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## Authors

Swaroop, D.
Huandra, R.
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D. Swaroop, R. Huandra<br>Texas A\&M University, College Station

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# Intelligent Cruise Control System Design based on a Traffic Flow Specification ${ }^{1}$ 

D. Swaroop ${ }^{2}$, R. Huandra ${ }^{3}$<br>Department of Mechanical Engineering, Texas A\&M University, College Station, TX - 77843-3123<br>e-mail: dswaroop@usha.tamu.edu<br>4 January 1999


#### Abstract

This paper investigates the problem of designing an Intelligent Cruise Control (ICC) algorithm for automated vehicles. An ICC algorithm, if implemented by every vehicle in the traffic, must guarantee that the density disturbances attenuate as they propagate upstream. Such a desirable property of the traffic is dependent on the spacing policy employed by automated vehicles and on the availability of information required to synthesize a string stable control law consistent with the employed spacing policy. The first part of the paper is concerned with the design of a spacing policy and the latter part is concerned with synthesizing an ICC algorithm for automated vehicles. Various other issues relating to the design of an ICC algorithm are also discussed.


## 1 Introduction

Congestion is seriously hampering the mobility of vehicles on freeway segments in urban areas. The toll of congestion is estimated at $\$ 100$ billion dollars annually due to productivity losses associated with increasing travel delays. In addition, significant costs are incurred due to environmental damage, human fatalities and fuel wastage. Intelligent Transportation Systems (ITS) envisage the use and integration of advances in sensing, communication, actuation and information processing technologies to provide safe and effective solutions to the current transportation problems [39, 34]. It is estimated that $90 \%$ of the accidents occur due to human errors. Automating human driving, therefore, is expected to lead to increased capacity and safety of the existing highways.

An important aspect of automating human driving is the design of an automatic vehicle following control system and it is the focus of this paper. Automatic vehicle following requires:

1. the design of a spacing policy which dictates how an automatically controlled vehicle's speed must be regulated as a function of the following distance.
2. the design of an automatic control system to regulate the vehicle's speed in response to any changes in the following distance.
[^0]In the literature, two types of spacing policies have been explored:

1. Constant spacing policy: In this spacing policy, the desired intervehicular spacing (or following distance) in a string of automated vehicles is constant and does not vary with the controlled vehicle's speed.
2. Variable spacing policy: In this spacing policy, the desired intervehicular spacing is a function of the controlled vehicle's speed. Two variable spacing strategies have received considerable attention - Constant Time headway policy and Constant Safety Factor Policy. In a Constant Time Headway policy, the desired intervehicular spacing increases linearly with the controlled vehicle's speed. In a constant safety factor policy, the desired intervehicular spacing is proportional to the controlled vehicle's stopping distance and hence, varies quadratically with the controlled vehicle's speed.

String stability is an issue in the design of automatic vehicle following control systems. Intuitively, string stability ensures that spacing errors in regulating the following distance according to the specified spacing policy do not amplify upstream from one vehicle to another. In a constant spacing policy, reference vehicle information is required to guarantee string stability [34, 33, 35]. If every controlled vehicle has access to the information of its position relative to a reference vehicle in the string, geometric attenuation of spacing errors can also be guaranteed [35]. In contrast, some variable spacing control strategies do not necessarily require external reference information and can be shown to guarantee string stability with on-board information only $[3,5,14]$.

The macroscopic behavior of traffic is adequately described by the speed and density (number of vehicles per unit length of a highway) of traffic on a section of a highway. The density of traffic is inversely proportional to the average distance between vehicles or simply the average following distance of vehicles on a highway. Since the implementation of a constant spacing policy is infrastructure intensive, it is imperative that near-term Intelligent Cruise Control (ICC) systems employ a variable spacing policy. In this paper, we will only consider such spacing policies for automated vehicles. As a result, the spacing policy used in ICC systems determines the equilibrium speed-following distance or equivalently equilibrium speed-density or equilibrium traffic volume-density relationship for the traffic. This relationship, also known as the fundamental traffic characteristic, governs the traffic capacity and flow behavior [7, 36]. Until recently [34], the design of a spacing policy for automated vehicles is considered as much a philosophical issue as it is a technical issue. This paper dispels such a myth and is concerned with designing an ICC algorithm for a specified traffic performance specification.

