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Author
Laslett, L. Jackson.

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L. Jackson Laslett

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L. Jackson Laslett

Lawrence Berkeley Laboratory
University of California
Berkeley, California

Introduction

The electron ring accelerator has appeared to constitute a particularly attractive, or at least a conceptually straightforward, means for the acceleration of ions by use of collective fields. The potential advantages of collective-field acceleration were emphasized for many years by Veksler.[1] The report by Kolomenskii at the 1967 Accelerator Conference in Cambridge of an active electron-ring development program at Dubna,[2] under the direction of V.P. Sarantsev, stimulated the initiation of similar experimental programs and related theoretical studies by several other groups. It was hoped by many at that time that the formation of a high-quality ring of relativistic electrons would provide a holding field as great as at least a few hundred MV per meter and that acceleration of the ring then would permit trapped positive ions to be brought to high energies in a relatively short distance.[2,3] Other potentially useful characteristics of such an accelerator were also recognized -- as the availability of output beam pulses of short duration and the ability to accelerate simultaneously heavy ions of diverse charge/mass ratio.[4]

Now that electron-ring programs have been in progress for several years at a number of laboratories, it is indeed timely to inquire concerning our present degree of understanding with respect to the basic physical phenomena that can act to limit the performance of an electron-ring accelerator. It would appear premature at this time to attempt to specify the competitive economic position that such an accelerator could occupy for various applications -- more relevant at the moment is a quantitative and confirmed understanding of ring behavior, and of the implications of this behavior. Much of the effort during the past years of course necessarily has been devoted to the assembly of experimental equipment and, on the theoretical side, to the examination of alternative concepts for ring-formation and use. Nevertheless, quantitative experiments have been performed (and are being continued), related theoretical and computational analyses have been carried through, and useful information concerning relevant phenomena also has become available from work with synchrotrons and storage rings. These activities have led, as I hope to indicate, to an apparently good degree of understanding concerning several phenomena of importance, while a considerable amount of joint experimental and theoretical effort with respect to other phenomena remains to be completed before the performance capabilities of electron-ring accelerators can be confidently forecast. We of course have been encouraged during this work by the reported success of the Dubna group[5] in achieving He^{++}-ions of 30 MeV energy by magnetic-expansion acceleration through a distance estimated as 0.4m.

Problems of Basic Interest

The problems for which basic understanding and control are the most critical appear to be the following:

The stability with respect to single-particle betatron-oscillation resonances,

The stability with respect to collective oscillations of the electron beam -- transverse and longitudinal,

The stability with respect to collective electron-ion oscillations,

The radiation of energy by an intense ring beam during passage through an electric-acceleration structure, and

The selection of design parameters that conform to fundamental physical restrictions and lead to "optimized" performance.

Single-Particle Resonances

Early electron-ring experiments[6] at several laboratories have revealed beam loss or a degradation of beam quality attributable to single-particle betatron resonances -- notably at field-index values n = 0.5, 0.36, 0.25, and 0.2.[7] It is noteworthy that even the difference resonances associated with n = 0.5 or 0.2 evidently can lead to unacceptably large axial amplitudes through the resonant coupling of axial and radial betatron oscillations.

Analytic studies of betatron resonances of course have been made[8] for application to more conventional devices, and the analyses can be directly adapted[9] to describe such resonant behavior in an electron-ring compressor. One's understanding of these phenomena should be considered sound, particularly since the analyses can be monitored and the results readily checked by means of computer calculations. The phenomena do exist, however, and should be controlled in practical compressor designs.

Significant beam deterioriation as a result of betatron resonances can be avoided by suitable design of the compression cycle, so as to obtain a variation of n with time ("n-trajectory") that neither leads to the crossing of resonant values immediately after injection nor to slow traversal of such values at any stage of the compression cycle. The reduction of large spatial non-linearities of the magnetic field can also be desirable. In the case of inhomogeneous resonances, which are driven by azimuthal variations of the field, reduction of azimuthal inhomogeneities also can be helpful -- as has been shown experimentally by, for example, some recent experiments[6c] at Garching relating to the n = 0.25 resonance.

