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USER INFORMATION MANUAL. SUBSID - A NONLINEAR, TOO DIMENSIONAL FINITE ELEMENT PROGRAM FOR STATIC EVALUATION OF MINING SUBSIDENCE

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USER INFORMATION MANUAL SUBSID — A NONLINEAR, TWO DIMENSIONAL FINITE ELEMENT PROGRAM FOR STATIC EVALUATION OF MINING SUBSIDENCE

J. L. Ratigan

June 1980

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

LBL-11356

USER INFORMATION MANUAL

SUBSID - A NONLINEAR, TWO DIMENSIONAL FINITE ELEMENT PROGRAM FOR STATIC EVALUATION OF MINING SUBSIDENCE

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June 30, 1980

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1. INTRODUCTION

In modified in-situ (MIS) oil shale retorting, the resource is processed in the ground. Large chambers of rubblized oil shale are formed by mining out about 20% to 40% of the in-place shale and blasting the balance into the created void. The mined-out material is brought to the surface and oil is recovered from it by surface retorting. The in-place material is pyrolyzed to recover oil, leaving large numbers of abandoned retort chambers underground.

This type of oil shale processing may result in a number of environmental problems, including in-situ leaching of the abandoned retorts, low resource recovery (large pillars are required to support the overburden), and subsidence. These problems may be mitigated by filling abandoned retorts with a grout that would fill the void space, thus improving retort structural strength and stiffness, and reducing retort permeability to groundwater flow. If sufficient strength and stiffness can be developed, it may be possible to design retorts so that the pillars can be retorted and resource recovery improved.

This document describes a computer program that was developed and used to evaluate the strength and stiffness required in a grouted retort to support the overburden and to improve resource recovery. The program SUBSID was developed to predict subsidence over a field of MIS retorts under various conditions of retort backfill. In MIS retorting the pillars between retorts form a continuous structure, separating individual retorts (Fig. 1). This is the reverse of conventional room-and-pillar mining, where the pillars are discontinuous in a continuous room. SUBSID models either case in two dimensions. In subsequent sections of this report, retorts are referred to as "rooms" to maintain consistency with usual mining terminology.

Where subsidence must be prevented or limited, backfilling retorts can permit retort design with a higher extraction ratio than would otherwise be possible. SUBSID can be used to relate the desired increase in extraction ratio to the required structural properties of the retorts after backfilling, thus allowing costs and benefits to be balanced against each other. These considerations are relevant to MIS oil shale retorting, because slurry backfilling or grouting MIS retorts with the surface-retorted spent shale or other grouts may be required to reduce groundwater pollution caused by leaching of the in-situ spent shale. The benefit here would come from reducing the permeability of the retort. Additional benefit may also result from developing structural strength in the retort if this permits the extraction ratio to be increased, or the pillars to be retorted in a second pass. For a complete discussion of the problems of retort abandonment and proposals for retort grouting, see Fox and Persoff (1980).

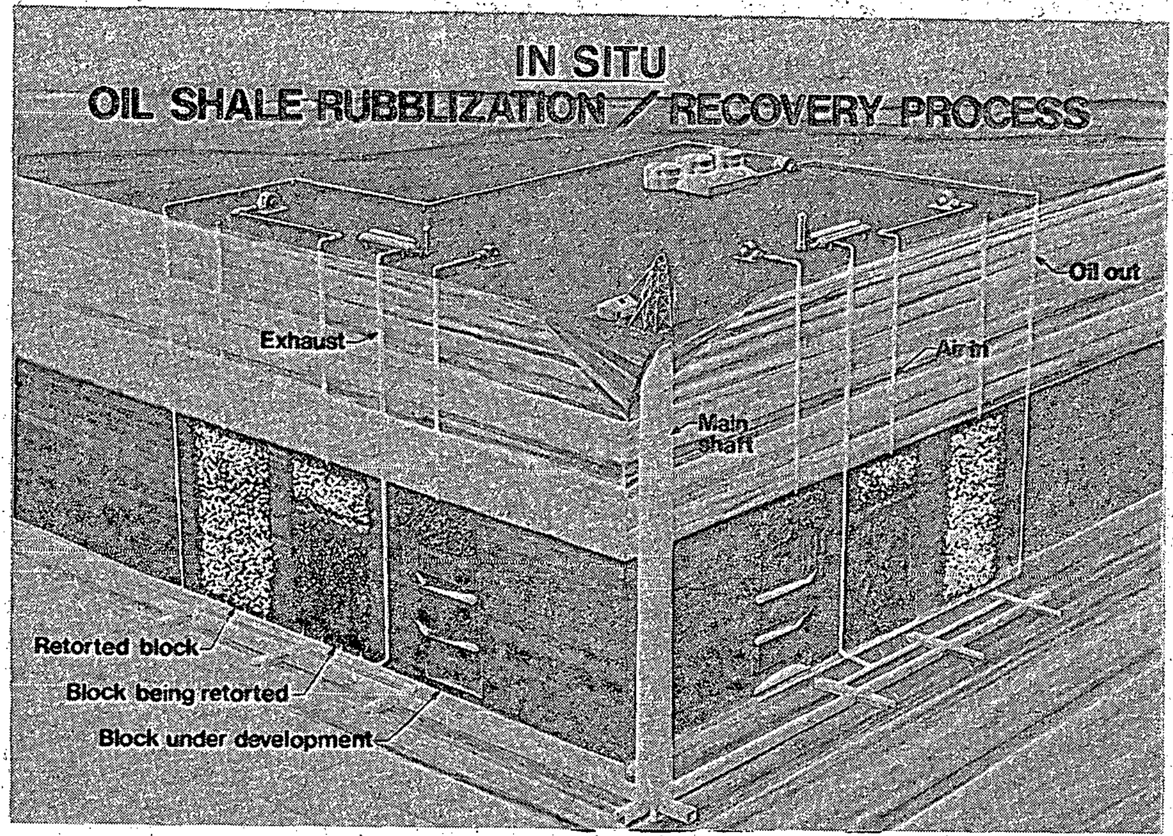

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Fig. 1. Schematic representation of modified in-situ (MIS) retorting. Six retorts are shown here; a commercial operation would have hundreds. (Source: Campbell et al., 1977.)

2. DESCRIPTION OF THE PROGRAM

SUBSID is a nonlinear, two-dimensional, finite-element computer program for evaluating static subsidence in a room-and-pillar or a lane-and-pillar configuration with homogeneous and isotropic overburden and homogeneous and isotropic pillars. The basic structural elements within the program are:

- overburden elements
- pillar elements
- room elements

The two-dimensional overburden elements possess six degress of freedom; specifically, horizontal and vertical translation and rotation at both ends. The behavior of these elements is governed by the slope deflection equations of classical beam theory (Wilson, 1978). Shear deflections are optional in the overburden elements.

The pillar elements possess three degrees of freedom, all at the overburden/pillar connection. The pillars may rotate or translate vertically or horizontally. Each pillar is considered to be rigidly attached to a foundation at its lower end (i.e., the lower end may not rotate or translate).

The room elements are similar to the pillar elements. These elements may represent totally excavated cavities by specifying a compressive strength of zero, or they may represent "backfilled" rooms with the appropriate input parameters. Room elements are also considered to be rigidly attached to a foundation at the lower end. Shear deflections are optional for the pillar and room elements.

The uniaxial pillar and room elements do not allow for horizontal variations in vertical stress and are only connected structurally via the overburden elements. The response of the pillar and room elements to load is characterized by an initial pore volume decrease and collapse followed by a monotonic strain hardening behavior asymptotically approaching the unconfined compressive strength. Only response to loading in compression is modeled in the program.

Program SUBSID is operational on the CDC 7600 computer at the Lawrence Berkeley Laboratory. The program is compatible with other CDC systems; however, the dynamic storage feature may require updating for other CDC installations. The program is written in FORTRAN IV. A listing of SUBSID is provided in Appendix A, and the job control language for execution on the LBL CDC-7600 is given in Appendix B.

The following sections of the report provide a description of the constitutive behavior of the structural elements, the numerical algorithm for the problem solution, and the organization of input and output data. An example of program usage is also provided.

3. CONSTITUTIVE RELATIONSHIPS

The uniaxial stress/strain behavior of the pillar and room elements is illustrated in Fig. 2. Region I of the stress/strain curve represents pore volume decrease or microcrack closing and Region II represents strain hardening of the matrix material. Mathematically:

$$\begin{aligned} \sigma &= k(\varepsilon/\varepsilon^*)^n && \text{for } \varepsilon \leqslant \varepsilon^* \\ \sigma &= (C_o - k)\,[1 - e^{-\lambda(\varepsilon-\varepsilon^*)}] + k && \text{for } \varepsilon \geqslant \varepsilon^* \end{aligned} \tag{1}$$

where σ = uniaxial stress,

ε = uniaxial strain,

C_o = unconfined compressive strength,

ε^*, n, λ = constants determined from laboratory stress/strain data.

Note that the maximum tangent modulus from Eq. (1) occurs at $\varepsilon = \varepsilon^*$. Mathematically, the maximum tangent modulus is:

$$\max \frac{\partial\sigma}{\partial\varepsilon} = \lambda(C_o - k) \quad .$$

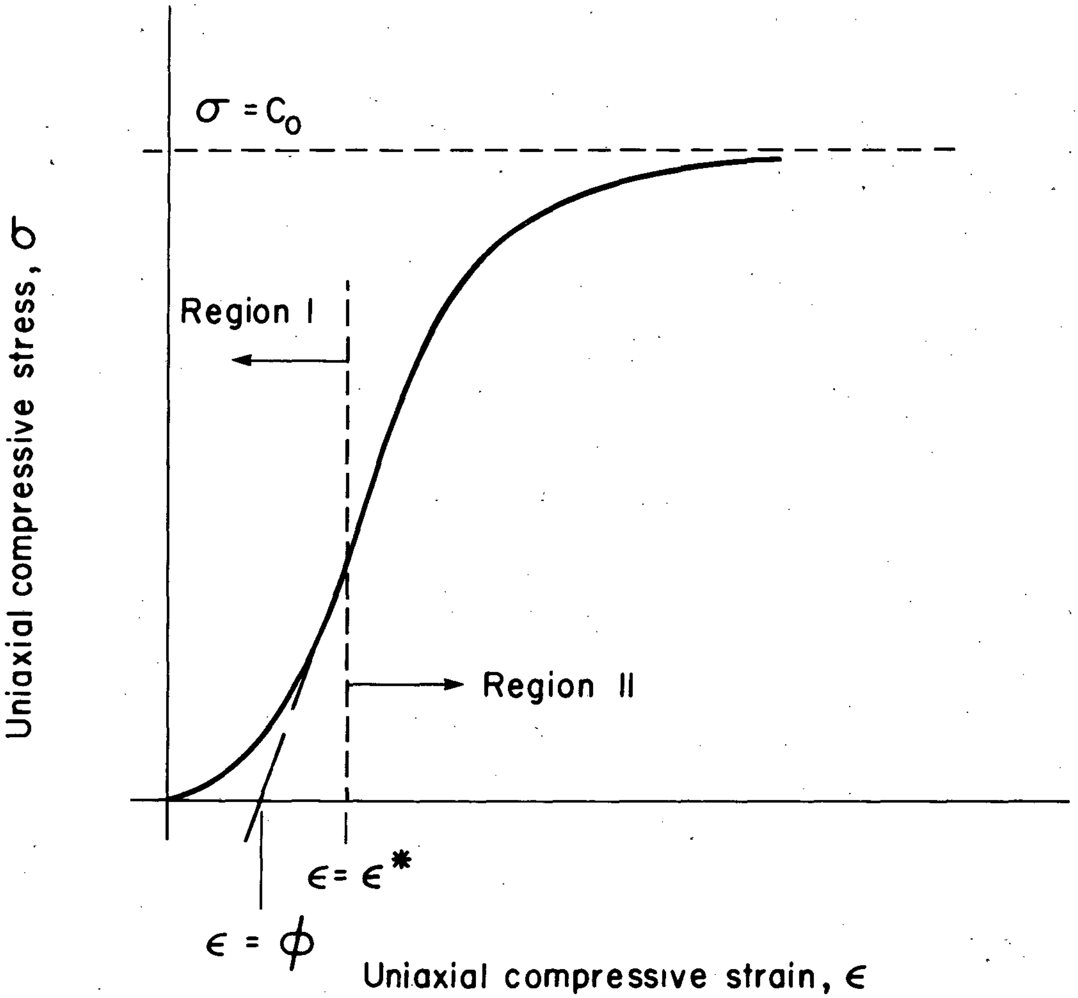
From the requirements of stress and stiffness continuity at $\varepsilon = \varepsilon^*$, the following relationship is obtained:

$$k = \frac{\lambda C_o \varepsilon^*}{n + \lambda\varepsilon^*} \quad . \tag{2}$$

If we consider that the pore structure has collapsed at ε^*, the permanent deformation which would be present from an elastic unloading from ε^* (with a tangent modulus at ε^*), would represent the pre-loading porosity, ϕ.

The resulting expression for pre-loading porosity is obtained:

$$\phi = \frac{\varepsilon^*(n-1)}{n} \quad . \tag{3}$$



XBL 809-1973

Fig. 2. Uniaxial stress/strain response for pillar and room elements.

Bending and flexural deformation within the pillar and room elements is considered to be linear with the applied load. The deformation modulus for these modes of loading is taken to be identical to the tangent modulus from the stress/strain curve of Fig. 2 at the present vertical compressive strain. Shear deformations in the pillar and room elements may be included.

The load/deformation response of the overburden elements is taken to be linear and constant. Classical beam theory applies with the addition of shear deformations.

4. ELEMENT STIFFNESS MATRICES

This section describes the form of the element stiffnesses. The general form of the force/displacement relations is:

$$[K^e]\{U^e\} = \{F^e\} , \tag{4}$$

where $[K^e]$ = the element stiffness matrix,

$\{U^e\}$ = element displacement vector,

$\{F^e\}$ = element force vector.

Discussion of the direct stiffness method and further explanation of Eq. (4) are given by Wilson (1978).

4.1 Overburden Elements

The forces, displacements, and appropriate sign convention for an overburden element are illustrated in Fig. 3. The corresponding form of Eq. (4) is:

$$\frac{E_bI_b}{L(1+2\beta_b)}\left[\begin{array}{cccccc}
\frac{A_b(1+2\beta_b)}{I_b} & & & & & \\
0 & \frac{12}{L^2} & & & \text{SYMMETRIC} & \\
0 & -\frac{6}{L} & 4+2\beta_b & & & \\
-\frac{A_b(1+2\beta_b)}{I_b} & 0 & 0 & \frac{A_b(1+2\beta_b)}{I_b} & & \\
0 & -\frac{12}{L^2} & -\frac{6}{L} & & \frac{12}{L^2} & \\
0 & \frac{6}{L} & 2-2\beta_b & 0 & -\frac{6}{L} & 4+2\beta_b
\end{array}\right]
\left[\begin{array}{c} X_1 \\ Y_1 \\ \theta_1 \\ X_2 \\ Y_2 \\ \theta_2 \end{array}\right]
=
\left[\begin{array}{c} F_{X_1} \\ F_{Y_1} \\ F_{\theta_1} \\ F_{X_2} \\ F_{Y_2} \\ F_{\theta_2} \end{array}\right] \tag{5}$$

where E_b = Young's modulus of the overburden element,

I_b = moment of inertia of the overburden element about the z axis,

L = length of the overburden element,

A_b = cross-sectional area of the overburden element,

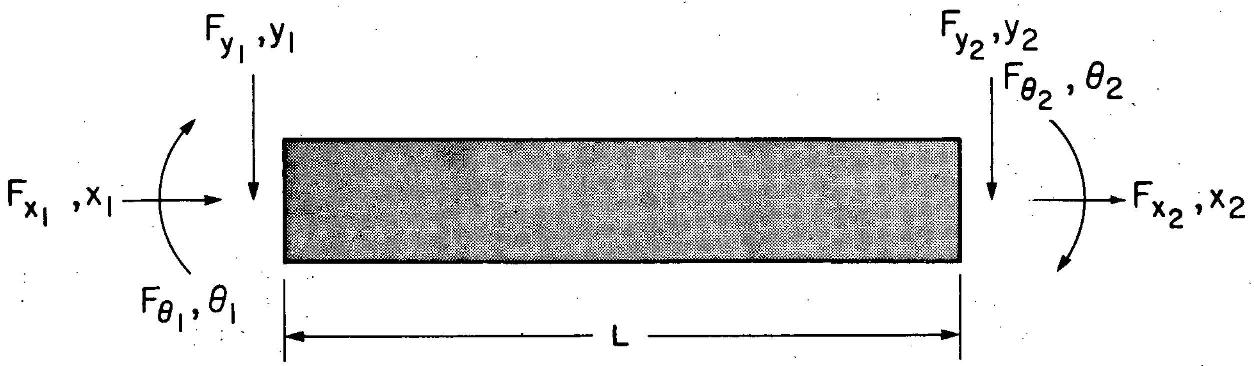


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Fig. 3. Overburden element forces, displacements, and positive sign convention.

$\beta_b = 6E_bI_b/L^2\bar{A}_pG_b = 12I_b(1+\nu)/L^2\bar{A}_p$,

$\bar{A}_p$ = effective area of the overburden element cross section ($5A_b/6$ for a rectangular cross-section; see any elementary structures text),

G_b = shear modulus of the overburden element,

ν = Poisson's ratio.

