $\delta_{13}$**

![Fraction of New in $\varepsilon$ vs. $\delta_{13}$](image2)

In order to display how the FCNC $Z^0$ exchange with the new phases in $U_{ds}$ can account for the $CP$ violation in $\epsilon_K$, we plot the ratio of the FCNC contribution to the root-mean-square of the SM and the FCNC contributions,

$$R_{FCNC}^{\epsilon_K} = \frac{\text{Im}(U_{ds}^2)}{[(A_{SM}^2)^2 + (\text{Im}(U_{ds}^2))^2]^{1/2}},$$

so that $-1 \leq R_{FCNC}^{\epsilon_K} \leq 1$. Here $A_{SM}^2 = \alpha \text{Im}(-E^{*}/(4\pi \sin^2 \theta_W)$ and $E$ is from Inami and Lim [20]. In Fig. 5 $R_{FCNC}^{\epsilon_K}$ is shown against the angle of $V_{ub}^*\theta$ which is $\delta_{13}$. In Fig. 5, for $\delta_{13}$ from $20^\circ$ to $150^\circ$, $R_{FCNC}^{\epsilon_K} = 0$ is allowed, i.e., the SM can account for $\epsilon_K$ in this analysis. At angles further outside that region, for $-180^\circ < \delta_{13} < 0$, only new physics contributions can give the imaginary part, where $R_{FCNC}^{\epsilon_K} = 1$.

In computing $\chi^2$ for a $(\sin(2\alpha), \sin(2\beta))$ contour plot for the four down quark model, we find that all pairs of $(\sin(2\alpha), \sin(2\beta))$ are individually allowed at $1-\sigma$. This is a much broader allowed region in $\sin(2\beta)$ than the standard model result from present data in Fig. 2. The allowed $1-\sigma$, 90% C.L. and 2-$\sigma$ contours in the $(\sin(2\alpha), \sin(2\beta))$ plot for the cases of the $B$ factory results with the four down quark model are very similar to the SM results shown in Fig. 2.

In terms of other experiments, the $(x, \sin(\gamma))$ plot for the four down quark model is shown in Fig. 6(a) with the allowed region from present data, with $1-\sigma$ and 90% C.L. contours. This allows all values of $\sin(\gamma)$ even in the extended region from $-1 \leq \sin(\gamma) \leq 1$ at the 90% C.L. At $1-\sigma$, $x$, lies between 13 and 48.

In Figs. 6(b), 6(c) and 6(d) are shown the cases $\sin(2\alpha) = -1.0$, and 1, respectively, at $1-\sigma$ and at 90% C.L. They reflect the same regions that appeared in the $(\rho, \eta)$ plots, Figs. 4(b), 4(c), and 4(d). The resemblance is increased if we recall that roughly $\sin(\gamma) = \eta$, and also that $x = x_d/|V_{ud}|$ where $|V_{ud}|$ is the distance from the point $\rho = 1, \eta = 0$ point. We see that experiments on $\sin(\gamma)$ and $x$, are necessary to resolve the
possible regions allowed by the four down quark model. For the case of \( \sin(2\alpha) = -1 \), the allowed values of \( \sin(g) \) in Fig. 6(b) are different than those for the standard model in Fig. 3(b). The \( \sin(2\alpha) = 0 \) case allows regions of \( \sin(g) \) lower than in the SM.

The extent of the non-zero value of \( A_{B_s} \) in the four down quark model is shown in Fig. 7 from present data with contours at 1-, 2-, and 3-\( \sigma \). Plots for the \( B \) factory cases are similar. We note that in the new full range plot \( A_{B_s} \) is roughly symmetric about zero, with the largest absolute values at 0.35 at 1-\( \sigma \), and 0.5-0.6 at 90% C.L. This is much different from the \(<0.025 \) value of \( A_{B_s} \) in the SM.

We now report on additional plots that are not shown here. We compared the limits on the four down quark FCNC amplitude \( |U_{db}| \) versus the standard model amplitude \( |U_{std-d_b}| \) for \( B_d^0 - \bar{B}_d^0 \) mixing, at present and after the \( B \) factory results. At present the constraints are such that \( |U_{db}| \) can go from zero up to as large as the magnitude of \( |U_{std-d_b}| \) at 1-\( \sigma \) [9]. \( |U_{sb}| \) is restricted to about half of \( |U_{std-b_s}| \). The total phase of \( B_d^0 - \bar{B}_d^0 \) mixing can range over all angles, while the SM phase is between \(-30^\circ\) and \(80^\circ\) when in combination with the FCNC amplitude. The magnitude of possible regions allowed by the four down quark model. For the case of \( \sin(2\alpha) = -1 \), the allowed values of \( \sin(g) \) in Fig. 6(b) are different than those for the standard model in Fig. 3(b). The \( \sin(2\alpha) = 0 \) case allows regions of \( \sin(g) \) lower than in the SM.

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\[ |U_{db}|/(V_{cd}V_{cb}) \]

in the unitarity triangle is \(<0.15 \) at 1-\( \sigma \).

The 90% C.L. limits on the three new quark mixing elements \( |V_{4d}|, |V_{4s}|, \) and \( |V_{4b}| \) are roughly equal to the mixing angles to the fourth down quark \( \theta_{4d}, \theta_{4s} \) and \( \theta_{4b} \), respectively. They are bounded by 0.05, 0.05, and 0.08, respectively. The values allowed in combination are much more restricted, since they are roughly bounded by hyperbolic curves, due to constraints acting on their products in \( U_{ds}, U_{sb}, \) and \( U_{bd} \).

VI. CONCLUSIONS

We have extended our analysis to the full range of the variables \( \eta, \sin(g) \) and \( A_{B_s} \), all of which are imaginary parts, to include all of their negative values. For the four down quark model, they all show remarkable and experimentally important new behaviors. From present constraints, the vertex of \( V_{ab} \) now is allowed in this model to be complete circular annuli about \((r, h) = (0, 0) \) at 90% C.L. due to the new phases \( \delta_{14} \) or \( \delta_{24} \) accounting for the presently observed \( CP \) violation in \( \epsilon, \sin(g) \) is now allowed in this model over its entire range from \(-1 \) to \(+1 \). The range of \( A_{B_s} \) is almost equally as large for its negative values as it is for its positive values, and perhaps large enough to be observed. Since it is almost null in the SM, this could be a dramatic evidence of new physics.

For the \( B \) factory cases, there are new multifold allowed regions as shown in the extended \((\rho, \eta)\) plots including for
negative $\eta$. This will require additional experiments on $x_s$ and $\sin(\gamma)$ to well define the four down quark model results, and eventually to verify or bound out the relevance of the model here. In the $(x_s, \sin(\gamma))$ plot for similar cases, there are new multiple regions for $\sin(\gamma)$ negative.

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[13] Here in $K_L \rightarrow \mu \mu$ the joint error from BNL and KEK is scaled up by a factor of 1.3 using the Particle Data Group method. After subtracting the $2\gamma$ unitarity contribution we have $|A_R|^2 = (0.23 \pm 0.54) \times 10^{-9}$. The latest experimental report from BNL is A. P. Heinson et al., Phys. Rev. D 51, 985 (1995). At 1-$\sigma$, the bound on $|A_R|$ is about the upper limit on the long distance contribution estimates.
[18] We take the 1-$\sigma$ fractional error on $R_b$ from the deviation of the combined average from the SM result, since if there is a real positive deviation, it would not come from FCNC, but from new physics not included here.