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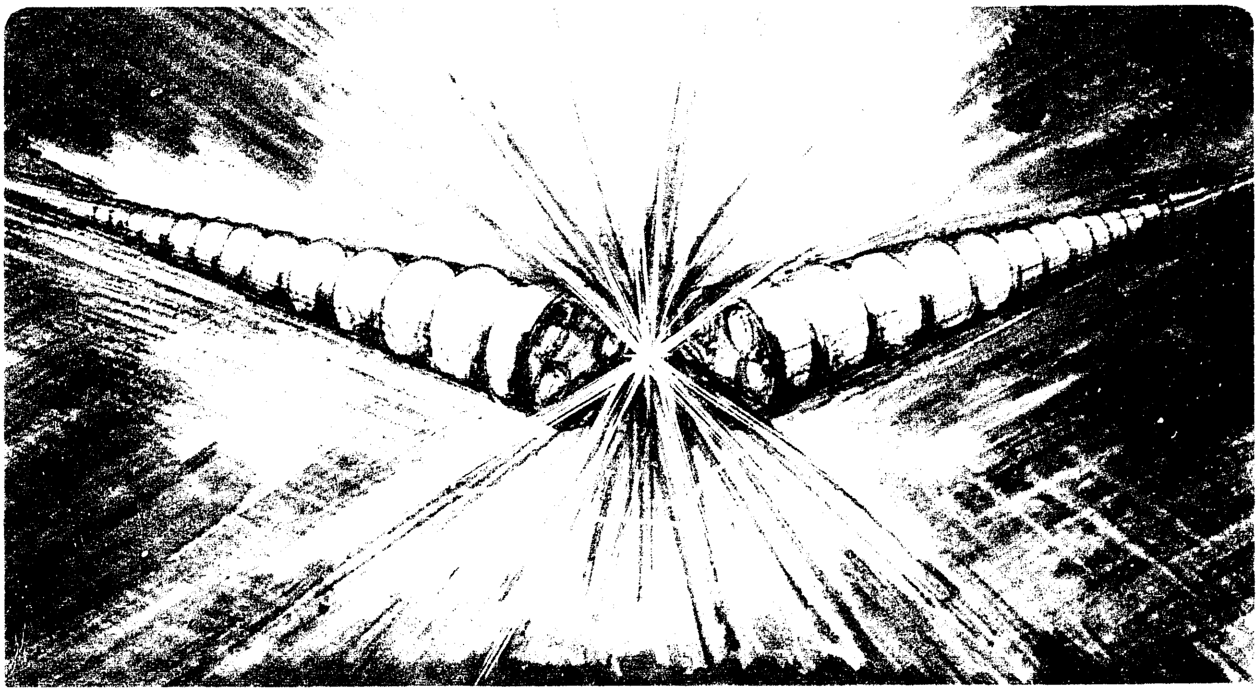

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Polarized nature of synchrotron radiation*

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ABSTRACT

Synchrotron radiation is inherently highly polarized, and this fact makes synchrotron radiation useful for probing the directional and/or helical nature of the matter via polarization sensitive experiment. Here we summarize briefly the nature of the polarization experiments and the polarization characteristics of synchrotron radiation.

1. EXPERIMENT WITH PARTIALLY POLARIZED RADIATION

Partially polarized radiation is characterized by the Stoke's vector $S = (S_0, S_1, S_2, S_3)$, where S_0 is the total intensity, S_1, S_2 are respectively the partial intensities polarized linearly along the x-direction (the propagation direction is assumed to be in the z-direction) and along the direction 45° from the x-direction in the x-y plane, and S_3 is the partial intensity with circular polarization.[1] The quantities $(P_1, P_2, P_3) = (S_1/S_0, S_2/S_0, S_3/S_0)$ are some times referred to as the degree of the polarization in the respective directions. In general, $P_1^2 + P_2^2 + P_3^2 \leq 1$.