4.2 Pillar Element

The forces, displacements, and appropriate sign convention for a pillar element are illustrated in Fig. 4. The force/displacement equations are:

$$
\begin{aligned}
F_{X_p} &= \frac{12E_pI_p}{H_p^{\,3}(1+2\beta_p)} \cdot X_p - \frac{6E_pI_p}{H_p^{\,2}(1+2\beta_p)} \cdot \theta_p \\
F_{Y_p} &= k\,A_p\left[\frac{Y_p}{H_p\varepsilon^*}\right]^n && \text{for } \frac{Y_p}{H_p} \leqslant \varepsilon^* \\
&= A_p(C_o-k)\left\{1-\exp\left[-\lambda\left(\frac{Y_p}{H_p}-\varepsilon^*\right)\right]\right\} + k\,A_p && \text{for } \frac{Y_p}{H_p} \geqslant \varepsilon^* \qquad (6) \\
F_{\theta_p} &= -\frac{6E_pI_p}{H_p^{\,2}(1+2\beta_p)} \cdot X_p + \left[\frac{4+2\beta_p}{1+2\beta_p}\right]\frac{E_pI_p}{H_p} \cdot \theta_p
\end{aligned}
$$

where E_p = tangent modulus of the pillar at Yp/Hp from Eq. (1),

I_p = moment of inertia of the pillar about the z axis,

H_p = height of the pillar,

A_p = cross-sectional area of the pillar,

β_p = shear strain parameter for the pillar (see Eq. (5)).

The following relations between the displacement and forces of the pillar connected to node "i" of an overburden element and the displacements and forces at the overburden element node "i" can be stated as:

$$
\begin{aligned}
X_p &= X_i - \theta_iH_b/2 & F_{X_i} &= F_{X_p} \\
Y_p &= Y_b & F_{X_i} &= F_{y_p} \\
\theta_p &= \theta_b & F_{\theta_i} &= F_{\theta_p} - \left(H_b \cdot F_{X_p}/2\right)
\end{aligned}
$$

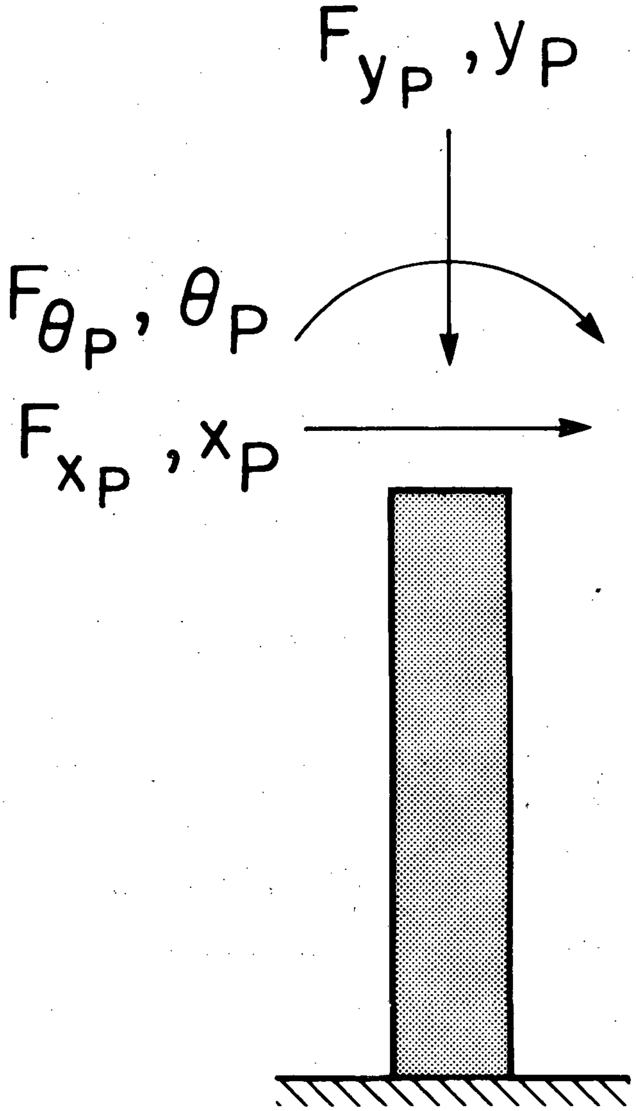

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Fig. 4. Pillar element forces, displacements, and positive sign convention.

where H_b = the height of the overburden element. Then, Eq. (6) becomes (in terms of the displacements and forces at the ith node):

$$\begin{aligned}
F_{X_i} &= \frac{12E_pI_p}{H_p^3(1+2\beta_p)} \cdot X_i - \frac{6E_pI_p}{H_p^2(1+2\beta_p)} \left(1 + \frac{H_b}{H_p}\right) \cdot \theta_i \\
F_{Y_i} &= k\,A_p \left[\frac{Y_i}{H_p\varepsilon^*}\right]^n & \text{for } \frac{Y_i}{H_p} \leqslant \varepsilon^* \\
&= A_p(C_o-k) \left\{1-\exp\left[-\lambda\left(\frac{Y_i}{H_p} - \varepsilon^*\right)\right]\right\} + k\,A_p & \text{for } \frac{Y_i}{H_p} \geqslant \varepsilon^* \\
F_{\theta_i} &= -\frac{6E_pI_p}{H_p^2(1+2\beta_p)} \left[1 + \frac{H_b}{H_p}\right] \cdot X_i + \left[\frac{4+2\beta_p}{1+2\beta_p}\right] \frac{E_pI_p}{H_p} \left[1 + \frac{3H_b}{4H_p}\left(2 + \frac{H_b}{H_p}\right)\right] \cdot \theta_i
\end{aligned} \tag{7}$$

Equation (7) cannot be written in the form of Eq. (4) directly due to the obvious nonlinearity. In other words, the stiffness of the pillar element depends explicitly on the vertical displacement of the pillar element. The solution procedure or numerical algorithm for the nonlinear equations is discussed in section 5 of this report.

4.3 Room Element

The forces, displacements, and appropriate sign convention for the room element are illustrated in Fig. 5. The force/displacement equations are identical to those of the pillar elements except that the subscript "p" is replaced with a subscript "r" indicating room material properties and geometry.

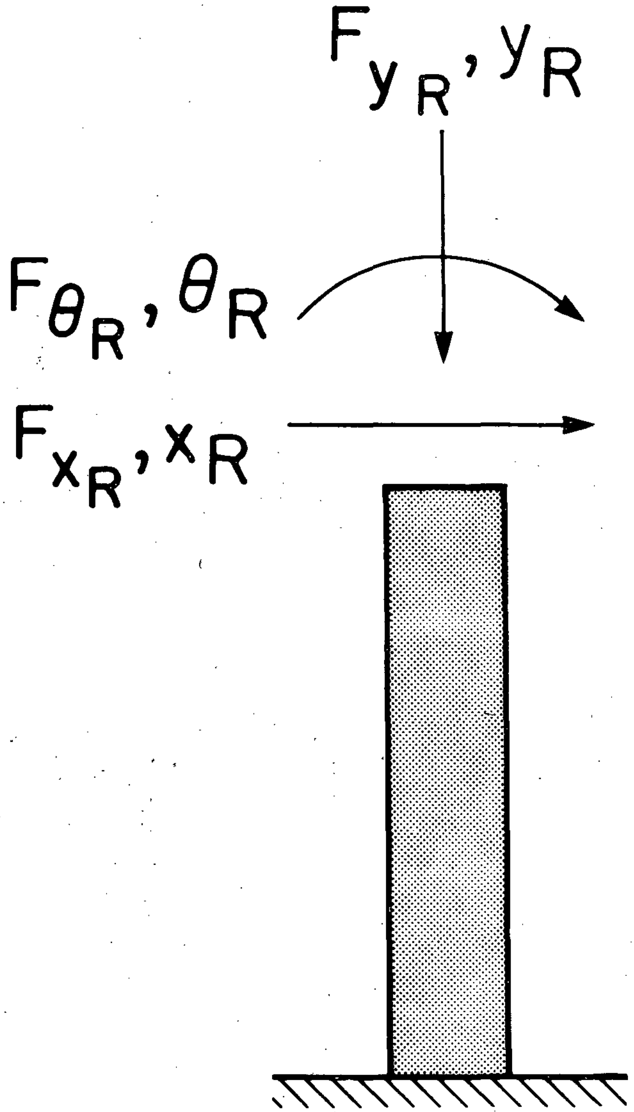

Fig. 5. Room element forces, displacements, and positive sign convention.

5. NUMERICAL ALGORITHM

If the nonlinearity of Eq. (7) did not exist, the element force/displacement relations (Eq. (4)) for each element in a given problem mesh or structure could be merely assembled by means of the direct stiffness method (c.f. Wilson, 1978) and the resulting set of equations solved. Mathematically:

$$[K]\ \{U\} = \{F\} \quad , \qquad (8)$$

where $[K] = \sum_e [K^e]$,

$\{U\}$ = 1 by 3N vector of displacements at each of the N nodes,

$\{F\}$ = 1 by 3N vector of forces at each of the N nodes.

However, due to the nonlinear equations, the solution procedure implied by Eq. (8) is recast as:

$$[K]\{U_i\} = \{F\} - [K^*(U_{i-1})]\ \{U_{i-1}\} \quad , \qquad (9)$$

where $[K]$ = the stiffness of the structure consisting of the linear portion of the stiffness for all element types,

$\{U_i\}$ = 1 by 3N vector of displacements at iteration i,

$[K^*]$ = the nonlinear part of the stiffness of the structure evaluated for displacements at the i-1 iteration.

The linear portion of the pillar and room stiffness is obtained by taking a power series representation of the exponential term of Eq. (7). The iteration procedure of Eq. (9) is repeated until a specified number of iterations has been performed or until a maximum relative tolerance on displacement change has been satisfied at every node. Relative tolerance for this program is defined as:

$$\text{Maximum} \left| \frac{U_i^j - U_{i-1}^j}{U_i^j} \right| \qquad j = 1,2,3,\ldots,3N \ , \qquad (10)$$

where j = the displacement equation number,

i = the iteration number.

Equation (9) is solved with a direct solver for banded symmetric matrices. Repeated iterations are accomplished by continually re-evaluating the right-hand side of Eq. (9) and performing the appropriate load vector modification and back-substitution for the new displacements. Further discussion of the direct solution procedure can be found in most texts on numerical methods or finite element methods (e.g., Zienkiewicz, 1977).

6. PROGRAM INPUT DATA

The input data for the finite element program SUBSID are grouped into card sets. In the following description, the variables for each card set are given, followed by the format in parentheses; then a description of each variable is given. When appropriate, the respective units of each variable are given at the end of the card-set description.

CARD SET 1 - NPR0B, TITLE (I5, 3X, 9A8)

NPR0B = the problem number. If NPROB is less than or equal to zero, the program stops. Thus all problems are eventually terminated with a blank card (see CARD SET 9).

TITLE = the title of the current problem set.

CARD SET 2 - NNP, NELB, NELP, NELR, NMATB, NMATP, NMATR, NB0DY (16I5)

NNP = number of nodes in the structure

NELB = number of overburden elements in the structure

NELP = number of pillar elements in the structure

NELR = number of room elements in the structure

NMATB = number of different overburden material types (≤10)

NMATP = number of different pillar material types (≤10)

NMATR = number of different room material types (≤10)

NB0DY = 0 if body forces are to be neglected
= 1 if body forces are to be included

CARD SET 3 - ITER, T0L (I5, F10.0)

ITER = maximum allowed iterations

T0L = desired relative tolerance for the maximum displacement change at any node

CARD SET 4 - N, IBC(N), X(N), Y(N), FX(N), FY(N), FTHETA(N) (2I5, 5F10.0)

N = node number

IBC(N) = boundary condition for node N

0	free
1	fixed in X direction
10	fixed in Y direction
100	fixed in θ direction
11	fixed in X and Y direction
101	fixed in X and θ direction
110	fixed in Y and θ direction
111	fixed in X, Y and θ direction

X(N) = horizontal coordinate of node N

Y(N) = vertical coordinate of node N (dummy variable within the program)

FX(N) = applied horizontal force or displacement at node N

FY(N) = applied vertical force or displacement at node N

FTHETA(N) = applied moment or rotation at node N

UNITS

X, Y - length
FX, FY, FTHETA - length or force

The first card in this set must be for node 1 and the last for node NNP.

Any equally spaced nodes may be generated by the program. For example, if the card for node 10 is followed by the card for node 20, nodes 11 through 19 will be generated with equal spacing between nodes. The boundary condition for the generated nodes will be equal to that for node 20. All applied forces for the generated nodes are set equal to zero.

CARD SET 5 - NNEL (IE(NNEL,J), J=1,4) (5I5)

NNEL = element number

IE(NNEL,1) = node 1 of element NNEL

IE(NNEL,2) = node 2 of element NNEL (equal to zero for pillar or room elements)

IE(NNEL,3) = material type for element NNEL

IE(NNEL,4) = element type

1 = overburden
2 = pillar
3 = room

The first card in this set must be for element 1 and the last card must be for element NEL = NELB + NELP + NELR. Elements may also be generated. However, material and element types of two element cards between which elements are to be generated must be identical.

CARD SET 6 - E(I), XI(I), AREAB(I), HØ(I), BETA(I), DENB(I), (6E10.0)

E(I) = modulus of the overburden material type I

XI(I) = moment of inertia about the z axis for overburden material type I

AREAB(I) = cross-sectional area of overburden material type I

HØ(I) = height of overburden material type I

BETA(I) = coefficient for shear deflection for overburden material type I (see Eq. (5))

DENB(I) = density of overburden material type I

UNITS

E - force/unit area

XI - (length)^4

AREAB - (length)^2

HØ - length

BETA - dimensionless

DENB - force/unit volume

CARD SET 6 is repeated NMATB times.

CARD SET 7 - PHATP(I), XLAMP(I), AREAP(I), PL(I), DENP(I), XIP(I) ,PØRØ(I), PØWP(I) BETAP(I) (8E10.0/E10.0) (optional card set, include if NMATP $\neq$ 0)

PHATP(I) = unconfined compressive strength of pillar material type I

XLAMP(I) = parameter λ (see Eq. (7)) of pillar material type I

AREAP(I) = cross-sectional area of pillar material type I

PL(I) = height of pillar material type I

DENP(I) = density of pillar material type I

XIP(I) = moment of inertia about the z axis for pillar material type I

PØRØ(I) = parameter ε^* (see Eq. (7)) of pillar material type I

P0WP(I) = parameter n (see Eq. (7)) of pillar material type I

BETAP(I) = coefficient for shear deflection for pillar material type I (see Eq. (5))

UNITS

PHATP - force/unit area

XLAMP - dimensionless

AREAP - $(\text{length})^2$

PL - length

DENP - force/unit volume

XIP - $(\text{length})^4$

P0R0 - dimensionless

P0WP - dimensionless

BETAP - dimensionless

Card set 7 (which consists of two cards) is repeated NMATP times.

CARD SET 8 - PHATR(I), XLAMR(I), AREAR(I), RL(I), DENR(I), XIR(I), V0ID(I), P0WR(I), BETAR(I) (8E10.0/E10.0)
(optional card set, include if NMATR $\neq$ 0)

PHATR(I) = unconfined compressive strength of room material type I

XLAMR(I) = parameter λ (see Eq. (7)) of room material type I

AREAR(I) = cross-sectional area of room material type I

RL(I) = height of room material type I

DENR(I) = density of room material type I

V0ID(I) = parameter ε^* (see Eq. (7)) of room material type I

XIR(I) = moment of inertia about z axis for room material type I

P0WR(I) = parameter n (see Eq. (7)) of room material type I

BETAR(I) = coefficient for shear deflection for room material type I (see Eq. (5))

UNITS

PHATR - force/unit area

XLAMR - dimensionless

AREAR - $(\text{length})^2$

RL - length

DENR - force/unit volume

V0ID - dimensionless

XIR - $(\text{length})^4$

P0WR - dimensionless

BETAP - dimensionless

Card set 8 (which consists of two cards) is repeated NMATR times.

CARD SET 9 - NPR0B, TITLE (I5, 3X, 9A8)

This card set is optional. It is not included if another problem set follows the problem just input. If another problem set follows, card sets 1 through 8 are repeated. Card set 9 (which consists of one blank card) is included only after the last problem set to be executed.

7. PROGRAM OUTPUT

The program output includes all input data plus any generated nodes or elements. Calculated results that are printed include the number of equations in the matrix system, the semi-bandwidth of the stiffness matrix, the words of computer memory for variable storage, the reason for problem termination, displacements and overburden element forces at all nodes, and pillar and room vertical stresses.

8. EXAMPLE PROBLEM

This section illustrates the input data and computer output for a typical example problem and demonstrates how the results may be interpreted. The example problem consists of three lanes 2 m high by 6 m wide located 20 m below the surface (see Fig. 6). The lanes or rooms are separated by 4 m wide pillars. By taking advantage of the mechanical symmetry about the vertical plane through the center of the central room, only one half of the configuration need be modeled. The discretization chosen for the evaluation of the subsidence is illustrated in Fig. 7. The model is extended in the positive horizontal distance to such a position that the right-hand end will not influence the results near the rooms. Due to the mechanical symmetry, the left-hand side of the model is not free to rotate or translate horizontally. In the region of the rooms, the "coarsest" discretization possible has been used. In the pillar region separating the two modeled rooms, several pillar elements could have been used rather than a single element with an appropriate increase in the number of overburden elements.