## 2 Background

The macroscopic dynamics of a traffic flow is described by differential equations that govern the evolution of traffic density and traffic speed. The conservation of mass equation together with a constitutive equation similar to the conservation of linear momentum describes the macroscopic dynamics of automated,
partially automated and non-automated traffic flows. By an automated traffic flow, we mean a traffic flow consisting entirely of automated vehicles. By a non-automated traffic flow, we mean a traffic flow consisting entirely of nonautomated vehicles and by a partially automated traffic flow, we mean a traffic flow consisting of both automated and non-automated vehicles. The constitutive equation is different for the three kinds of traffic flows. Researchers have obtained the constitutive equation for non-automated traffic flows by using either car-following models of traffic flow or continuum models of traffic flow [38].

Extensive experimentation has shown that human distance regulation on existing highways leads to traffic flow instability when the traffic density exceeds a critical value $[7,21]$. Attempts to explain the traffic flow behavior have resulted in the development of several traffic flow models. The first known car following models are due to Reuschel [28] and Pipes [26]. In their work, it is hypothesized that each driver maintains a separation distance proportional to the speed of their vehicle plus a distance headway at standstill that includes the length of lead vehicle. In other words, each driver adopts a constant time headway spacing policy. Greenshields [12] hypothesized the following steady state relationship between traffic density, $\rho$ and speed, $v$ :

$$
v=v_{f}\left(1-\frac{\rho}{\rho_{\max }}\right) \Rightarrow q=v_{f} \rho\left(1-\frac{\rho}{\rho_{\max }}\right)
$$

Here $\rho_{\max }$ is the jam density and $v_{f}$ is the free speed of the traffic. Since the traffic volume, $q$, is a quadratic function of $\rho, q$ increases with increasing density upto a critical density, $\rho_{p e a k}=\frac{\rho_{\max }}{2}$ and the corresponding traffic volume, $q_{p e a k}=\frac{v_{f} \rho_{\max }}{4}$. It can be shown that the traffic flow is stable upto the critical density and is unstable thereafter.

Car following modeling is a microscopic approach to describing the aggregate traffic speed dynamics. Car following algorithms describe in detail how one vehicle follows another. With such a description, the macroscopic behavior of a single lane of traffic can be approximated, if string stability is guaranteed [36]. The steady state traffic volume-density relationship is obtained by using the conservation of mass equation in conjunction with the car following model.

Analytical and experimental studies detailing vehicle following performed by humans can be found in $[30,13,27,9,16]$. Inherent to this approach is the assumption [30] that each driver in a following vehicle is an active and consistently predictable control element. The macroscopic approximation of traffic flow dynamics is seriously affected by this assumption. However, the reduction/elimination of human role in automated vehicle following makes it reliable to obtain the aggregate speed dynamics from car-following models.

In modeling traffic as a continuum, traffic flow is compared with fluid flow. The density evolution is governed by the conservation of mass principle. The simplest of continuum models for freeway traffic is due to Lighthill and Whitham, [18]. In their work, Lighthill and Whitham employ a static constitutive traffic volume - density relation, $q=q(\rho)$ in the conservation of mass equation. As a result,

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{\partial q(\rho)}{\partial x}=0 \tag{1}
\end{equation*}
$$

Here, $x$ is the spatial variable, $t$ is the time. In this traffic flow model, traffic velocity dynamics is neglected. Since the time scale for traffic density evolution is much slower than the time scale for aggregate traffic speed dynamics, this is a reasonable assumption.