A general light scattering experiment can be thought of as a study of the transformation of the Stoke's vector from the initial value S to the final value S';

$$S' = MS$$

Here M is a 4x4 matrix known as the Mueller matrix, which is the properties of the sample. An experiment leading to a determination of every element M_{ij} of the Mueller matrix (there are ten independent elements[2]) may be termed as the "complete" experiment. An example of the simpler experiment is to measure the final intensity

$$S_0' = M_{00}S_0 + M_{01}S_1 + M_{01}S_2 + M_{02}S_3$$

The element M_{03} represents the helical characteristics of the sample, and can be determined by measuring the final intensities $S'_{0\pm}$ corresponding to the incident radiation of opposite helicities, $S_3 = \pm P_3 S_0$. This type of experiment is called the circular intensity differential scattering (CIDS).

These discussions show that a determination of the Mueller matrix requires, in general, the ability to modulate the polarization.[3] In particular, to determine the helical properties it is necessary to have circularly polarized light which can be modulated between opposite helicities. Synchrotron radiation provides such capabilities in wavelength regions not available with conventional lasers.

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2. POLARIZATION CHARACTERISITCS OF BENDING MAGNET SOURCES

The radiation from a bending magnet is of the form

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} \alpha \begin{pmatrix} K_{2/3}(\eta) \\ \dfrac{i\gamma\psi}{\sqrt{1+(\gamma\psi)^2}} K_{1/3}(\eta) \end{pmatrix},$$

where E_x (E_y) is the field amplitude in the horizontal (vertical) direction, γ is the electron's kinetic energy in unit of the rest energy, ψ is the vertical angle, K's are the modified Bessel functions, and

$\eta = \left(\frac{\omega}{2\omega_c}\right)\left(1+(\gamma\psi)^2\right)^{3/2}$, ω = radiation frequency and ω_c = the critical frequency. We see from this

that,when $\psi \neq 0$, the vertical component is non-vanishing and is 90° out of phase with respect to the horizontal axis. This implies that the radiation is elliptically polarized, the ratio between the minor and the major axis of the polarization ellipse being given by $r = E_y/iE_x$. The corresponding Stoke's vector is

$$P_1 = \frac{1-r^2}{1+r^2},\quad P_2 = 0,\quad P_3 = \frac{2r}{1+r^2}$$

The fact that $P_2 = 0$ implies that the polarization ellipse lies along the horizontal direction. The sign of r, and hence P_3, can be reversed by reversing the sign of ψ. Therefore the circular polarization can be modulated by collecting the radiation alternately from above and below with respect to the orbit plane. However, the sign of the linear polarization P_1 can not be reversed since $0 \leq r^2 < 1$.

The double-headed Dragon monochronometer developed by C. T. Chen[4] provides an efficient way to utilize the bending magnet polarization.

Figure (1) shows the degree of circular polarization, P_3, for the bending magnet radiation for several values of the ratio $\varepsilon/\varepsilon_c$, where ε and ε_c are respectively the photon energy and the critical photon energy (ε_c(keV) = 0.665 E_e [GeV] B[T] as a function of the vertical angle $\gamma\psi$. The corresponding angular density of the flux is shown in Fig. (2). For $\varepsilon/\varepsilon_c = 0.1$, the radiation collected at $\gamma\psi = 1.5$ will provide a high degree of polarization, $P_3 = 0.9$, without a significant reduction in the flux density. For $\varepsilon/\varepsilon_c = 1$ (3), the angle $\gamma\psi = 1(0.5)$ corresponds to $P_3 = 0.9$ (0.7) and an angular density of flux reduced by a factor 3.5 (2.5) from the peak density at $\psi = 0$.

3. SPECIAL DEVICES FOR ENHANCED POLARIZATION CAPABILITIES

Recently, several novel types of synchrotron radiation sources have been developed for higher flux and for more versatile polarization control. Table 1 gives a list of operating devices. The devices are classified according to the usual classification scheme; bending magnet, wigglers and undulators. In the following, these devices are discussed briefly, referring to the literature[11] for more details.