The nodal and elemental numbering for the model is given in Fig. 8. Note that the overburden elements need not be numbered consecutively as they appear in the model. The same is true for the pillar and room elements.

Two cases will be presented for this model. Firstly, the model is evaluated with fully excavated rooms (i.e., the room elements have zero strength). Secondly, the rooms will be assumed to be backfilled with a low-strength material. The assumed stress/strain response of the overburden and room and pillar materials is shown in Fig. 9 and the remaining material properties are given in Table 1. Note that the pillar and room material has been given a density of zero. Within the program the vertical stress due to the weight of the pillar and room elements is not included in the force/displacement equations. Rather, the mean vertical stress due to the weight of the pillar or room (one half the height of the element times the density) is merely added to the vertical stress induced by the overburden prior to printing of the output.

Since horizontal forces between pillars are not explicitly taken into account in the program, the influence of confinement can be approximated as follows. In pillar elements that will experience little horizontal confinement (elements between rooms), input the uniaxial stress/strain response of the material. In pillar elements that will experience horizontal confinement (elements removed from the rooms), input the stress/strain response of the material at the estimated confining pressure.

The input data for the first case (fully excavated rooms) is given in Appendix C. The input data for the second case is identical except that the yield of the room material is changed from 0 to 1.5 (variable PHATR of card set 8). The computer output for both cases is given in Appendix D. Note that, in the output, the label "RETORT" is used instead of "ROOM". This is due to the original problem for which the program was developed (Ratigan, 1980). The remainder of this section will deal with the presentation and interpretation of the output.

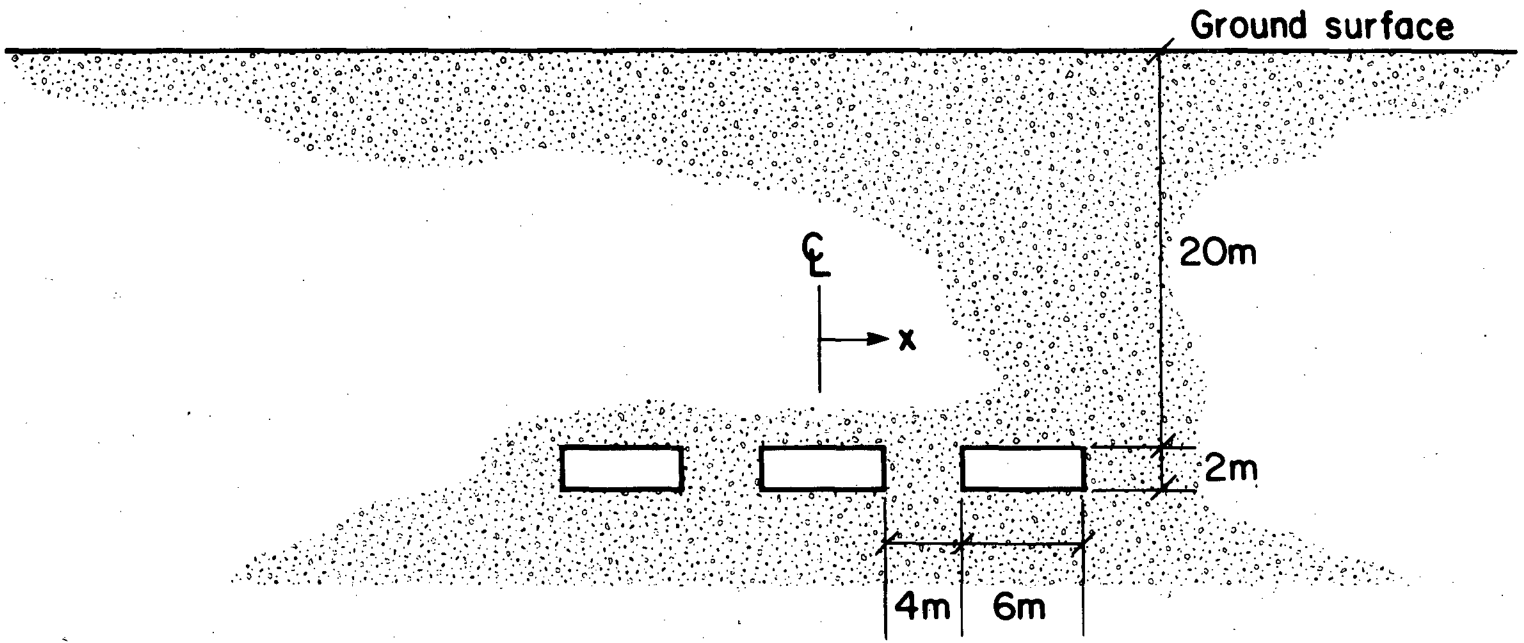

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Fig. 6. Underground lane section.

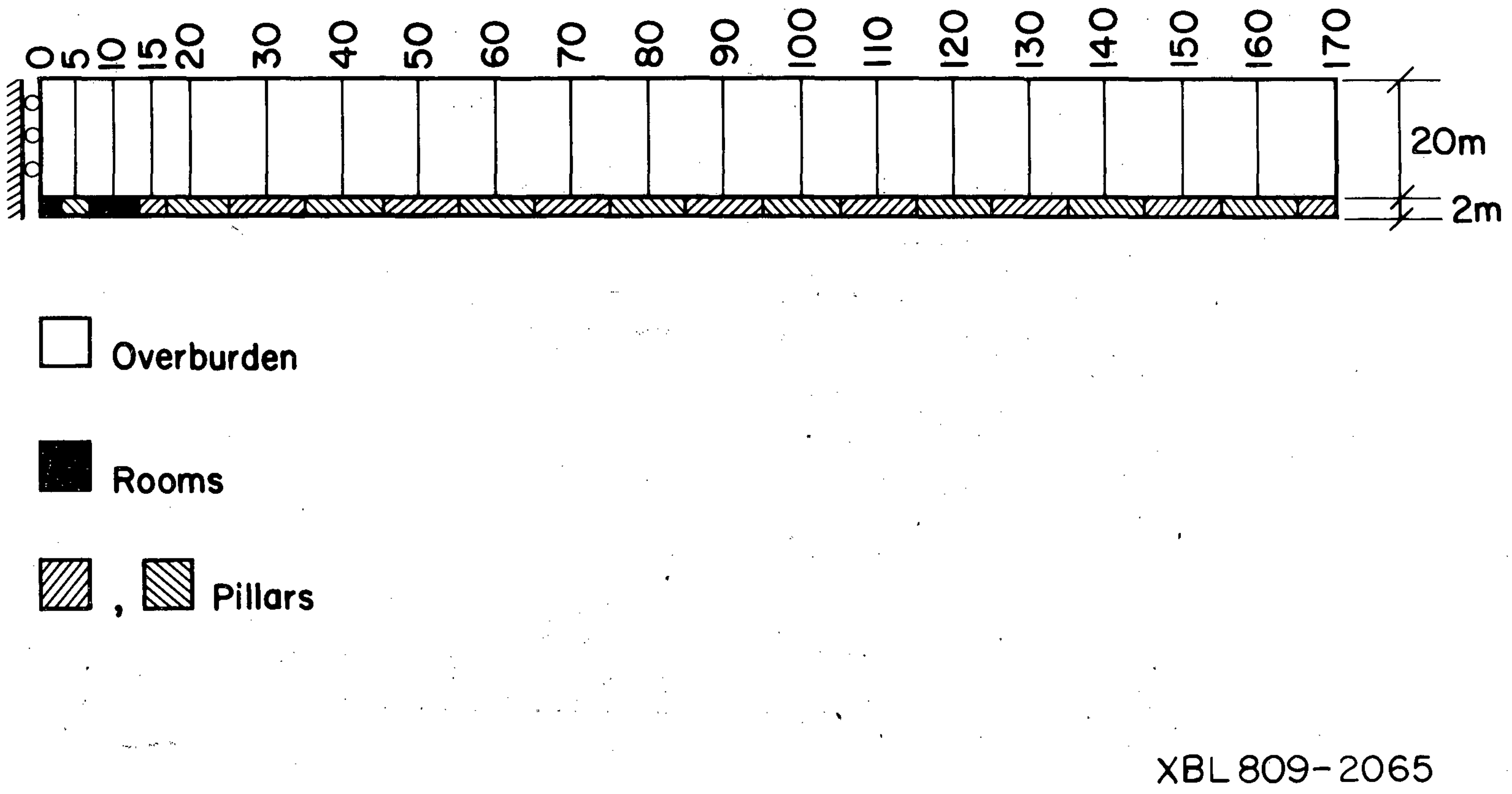

Fig. 7. Discretization of underground lanes for analyses with SUBSID.

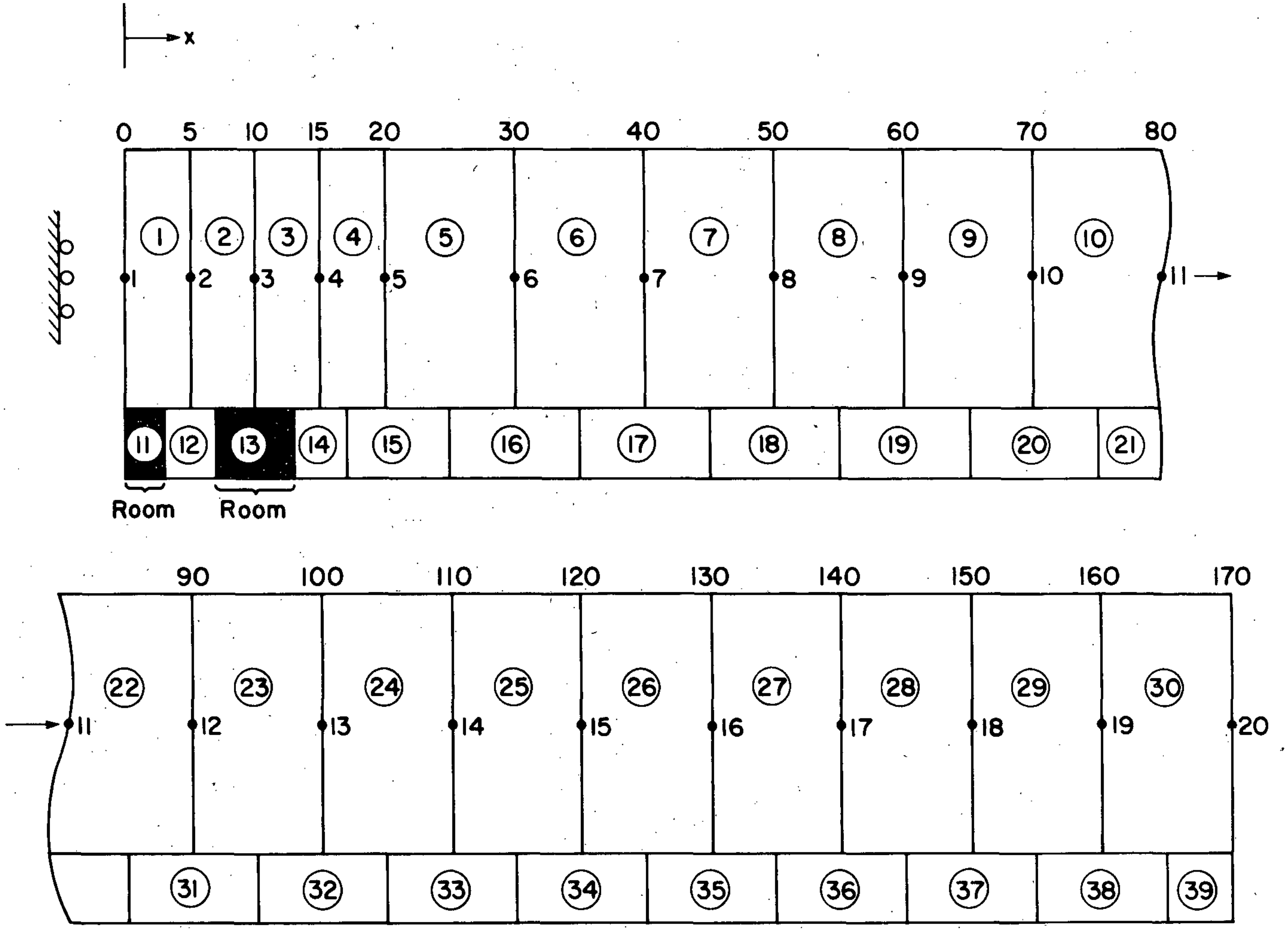

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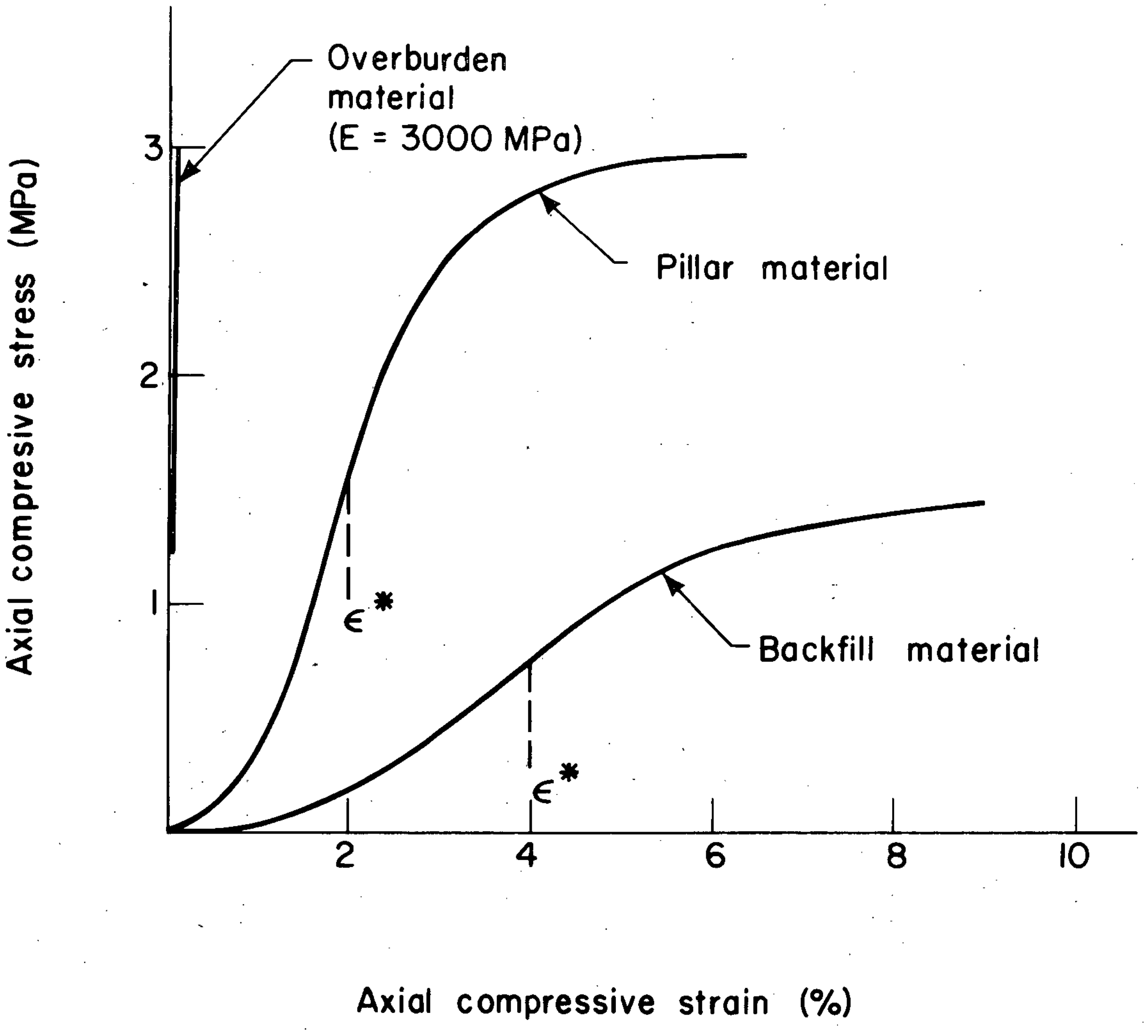
Fig. 8. Node and element identification.



XBL 809-1971

Fig. 9. Stress/strain response for model materials.

Table 1. Material properties for example problem.

Region	C_o (MPa)	E (MPa)	G (MPa)	ν	λ	ρ (MPa/m)	n	ε^*
Overburden	-	3000	1200	0.25	-	0.027	-	-
Pillar	3	-	-	0.25	100	-	2	0.02
Room	0/1.5	-	-	0.25	50	-	2	0.04

The surface displacements for the two cases are shown in Fig. 10. The vertical displacements from the computer output are reduced by the amount of the vertical displacement over the "pillar" region without rooms beneath. In other words, the vertical displacement in Fig. 10 is that which would result only from the excavation of the rooms. Since the computer output displacements are for nodal locations that are at a depth of one-half the height of the overburden element, the horizontal surface displacements are calculated as:

$$X_s = X_i + \theta_i H_b/2 \quad ,$$

where X_s = horizontal displacement at the top surface of the overburden element,

X_i = horizontal displacement at the nodal location,

θ_i = rotation at the nodal location (positive clockwise).

