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COMMENT ON A LETTER RELATED TO BELL'S THEOREM*

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ABSTRACT

A conclusion asserted in a recent letter is analyzed and shown not to follow from the arguments given. Also, Bell's theorem is formulated as a nonlocality property of quantum theory itself, with no explicit or implicit reference to determinism or hidden variables.

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In a recent letter[1] related to Bell's theorem[2] Arthur Fine proved several propositions, and asserted the following conclusion: "Proposition (2) shows that, despite appearances, no significant generality is achieved by those derivations of the Bell/CH inequalities that dispense with explicit reference to hidden variables and/or determinism.[9] The assumptions of such derivations imply the existence of deterministic hidden variables for any experiment to which they apply."

This conclusion consists of two assertions, which must be distinguished. The second is meant to be a rephrasing of Proposition (2), and, as such, is technically correct. However, it is misleading due to two semantic irregularities: (1) Fine leaves the word "local" out of his name "deterministic hidden variable models." Usually this word is inserted to remind the reader that the models in question have an important factorization property that normally arises from the idea that the deterministic hidden variables are separated into two local parts, each of which determines those results of the experiment that occur in one of two separated regions. (2) Fine leaves the word "model" out of the rephrasing. This creates the impression that what was proved was the actual existence of deterministic hidden-variables, rather than the existence of a certain kind of factorizable model.

The first part of Fine's conclusion is incorrect: Proposition (2) shows that any assumption "LOC" that entails the Bell/CH inequalities entails also that any probabilities conforming to LOC can be modeled, or reproduced, by what is called by Fine a deterministic hidden variable model. This true fact has no logical bearing on the issue of the generality achieved by the cited works that dispense with determinism and hidden variables. For the correct and proper aim of those works is to dispense with all explicit and implicit reference to determinism and hidden variables in forming the <u>definition</u> and <u>physical justification</u> of a locality property that is incompatible with the statistical predictions of quantum theory. It is well known that in these works the actual arithmetic proof of the incompatibility with quantum theory is essentially the same as in Bell's earlier work based on local deterministic hidden variable theories. Thus it is important that the general conception of nonlocality, though leaning in no way on determinism or hidden variables for its definition or physical justification, nevertheless leads to conditions on the conceivable results of experiments that are essentially equivalent to those that Bell showed incompatible with the statistical predictions of quantum theory. Then, because the general conception of nonlocality is defined and physically justified with no explicit or implicit reference to determinism or hidden variables, Bell's nonlocality theorem can be extended to theories, such as quantum theory itself, that make no assumption about determinism or hidden variables.

To make this point absolutely clear a concrete example of a generalization of the kind under discussion is needed. Rather than restating an existing work, I use the opportunity to present a modified, and intrinsically interesting, version of the theorem of Ref. 3 that makes weaker assumptions and shows quantum theory itself to be nonlocal in a physically reasonable sense that is formulated

with no explicit or implicit reference to determinism or hidden variables.

The point of departure is Bell's theorem, which says that any theory compatible with the statistical predictions of quantum theory is nonlocal, provided the theory is a deterministic hidden-variable theory. The aim of the generalization is to remove this proviso.

The experiment used to demonstrate the result is well known.[2,3] I add one extra feature. The particles entering the original scattering experiment are monitored by fast electronics that allow the individual pairs to be identified. Those scattered pairs i that pass through two polar escape holes in a spherical array of counters are numbered i = (1, ..., n). The fast electronics and known geometry allows the individual arrival times t_i at two Stern-Gerlach devices A and B to be placed in separate and known time windows.

The <u>result</u> of the experiment is specified by

$$r = (r_A; r_B)$$

$$= (r_{A1}, \ldots, r_{An}; r_{B1}, \ldots, r_{Bn}), \qquad (1)$$

where each r_{Ai} and r_{Bi} takes a value of either +1 or -1, corresponding to a deflection along the direction D_A or D_B, or against this direction, respectively.

There are two alternative possible settings D_A' and D_A'' of the direction D_A, and two alternative possible settings D_B' and D_B'' of D_B.

The experiments are set-up so that both the choice between D'_A and D''_A and the subsequent deflections and recordings of the results r_{iA} $i = (1, 2, \ldots n)$ will occur in a spacetime region R_A, and similarly for B, where R_A and R_B are spacelike separated.