Each device in the Table has different merits. Thus the wiggler type of the devices will produce intense flux of elliptically polarized radiation over a broad spectral range. However, in wiggler type

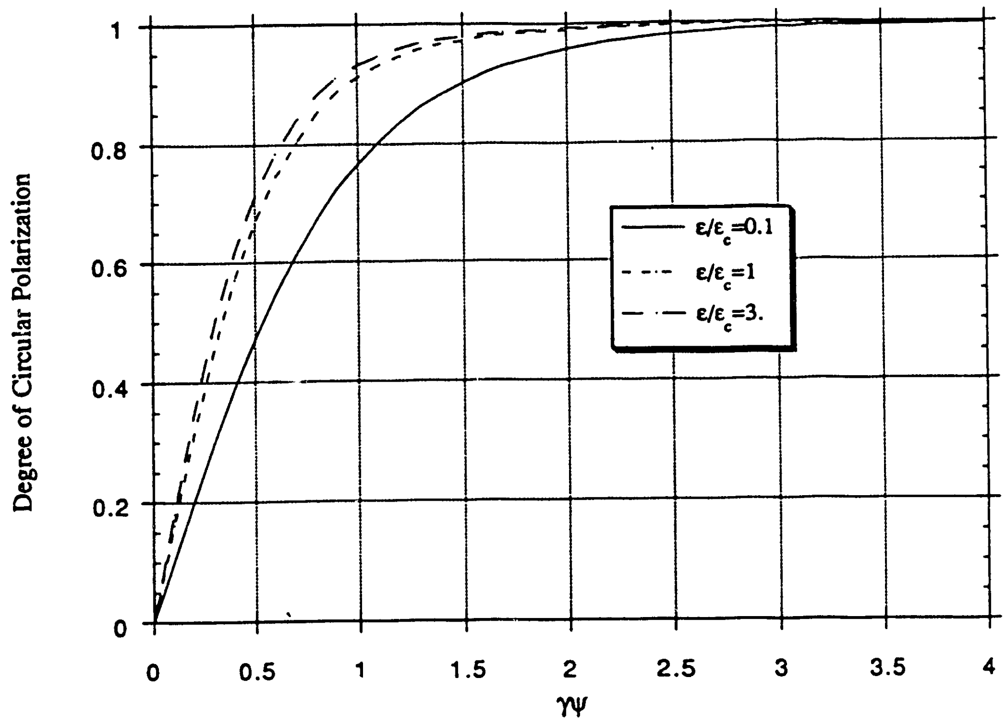

Fig. 1. Degree of Circular Polarization of Bending Magnet Radiation

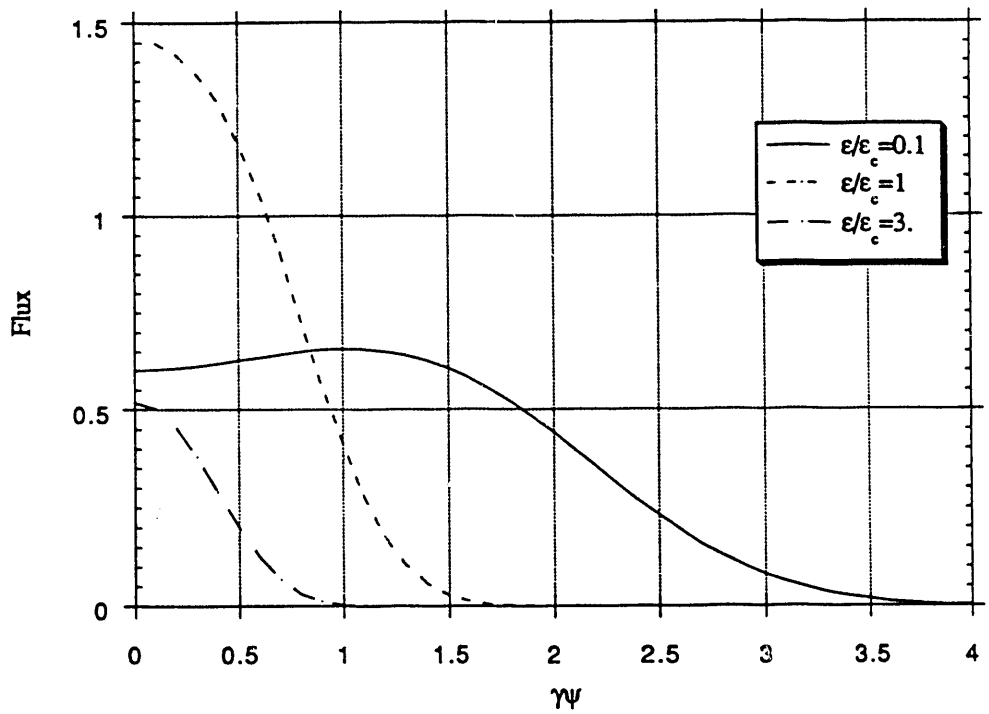

Fig. 2. Angular density of Bending Magnet Flux (#/(sec)(0.1% B.W.)(mr^2) in units of 1.33×10^{13} I[A] E^2[GeV])

devices, the direction of the linear component of the polarization can not be modulated, being fixed in the horizontal direction. This could be a drawback for some experiments aiming for a complete determination of the Mueller matrix.