Let $\rho^{*}(x)$ and $q\left(\rho^{*}\right)$ denote a basic solution to the conservation of mass equation with the constitutive relation, $q=q(\rho)$. In order to study the stability of the basic solution, consider a linearization about the basic solution:

$$
\begin{equation*}
\frac{\partial \tilde{\rho}}{\partial t}+\frac{d q}{d \rho}\left(\rho^{*}\right) \frac{\partial \tilde{\rho}}{\partial x}=0 . \tag{2}
\end{equation*}
$$

Here, $\tilde{\rho}=\rho-\rho^{*}$, is the deviation of the traffic density from the basic solution. Call $\frac{d q}{d \rho}\left(\rho^{*}\right)=c\left(\rho^{*}\right) . c\left(\rho^{*}\right)$ is the characteristic speed of the traffic wave and is the slope of the equilibrium traffic flow-density relationship. The solution of this partial differential equation is $\tilde{\rho}(x, t)=F(x-c t)$ around the vicinity of the basic or equilibrium solution, where $F$ is any differentiable function determined from the value of disturbance at some initial time.

It is important to understand the desirable characteristics of traffic flow at this stage. If there are density disturbances to the traffic flow, say due to a burst of vehicles after a football game, it should not propagate downstream unattenuated in space and time. Otherwise, for a traffic headed towards point $B$ from point $A$, any density disturbances at point $B$ will be felt at point $A$.

The sign of $c$ [21] determines how density disturbances are propagated. If $c>0$, the density disturbance propagates as a forward traveling wave. This is acceptable, and this implies that the density disturbances due to, say, a burst of vehicles are actually traveling forward in space and contributing to the throughput. If $c<0$, the density disturbance propagates upstream and is a backward traveling wave. Therefore, arbitrarily small density disturbances in traffic propagate upstream unattenuated and is undesirable of a traffic on a freeway. It should be pointed out that the both the forward and backward traveling wave solutions are stable by the conventional stability definitions; however, the backward traveling wave solution represents an undesirable traffic flow behavior. If $c<0$, inertial effects, i.e. cruise control system dynamics aggravate this behavior further, and result in traffic flow instability, see Swaroop and Rajagopal [36].

Payne [24] considered a non-automated traffic flow and developed a macroscopic traffic flow model incorporating the dynamics of aggregate velocity of vehicles. The speed density relationship at equilibrium (or steady state) is embedded in this model. This model is a basis for a considerable number of freeway traffic control algorithms [24,21] and is given by:

$$
\begin{equation*}
\frac{\partial v}{\partial t}+v \frac{\partial v}{\partial x}=\frac{1}{\tau}[h(\rho)-v]+\frac{1}{2} \frac{\partial h}{\partial \rho} \frac{\partial \rho}{\partial x} . \tag{3}
\end{equation*}
$$

In the above equation, $h(\rho)$ is the equilibrium velocity at an equilibrium density, $\rho$.

A continuum approximation of traffic flow behavior does away with human element in non-automated vehicle following. As a result, the constitutive equation represents a crude approximation of the aggregate traffic speed dynamics. Parameters to this model are obtained empirically [21, 24].

The problem of obtaining a constitutive equation, similar to the balance of linear momentum in continua or Newton's second law for a system of particles, for the automated traffic is addressed in [36]. This model is used to study the behavior of automated traffic in this report. This is in contrast to the model developed above in equation 3 and adopted for analysis in automated highway systems by Karaslaan and Varaiya [15], and by Chien, Ioannou and Stotsky [4]. The relationship between ICC systems and traffic flow behavior recently investigated by Swaroop and Rajagopal [36] is used in this paper to demonstrate the effectiveness of proposed variable spacing control algorithm.

### 2.1 Automated Traffic Flow Behavior with Variable Spacing algorithms

At this stage, it is important to delineate the difference between string stability and desired traffic flow behavior in an automated traffic. The behavior of an automated traffic flow is governed not only by the individual vehicle following control laws, but also constrained by the conservation of mass principle. Thus far, the conservation of mass principle has never been considered in the string stability analyses performed for automated vehicle following in the literature. Therefore, guaranteeing string stability is by no means sufficient to guarantee that a desired traffic flow behavior can be achieved. For a further discussion, the reader is referred to Swaroop and Rajagopal [36]. For the sake of completeness, analysis of automated (or non-automated) traffic flow with variable spacing policies in [36] is presented here.