The three forces which act on the overburden element at each end (see Fig. 3) each produce a horizontal stress at the top surface of the overburden element. This horizontal stress at end "1" of the element may be calculated as:

$$\sigma_{X_1} = \frac{F_{\theta_1} \cdot H_b}{2 \cdot I_b} - \frac{F_{X_1}}{A_b} - \frac{F_{Y_1} \cdot H_b \cdot L_b}{4 \cdot I_b}$$

and at end "2" of the element as:

$$\sigma_{X_2} = - \frac{F_{\theta_2} \cdot H_b}{2 \cdot I_b} + \frac{F_{X_2}}{A_b} - \frac{F_{Y_2} \cdot H_b \cdot L_b}{4 \cdot I_b}$$

where compression is taken as positive. The horizontal stress at the top surface of the overburden element will be numerically the same for either end of the element.

The components of the horizontal stress arising form each of the three forces is illustrated in Fig. 11 for both cases of the example problem. The superposed total horizontal stress is shown in Fig. 12. Note that a tensile stress of about 0.1 MPa occurs at a distance of about 18 m from the center of the three-room configuration.

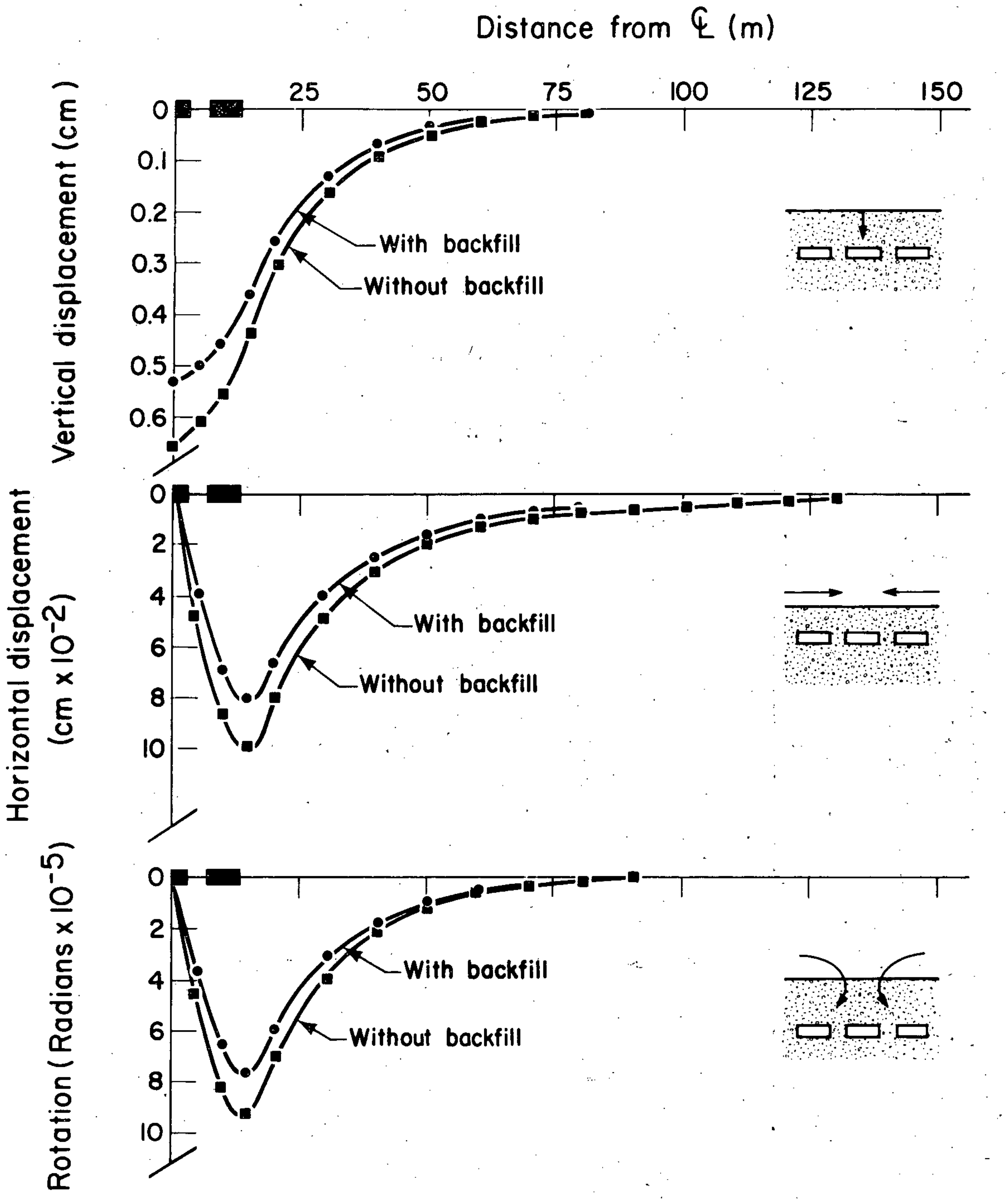


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Fig. 10. Surface displacements.

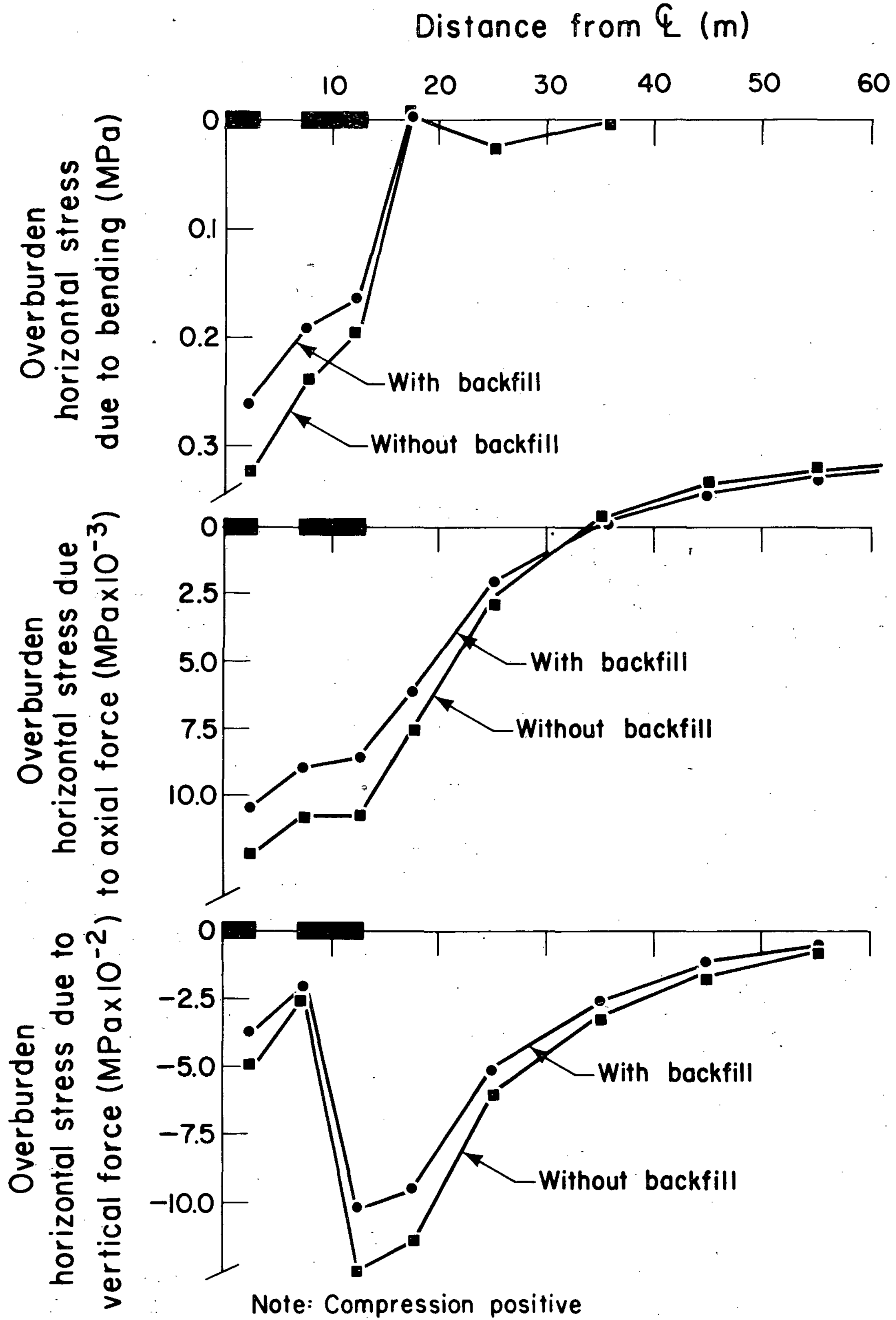

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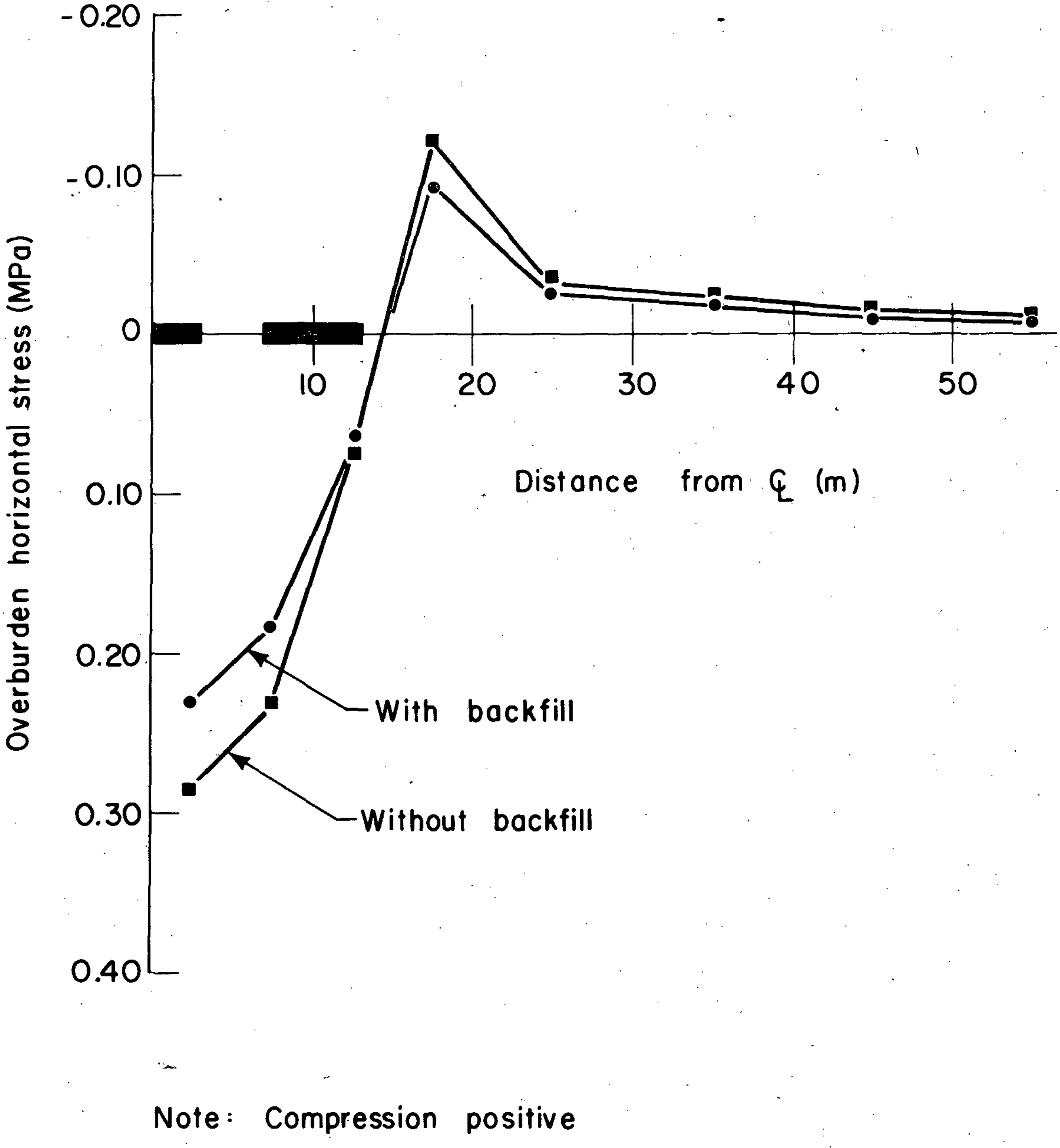
Fig. 11. Bending, axial and flexural stresses in the overburden at the surface.



XBL 809-1974

Fig. 12. Total horizontal stress at the overburden surface.