The four alternative possible experiments are labeled by the four values of (D_A, D_B). For each alternative value of (D_A, D_B) there are $(2^n)^2$ conceivable results r. To each conceivable result r of each of the four alternative possibilities (D_A, D_B) quantum theory assigns a probability P.

Consider the set S consisting of all conceivable combinations of the conceivable results of all four alternative possible experiments. The different elements of S correspond to the different possible functions

$$r(D_A, D_B)$$

$$= (r_{A1}(D_A, D_B), \ldots, r_{An}(D_A, D_B); r_{B1}(D_A, D_B), \ldots, r_{Bn}(D_A, D_B)), \quad (2)$$

where the possible values of each function $r_{Ai}(D_A, D_B)$ and $r_{Bi}(D_A, D_B)$ are +1 and -1.

A general theory T that makes statistical predictions for all four possible experiments of the kind under consideration here will be said to entail a <u>nonlocal connection</u> (or be <u>nonlocal</u>) if, as n tends to infinity, there is no conceivable combination of conceivable results of the four alternative possible measurements that is compatible with both the statistical predictions of T and the locality conditions that the results in each region be independent of the choice made in the

other:

$$r_{Ai}(D_A, D_B) = r_{Ai}(D_A), \; r_{Bi}(D_A, D_B) = r_{Bi}(D_B). \quad (3)$$

Quantum theory predicts that, whichever of the four experiments (D_A, D_B) is performed, the correlation parameter

$$c[r(D_A, D_B)]$$

$$= \frac{1}{n} \sum_{i=1}^{n} r_{iA}(D_A, D_B)\, r_{iB}(D_A, D_B) \quad (4a)$$

will, as n tends to infinity, come to satisfy

$$|c[r(D_A, D_B)] - \bar{c}(D_A, D_B)| < .03, \quad (4b)$$

where $\bar{c}(D_A, D_B)$ is a number specified by quantum theory. But Bell's arithmetic shows[3] that there is no conceivable combination of conceivable results that satifies both (3) and (4). Thus any theory T that gives the prediction (4) is nonlocal. One such theory is quantum theory itself.

What do Fine's arguments and results show. As delicate issues are involved it is best to state things precisely. Consider a couple (E, T) consisting of an experiment E, and a theory T that makes predictions about E. Each experiment E consists of a set of four alternative possibilities of the kind being discussed.

Some theories predict probabilities and some predict individual results. Let P(E, T) represent the probabilities predicted for E by T, if such predictions are made. Let (F) represent the conditions imposed on P(E, T) by the requirements on Fine's class of deterministic hidden-variable models. Let R(E) represent a conceivable combination of conceivable results of E.

Two classes of couples (E, T) may now be defined:

$$C_{FD} \equiv \{(E, T);\ P(E, T) \text{ is defined and satisfies } (F)\} \qquad (4)$$

$$C_{NL} \equiv \{(E, T);\ P(E, T) \text{ is defined, and no conceivable } R(E) \text{ is compatible with both } P(E, T) \text{ and } (3)\} \qquad (5)$$

The subscripts FD and NL stand for Factorized Deterministic (as defined by Fine's equations) and Nonlocal (as defined by the present work). Two semi-complementary classes C_{NFD} and $C_{NNL} \equiv C_{LOC}$ are defined by changing "satisfies" to "does not satisfy" and "no" to "some", respectively.

Two conceivable definitions of nonlocal theories are identified by the following two classes of theories:

$$\tau_{NFD} \equiv \{T;\ \text{for some } E,\ (E, T) \in C_{NFD}\}, \qquad (6)$$

$$\tau_{NL} \equiv \{T;\ \text{for some } E,\ (E, T) \in C_{NL}\}. \qquad (7)$$

The final class is the one defined in this work. The other possibility uses the equations of Fine.