An analysis of the traffic flow behavior begins with the conservation of mass equation:

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0 . \tag{4}
\end{equation*}
$$

Here, $v$ is the aggregate speed of the automated traffic, and $\rho$ is the aggregate density.

In the derivation of aggregate speed dynamics, it is assumed that the acceleration of every vehicle can be controlled, i.e., it can be assigned any value. Let $L_{c}$ be the length of a car, and $\Delta$ be the following distance, and let $\bar{h}(\Delta)$ be the desired speed of a vehicle, $v_{d e s}$ at the following distance, $\Delta$. Define an error, $e_{v}$ in the velocity as:

$$
\boldsymbol{e}_{v}:=v-\bar{h}(\Delta)=v-h(\rho) .
$$

In the above equation, $\rho=\frac{1}{\Delta+L_{c}}$ and this equation holds if string stability is guaranteed. The following control law:

$$
a_{d e s}=\dot{v}=\bar{h}(\dot{\Delta})-\frac{1}{\tau} e_{v}
$$

guarantees that the error in velocity, $e_{v}$ decays to zero exponentially with a time constant, $\tau$.

Since

$$
\frac{d v}{d t}=\frac{\partial v}{\partial t}+v \frac{\partial v}{\partial x},
$$

$$
\frac{\text { and }}{d t}=\frac{\partial \rho}{\partial t}+v \frac{\partial \rho}{\partial x}
$$

it follows from the vehicle following control law, that

$$
\begin{equation*}
\frac{\partial v}{\partial t}+\left(v+\rho \frac{\partial h(\rho)}{\partial \rho}\right) \frac{\partial v}{\partial x}=\frac{1}{\tau}[h(\rho)-v] . \tag{5}
\end{equation*}
$$

A stationary solution admitted by the equations 4 and 5 is $\rho=\rho_{0}$ and $v=h\left(\rho_{0}\right)$. To study the linearized stability of such a stationary solution, consider the following possible perturbed backward traveling wave solutions,

$$
\begin{aligned}
\rho & =\rho_{0}+\tilde{\rho} e^{i k(x+c t)} e^{\lambda t} \\
v & =h\left(\rho_{0}\right)+\tilde{v} e^{i k(x+c t)} e^{\lambda t} .
\end{aligned}
$$

In the above equations, $k, c, \lambda$ are real numbers. In fact, only if $c>0$, does the solution represent a backward traveling wave. In such a case, $c$ is the speed of the backward traveling wave. Define $q_{c}^{\prime}(\rho):=\rho \frac{d h(\rho)}{d \rho}+h(\rho)$. $\lambda$ is indicative of the attenuation or non-attenuation of the traveling wave.

If one ignores higher order terms in $\tilde{v}, \tilde{\rho}$ when the above set of equations is substituted into the equations 4 and 5 , it follows that for a non-trivial solution, the following conditions must hold:

$$
\begin{aligned}
k^{2}(c+h(\rho))\left(c+q_{c}^{\prime}\left(\rho_{0}\right)\right) & =\lambda\left(\lambda+\frac{1}{\tau}\right), \\
\lambda\left(c+q_{c}^{\prime}\left(\rho_{0}\right)\right)+\left(\lambda+\frac{1}{\tau}\right)\left(c+h\left(\rho_{0}\right)\right)+\frac{q_{c}^{\prime}\left(\rho_{0}\right)-h\left(\rho_{0}\right)}{\tau} & =0 .
\end{aligned}
$$

Note that $\lambda=0, c=-q_{c}^{\prime}(\rho)$ and $k$ is any non-trivial real number satisfies the above set of algebraic equations. In other words, the perturbed solutions agree quite closely with the traveling wave solutions when the deviation of the perturbed solution from the nominal solution is sufficiently small. Such an observation is in concurrence with the simulation results and with the results of Lighthill and Whitham [18].