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APPENDIX A

FORTRAN LISTING OF THE PROGRAM - SUBSID

```
      **PROGRAM SUBSID(INPUT,OUTPUT)**

      PROGRAM SUBSID(INPLT,OUTPUT)
      CCMMON /N1/ NEQ, IBAND, IT
      COMMON /N2/ NNP, NEL, NELB, NELP, NELR
      COMMON /N3/ NMATB, NMATP, NMATR, NBODY
      CCMMON/N4/E(10), XI(10), AREAB(10), BETA(10), DENB(10), HO(10)
      COMMON /N5/ PHATP(10), XLAMP(10), AREAP(10), PL(10), DENP(10)
     1, XIP(10), PCWP(10), PORO(10)
     2, BETAP(10)
      COMMON/N6/PHATR(10),XLAMR(10),AREAR(10),RL(10),DENR(10),VOID(10)
     1 , XIR(10), PCWR(10)
     2 , BETAR(10)
      COMMON/QJOBSFL/ISIZE,ILCM
      CCMMON A(1)
      INTEGER TITLE(9)
C
C          PRCGRAM SUBSIDE IS SPECIFICALLY FCR EVALUATING
C
C               (1) SUBSIDENCE IN OVERBURDEN ABCVE A LANE-AND-PILLAR PLAN
C               (2) SHEAR AND FLEXURAL STRESSES IN THE OVERBURDEN
C               (3) PILLAR STRESSES (AVERAGE VERTICAL)
C               (4) RETCRT STRESSES (IF RETORTS OR BACKFILL IS IN ROOMS)
C
C          THE PRCGRAM ASSUMES THAT THE STRESS/STRAIN BEHAVIOR OF THE
C          SUPPORT PILLARS CAN BE REPRESENTED AS
C
C               STRESS = YIELD( 1.-EXP(-LAMDA(STRAIN-VOID RATIO)))
C                        + K(EXP(-LAMDA(STRAIN-VCID RATIC)))
C                        FOR STRAIN ≥ VOID RATIO
C               STRESS = (K)*(STRAIN/VCID RATIO)**N
C                        FOR STRAIN ≤ VOID RATIO
C                        WHERE K = (STRESS AT STRAIN = VOID RATIO)/ YIELD
C
C
C          SEPARATE RELATICNS FCR RETORTS CAN ALSC BE INPUT
C
C          THE VARIABLES FOR THIS PROGRAM ARE AS FOLLOWS
C
C               VARIABLE       DESCRIPTION
C               ----------     -------------
C               NNP            NUMBER CF NCDES
C
C               NELB           NUMBER OF OVERBURDEN ELEMENTS
C
C               NELP           NUMBER OF PILLAR ELEMENTS
C
C               NELR           NUMBER OF RETORT ELEMENTS
C
C               NBODY          1 FOR BODY FORCES
C                              0 FOR NO BODY FORCES
C
C               NEL            NELB + NELP + NELR
C
C               X              X COORDINATE CF NCDES
C
C               Y              Y COORDINATE CF NCDES
C
C               FX             X FORCE APPLIED TO NODES
```

```
      **PROGRAM SUBSID( INPUT,OUTPUT)**
C
C         FY            Y FORCE APPLIED TO NODES
C
C         FTHETA        THETA FORCE APPLIED TO NODES
C
C         IE            ELEMENT CONECTIONS
C
C         APLOAD        JOINT LOADS (AT NODES)
C
C         YIELD         LOAD VECTOR FOR PILLARS AND RETORTS
C
C         DISPL         DISPLACEMENT AT I-1 ITERATION
C
C         TLOAD         TOTAL LOAD VECTOR
C
C         IBC           BOUNDARY CONDITION
C
C                            1 - FIXED X
C                           10 - FIXED Y
C                          100 - FIXED THETA
C                           11 - FIXED X AND Y
C                          101 - FIXED X AND THETA
C                          110 - FIXED Y AND THETA
C                          111 - FIXED X, Y, AND THETA
C
C         AK            STIFFNESS MATRIX
C
C         E             OVERBURDEN MODULUS
C
C         NMATB         NUMBER OF OVERBURDEN MATERIALS
C
C         NMATP         NUMBER OF PILLAR MATERIALS
C
C         NMATR         NUMBER OF RETORT MATERIALS
C
C         XI            INERTIA MOMENT - OVERBURDEN
C
C         AREAB         OVERBURDEN CROSS SECTION AREA
C
C         BETA          SHEAR STRAIN PARAMETER
C
C         DENB          DENSITY OF THE OVERBURDEN
C
C         HC            HEIGHT OF THE OVERBURDEN ELEMENT
C
C         PHATP         YIELD FOR PILLAR
C
C         XLAMP         EXPONENTIAL CONSTANT FOR PILLAR
C
C         AREAP         CROSS - SECTION AREA OF PILLAR
C
C         PL            PILLAR HEIGHT
C
C         DENP          DENSITY OF THE PILLARS
C
C         XIP           INERTIA MOMENT OF PILLARS
C
C         PORO          VOID RATIO OF PILLARS
```

```
      **PROGRAM SUBSID(INPUT,OUTPUT)**
C
C             POWP              POWER ON STRAIN FOR PILLARS IN CONSTITUTIV
C                               RELATION FOR STRAIN LESS THAN POROSITY
C
C             BETAP             PILLAR SHEAR STRAIN PARAMETER
C
C             PHATR             YIELD OF RETORTS
C
C             XLAMR             EXPONENTIAL CONSTANT FOR RETORT
C
C             AREAR             RETORT CROSS-SECTION AREA
C
C             RL                RETORT HEIGHT
C
C             DENR              DENSITY OF THE RETORTS
C
C             VOID              INITIAL VOID RATIO IN RETORTS
C
C
C             XIR               INERTIA MOMENT FOR RETORTS
C
C
C             POWR              POWER ON STRAIN FOR RETORTS IN CONSTITUTIV
C                               RELATION FOR STRAIN LESS THAN VOID RATIO
C
C             BETAR             RETORT SHEAR STRAIN PARAMETER
C
C        DYNAMIC STORAGE ALLOCATION IS USED IN THIS PROGRAM. THE
C        STORAGE LOCATIONS ARE ALLOCATED AS FOLLOWS,
C
C
C        FROM       TO          VARIABLE
C       ------     ----        ----------
C        N1         N2-1        X
C
C        N2         N3-1        Y
C
C        N3         N4-1        FX
C
C        N4         N5-1        FY
C
C        N5         N6-1        FTHETA
C
C        N6         N7-1        IE
C
C        N7         N8-1        APLOAD
C
C        N8         N9-1        YIELD
C
C        N9         N10-1       DISPL
C
C        N10        N11-1       TLOAD
C
C        N11        N12-1       IBC
C
C        N12        NTOT        AK
C
C-------------------------------------
```

```
      **PROGRAM SUBSID( INPUT,OUTPUT)**
C
   10 READ 1000, NPRCB, TITLE
 1000 FORMAT(I5,3X,9A8)
      IF(NPRCB.EQ.0) CALL EXIT
      READ 2000,NNP, NELB, NELP, NELR, NMATB, NMATP, NMATR, NBODY
      IF(NMATB.LE.10.AND.NMATP.LE.10.AND.NMATR.LE.10) GO TO 13
      IF(NMATB.LE.10) GO TC 11
      PRINT 9000, NMATB
      CALL EXIT
   11 IF(NMATP.LE.10) GO TO 12
      PRINT 10000, NMATP
      CALL EXIT
   12 PRINT 11000, NMATR
      CALL EXIT
 9000 FORMAT(1H1,10X,*NMATB*,I10,* GREATER THAN ALLOWED (10), PROGRAM ST
     1OPS....*,)
10000 FCRMAT(1H1,10X,*NMATP*,I10,* GREATER THAN ALLOWED (10), PROGRAM ST
     1OPS....*,)
11000 FORMAT(1H1,10X,*NMATR*,I10,* GREATER THAN ALLOWED (10), PRCGRAM ST
     1OPS....*,)
   13 CONTINUE
 2000 FORMAT(16I5)
      READ 3000, ITER, TOL
 3000 FORMAT(I5,F10.0)
      PRINT 4000, TITLE
      PRINT 5000, NNP, NELB, NELP, NELR, NBODY, ITER, TOL
      NEL = NELB + NELR + NELP
 4000 FORMAT(1H1,1CX,SA8,//, 9X,* PRCGRAM PARAMETERS*,/,10X,18(*-*),/ )
 5000 FORMAT(10X,*NUMBER OF NODES . . . *,I20,/,
     1         10X,*NUMBER OF OVERBURDEN ELEMENTS . . . *,I6, /,
     2         10X,*NUMBER OF PILLAR ELEMENTS . . . *,I10,/,
     3         10X,*NUMBER OF RETORT ELEMENTS . . . *,I10,/,
     3         10X,*BODY FORCES (1-YES,0-NO) . . . *,I11,/,
     4         10X,*NUMBER OF ITERATIONS ALLOWED . . . *, I7,/,
     5         10X,*RELATIVE TOLERANCE REQUESTED . . . *, F7.5, /)
      N1 = 1
      N2 = N1 + NNP
      N3 = N2 + NNP
      N4 = N3 + NNP
      N5 = N4 + NNP
      N6 = N5 + NNP
      N7 = N6 + 4*NEL
      N8 = N7 + 3*NNP
      N9 = N8 + 3*NNP
      N10 = N9 + 3*NNP
      N11 = N10 + 3*NNP
      N12 = N11 + NNP
C
C     THE FOLLOWING ARE SYSTEM DEPENDENT OPERATIONS FOR SETTING THE
C     COMPUTER CORE REQUIREMENTS FOR THE CODE AND STORAGE
C
      ISIZE = 0
      CALL SETFLS(ISIZE)
      ISIZE = ISIZE + N12 + 1
      CALL SETFLS(ISIZE)
C
C           TOTAL CORE REQUIREMENTS
C
```

```
      **PROGRAM SUBSID(INPUT,CUTPUT)**
C                      18*NNP + 4*NEL + IBAND*NEQ
C
      CALL INPUT(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N11),NEL)
      CALL BANDWD(A(N6), NEL, IBAND, NEQ)
      NTOT = N12 + IBAND*NEQ
      ISIZE = ISIZE + IBAND*NEQ
      CALL SETFLS(ISIZE)
      PRINT 6000, NEQ, IBAND, NTOT
 6000 FCRMAT(//,10X,I5,2X,*EQUATIONS WITH A BANDWIDTH OF*,I6,2X,*REQUIRE
     1 *,I5,2X,*WORDS OF STCRAGE*,/)
      CALL ASEMBL(A(N1),A(N2),A(N6),A(N12),NEL,NEQ,IBAND,A(N4),A(N10),A(
     1 N11))
      CALL BCUNDC(A(N3),A(N4),A(N5),A(N11),A(N12),A(N10),IBAND,NEQ)
      IT = 0
C
C     THE ITERATIVE PROCEEDURE INITIATES AT STATEMENT 20
C
   20 CALL ADDLOD(A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),A(N10),NEL,
     1 A(N11))
      IF(IT.GT.0) GC TC 30
      CALL SOLVE( 1, A(N12), A(N10), NEQ, IBAND)
   30 CALL SCLVE( 2, A(N12), A(N10), NEQ, IBAND)
      CALL CONVG(A(N1C), A(N9), TCL ,IT, ITER, ISTOP)
      IF(ISTOP .EQ. 0) GO TO 20
      GC TO (40,50), ISTCP
   40 PRINT 7000, TOL
      GC TO 60
   50 PRINT 8000, IT
 7000 FORMAT(1HC,10X,*MAXIMUM ITERATIONS PERFORMED. RELATIVE TOLERANCE O
     1BTAINED..*,/,20X,*TOL = *, E12.4, /)
 8C00 FCRMAT(1HC,10X,*DESIRED TCLERANCE CBTAINED AFTER*,I7,2X,*ITERATION
     1S*,/)
C
C     THE FOLLOWING SUBROUTINE CALLS RESULT IN OUTPUT OF THE RESULTS
C
   6C CALL DISPL(A(N1C))
      CALL FORCE(A(N1),A(N6),A(N10),NEL)
      IF(NMATP.EQ.0) GC TC 70
      CALL PSTRES (A(N10),A(N6),NEL)
   70 IF(NMATR.EQ.0) GO TO 10
      CALL RSTRES (A(N10),A(N6),NEL)
      GO TO 10
      END

- MNF4 LEVEL 5.24      27 JUN 80 11.31.24
```

```
      **SUBROUTINE INPUT(X, Y, FX, FY, FTHETA, IE, IBC, NEL)**

      SUBROUTINE INPUT(X, Y, FX, FY, FTHETA, IE, IBC, NEL)
C
C     THIS SUBROUTINE READS AND PRINTS APPROPRIATE INPUT DATA
C
      COMMON /N2/NNP, NX, NELB, NELP, NELR
      COMMON /N3/NMATB, NMATP, NMATR
      COMMON/N4/E(1C), XI(10), AREAB(10), BETA(10), DENB(10), HO(10)
      COMMON /N5/ PHATP(10), XLAMP(10), AREAP(10), PL(10), DENP(10)
     1 , XIP(10), PCWP(10), PCRO(10)
     2 , BETAP(10)
      CCMMCN/N6/PHATR(10),XLAMR(10),AREAR(10),RL(10),DENR(10),VOID(10)
     1 , XIR(10), POWR(10)
     2 , BETAR(10)
      DIMENSION X(1), Y(1), FX(1), FY(1), FTHETA(1), IE(NEL,4), IBC(1)
      PRINT 2000
 2000 FORMAT(10X,*NCDAL COORDINATES*,/,10X,17(*-*),/,10X,*NODE*, 6X,*IBC
     1*  7X,*X*,14X,*Y*
     1 ,13X,*F-X*,12X,*F-Y*,10X,*F-THETA*,/)
      IGC = 0
      KC = 1
   10 READ 1000, N, IBC(N), X(N), Y(N), FX(N), FY(N), FTHETA(N)
 1000 FORMAT(2I5,5E10.0)
      IGC = IGC + 1
      IF(IGC.LE.N) GC TO 15
      PRINT 1001, IGC
 1001 FORMAT(/,*DATA OUT OF ORDER AT NODE *,I5)
   15 DNOD = FLCAT(N+1-KC)
      IF(N-KC)4C,40,17
   17 MC = KC - 1
      DX = (X(N) - X(MC))/DNOD
      DY = (Y(N) - Y(MC))/DNOD
   35 IBC(KC) = IBC(N)
      IGC = IGC + 1
      MC = KC - 1
      X(KC) = X(MC) + DX
      Y(KC) = Y(MC) + DY
      FX(KC) = FY(KC) = FTHETA(KC) = 0.
   40 KC = KC + 1
      IF(N-KC)50,40,35
   50 IF(NNP+1-KC)60,60,10
   60 CONTINUE
      PRINT 3000,(I,IBC(I),X(I),Y(I),FX(I),FY(I),FTHETA(I), I=1,NNP)
 3000 FORMAT(3X, 2I1C, F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X)
      LEN = 0
      IHH = 0
      NNEL = 0
      DO 140 N = 1, NEL
      IHH = IHH + 1
      IF(NNEL-N)70,14C,110
   70 READ 4000, NNEL,(IE(N,J), J = 1, 4)
 4000 FORMAT(5I5)
      IF(IHH.LE.NNEL) GO TO 80
      PRINT 4001, IHH
 4001 FORMAT(/,*DATA OUT OF ORDER AT ELEMENT*,I5)
      IHH = NNEL
   80 IF(NNEL-1-N) 140,90,90
   90 IF(LEN.EQ.NNEL) GO TO 110
```

```
      **SUBRCUTINE INPUT(X, Y, FX, FY, FTHETA, IE, IBC, NEL)**

      DC 100 I = 1, 4
  10C IE(NNEL,I) = IE(N,I)
      LEN = NNEL
  110 M = N - 1
  120 IE(N,1) = IE(M,1) + 1
      IE(N,2) = 0
      IF(IE(M,2).NE.0) IE(N,2) = IE(M,2) + 1
      IE(N,3) = IE(M,3)
      IE(N,4) = IE(M,4)
      IF(NEL-N-1) 150,150,130
  130 CCNTINUE
  140 CONTINUE
  15C CONTINUE
      PRINT 5000
 5000 FORMAT(10X,*ELEMENT CCNNECTIONS*,/,10X,19(*-*),/,10X,*ELEMENT*, 6X
     1,*I*, 9X,*J*, 5X, *MATERIAL*,  2X,*ELEM TYPE*,5X,*(1=OVERBURDEN, 2
     2=PILLAR, 3=RETCRT)*,/)
 6000 FORMAT( 4X, 5I10 )
      PRINT 6000, (I, ( IE(I,J), J = 1, 4 ), I = 1, NEL)
      READ 8000, ( E(I), XI(I), AREAB(I), HO(I), BETA(I), DENB(I),I = 1,
     1 NMATB)
      PRINT 9000
      PRINT 10000, (I,E(I), XI(I), AREAB(I), HO(I), BETA(I), DENB(I), I
     1 = 1, NMATB)
 8000 FCRMAT( 6E10.0)
 9000 FORMAT(/,10X,*MATERIAL PROPERTIES*,/,10X,19(*-*), /,15X,*OVERBURDE
     1N*,/,15X,10(*-*),/,15X,*MAT*,11X,*E*,15X,*I*,12X,*AREA*,9X,*HEIGHT
     1*,9X,*BETA*,9X,*DENSITY*,/)
10000 FORMAT(15X,I3,5X,F10.2,5X,F12.2,3X,F10.2,5X,F10.2,5X,F10.2,5X,F10.
     12)
      IF(NMATP.EQ.C) GC TC 20
      DO 160 I = 1, NMATP
      READ 8010, (PHATP(I), XLAMP(I), AREAP(I), PL(I), DENP(I), XIP(I),
     1 PORC(I), POWP(I))
      READ 8020, BETAP(I)
  160 CONTINUE
 8020 FORMAT(E1C.0)
 8010 FORMAT(8E10.0)
      PRINT 110C0
      PRINT 12000
      PRINT 13000, (I,PHATP(I),XLAMP(I),AREAP(I),PL(I),DENP(I),XIP(I),
     1 I = 1, NMATP)
      PRINT 12001
      PRINT 13001, (I, PORO(I), POWP(I), BETAP(I), I = 1, NMATP)
12001 FORMAT(/,15X,*MAT*,5X,*POROSITY*,8X,*POWER ON STRAIN*,3X,*BETA-P*
     1 , /)
13001 FORMAT(15X,I3,5X,F10.5,5X,F10.3,6X,F10.2)
11000 FORMAT(15X,*PILLARS*,/,15X, 7(*-*),/)
12000 FCRMAT(15X,*MAT*,10X, *YIELD*, 10X, *LAMDA*, 10X,*AREA*,10X,*HEIGH
     1T*, 8X,*DENSITY*, 12X,*I*,/)
   20 IF(NMATR.EQ.0) GC TC 30
      DO 170 I = 1, NMATR
      READ 15000, (PHATR(I), XLAMR(I), AREAR(I), RL(I), DENR(I),XIR(I),
     1 VCID(I), POWR(I))
      READ 8020, BETAR(I)
  170 CONTINUE
15000 FCRMAT(8E10.C)
      PRINT 14000
```

```
      **SUBROUTINE INPUT(X, Y, FX, FY, FTHETA, IE, IBC, NEL)**

      PRINT 16000
      PRINT 17000, (I, PHATR(I),XLAMR(I),AREAR(I),RL(I),DENR(I),VOID(I),
     1 XIR(I), I = 1, NMATR)
      PRINT 16001
      PRINT 17001, (I, POWR(I), BETAR(I), I = 1, NMATR)
16001 FORMAT(/,15X,*MAT*,5X,*POWER ON STRAIN*,4X,*BETA-R*,/)
17001 FORMAT(15X,I3,7X,F10.3,4X,F10.2)
16000 FORMAT(15X,*MAT*,10X,*YIELD*,10X,*LAMDA*,10X,*AREA*,10X,
     1*HEIGHT*,8X,*DENSITY*,8X,*VOID RATIO*,9X,*I*,/)
17000 FORMAT(15X,I3,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.
     12,3X,F15.2)
   30 CONTINUE
13000 FORMAT(15X,I3,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,5X,F10.2,3X,
     1F15.2)
14000 FORMAT(15X,*RETORTS*,/,15X,7(*-*),/)
      RETURN
      END

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```

```
      **SUBROUTINE BANDWD( IE, NEL, IBAND, NEQ )**

      SUBROUTINE BANDWD( IE, NEL, IBAND, NEQ )
      COMMON /N2/NNP
      DIMENSION IE(NEL, 4)
C
C     THIS SUBROUTINE DETERMINES THE SEMI-BANDWIDTH OF THE
C     STIFFNESS MATRIX FOR THE MODEL TO BE ANALYZED.  THE BANDWIDTH
C     SHOULD NORMALLY BE 6.
C
      IBAND = 0
      DO 10 I = 1, NEL
C
C     SKIP RETORTS OR PILLARS
C
      IF( IE(I,2).EQ. 0) GO TO 10
      IB = IABS ( IE(I,1) - IE(I,2))
      IF(IB.GT. IBAND) IBAND = IB
   10 CONTINUE
      IBAND = 3*(IBAND + 1 )
      NEQ = 3* NNP
      RETURN
      END