Attention to the points just mentioned should permit suppression of resonant beam loss and preclude any significant degradation of beam quality. Rapid compression cycles, which can be attractive for other reasons, have been favored by some workers[10] as an effective technique for reducing resonant effects. It should be noted that betatron resonances other than those mentioned here may warrant attention if the field at the orbit does not exhibit median-plane symmetry or if an azimuthal component of magnetic field is present. Thus if a supplemental azimuthal magnetic field ("H_ϕ-field") is provided, as has been advocated in some reports,[11] the $\nu_z = 1/3$ single-particle resonance may become prominent[12] and require study. During acceleration of the ring, moreover, when some ion focusing will be present but the axial magnetic field will provide negligible axial focusing and orbit curvature introduces some defocusing,[5,13] supplemental image focusing (e.g., by a suitably slotted cylinder-like screen or "squirrel cage"[11a]) or use of an H_ϕ-field[11] may be desirable.

Coherent Oscillations

Coherent beam oscillations have been observed, and in many cases stabilized, in a large number of particle-handling devices. There also has been a great amount of analytic work devoted to this topic, for which the results considered most relevant to electron-ring performance have been summarized in a recent paper.[14] The collective phenomena of course are driven by self-generated electro-magnetic fields, whose magnitude and phase will be significantly influenced by the surroundings and that possibly may be resonant with the vacuum chamber within which the beam is situated. It has been found convenient in recent work to characterize the effective $\vec{\mathcal{E}} + \vec{v} \times \vec{B}$ driving field that arises in this way by a complex "coupling impedance",[15] defined as $Z_M = - C\,\mathcal{E}_{eff}/I_M$, where C is the orbit circumference and I_M is the current of the associated collective-oscillation mode. In some instances measurements of Z_M can conveniently be made by electrical means.[16]

The dynamical analysis of a potential collective instability customarily commences with the Vlasov equation, which normally is linearized about a simple (e.g., unmodulated) equilibrium state, and the nature of complex solutions ω to the resulting dispersion equation are then sought for various assumed distribution functions that can provide some Landau damping.[17] In the event that any of the simplifying assumptions required for the analysis appear to be unrealistic, recourse may be had to alternative techniques of a computational nature and simulation programs[18] relating to phenomena occurring in electron-ring devices have begun to be applied for this purpose.

Transverse Coherent Stability -- The transverse stability of a coasting (unmodulated) beam, moving in the presence of walls of a high conductivity and a thickness at least as great as several skin depths, has been investigated analytically and the results applied in assessing the possible performance characteristics of electron-ring accelerators.[14] The results indicate that, for the assumed conditions, the transverse collective stability of a high quality ring beam can easily be achieved by the Landau damping normally associated with only a modest amount of energy spread.

Some experiments during the latter part of 1970 with Compressor 4 at Berkeley did show, however, a radial instability of this nature at certain radii and the associated radio-frequency signals were observed to be of frequency $(M-\nu_r)\omega_\odot/2\pi$ with $M = 1$.[19] The circumstances of these particular experiments were somewhat special in that, as one feature, the thin metallic side walls of the chamber, situated about 3.5 cm to either side of the ring beam, had a surface resistance $R_s \cong 50\Omega/\square$ to permit virtually unimpeded penetration by the pulsed inflector field. Recognizing this special nature of the side walls, Lambertson pointed out that radial oscillations of the beam current could induce wall currents whose magnetic fields would act regeneratively on the oscillating beam and that a specific analysis (valid for large R_s) would be desirable. Neglecting the somewhat smaller effect of wall currents arising from the motion of electrostatically induced charges, the e-folding growth rate of a collective radial oscillation in the absence of Landau damping could be estimated as[20]

$$\frac{1}{\tau_G} \cong \frac{N\, r_e\, \beta^2\, c\, (M-\nu_r)}{4\pi\, \gamma\, \nu_r\, h\, R\, R_s}\, Z_o\, f(h/R)$$

for a ring of radius R containing N particles, where r_e is the classical electron radius, $Z_o = \sqrt{\mu_o/\epsilon_o} = 377$ ohms, and the curvature factor f is of the order of 1/2. With the parameters characteristic of Compressor 4 at that time, a ring beam of no more than 5×10^{11} electrons thus could be expected to lead to growth rates of the order of 1.6 μsec^{-1} (for $M = 1$) unless suppressed by Landau damping, and for more intense beams ($N \cong 10^{13}$) the growth rate to be suppressed would be correspondingly greater (e.g., 32 μsec^{-1}).