With a constant spacing policy, the slope, $c(\rho)$, of the fundamental traffic characteristic at a given density, $\rho$, is always constant and equals $-\frac{L_{c}}{h_{w}}$, where $L_{c}$ is the car length and $h_{w}$ is the value of the time headway employed by the ICC system. Therefore, small density disturbances propagate upstream without any attenuation and the constant time headway policy is unstable according to the definition given in [36].

## 3 ICC Algorithm Design and Traffic Flow Stability

An ICC algorithm is constrained by the following [36]: Any spacing policy chosen for cruise control applications must be such that the desired following distance (and consequently the inverse of equilibrium density) is a nondecreasing continuous function of velocity. As a result, the minimum value of desired following distance (and hence, maximum equilibrium or jam density)
occurs when the velocity is zero. Since the steady state traffic volume is the product of steady state traffic density and velocity, it follows that the traffic volume is very small at very low densities and zero at the jam density. By an application of Rolle's Theorem, it follows that the fundamental characteristic obtained with any spacing policy must have a density regime where its slope is negative. By the results from the above section, it follows that no ICC scheme can guarantee traffic flow stability through the entire density regime.

The spacing policy used in an ICC algorithm can be synthesized from the specification of the fundamental traffic characteristic. A typical fundamental characteristic of automated traffic equipped with autonomous ICC is shown in the figure below.


Figure 1: Typical Fundamental Traffic Characteristic
The impossibility of obtaining a stable traffic flow behavior over the entire density regime suggests that a specification on the peak traffic capacity, $C_{\text {max }}$ and on critical density, $\rho_{c r}$ is a reasonable one. Such a specification is subject to the following reality checks:

1. Vehicle capabilities: The spacing policy synthesized from the specifications dictates how a controlled vehicle must change its speed with respect to the following distance. If the spacing policy requires a significant change in the speed corresponding to a small change in the following distance, vehicle capabilities may not be adequate.
2. Speed of upstream propagation of traffic waves: The slope of the unstable regime of the fundamental traffic characteristic dictates the speed of upstream propagation of traffic waves. Hence, requiring a high critical density and a high traffic capacity, will result in a faster upstream, undesirable propagation of traffic waves in the unstable regime and this can potentially result in collisions between vehicles.
It is appropriate to discuss the synthesis of ICC algorithms now. A feedback linearized vehicle model is given by

$$
\ddot{x}=u,
$$

where $u$ is the synthetic control effort and can be translated into throttle and brake commands, see [37].

### 3.1 ICC algorithm based on traffic flow specification

Suppose that $v_{f}$ can be changed in accordance with the specification on maximum traffic capacity. With Greenshield's relation,

$$
v=v_{f}\left(1-\frac{\rho}{\rho_{\max }}\right),
$$

and therefore, in equilibrium

$$
q=v_{f} \rho\left(1-\frac{\rho}{\rho_{\max }}\right)=v_{f} \rho\left(1-\rho L_{0}\right),
$$

where $L_{0}=\frac{1}{\rho_{\max }}$ is the distance headway at standstill plus the vehicle length. $c=\frac{d q}{d \rho}$ is positive for all $\rho<\rho_{c r}=\frac{\rho_{\max }}{2}$. Therefore, in order to achieve a maximum traffic capacity, $C_{\max }, v_{f}=\frac{4 C_{\max }}{\rho_{\max }}$. In order to synthesize a control law that produces Greenshield's traffic characteristic, the following spacing policy must be adopted:

$$
\Delta_{d e s}=\frac{L_{0}}{1-\frac{v}{v_{f}}}-L_{c},
$$

where $\Delta_{\text {des }}$ is the desired following distance of the controlled vehicle. A control law which regulates spacing according to this policy is given by:

$$
u=k_{1}\left(\Delta+L_{c}-\frac{L_{0}}{1-\frac{v}{v_{f}}}\right)+k_{2} \dot{\Delta} .
$$