Table 1

Operational Synchrotron Radiation Devices Producing Variable/Circular Polarization

Devices	Institute
Bending magnet	
off-plane observation	BESSY, NSLS
Wiggler	
Asymmetric Wiggler [5]	HASYLAB, LURE
Elliptical Wiggler [6]	Photon Factory
Undulator	
Based on helical field	
Crossed-Overlapped Undulator [7]	ETL
Planar-Helical Undulator [8]	ESRF
Based on interference effect	
Crossed Undulator [9], [10]	BESSY

A versatile version of the wiggler type device is the modified elliptical wiggler,[12] similar to the elliptical wiggler[6] except that the horizontal field component is generated by an a.c. electromagnet. The polarization of an elliptical wiggler can be determined from Fig. (1) by replacing the vertical angle by $\gamma\psi \rightarrow K_x = 0.934\ \lambda_w\,[\mathrm{cm}]\ B_x[\mathrm{T}]$, where λ_w and B_x are the period length and the peak horizontal magnetic field. The flux is 2 x number of periods times that determined by Fig. (2).

The undulator type of devices will provide higher brightness (in a small phase space area). The spectrum is concentrated in narrow peaks, but the peaks can be tuned by changing the magnet gaps. The undulator devices can be further classified into two groups; those based on the helical field (helical undulators) and the crossed undulator based on the interference principle.

The Stoke's vector for the undulator type of devices listed in Table 1 is of the form

$$P_1 = 0, \qquad P_2 = P \cos \alpha, \qquad P_3 = P \sin \alpha.$$

Here P is the degree of the polarization and α is the polarization phase determined by the relative position of the magnets in the case of helical undulators and by the modulator current level in the case of the crossed undulator. Note that the sign of either P_2 or P_3 can be reversed by changing α; thus both the linear and the circular polarization can be modulated. Undulators based on helical field produce a high degree of polarization ($P \sim 1$). A clever scheme for a helical undulator due to P. Elleaume[8] is to separate the magnet producing the vertical and the horizontal field on two different (top and bottom) jaws, making it possible to accommodate a conventional flat vacuum chamber.

For a crossed undulator, there is some reduction in the degree of polarization P due to the angular smearing from the electron beam emittance and the opening angle of the pinhole. However, it turns out that the crossed undulator produces a significant amount of polarization even at very high photon energies where the angular effect is expected to be maximum. Figure 3 shows the performance of the crossed undulator in the limiting case of the vanishing photon wavelength and a large angular opening, assuming that N is large and that the effect of the modulator section can be neglected. See the Appendix for calculational details.

The crossed undulator can be operated at higher harmonics in contrast to the helical undulators. This is an important feature to reach high photon energies with low energy electron machines. Also, the polarization of the crossed undulators can be modulated electromagnetically by modulating the current in the modulator magnet between two planar undulators,[10] while the modulation is mechanical for the helical undulators.