Fine's argument claims that C_{LOC} is contained in C_{FD}: $C_{LOC} \subset C_{FD}$. This result is true: it follows immediately from the fact that if a set of conceivable results R(E, T) satisfies the independence property (3), then the probabilities generated by those results will satisfy the crucial factorization property imposed by Fine's equations.[4] (This is the property that each of the four four-valued functions (AB(λ), AB'(λ), A'B(λ), A'B'(λ)) normally required to model such an experiment be factorized into a product of two two-valued functions: $AB(\lambda) = A(\lambda)B(\lambda)$, $AB'(\lambda) = A(\lambda)B'(\lambda)$, $A'B(\lambda) = A'(\lambda)B(\lambda)$, $A'B'(\lambda) = A'(\lambda)B'(\lambda)$). It is easy to prove also that $C_{FD} \subset C_{LOC}$, and thus derive $C_{LOC} = C_{FD}$, and hence conclude that $\tau_{NL} = \tau_{NFD}$. Thus the definition of nonlocal theories introduced in this work is equivalent to a similar one that could be defined by using Fine's equations.

The equivalence of these two alternative possible definitions of nonlocality, which is the essential basis of Fine's claim, has no effect on the generality achieved by definition (7). For simply defining a theory T to be nonlocal if it belongs to class τ_{NFD} would not permit any claim of having derived a nonlocality property of, say, quantum theory, with no explicit or implicit reference to determinism or hidden variables. For this definition depends on the concept of deterministic hidden variables. What is needed is a conception of nonlocality that makes no explicit or implicit reference to determinism or hidden variables, and which leads, via the conflict between (3) and (4) discovered by Bell, to a conflict between locality and any theory that gives the quantum predictions (4). Such a concept of nonlocality is embodied in definition (7).

The fact that this conflict between (3) and (4) can also be formulated, as it originally was, by using deterministic hidden variables has no bearing on the fact that is essential for the kind of generalization being sought, namely that it is not necessary to invoke determinism or hidden variables in order to exploit the conflict between (3) and (4).

The essential point is that there are no actual mathematical conditions on the equations of Bell from which the contradiction with quantum theory arises that demand that the functions $r_{Ai}(D_A, D_B)$ and $r_{Bi}(D_A, D_B)$ in his proof represent the results of the alternative possible experiments determined beforehand by some invisible variables. Thus, from a mathematical point of view, the content of his result is not well represented by the words "deterministic hidden variable": these words are present, but there are no corresponding mathematical conditions of "beforehandeness" and "invisibility." The aim or the generalization is to exploit this fact, and show how to use Bell's mathematics without getting embroiled with these irrelevant concepts of determinism and hidden variables.

The formulation of nonlocality used here avoids having to introduce the concept that all four alternative possible results of the experiment be determined beforehand by hidden variables. This concept assigns definite results to experiments that "could have been performed but were not." The need to use this contrafactual concept severely limits the

scope of the theorem, in the form originally put forth by Bell. The present formulation asserts that a theory entails a nonlocal connection if it makes statistical predictions, and these predictions, by themselves, entail (in some cases) that there is no way within the set of all conceivable combinations of conceivable results for the results in each region to be independent of the choice made in the other region. Quantum theory has such a nonlocal connection: that is what Bell actually discovered. Tying this discovery to the mathematically irrelevant concepts of beforehandeness and invisibility obscures its logical essence, and needlessly curtails its significance. For the functions $r_{Ai}(D_A, D_B)$ and $r_{Bi}(D_A, D_B)$ can more rationally be viewed as defining the set of all conceivable combinations of conceivable results.

The nonlocality property of quantum theory discussed here does not conflict with the microcausality property of quantum theory, which prevents faster-than-light communication by means of quantum observables.

As stressed in reference 3 the nonlocality property of quantum theory does not necessarily entail nonlocal influences: there appear to be two alternatives. The first is a superdeterminism, in which the choice of the experimenter is not effectively free: some tight connection from their common past binds the results in one region to the choice of experiment in the other. The second alternative, exemplified by the many-worlds (or many-minds) interpretation of quantum theory, exploits the fact that experienced worlds in which the results in both regions are definite are confined to the intersection of the forward light cones from the two regions. The third alternative is that the manifestly nonlocal character of von Neumann's process 1, unlike that of its counterpart in classical statistical mechanics, reflects the existence of subtle nonlocal influences that are not evident at the level of probabilities and averages normally dealt with by pragmatic quantum theory and classical mechanics.

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2. J.S. Bell, Physics (NY) 1, 195 (1964).
3. H.P. Stapp, Phys. Rev. D3, 1303 (1971).
4. Here n is assumed to be large enough so that 1/n is negligible.

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