Here $\Delta$ and $\dot{\Delta}$ are the range (following distance) and the range rate of the controlled vehicle. The measurements of $\Delta$ and $\dot{\Delta}$ are obtained from on-board sensors such as a sonar or a radar. In the above control law, $k_{1}, k_{2}$ are associated control gains. An obvious troublesome point is when vehicle velocity as measured by sensors is $v_{f}$. In order to overcome such a problem, let

$$
v_{d e s}=v_{f}\left(1-\left(\frac{L_{0}}{\Delta+L_{c}}\right)\right) .
$$

Consider the following control law:

$$
u=\dot{v}_{\text {des }}-K\left(v-v_{d e s}\right)=v_{f} \frac{L_{0}}{\left(\Delta+L_{c}\right)^{2}} \dot{\Delta}-K\left(v-v_{f}\left(1-\frac{L_{0}}{\Delta+L_{c}}\right)\right) .
$$

In the above equation, $K>0$ is the control gain. The disadvantage with the above policy is that the critical density is always fixed at half the jam density.

A simple modification of the above spacing policy, available in the literature [7], can be used to overcome the above disadvantage. Consider the following equilibrium traffic speed-density characteristic:

$$
v=v_{f}\left(1-\left(\frac{\rho}{\rho_{\max }}\right)^{l}\right)^{m}
$$

where $v_{f}, l, m$ are design parameters for the spacing policy and can be chosen to meet the specifications on peak traffic volume and critical density. As a result, equilibrium traffic volume is given by

$$
q=v_{f} \rho\left(1-\left(\frac{\rho}{\rho_{\max }}\right)^{l}\right)^{m}
$$

The critical density for this characteristic is given by:

$$
\rho_{c r}=\rho_{\max }\left(\frac{1}{1+l m}\right)^{\frac{1}{l}}
$$

Given any $\rho^{*}$ as a specification for critical density, there exist a choice of $l, m$ that satisfy this specification. For example, a choice of $l=1, m=\frac{\rho_{\max }-\rho^{*}}{\rho^{*}}$ yield $\rho_{c r}=\rho^{*}$. Once the exponents $l, m$ are chosen, a specification on the peak traffic volume, $C_{\max }$ can be met by the following choice of $v_{f}$

$$
v_{f}=\frac{C_{\max }(1+l m)^{\frac{1+l m}{l}}}{\rho_{\max }(l m)^{m}}
$$

Corresponding to this specification, the desired velocity, $v_{d e s}$ must vary according to the following relation:

$$
v_{d e s}=v_{f}\left(1-\left(\frac{L_{0}}{\Delta+L_{c}}\right)^{l}\right)^{m}
$$

An implementable ICC law that yields the specified traffic flow characteristic is

$$
u=\dot{v}_{d e s}-K\left(v-v_{d e s}\right)
$$

### 3.2 String Stability in Variable Spacing algorithms

String stability can only be guaranteed for small deviations of the lead vehicle's speed from the operating speed. However, simulations with an ICC algorithm employing Greenshield's policy indicate the absence of any string instability for any lead vehicle's maneuvers. Figures 2 and 3 indicate that string stability is achieved. Figure 2 shows how the spacing between vehicles change and figure 3 indicates the evolution of velocity of vehicles in a string.

### 3.3 Sensitivity Issues

The sensitivity of the desired velocity as a function of the actual following distance is important in the design of Intelligent Cruise Control Systems. It provides a measure of vehicle capabilities required to implement a cruise control system. In this paper, $S(\Delta)$ represents the sensitivity and is defined as:

$$
S(\Delta):=v_{d e s} \frac{d v_{d e s}}{d \Delta}
$$

This measure of sensitivity is indicative of the desired reduction in kinetic energy for a unit change in the following distance and directly relates to the vehicle's braking capability. A higher value of sensitivity is required for higher traffic throughput and demands higher vehicle braking and acceleration capabilities.