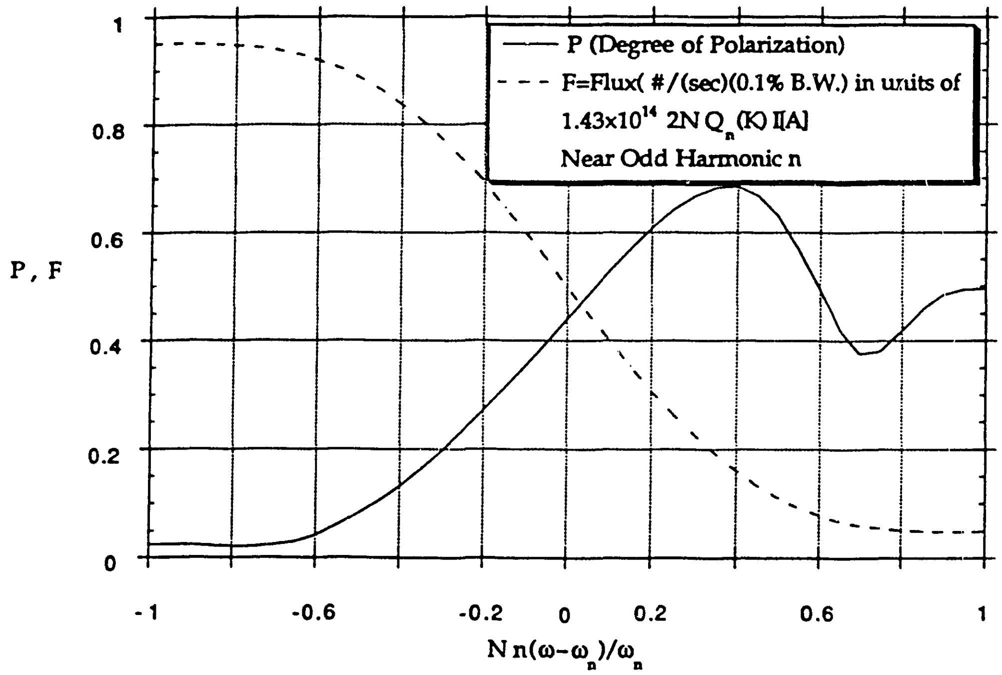

Fig. 3. Crossed Undulator (N+N) Performance in the Short Wavelength Limit (angular opening >> $(\lambda/L)^{1/2}$)

4. APPENDIX: CALCULATION LEADING TO FIGURE (3)

We consider an idealized crossed undulation consisting of two identical N-period undulators, one in the x-direction and one in the y-direction. We assume that the effect of the modulator on the degree of the polarization is negligible. The angle dependent part of the radiation amplitudes in the x- and y-directions when N is large are, respectively,

$$A_x = \frac{\sin\left(N\,\Phi/2\right)}{N\,\Phi/2} \quad ,$$

$$A_y = A_x\, e^{iN\Phi} \, ,$$

where $\Phi = 2\pi\left(\Delta\nu + n^2\gamma^2\theta^2/(1 + K^2/2)\right)$, $\Delta\nu = (\omega - n\omega_1)/\omega_1$, $\omega_1 =$ fundamental harmonic frequency, θ = angle from the forward direction, K = the deflection parameter. Here we are assuming that the frequency is near one of the harmonic, $\omega_n = n\omega_1$. The coherency matrix when the radiation is observed through an angular aperture (pinhole) is given by

$$J = \text{const}\begin{bmatrix} \int A_x^* A_x d\Omega \ , & \int A_x^* A_y d\Omega \ , \\ \int A_y^* A_x d\Omega \ , & \int A_y^* A_y d\Omega \ , \end{bmatrix} ,$$

where the integration $d\Omega = \pi\, d\theta^2$ is over the angular aperture. The degree of polarization is determined from the following relation:

$$J = \text{const}\left(\int A_x^* A_x \, d\Omega\right) \cdot \begin{bmatrix} 1 \, , & Pe^{i\alpha} \\ Pe^{-i\alpha}, & 1 \end{bmatrix} ,$$

where α is the polarization phase. An explicit expression for P is

$$P = \sqrt{P_2^2 + P_3^2} \ ,$$

$$\begin{pmatrix} P_2 \\ P_3 \end{pmatrix} = \int_{\xi_1}^{\xi_2} \left[\frac{\sin(\xi/2)}{(\xi/2)}\right]^2 \begin{pmatrix} \cos\xi \\ \sin\xi \end{pmatrix} d\xi \quad ,$$

$$\xi_1 = 2\pi N \Delta\nu, \ \xi_2 = 2\pi\left[N\Delta\nu + (\Delta\theta/2\sigma_N)^2\right] \ ,$$

$$\sigma_N = \sqrt{\lambda_n/2\, L_N} \ ,$$

In the above $\Delta\theta$ is the radius of the angular aperture, λ_n is the wavelength corresponding to $n\omega_1$, and L_N is the length of the N-undulator. Figure (3) corresponds to the case $\Delta\theta/2\sigma_N \rightarrow \infty$. The polarization property of this case should be the same as that of the large emittance case.

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