-- MNF4 LEVEL 5.24      27 JUN 80 11.31.24
```

```
      **SUBRCUTINE ASEMBL (X,Y,IE,AK,NEL,NEQ,IBAND,R,TLOAD,IBC)**

      SUBROUTINE ASEMBL (X,Y,IE,AK,NEL,NEQ,IBAND,R,TLOAD,IBC)
C
C     THIS SUBROUTINE FORMS THE LINEAR STIFFNESS OF THE OVERBURDEN
C     AND STORES IT IN  AK .
C
      COMMON /N3/ NMATB, NMATP, NMATR, NBODY
      COMMON/N4/E(10), XI(10), AREAB(10), BETA(10), DENB(10), HO(10)
      COMMON/N5/PHATP(10),XLAMP(10),AREAP(10),PL(10),DENP(10)
     1 , XIP(10),POWP(10),PORO(10),BETAP(10)
      COMMON/N6/PHATR(10),XLAMR(10),AREAR(10),RL(10),DENR(10),VOID(10)
     1 , XIR(10), POWR(10), BETAR(10)
      DIMENSION X(1), Y(1), IE(NEL, 4), AK(NEQ, IBAND), A(6,6), LP(6)
     1   ,R(1), TLOAD(1), IBC(1)
      REAL L
      DO 10 I = 1, NEQ
      TLOAD(I) = 0.
      DO 10 J = 1, IBAND
   10 AK(I,J) = 0.
      DO 50 NN = 1, NEL
C
C     SKIP ALL NON-OVERBURDEN ELEMENTS
C
      IF (IE(NN,4).NE. 1) GC TC 50
      IF(NBODY.EQ.0) GO TO 15
      MTYPE = IE(NN,3)
      INDEX = IE(NN,1)
      JNDEX = IE(NN,2)
      L = ABS( X(IE(NN,1)) - X(IE(NN,2)))
C
C     THE BODY FORCES (IF ANY) ARE ASEMBLED IN THE FOLLOWING 2
C     STATEMENTS
C
      R(INDEX) = R(INDEX) + DENB(MTYPE)*L*AREAB(MTYPE)*BODYF(IBC(INDEX))
      R(JNDEX) = R(JNDEX) + DENB(MTYPE)*L*AREAB(MTYPE)*BODYF(IBC(JNDEX))
   15 CONTINUE
      CALL BEAM (NN, X, Y, IE, A, NEL)
      DO 20 I = 3, 6, 3
      IJ = I/3
      LP(I) = 3*IE(NN,IJ)
      LP(I-1) = 3*IE(NN, IJ) - 1
   20 LP(I-2) = 3*IE(NN, IJ) - 2
      DO 40 LL = 1, 6
      I = LP(LL)
      DO 30 MM = 1, 6
      J = LP(MM) - I + 1
      IF (J.LE.0 ) GO TO 30
      AK(I,J) = AK(I,J) + A(LL,MM)
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
C
C     THE LINEAR PORTION OF THE AXIAL AND BENDING STIFFNESS
C     FOR THE SECCNDARY PCRTION OF THE CONSTITUTIVE EQUATICN(I.E. FOR
C     STRAINS GREATER THAN THE VOID RATIO OR POROSITY)
C     OF THE PILLARS AND RETORTS ARE ASSEMBLED IN THE STIFFNESS
C     MATIX IN THE FOLLCWING STATEMENTS.
C
```

```
      **SUBROUTINE ASEMBL (X,Y,IE,AK,NEL,NEQ,IBAND,R,TLOAD,IBC)**

      IF(NMATP.EQ.0) GO TO 70
      DO 60 NN = 1, NEL
      IF(IE(NN,4).NE.2) GO TO 60
      MTYPE = IE(NN,3)
      FACT = PHATP(MTYPE)*AREAP(MTYPE)*XLAMP(MTYPE)/PL(MTYPE)
      INDEX = 3*IE(NN,1) - 1
      AK(INDEX,1) = AK(INDEX,1) + FACT
      DO 51 NOB = 1, NEL
      IF(IE(NOB,4).NE.1) GO TO 51
      IF(IE(NN,1).EQ.IE(NOB,1)) IOB = NOB
      IF(IE(NN,1).EQ.IE(NOB,2)) IOB = NOB
   51 CONTINUE
      HB = HO(IE(IOB,3))
      FACT = FACT*XIP(MTYPE)/AREAP(MTYPE)/(1.+2.*BETAP(MTYPE))
C
C      PUT IN THE X ROW STIFFNESS
C
      INDEX = INDEX + 1
      HP = PL(MTYPE)
      AK(INDEX,1) = AK(INDEX,1) + 12.*FACT/(HP*HP)
C
C     PUT IN THE THETA ROW STIFFNESS
C
C
      INDEX = INDEX - 2
      AK(INDEX,1) = AK(INDEX,1) + (4.+2.*BETAP(MTYPE))*FACT*(1.+.75*HB/
     1 HP*(2.+HB/HP))
      AK(INDEX,3) = AK(INDEX,3) - 6.*FACT/HP*(1.+HB/HP)
   60 CONTINUE
   70 IF(NMATR.LE.0) RETURN
      DO 80 NN = 1, NEL
      IF(IE(NN,4).NE.3) GO TO 80
      MTYPE = IE(NN,3)
      FACT = PHATR(MTYPE)*AREAR(MTYPE)*XLAMR(MTYPE)/RL(MTYPE)
      INDEX = 3*IE(NN,1) - 1
      AK(INDEX,1) = AK(INDEX,1) + FACT
      DO 71 NOB = 1, NEL
      IF(IE(NOB,4).NE.1) GO TO 71
      IF(IE(NN,1).EQ.IE(NOB,1)) IOB = NOB
      IF(IE(NN,1).EQ.IE(NOB,2)) IOB = NOB
   71 CONTINUE
      HB = HO(IE(IOB,3))
      FACT = FACT*XIR(MTYPE)/AREAR(MTYPE)/(1.+2.*BETAR(MTYPE))
C
C      PUT IN THE X ROW STIFFNESS
C
      INDEX = INDEX + 1
      HP = RL(MTYPE)
      AK(INDEX,1) = AK(INDEX,1) + 12.*FACT/(HP*HP)
C
C     PUT IN THE THETA ROW STIFFNESS

      INDEX = INDEX - 2
      AK(INDEX,1) = AK(INDEX,1) + (4.+2.*BETAR(MTYPE))*FACT*(1.+.75*HB/
     1 HP*(2.+HB/HP))
      AK(INDEX,3) = AK(INDEX,3) - 6.*FACT/HP*(1.+HB/HP)
   80 CONTINUE
      RETURN
```

```
        **SUBROUTINE ASEMBL (X,Y,IE,AK,NEL,NEQ,IBAND,R,TLOAD,IBC)**
        END
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```

```
      **FUNCTION BODYF(II)**

      FUNCTION BODYF(II)
      DIMENSION IB(4)
      DATA IB/10,11,110,111/
C
C     THIS FUNCTION IS ZERO IF A NODE IS FIXED IN THE Y DIRECTION.
C     OTHERWISE THE FUNCTION IS 0.5
C
      FACT = .5
      DO 10 I = 1, 4
      IF(IB(I) .EQ. II) FACT = 0.
   10 CONTINUE
      BODYF = FACT
      RETURN
      END

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```

```
      **SUBROUTINE BEAM ( N, X, Y, IE, A, NEL)**

      SUBROUTINE BEAM ( N, X, Y, IE, A, NEL)
      COMMON/N4/E(10), XI(10), AREAB(10), BETA(10), DENB(10), HO(10)
      DIMENSION X(1 ), Y(1), IE(NEL,4),A(6,6)
      REAL L, LL
C
C     THIS SUBROUTINE ASSEMBLES THE CONVENTIONAL LINEAR 6 X 6 BEAM
C     STIFFNESS (SLOPE-DEFLECTION EQUATIONS) WITH AXIAL DEFORMATIONS.
C     SHEARING DEFORMATICN IS ACCOUNTED FOR IN THE BETA TERM.
C
      DO 10 I = 1, 6
      DO 10 J = 1, 6
   10 A(I,J) = 0.
      MTYPE = IE(N,3)
      L = ABS ( X(IE(N,1)) - X(IE(N,2)))
      LL = L*L
      BATA = BETA(MTYPE)
      CONS = 1. + 2.*BATA
      A(1,1) = A(4,4) = (4. + 2.*BATA) / CONS
      A(1,4) = (2.- 2.*BATA) / CONS
      A(1,2) = A(2,4) =  6. / (L*CONS)
      A(1,5) = A(4,5) = -A(1,2)
      A( 2,2) = A(5,5) = 12./(LL*CONS)
      A(2,5) = - A( 2, 2)
      A(3,3) = A(6,6) = AREAB(MTYPE)/XI(MTYPE)
      A( 3,6) = -A( 3,3)
      C = E(MTYPE) * XI(MTYPE)/L
      DO 15 I = 1, 6
      DO 15 J = I, 6
   15 A(J,I) = A(I,J)
      DO 20 I = 1, 6
      DO 20 J = 1, 6
   20 A(I,J) = A(I,J)* C
      RETURN
      END

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```

```
      **FUNCTION FNCNLN(A, L, P, LAMDA, U, N, V)**

      FUNCTICN FNCNLN(A, L, P, LAMDA, U, N, V)
C
C     THIS FUNCTICN CALCULATES THE FORCE IN THE NONLINEAR MATERIAL
C     DESCRIBED BY
C
C           STRESS = YIELD(1.-EXP(-LAMDA(STRAIN-VCID RATIO)))
C                    + (K)*EXP(-LAMDA(STRAIN-VOID RATIO))
C                    FCR STRAIN ≥ VOID RATIO
C           STRESS = (K)*(STRAIN/VOID RATIO)**N
C                    FOR STRAIN ≤ VOID RATIC
C                    WHERE K = (STRESS AT STRAIN = VCID RATIO)/YIELD
C
C     THE MATERIAL MAY BE EITHER A PILLAR OR A RETCRT
C
      REAL L, LAMDA, N
      FNONLN = P*A
      FACT = LAMDA*(U/L - V)
      FACT2 = 0.
      IF( V.EQ.0.) GC TC 10
      FACT2 = LAMDA*V*P*A/(N + LAMDA*V)
   10 CONTINUE
      IF(ABS(FACT).GE.5CO.) RETURN
      FNONLN = P*A*(1.-EXP(-FACT)) + FACT2*EXP(-FACT)
      IF((V.EQ.0.).OR.(V.LE.(U/L))) RETURN
      FNONLN = LAMDA*V*P*A/(N+LAMDA*V)*(U/L/V)**N
      RETURN
      END

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```

```
      **SUBROUTINE DISPL ( R )**

      SUBROUTINE DISPL ( R )
C
      COMMON /N2/NNP
      DIMENSION R(1)
      PRINT 1000
      DO 10 I = 1, NNP
      LS = 3*(I -1 ) + 1
      LT = LS + 2
   10 PRINT 2000, ( I, ( R(J), J = LS, LT) )
 1000 FORMAT(//,10X,*DISPLACEMENTS*,/, 10X,13(*-*),/, 15X,*NODE*,10X,*U-
     1THETA*,10X,*U-Y*,10X,*U-X*,/)
 2000 FORMAT(10X,I8,3X,3(E12.4,5X))
      RETURN
      END

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```

```
      **SUBROUTINE PSTRES (U , IE, NEL )**

      SUBROUTINE PSTRES (U , IE, NEL )
      COMMON /N3/ NMATB, NMATP, NMATR, NBODY
      COMMON /N5/ PHATP(10), XLAMP(10), AREAP(10), PL(10), DENP(10)
     1 , XIP(10), PCWP(10), PORO(10)
      DIMENSION  U(1), IE(NEL,4)
C
C     THIS SUBROUTINE CALCULATES THE APPLIED AND GRAVITATIONAL PILLAR
C     STRESSES.  BENDING STRESSES ARE NOT INCLUDED.
C
      PRINT 1000
 1000 FORMAT(//,10X,*PILLAR STRESSES*,/,10X,15(*-*), /,10X,*PILLAR ELEME
     1NT*,10X,*STRESS*, /)
      DO 10 N = 1, NEL
      IF( IE(N,4) .NE. 2 ) GO TO 10
      MTYPE = IE (N,3 )
      BODYS = DENP(MTYPE)*PL(MTYPE)*0.5*FLOAT(NBODY)
      S = FNONLN(AREAP(MTYPE), PL(MTYPE), PHATP(MTYPE), XLAMP(MTYPE),
     1    U(3*IE(N,1)-1),PCWP(MTYPE), PORO(MTYPE))
     2   /AREAP(MTYPE) + BODYS
      PRINT 2000, N, S
 2000 FORMAT(16X, I2,14X ,F10.2 )
   10 CONTINUE
      RETURN
      END

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```

```
        **SUBROUTINE RSTRES ( U, IE, NEL)**

        SUBROUTINE RSTRES ( U, IE, NEL)
        COMMON /N3/ NMATB, NMATP, NMATR, NBODY
        COMMON /N6/ PHATR(10), XLAMR(10), AREAR(10), RL(10), DENR( 10)
       1 , VOID( 10), XIR( 10), POWR(10)
        DIMENSION U(1), IE(NEL, 4)
 C
 C      THIS SUBROUTINE CALCULATES THE APPLIED AND GRAVITATIONAL STRESSES
 C      IN THE RETORTS.  BENDING STRESSES ARE NOT INCLUDED.
 C
        PRINT 1000
  1000 FORMAT(//,10X, *RETORT STRESSES*,/,10X,15(*-*), /,10X,*RETORT ELEM
       1ENT*,10X,*STRESS*,/ )
        DO 10 N = 1,NEL
        IF( IE(N,4).NE. 3) GO TO 10
        MTYPE = IE(N,3)
        BODYS = DENR(MTYPE)*RL(MTYPE)*0.5*FLOAT(NBODY)
        S = FNONLN(AREAR(MTYPE), RL(MTYPE), PHATR(MTYPE), XLAMR(MTYPE),
       1        U(3*IE(N,1)-1),POWR( MTYPE),VOID(MTYPE))
       2        / AREAR(MTYPE) + BODYS
        PRINT 2000, N, S
  2000 FORMAT(16X, I2, 14X, F10.2 )
    10 CONTINUE
        RETURN
        END

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```

```
      **SUBROUTINE FORCE ( X, IE, DISPL, NEL)**

      SUBROUTINE FORCE ( X,  IE, DISPL, NEL)
      COMMON /N2/ NNP
      COMMON/N4/E(10), XI(10), AREAB(10), BETA(10), DENB(10), HO(10)
      DIMENSION X(1), IE(NEL, 4), DISPL(1),A(6,6),FORCX(6),Y(1),LP(6)
C
C     THIS SUBROUTINE CALCULATES THE INTERNAL MEMBER FORCES IN THE
C     OVERBURDEN FROM THE BEAM STIFFNESS AND THE ASSOCIATED
C     DISPLACEMENTS.
C
      PRINT 1000
 1000 FORMAT(//,10X,*OVERBURDEN FORCES*,/,10X,17(*-*),/)
      DO 30 N = 1, NEL
      IF(IE(N,4).NE.1) GO TO 30
      PRINT 2000, IE(N,1),IE(N,2)
      CALL BEAM(N,X,Y,IE,A,NEL)
      DO 10 I = 3, 6, 3
      IJ = I/3
      LP(I) = 3*IE(N,IJ)
      LP(I-1) = LP(I) - 1
   10 LP(I-2) = LP(I) - 2
      DO 20 I = 1, 6
      FORCX(I) = 0.
      DO 20 J = 1, 6
   20 FORCX(I) = FORCX(I) + A(I,J)*DISPL(LP(J))
      PRINT 3000, (N,(FORCX(I),I = 1, 6))
   30 CONTINUE
 2000 FORMAT(/,10X,*ELEMENT*,21X,*NODE*, I4, 42X, *NODE*, I4,/,
     1 24X,*P-THETA*,10X,*P-Y*,10X,*P-X*,15X,*P-THETA*,10X,*P-Y*,10X,
     2 *P-X*)
 3000 FORMAT(10X,I4,5X,E12.3,3X,E12.3,3X,E12.3,7X,E12.3,3X,E12.3,3X,E12.
     13)
      RETURN
      END

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```

```
      **SUBRCUTINE CCNVG(TLCAD, DISPL, TOL, IT, ITER, ISTCP)**

      SUBROUTINE CONVG(TLOAD, DISPL, TOL, IT, ITER, ISTCP)
      COMMON /N2/ NNP
      DIMENSION TLOAD(1), DISPL(1)
C
C     THIS SUBROUTINE EVALUATES WHETHER THE ITERATIVE METHOD HAS
C     CCNVERGED.
C
C     THE VARIABLE RETURNED IS ISTOP, WHERE
C
C              ISTOP
C
C                0             RELATIVE TOLERANCE NOT YET OBTAINED,
C                              ITERATIONS REMAINING
C
C                1             MAXIMUM ITERATICNS REACHED
C
C                2             RELATIVE TOLERANCE REACHED
C
      ISTOP = 0
      TT = 0.
      ICHECK = 0
      NEQ = 3*NNP
      DO 10 N = 1, NEQ
      IF( DISPL(N) .EQ. 0. ) GO TO 10
      TA = ABS( (DISPL(N) - TLOAD(N))/ DISPL(N))
      ICHECK = 1
      IF (TA.GT.TT) TT = TA
   10 CONTINUE
      IT = IT + 1
      IF( IT .LT. ITER) GC TC 20
      TOL = TT
      ISTOP = 1
      RETURN
   20 IF(TT.GT.TCL .CR. ICHECK.EQ. 0 ) GO TO 30
      ISTOP = 2
   30 CONTINUE
      RETURN
      END