For the proposed spacing policy, the sensitivity is given by

$$
S(\Delta)=\operatorname{lm} v_{f}^{2}\left(1-\left(\frac{L_{0}}{\Delta+L_{c}}\right)^{l}\right)^{2 m-1}\left(\frac{L_{0}}{\Delta+L_{c}}\right)^{l} \frac{1}{\Delta+L_{c}}
$$



Figure 2: Evolution of spacing between vehicles in a string with the proposed variable spacing policy

A plot showing the sensitivity for the Greenshield's policy is shown in Figure 4. It is interesting to see that sensitivity has a maximum value, $S_{\max }$, at a nonzero following distance and tapers to zero for higher following distances. For this spacing policy, maximum sensitivity occurs at

$$
\Delta=L_{0}\left(\frac{2 l m+1}{l+1}\right)^{\frac{1}{\imath}}-L_{c},
$$

and the maximum value of sensitivity is given by:

$$
S_{\max }(\Delta)=\frac{v_{f}^{2}}{L_{0}} \operatorname{lm}\left(\frac{2 l m-l}{2 l m+1}\right)^{2 m-1}\left(\frac{l+1}{2 l m+1}\right)^{\frac{l+1}{l}} .
$$

To summarize the design, there are three design parameters, $v_{f}, l$ and $m$. There are two specifications - $C_{m a x}$, the desired maximum throughput and $\rho_{c r}$, the critical traffic density. The extra design parameter can be used to minimize the maximum sensitivity so that this spacing policy does not demand high acceleration/braking performance of vehicles.

## 4 Simulation Results

The following specifications on traffic flow are chosen for simulation purposes: $C_{\max }=3240$ vehicles/lane/hour, $\rho_{c r}=\frac{\rho_{\max }}{2}$. An average vehicle length of 5 m and a distance between vehicles at standstill of 5 m is chosen for this simulation. One can verify that the following values of parameters, $v_{f}=36 \mathrm{~m} / \mathrm{s}$ in Greenshield's spacing policy ( $l=1, m=1$ ) satisfy the traffic flow specification. Correspondingly, the maximum sensitivity value is $S_{\max }(\Delta)=19.2 \mathrm{~m} / \mathrm{s}^{2}$ and


Figure 3: Evolution of velocity of vehicles in a string with the proposed variable spacing policy
the maximum sensitivity occurs when the following distance is twice the vehicle length.

The simulations obtained with an ICC system based on Greenshield's spacing policy will be compared with an AICC [14] system based on a constant time headway policy. In a constant time headway spacing policy, the desired velocity is linearly proportional to the following distance, i.e

$$
v_{d e s}=\frac{1}{h_{w}}\left[\Delta+L_{c}-L_{0}\right] .
$$

For simulations, $h_{w}$ is chosen as 1 sec and $L_{0}$ as 10 m .
A section of length 1000 m of a single lane of freeway is considered. An on-ramp is located at 350 m from the downstream end of this section, where vehicles merge into the traffic. The simulations are set up such that the mainline traffic volume and the merging traffic volume is the same with both spacing policies. The mainline traffic volume considered in this simulation is 2700 vehicles/lane/hour. In other words, three vehicles enter the upstream end of the section every four seconds. Vehicles entering the upstream end of the section are set up such that they do not have any velocity error (their speed matches that of their immediate predecessor). However, they may have a spacing error, i.e the actual following distance may not necessarily match the desired following distance.

Traffic from on-ramp starts only after $t=50$ seconds in these simulation. Vehicles merge in between vehicles that are closest to the ramp, with its position and velocity as the mean position and velocity of the vehicles closest to the ramp. Two types of merging traffic are considered: bursty (or pulse) and continuous stream. A bursty on-ramp traffic may be thought of as a pulse


Figure 4: Sensitivity plot of the proposed spacing policy
disturbance in the traffic demand. In this simulation, a total of eight vehicles from the on-ramp merge into the traffic 20 seconds apart (i.e from $t=55 \mathrm{sec}$ to $t=195 \mathrm{sec}$ ). In the second scenario, a continuous stream of on-ramp traffic with a traffic volume of 180 vehicles/hour is considered.

There are four cases in all: Greenshield's policy with bursty on-ramp traffic, constant time headway policy with bursty on-ramp traffic, Greenshield's policy with continuous stream of on-ramp traffic, constant time headway policy with continous stream of on-ramp traffic. Corresponding to each case, the following statistics are collected every half second: the mean velocity of all the vehicles in the section, the cumulative number of vehicles that have entered the section either from the mainline or from the on-ramp, the total number of vehicles in the section and the position of all vehicles in the section at the sampled time instant.