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```

```
      **SUBROUTINE ACDLOC(FX, FY, FTHETA, IE, APLOAD, YIELD, DISPL,**

      SUBROUTINE ADDLOD(FX, FY, FTHETA, IE, APLCAD, YIELD, DISPL,
     1          TLOAD,NEL,IBC)
      COMMON /N1/ NEQ, IBAND, IT
      COMMON /N2/ NNP
      COMMCN /N4/ZZ(20),AREAB(10),BB(20),HO(10)
      COMMON /N5/ PHATP(10), XLAMP(10), AREAP(10), PL(10)
     1, DENP(10), XIP(10), POWP(10), PORO(10)
     2, BETAP(10)
      COMMON/N6/PHATR(10),XLAMR(10),AREAR(1C),RL(10),DENR(10),VCID(10)
     1, XIR(10), POWR(10),
     2 BETAR(10)
      DIMENSICN FX(1), FY(1), FTHETA(1), IE(NEL,4), APLOAD(1), YIELD(1
     1          ), DISPL(1), TLOAD(1)
     2 , IBC(1)
C
C     THIS SUBROUTINE ASSEMBLES THE LOAD VECTOR WITH CONTRIBUTIONS FROM
C
C              (1) APPLIED NODAL LOADS
C
C              (2) LOAC FRCM NON-LINEAR YIELDING PILLARS AND RETORTS
C
C
      IF(IT.GT.0) GC TO 20
      DO 10 N = 1, NEQ
      APLCAD(N) = 0.
   10 TLOAD(N) = 0.
   20 DC 30 N = 1, NEG
      YIELD(N) = 0.
   30 DISPL(N) = TLOAD(N)
      IF (IT.GT.0) GC TO 50
C
C     APPLIED NODAL LOADS (INCLUDING GRAVITATIONAL) ARE ASSEBLED INTO
C     APLCAD.
C
      DC 40 N = 1, NNP
      IS = 3*(N-1) + 1
      APLOAD(IS) = FTHETA(N)
      APLOAD(IS+1) = FY(N)
   40 APLOAD(IS+2) = FX(N)
   50 DO 80 N = 1, NEL
C
C     SKIP OVERBURDEN ELEMENTS
C
C
C     THE NCN LINEAR PORTICN OF THE PILLAR AND RETORT BENDING AND
C     AXIAL STIFFNESS IS ASSEMBLED INTO THE LOAD VECTOR, YIELD.
C
      IP = IE(N,4)
      MTYPE = IE(N, 3)
      GO TO (80, 60, 70) , IP
   60 NOD = IE(N, 1)
      IC = IBC(NOD)
      U = DISPL(3*NCD -1)
      A = AREAP(MTYPE)
      H = PL(MTYPE)
      P = PHATP(MTYPE)
      X = XLAMP(MTYPE)
```

```
      V = PORO(MTYPE)
      XN = POWP(MTYPE)
      XK = 0.
      B = BETAP(MTYPE)
      IF((XN+X*V).EQ.0.) GO TO 600
      XK = X*P*V/(XN + X*V)
  600 XI = XIP(MTYPE)
      DO 61 NOB = 1, NEL
      IF(IE(NOB,4).NE.1) GO TO 61
      IF(IE(N,1).EQ.IE(NOB,1)) IOB = NOB
      IF(IE(N,1).EQ.IE(NCB,2)) IOB = NOB
   61 CONTINUE
      HB = HC(IE(IOB,3))
C
C     EVALUATE THE TANGENT MODULAUS FOR THE PILLAR AT THE
C     CURRENT VERTICAL STRAIN
C
      IF(V.LE.ABS(U/H)) GO TO 66
      PWR = 1.- XN
      IF(PWR)62,63,64
   62 E = XK*XN*(U/H/V)**(XN-1.)/V
      GO TO 65
   63 E = XK/V
      GO TO 65
   64 E = XK*XN*(V/U*H)**(1.-XN)/V
   65 GO TO 67
   66 E = X*EXP(-X*(U/H-V))*(P-XK)
C
C     APPLY THE X FORCES WHERE APPLICABLE TO THE PILLAR
C
   67 IF(IC.NE.10.AND.IC.NE.100.AND.IC.NE.110.AND.IC.NE.0) GO TC 68
      YIELD(3*NOD) = (2./H*DISPL(3*NOD)-(1.+HB/H)*DISPL(3*NOD-2))*
     1 6.*XI/H/H*(E-X*P)/(1.+2.*B)
C
C     APPLY THE THETA FORCES WHERE APPLICABLE TC THE PILLAR
C
   68 IF(IC.NE.1.AND.IC.NE.10.AND.IC.NE.11.AND.IC.NE.0) GO TO 69
      YIELD(3*NOD-2) = ((4.+2.*B)*(1.+.75*HB/H*(2.+HB/H))*DISPL(3*NOD-2)
     1 - 6./H*(1.+HB/H)*DISPL(3*NOD))*XI/H*(E-X*P)/(1.+2.*B)
C
C     APPLY THE Y FORCES WHERE APPLICABLE TO THE PILLAR
C
   69 IF(IC.NE.1.AND.IC.NE.100.AND.IC.NE.101.AND.IC.NE.0) GC TO 80
      YIELD(3*NCD-1) = FNONLN(A,H,P,X,U,XN,V) - P*A*X*U/H
      GO TO 80
   70 NOD = IE(N,1)
      IC = IBC(NCD)
      U = DISPL(3*NOD -1)
      A = AREAR(MTYPE)
      H = RL(MTYPE)
      P = PHATR(MTYPE)
      X = XLAMR(MTYPE)
      V = VOID(MTYPE)
      XN = PCWR(MTYPE)
      XK = 0.
      B = BETAR(MTYPE)
      IF((XN+X*V).EC.0.) GC TC 700
      XK = X*P*V/(XN + X*V)
```

```
       **SUBRCUTINE ACDLOD(FX, FY, FTHETA, IE, APLOAD, YIELD, DISPL,**

   700 XI = XIR(MTYPE)
       DO 71 NOB = 1, NEL
       IF(IE(NOB,4).NE.1) GC TC 71
       IF(IE(N,1).EQ.IE(NCB,1)) IOB = NOB
       IF(IE(N,1).EQ.IE(NOB,2)) IOB = NOB
    71 CONTINUE
       HB = HO(IE(IOB,3))
 C
 C     EVALUATE THE TANGENT MODULAUS FCR THE RETORT AT THE
 C     CURRENT VERTICAL STRAIN
 C
       IF(V.LE.ABS(U/H)) GO TO 76
       PWR = 1.- XN
       IF(PWR)72,73,74
    72 E = XK*XN*(U/H/V)**(XN-1.)/V
       GO TO 75
    73 E = XK/V
       GO TO 75
    74 E = XK*XN*(V/U*H)**(1.-XN)/V
    75 GO TO 77
    76 E = X*EXP(-X*(U/H-V))*(P-XK)
 C
 C     APPLY THE X FORCES WHERE APPLICABLE TO THE RETORT
 C
    77 IF(IC.NE.10.AND.IC.NE.100.AND.IC.NE.110.AND.IC.NE.0) GO TO 78
       YIELD(3*NCD) = (2./H*DISPL(3*NOD)-(1.+HB/H)*DISPL(3*NOD-2))*
      1 6.*XI/H/H*(E-X*P)/(1.+2.*B)
 C
 C     APPLY THE THETA FCRCES WHERE APPLICABLE TO THE RETORT
 C
    78 IF(IC.NE.1.AND.IC.NE.10.AND.IC.NE.11.AND.IC.NE.0) GO TO 79
       YIELD(3*NOD-2) = ((4.+2.*B)*(1.+.75*HB/H*(2.+HB/H))*DISPL(3*NOD-2)
      1 - 6./H*(1.+HB/H)*DISPL(3*NOD))*XI/H*(E-X*P)/(1.+2.*B)
 C
 C     APPLY THE Y FORCES WHERE APPLICABLE TC THE RETORT
 C
    79 IF(IC.NE.1.AND.IC.NE.100.AND.IC.NE.101.AND.IC.NE.0) GO TO 80
       YIELD(3*NOD-1) = FNONLN(A,H,P,X,U,XN,V) - P*A*X*U/H
    80 CONTINUE
 C
 C     SUM THE APPLIED AND NONLINEAR LOADS
 C
       DO 90 N = 1, NEQ
    90 TLCAD(N) = APLCAD(N) - YIELD(N)
       RETURN
       END

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```
      **SUBROUTINE SOLVE ( KK, AK, R, NEQ, IBAND )**

      SUBROUTINE SOLVE ( KK, AK, R, NEQ, IBAND )
      DIMENSION AK(NEQ, IBAND) , R(1)
C
C     THIS SUBROUTINE SOLVES A SET OF NEQ SYMMETRIC
C     EQUATIONS IN A BANDED SYSTEM WITH A HALF-BANDWIDTH
C     OF IBAND.  THE COEFFICIENTS ARE STORES IN AK AND THE
C     LOADS ARE STORED IN R.  THE SOLUTION IS RETURNED IN R.
C     KK = 1 TRIANGULARIZES THE AK MATRIX.  KK = 2 PERFORMS THE
C     BACK-SUBSTITUTION.
C
      NRS = NEQ - 1
      NR = NEQ
      IF(KK .EQ. 2) GO TO 200
      DO 120 N = 1, NRS
      M = N - 1
      MR = MINO(IBAND, NR-M)
      PIVOT = AK(N, 1)
      DO 120 L = 2, MR
      CP = AK(N, L) /PIVOT
      I = M + L
      J = 0
      DO 110 K = L, MR
      J = J + 1
  110 AK(I,J) = AK(I,J) - CP*AK(N,K)
  120 AK(N,L) = CP
      GO TO 400
  200 DO 220 N = 1, NRS
      M = N - 1
      MR = MINO(IBAND, NR-M)
      CP = R(N)
      R(N) = CP/AK(N,1)
      DO 220 L = 2, MR
      I = M + L
  220 R(I) = R(I) - AK(N,L)*CP
      R(NR) = R(NR)/AK(NR,1)
      DO 320 I = 1, NRS
      N = NR - I
      M = N - 1
      MR = MINO(IBAND, NR-M)
      DO 320 K = 2, MR
      L = M + K
  320 R(N) = R(N) - AK(N,K)*R(L)
  400 RETURN
      END

-- MNF4 LEVEL 5.24      27 JUN 80 11.31.24
```

```
        **SUBROUTINE GECMBC( N, AK, R, U, IBAND, NEQ )**

        SUBROUTINE GEOMBC( N, AK, R, U, IBAND, NEQ )
        DIMENSION AK(NEQ, IBAND), R(1)
C
C       GEOMBC ZEROS THE ROWS AND COLUMNS OF AN EQUATION WHICH IS A
C       GECMETRIC BOUNCARY CONDITION, PLACES A VALUE OF UNITY ON THE
C       DIAGONAL AND PLACES THE BOUNDARY CONDITICN INTO THE LOAD VECTOR.
C
        DO 100 M = 2, IBAND
        K = N - M + 1
        IF( K.LE.0 ) GC TO 50
        R(K) = R(K) - AK(K,M)*U
        AK(K,M) = 0.
     50 K = N + M - 1
        IF( K.GT.NEQ ) GO TC 100
        R(K) = R(K) - AK(N,M)*U
        AK(N,M) = 0.
    100 CONTINUE
        AK(N,1) = 1.
        R(N) = U
        RETURN
        END

-- MNF4 LEVEL 5.24      27 JUN 80 11.31.24
```

```
        **SUBROUTINE BOUNDC (FX,FY,FTHETA,IBC,AK,TLOAD,IBAND,NEQ)**

        SUBROUTINE BOUNDC (FX,FY,FTHETA,IBC,AK,TLOAD,IBAND,NEQ)
        COMMON /N2/ NNP
        DIMENSION FX(1), FY(1), FTHETA(1), IBC(1), AK(NEQ,IBAND), TLOAD(1)
C
C       THIS SUBROUTINE CALLS UPON GEOMBC TO APPLY GEOMETRIC
C       BOUNDARY CONDITIONS
C
        DO 60 N = 1, NNP
        IF( IBC(N).LT.100) GO TO 10
        MM = 3*(N - 1) + 1
        CALL GEOMBC(MM, AK, TLOAD, FTHETA, IBAND, NEQ)
     10 IF ( IBC(N).LT.10  .OR. IBC(N).EQ.100.OR.IBC(N).EQ.101) GO TO 20
        MM = 3*(N - 1) + 2
        CALL GEOMBC(MM, AK, TLOAD, FY     , IBAND, NEQ)
     20 IF ( IBC(N).NE. 1) GO TO 30
        MM = 3*N
        CALL GEOMBC(MM, AK, TLOAD, FX     , IBAND, NEQ)
     30 IF ( IBC(N).NE.11) GO TO 40
        MM = 3*N
        CALL GEOMBC(MM, AK, TLOAD, FX     , IBAND, NEQ)
     40 IF ( IBC(N).NE.101) GO TO 50
        MM = 3*N
        CALL GEOMBC(MM, AK, TLOAD, FX     , IBAND, NEQ)
     50 IF(IBC(N).NE.111) GO TO 60
        MM = 3*N
        CALL GEOMBC(MM, AK, TLOAD, FX     , IBAND, NEQ)
     60 CONTINUE
        RETURN
        END

-- MNF4 LEVEL 5.24      27 JUN 80 11.31.24
```

APPENDIX B

JOB CONTROL LANGUAGE FOR EXECUTING SUBSID ON THE LAWRENCE BERKELEY LABORATORY CDC-7600

Job control language for executing SUBSID in two different modes is given below. The JCL is only applicable to the Lawrence Berkeley Laboratory CDC-7600.

1. Executing SUBSID from an object deck (running the program without any changes:

```
Jobcard...

FETCHPS,SUBSID,ØBJECT,BEAM.

LINK, F=ØBJECT,R=LGØ,X.

EXIT.

DUMP,0.

7-8-9-

     -- Data for SUBSID --

end-of-file card
```

2. Executing SUBSID from the UPDATE source deck (running the program with temporary changes):

```
Jobcard...

FETCHPS,SUBSID,ØLDPL,SBEAM.

UPDATE,F.

MNF4,I=CØMPILE,L=0,E=4.

LINK,X.

EXIT.

DUMP,0.

7-8-9-

     -- UPDATE changes for SUBSID --

7-8-9-

     -- Data for SUBSID --

end-of-file card
```

FORTRAN Coding Form

PROGRAM EXAMPLE PROBLEM FOR SUBSID

PROGRAMMER

DATE

PUNCHING INSTRUCTIONS GRAPHIC

PUNCH

PAGE 1 OF 2

CARD ELECTRO NUMBER

```
    1   EXAMPLE PROBLEM WITH FULLY EXCAVATED LANES
   20   19   18    2    2    5    2    1
   50     .001
    1  101
    5    0       20.
   20           170.
    1    1    2    1    1
    4    4    5    1    1
    5    5    6    2    1
   10   10   11    2    1
   11    1    0    1    3
   12    2    0    1    2
   13    3    0    2    3
   14    4    0    2    2
   15    5    0    3    2
   16    6    0    4    2
   21   11    0    4    2
   22   11   12    2    1
   30   19   20    2    1
   31   12    0    4    2
   38   19    0    4    2
   39   20    0    5    2
     3000.      667.       20.       20.       24.      .027
     3000.      667.       20.       20.        6.      .027
```