There are four plots, one for each statistic: The first one depicts the evolution of traffic density. The discrete nature of counting along with the fact that the section length cannot always be an integral multiple of the intervehicular distance leads to the spikes in the plot. Depending on the frequency of vehicle count, there are either upward spikes or downward spikes in a given interval of time. For example, in figure 3, the frequency of a count of 31 vehicles is much higher than the frequency of a count of 30 vehicles in the highway section in the time interval from 75 to 200 seconds. Hence you see downward spikes during that time interval.

The second plot depicts the aggregate traffic speed evolution. Chattering observed in this plot is also due to the discrete nature of counting vehicles and averaging them. As a result of averaging, this plot is smoother than the density evolution plot.

The third plot shows how the input and output traffic volumes of the section vary as a function of time.

The fourth plot is the space-time plot. Every single dot in this plot is described by two coordinates. The horizontal coordinate represents time and the vertical coordinate represents the position of a vehicle at that given time. A lower value of the vertical coordinate implies that the vehicle is close to the upstream end of the section at that time. The number of vehicles in the section at any time can be determined by counting the number of dots corresponding to that time. A higher density of dots in a part of the section at any time implies that the traffic is congested in that part of the section at that time. One can also infer the intervehicular spacing distribution at any given time from this plot similarly.

Figures 5 to 8 describe the behavior of traffic with a Greenshield's spacing policy and with a burst of vehicles entering the highway section from the on-ramp. Figures 9 to 12 describe the corresponding traffic behavior with a constant time headway policy. Figures 13 to 16 describe the traffic behavior with Greenshield's spacing policy and with a steady stream of vehicles entering the highway section from the on-ramp. Figures 17 to 20 describe the corresponding behavior with a constant time headway policy.

Clearly, one can see the backward propagation of traffic density from the space-time charts with a constant time headway policy and a forward propagation of traffic density with an ICC system based on Greenshield's spacing policy. The results agree with the theory. From the figures, it is clear that an ICC system based on Greenshield's spacing policy is much superior to the AICC system based on a constant time headway policy.


Figure 5: Greenshields policy-Bursty on-ramp traffic-Traffic density evolution plot


Figure 6: Greenshields policy-Bursty on-ramp traffic-Traffic speed evolution plot

## 5 Summary and Conclusions

In the presence of favorable density and velocity gradients, traffic flow can be locally stable past the critical density value. Conversely, in the presence of unfavorable density and velocity gradients, traffic flow can become unstable for basic flow densities smaller than the critical density. This is consistent with widely observed traffic data. From the analysis, it has been demonstrated that a well designed spacing policy for an ICC system must be such that the slope of the corresponding fundamental traffic characteristic must always be positive. Numerical simulations verify this fact.

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Figure 7: Greenshields policy-Bursty on-ramp traffic-Input and Output traffic volumes
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Figure 12: AICC -Bursty on-ramp traffic-Space-time chart


Figure 13: Greenshields policy-Steady on-ramp traffic-Traffic density evolution plot


Figure 14: Greenshields policy-Steady on-ramp traffic-Traffic speed evolution plot


Figure 15: Greenshields policy-Steady on-ramp traffic-Input and Output traffic volumes


Figure 16: Greenshields policy-Steady on-ramp traffic-Space-time chart


Figure 17: AICC-Steady on-ramp traffic-Traffic density evolution plot


Figure 18: AICC-Steady on-ramp traffic-Traffic speed evolution plot


Figure 19: AICC-Steady on-ramp traffic-Input and Output traffic volumes


Figure 20: AICC-Steady on-ramp traffic-Space-time chart


[^0]:    ${ }^{1}$ accepted for publication in the Special Issue on Intelligent Transportation Systems, Vehicle System Dynamics Journal
    ${ }^{2}$ Assistant Professor
    ${ }^{3}$ Graduate Student