FORTRAN Coding Form

PROGRAM

PROGRAMMER

DATE

PUNCHING INSTRUCTIONS GRAPHIC

PUNCH

PAGE 2 OF 2

CARD ELECTRO NUMBER

```
        3.      100.        4.        2.        0.      5.33       .02        2.
        6.
        3.      100.        4.        2.        0.      5.33       .02        2.
        6.
        3.      100.        8.        2.        0.      50.7       .02        2.
      28.5
        3.      100.       10.        2.        0.      83.3       .02        2.
      37.5
        3.      100.        5.        2.        0.      83.3       .02        2.
      37.5
        0.       50.        3.        2.        0.        9.       .04        2.
      13.5
        0.       50.        6.        2.        0.       18.       .04        2.
      13.5
    0
```

```
EXAMPLE PROBLEM WITH FULLY EXCAVATED LANES

PROGRAM PARAMETERS
------------------

NUMBER OF NODES . . .                          20
NUMBER OF OVERBURDEN ELEMENTS . . .            19
NUMBER OF PILLAR ELEMENTS . . .                18
NUMBER OF RETORT ELEMENTS . . .                 2
BODY FORCES (1-YES,0-NO) . . .                  1
NUMBER OF ITERATIONS ALLOWED . . .             50
RELATIVE TOLERANCE REQUESTED . . .         .00100

NODAL COORDINATES
-----------------
NODE      IBC        X               Y              F-X            F-Y            F-THETA

  1       101       -0.             -0.             -0.            -0.             -0.
  2         0        5.00            0.              0.             0.              0.
  3         0       10.00            0.              0.             0.              0.
  4         0       15.00            0.              0.             0.              0.
  5         0       20.00           -0.             -0.            -0.             -0.
  6        -0       30.00            0.              0.             0.              0.
  7        -0       40.00            0.              0.             0.              0.
  8        -0       50.00            0.              0.             0.              0.
  9        -0       60.00            0.              0.             0.              0.
 10        -0       70.00            0.              0.             0.              0.
 11        -0       80.00            0.              0.             0.              0.
 12        -0       90.00            0.              0.             0.              0.
 13        -0      100.00            0.              0.             0.              0.
 14        -0      110.00            0.              0.             0.              0.
 15        -0      120.00            0.              0.             0.              0.
 16        -0      130.00            0.              0.             0.              0.
 17        -0      140.00            0.              0.             0.              0.
 18        -0      150.00            0.              0.             0.              0.
 19        -0      160.00            0.              0.             0.              0.
 20        -0      170.00           -0.             -0.            -0.             -0.
ELEMENT CONNECTIONS
-------------------
ELEMENT      I         J      MATERIAL   ELEM TYPE     (1=OVERBURDEN, 2=PILLAR, 3=RETORT)

   1         1         2         1          1
   2         2         3         1          1
   3         3         4         1          1
   4         4         5         1          1
   5         5         6         2          1
   6         6         7         2          1
   7         7         8         2          1
   8         8         9         2          1
   9         9        10         2          1
  10        10        11         2          1
  11         1         0         1          3
  12         2         0         1          2
  13         3         0         2          3
  14         4         0         2          2
  15         5         0         3          2
  16         6         0         4          2
  17         7         0         4          2
  18         8         0         4          2
  19         9         0         4          2
  20        10         0         4          2
  21        11         0         4          2
```

```
         22        11        12        2        1
         23        12        13        2        1
         24        13        14        2        1
         25        14        15        2        1
         26        15        16        2        1
         27        16        17        2        1
         28        17        18        2        1
         29        18        19        2        1
         30        19        20        2        1
         31        12         0        4        2
         32        13         0        4        2
         33        14         0        4        2
         34        15         0        4        2
         35        16         0        4        2
         36        17         0        4        2
         37        18         0        4        2
         38        19         0        4        2
         39        20         0        5        2

        MATERIAL PROPERTIES
        -------------------
           OVERBURDEN
           ----------
           MAT          E            I          AREA        HEIGHT        BETA        DENSITY

             1     3000.00        667.00       20.00        20.00        24.00          .03
             2     3000.00        667.00       20.00        20.00         6.00          .03
           PILLARS
           -------

           MAT        YIELD        LAMDA        AREA        HEIGHT      DENSITY          I

             1         3.00       100.00        4.00         2.00         0.            5.33
             2         3.00       100.00        4.00         2.00         0.            5.33
             3         3.00       100.00        8.00         2.00         0.           50.70
             4         3.00       100.00       10.00         2.00         0.           83.30
             5         3.00       100.00        5.00         2.00         0.           83.30

           MAT    POROSITY       POWER ON STRAIN   BETA-P

             1      .02000          2.000          6.00
             2      .02000          2.000          6.00
             3      .02000          2.000         28.50
             4      .02000          2.000         37.50
             5      .02000          2.000         37.50
           RETORTS
           -------

           MAT        YIELD        LAMDA        AREA        HEIGHT      DENSITY      VOID RATIO      I

             1         0.          50.00        3.00         2.00         0.           .05          9.00
             2         0.          50.00        6.00         2.00         0.           .05         18.00

           MAT    POWER ON STRAIN    BETA-R

             1         2.000         13.50
             2         2.000         13.50

          60  EQUATIONS WITH A BANDWIDTH OF      6  REQUIRE   877  WORDS OF STORAGE
```

```
   CESIRED TCLERANCE CBTAINED AFTER      32  ITERATIONS


  DISPLACEMENTS
  -------------
      NODE           U-THETA              U-Y            U-X

         1     0.                      .3052E-01      0.
         2     -.4547E-04              .3006E-01      -.2049E-04
         3     -.8216E-04              .2958E-01      -.3821E-04
         4     -.9373E-04              .2828E-01      -.5593E-04
         5     -.7249E-04              .27C8E-01      -.6836E-04
         6     -.4026E-04              .2565E-C1      -.77C2E-04
         7     -.2222E-04              .2488E-01      -.7605E-04
         8     -.1216E-C4              .2446E-01      -.7077E-04
         9     -.6624E-05              .2425E-01      -.6391E-04
        10     -.3606E-05              .2413E-01      -.5683E-04
        11     -.1975E-05              .24C7E-01      -.5017E-04
        12     -.1098E-05              .2403E-01      -.4422E-04
        13     -.6261E-C6              .2402E-01      -.3906E-04
        14     -.3722E-06              .2401E-01      -.3470E-04
        15     -.2349E-06              .2400E-01      -.3109E-04
        16     -.1608E-06              .24C0E-01      -.2818E-04
        17     -.1217E-06              .2400E-01      -.2593E-04
        18     -.1032E-06              .2400E-01      -.2428E-04
        19     -.9773E-07              .24C0E-01      -.2320E-04
        20     -.9896E-07              .2400E-01      -.2267E-04


  CVERBURDEN FCRCES
  -----------------

  ELEMENT                    NODE  1                                         NODE  2
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       1       .216E+02     .135E+01      .246E+00          -.148E+02       -.135E+01     -.246E+00

  ELEMENT                    NCDE  2                                         NODE  3
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       2       .163E+02     .661E+00      .213E+00          -.130E+02       -.661E+00     -.213E+00

  ELEMEKT                    NCDE  3                                         NODE  4
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       3       .130E+02     .336E+01      .213E+0C           .377E+01       -.336E+01     -.213E+00

  ELEMENT                    NODE  4                                         NODE  5
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       4      -.845E+C0     .306E+01      .149E+00           .162E+02       -.3C6E+01     -.149E+00

  ELEMEKT                    NCDE  5                                         NODE  6
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       5       .161E+01     .161E+01      .520E-01           .145E+02       -.161E+01     -.520E-01

  ELEMENT                    NODE  6                                         NODE  7
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       6       .614E+00     .845E+C0     -.585E-02           .783E+01       -.845E+00      .585E-02

  ELEMENT                    NODE  7                                         NODE  8
                P-THETA       P-Y           P-X              P-THETA          P-Y           P-X
       7       .203E+00     .443E+00     -.317E-01           .423E+01       -.443E+00      .317E-01
```

```
ELEMENT                   NODE   8                                     NODE   9
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
   8       .498E-01       .232E+00    -.412E-01         .227E+01      -.232E+00     .412E-01

ELEMENT                   NODE   9                                     NODE  10
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
   9      -.125E-02       .121E+00    -.425E-01         .121E+01      -.121E+00     .425E-01

ELEMENT                   NODE  10                                     NODE  11
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  10      -.156E-01       .622E-01    -.399E-01         .637E+00      -.622E-01     .399E-01

ELEMENT                   NODE  11                                     NODE  12
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  22      -.177E-01       .316E-01    -.357E-01         .333E+00      -.316E-01     .357E-01

ELEMENT                   NODE  12                                     NODE  13
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  23      -.163E-01       .156E-01    -.309E-01         .172E+00      -.156E-01     .309E-01

ELEMENT                   NODE  13                                     NODE  14
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  24      -.142E-01       .732E-02    -.262E-01         .874E-01      -.732E-02     .262E-01

ELEMENT                   NODE  14                                     NODE  15
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  25      -.121E-01       .307E-02    -.217E-01         .428E-01      -.307E-02     .217E-01

ELEMENT                   NODE  15                                     NODE  16
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  26      -.102E-01       .922E-03    -.174E-01         .194E-01      -.922E-03     .174E-01

ELEMENT                   NODE  16                                     NODE  17
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  27      -.837E-02      -.109E-03    -.135E-01         .728E-02       .109E-03     .135E-01

ELEMENT                   NODE  17                                     NODE  18
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  28      -.636E-02      -.532E-03    -.989E-02         .104E-02       .532E-03     .989E-02

ELEMENT                   NODE  18                                     NODE  19
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  29      -.399E-02      -.579E-03    -.646E-02        -.180E-02       .579E-03     .646E-02

ELEMENT                   NODE  19                                     NODE  20
            P-THETA         P-Y          P-X            P-THETA          P-Y          P-X
  30      -.133E-02      -.315E-03    -.319E-02        -.182E-02       .315E-03     .319E-02


PILLAR STRESSES
---------------
PILLAR ELEMENT          STRESS

      12                  .85
      14                  .75
      15                  .69
      16                  .62
      17                  .58
      18                  .56
      19                  .55
      20                  .55
      21                  .54
```

```
           31                 .54
           32                 .54
           33                 .54
           34                 .54
           35                 .54
           36                 .54
           37                 .54
           38                 .54
           39                 .54

       RETORT STRESSES
       ---------------
       RETORT ELEMENT        STRESS

           11                 0.
           13                 0.
```

```
EXAMPLE PROBLEM WITH BACKFILLED LANES

PRCGRAM PARAMETERS
------------------

NUMBER OF NODES . . .                          20
NUMBER OF CVERBURDEN ELEMENTS . . .            19
NUMBER OF PILLAR ELEMENTS . . .                18
NUMBER OF RETCRT ELEMENTS . . .                 2
BODY FORCES (1-YES,0-NO) . . .                  1
NUMBER OF ITERATIONS ALLOWED . . .             50
RELATIVE TCLERANCE REQUESTED . . .         .00100

NODAL COCRDINATES
-----------------
NODE      IBC        X              Y              F-X            F-Y            F-THETA

  1       101       -0.            -0.             -0.            -0.             -0.
  2         0        5.00           0.              0.             0.              0.
  3         0       10.00           0.              0.             0.              0.
  4         0       15.00           0.              0.             0.              0.
  5         0       20.00          -0.             -0.            -0.             -0.
  6        -0       30.00           0.              0.             0.              0.
  7        -0       40.00           0.              0.             0.              0.
  8        -0       50.00           0.              0.             0.              0.
  9        -0       60.00           0.              0.             0.              0.
 10        -0       70.00           0.              0.             0.              0.
 11        -0       80.00           0.              0.             0.              0.
 12        -0       90.00           0.              0.             0.              0.
 13        -0      100.00           0.              0.             0.              0.
 14        -0      110.00           0.              0.             0.              0.
 15        -0      120.00           0.              0.             0.              0.
 16        -0      130.00           0.              0.             0.              0.
 17        -0      140.00           0.              0.             0.              0.
 18        -0      150.00           0.              0.             0.              0.
 19        -0      160.00           0.              0.             0.              0.
 20        -0      170.00          -0.             -0.            -0.             -0.
ELEMENT CONNECTIONS
-------------------
ELEMENT      I         J      MATERIAL   ELEM TYPE     (1=OVERBURDEN, 2=PILLAR, 3=RETORT)

   1         1         2         1          1
   2         2         3         1          1
   3         3         4         1          1
   4         4         5         1          1
   5         5         6         2          1
   6         6         7         2          1
   7         7         8         2          1
   8         8         9         2          1
   9         9        10         2          1
  10        10        11         2          1
  11         1         0         1          3
  12         2         0         1          2
  13         3         0         2          3
  14         4         0         2          2
  15         5         0         3          2
  16         6         0         4          2
  17         7         0         4          2
  18         8         0         4          2
  19         9         0         4          2
  20        10         0         4          2
  21        11         0         4          2
```

```
       22          11          12           2           1
       23          12          13           2           1
       24          13          14           2           1
       25          14          15           2           1
       26          15          16           2           1
       27          16          17           2           1
       28          17          18           2           1
       29          18          19           2           1
       30          19          20           2           1
       31          12           0           4           2
       32          13           0           4           2
       33          14           0           4           2
       34          15           0           4           2
       35          16           0           4           2
       36          17           0           4           2
       37          18           0           4           2
       38          19           0           4           2
       39          20           0           5           2

     MATERIAL PROPERTIES
     -------------------
          OVERBURDEN
          ----------
          MAT           E              I            AREA        HEIGHT         BETA        DENSITY

            1       3000.00         667.00         20.00         20.00         24.00            .03
            2       3000.00         667.00         20.00         20.00          6.00            .03
          PILLARS
          -------

          MAT          YIELD        LAMDA          AREA         HEIGHT      DENSITY            I

            1           3.00       100.00          4.00          2.00        0.               5.33
            2           3.00       100.00          4.00          2.00        0.               5.33
            3           3.00       100.00          8.00          2.00        0.              50.70
            4           3.00       100.00         10.00          2.00        0.              83.30
            5           3.00       100.00          5.00          2.00        0.              83.30

          MAT      POROSITY        POWER ON STRAIN   BETA-P

            1        .02000          2.000            6.00
            2        .02000          2.000            6.00
            3        .02000          2.000           28.50
            4        .02000          2.000           37.50
            5        .02000          2.000           37.50
          RETORTS
          -------

          MAT          YIELD        LAMDA          AREA         HEIGHT      DENSITY       VOID RATIO         I

            1           1.50        50.00          3.00          2.00        0.             .04            9.00
            2           1.50        50.00          6.00          2.00        0.             .04           18.00

          MAT      POWER ON STRAIN    BETA-R

            1            2.000          13.50
            2            2.000          13.50

       60  EQUATIONS WITH A BANDWIDTH OF      6  REQUIRE    877  WORDS OF STORAGE
```

```
DESIRED TCLERANCE CBTAINED AFTER      31   ITERATIONS


DISPLACEMENTS
-------------
    NCDE           U-THETA            U-Y              U-X

       1      0.                    .2937E-01        0.
       2      -.3661E-04            .2901E-01        -.1724E-04
       3      -.6592E-04            .2860E-01        -.3232E-04
       4      -.7656E-04            .2755E-01        -.4666E-04
       5      -.6000E-04            .2655E-01        -.5679E-04
       6      -.3352E-04            .2537E-01        -.6393E-04
       7      -.1851E-04            .2473E-01        -.6312E-04
       8      -.1012E-04            .2438E-01        -.5874E-04
       9      -.5506E-05            .2420E-01        -.5305E-04
      10      -.2994E-05            .2411E-01        -.4717E-04
      11      -.1639E-05            .24C6E-01        -.4165E-04
      12      -.9102E-06            .2403E-01        -.3671E-04
      13      -.5190E-06            .24C1E-01        -.3243E-04
      14      -.3085E-06            .2401E-01        -.2880E-04
      15      -.1948E-06            .2400E-01        -.2581E-04
      16      -.1334E-06            .2400E-01        -.2340E-04
      17      -.1010E-06            .2400E-01        -.2152E-C4
      18      -.8564E-07            .24C0E-01        -.2016E-04
      19      -.8111E-07            .2400E-01        -.1926E-04
      20      -.8214E-07            .2400E-01        -.1882E-04


OVERBURDEN FORCES
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ELEMENT                      NODE   1                                                NODE   2
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   1          .173E+02       .1C5E+01         .2C7E+00           -.120E+02            -.105E+01      -.207E+00

ELEMENT                      NODE   2                                                NODE   3
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   2          .132E+02       .591E+00         .181E+00           -.103E+02            -.591E+00      -.181E+00

ELEMENT                      NODE   3                                                NODE   4
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   3          .110E+02       .272E+01         .172E+00           .253E+01             -.272E+01      -.172E+00

ELEMENT                      NODE   4                                                NODE   5
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   4          -.204E+00      .257E+01         .122E+00           .131E+02             -.257E+01      -.122E+00

ELEMENT                      NCDE   5                                                NODE   6
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   5          .136E+01       .133E+01         .428E-01           .120E+02             -.133E+01      -.428E-01

ELEMENT                      NODE   6                                                NODE   7
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   6          .493E+00       .699E+00         -.485E-02          .650E+01             -.699E+00      .485E-02

ELEMENT                      NODE   7                                                NODE   8
              P-THETA         P-Y              P-X               P-THETA              P-Y            P-X
   7          .156E+00       .367E+00         -.263E-01          .351E+01             -.367E+00      .263E-01
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ELEMENT                 NODE   8                                   NODE   9
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
   8     .354E-01     .192E+00     -.342E-01     .188E+01     -.192E+00     .342E-01

ELEMENT                 NODE   9                                   NODE  10
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
   9    -.352E-02     .998E-01     -.353E-01     .100E+01     -.998E-01     .353E-01

ELEMENT                 NODE  10                                   NODE  11
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  10    -.139E-01     .515E-01     -.332E-01     .529E+00     -.515E-01     .332E-01

ELEMENT                 NODE  11                                   NODE  12
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  22    -.151E-01     .261E-01     -.296E-01     .276E+00     -.261E-01     .296E-01

ELEMENT                 NODE  12                                   NODE  13
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  23    -.137E-01     .129E-01     -.257E-01     .143E+00     -.129E-01     .257E-01

ELEMENT                 NODE  13                                   NODE  14
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  24    -.118E-01     .606E-02     -.217E-01     .724E-01     -.606E-02     .217E-01

ELEMENT                 NODE  14                                   NODE  15
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  25    -.101E-01     .254E-02     -.180E-01     .354E-01     -.254E-02     .180E-01

ELEMENT                 NODE  15                                   NODE  16
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  26    -.849E-02     .760E-03     -.145E-01     .161E-01     -.760E-03     .145E-01

ELEMENT                 NODE  16                                   NODE  17
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  27    -.695E-02    -.932E-04     -.112E-01     .601E-02      .932E-04     .112E-01

ELEMENT                 NODE  17                                   NODE  18
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  28    -.528E-02    -.443E-03     -.821E-02     .853E-03      .443E-03     .821E-02

ELEMENT                 NODE  18                                   NODE  19
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  29    -.331E-02    -.481E-03     -.537E-02    -.150E-02      .481E-03     .537E-02

ELEMENT                 NODE  19                                   NODE  20
          P-THETA        P-Y          P-X         P-THETA        P-Y          P-X
  30    -.110E-02    -.261E-03     -.265E-02    -.151E-02      .261E-03     .265E-02


PILLAR STRESSES
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PILLAR ELEMENT          STRESS

      12                  .79
      14                  .71
      15                  .66
      16                  .60
      17                  .57
      18                  .56
      19                  .55
      20                  .54
      21                  .54
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          31                    .54
          32                    .54
          33                    .54
          34                    .54
          35                    .54
          36                    .54
          37                    .54
          38                    .54
          39                    .54

      RETORT STRESSES
      ---------------
      RETORT ELEMENT        STRESS

          11                    .10
          13                    .10
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