Landau damping of the transverse collective oscillations is provided most effectively by energy spread in the circulating beam and stability requires an energy spread sufficiently great that[21]

$$\left|(E\, \partial S/\partial E)(\Delta E/E)\right| > 1/\tau_G,$$

where

$$S \equiv (M - \nu_r)\omega_\odot$$

and

$$E\, \partial S/\partial E = - \frac{1}{\beta^2} \left[(M-\nu_r)(1 - \frac{1-n}{\gamma^2}) + R\, \frac{\partial \nu_r}{\partial R}\right] \frac{\omega_\odot}{1-n}$$

with $\partial\nu_r/\partial R$ typically a negative quantity. Following a suggestion of Sessler concerning the possibility of a strong cancellation of terms within the square bracket of the equation for the damping coefficient $E\,\partial S/\partial E$, it was found[22] that eddy currents induced in the windings of the Stage-2 compression coils indeed did result in the Landau damping coefficient for the radial mode $M = 1$ becoming essentially zero some 20 μsec following injection. Substitution of a coil formed of stranded conductors served to suppress the eddy currents and raised the damping coefficient sufficiently that the radial instability was no longer observed. A corresponding re-design of the Stage-2 and Stage-3 coils of Compressor 5 (through use of thin-walled stainless-steel tubing) has likewise removed the similar undesired suppression of $E\,\partial S/\partial E$ at certain radii in that device, with the result that the calculated value for this quantity remains above 550 μsec^{-1} throughout the compression cycle.[23] Thus, although the nature of the walls of a compression chamber may be modified in future designs, the effectiveness of the Landau damping coefficient for suppressing collective radial instabilities appears to be understood and reasonably effective. Further control of this damping coefficient can be obtained, if desired, through the introduction of a supplemental azimuthal magnetic field.[24]

An alternative way of examining the effect of resistive walls may be useful for application to an injected beam with considerable radial or azimuthal structure -- especially when investigating the dynamical consequences computationally. In introducing this alternative approach, it is helpful first to think of a line of charge and current suddenly established in the mid-plane between two parallel resistive walls. At that instant full strength electric and current images should arise (since the side plates will momentarily shield the exterior region from the beam), and the effective $\vec{F} = \vec{\mathcal{E}} + \vec{v} \times \vec{B}$ image field in the interior will be small ($\propto 1/\gamma^2$) for β close to unity. Due to the resistance of the side plates, however, the induced currents will re-distribute and attenuate in a calculable way, with the result that the effective $\vec{\mathcal{E}} + \vec{v} \times \vec{B}$ field at any point in the interior will grow in magnitude to approach the field $\vec{\mathcal{E}}$. The changing magnetic image field does not exhibit an identical time behavior at all points, but for computational purposes the change of the image-field focusing coefficient may be conveniently characterized by a time constant that calculations indicate is of the order of $\tau_W = \mu_o h/2R_s$ (MKS). In orbit computations with a beam that is shifting its location, the effective image field at each of many sampling points in the interior then may be continually updated by means of differential equations that contain the "time constant" τ_W as a parameter:

$$\frac{dF(\vec{r})}{dt} = \frac{1}{\tau_W}\left[\sum_k \lambda_k E_1(\vec{r},\vec{r}_k) - F(\vec{r})\right] ,$$

where $E_1(\vec{r},\vec{r}_k)$ is the (electric) field initially developed at $\vec{r}$ by a unit line charge at $\vec{r}_k$.

This type of analysis of the electromagnetic problem, if applied to a line current oscillating transversely about the axis of a resistive circular tube and the results expressed in a Fourier form, appears to give agreement (for $R_s < Z_o/2$ and wall thickness $< 1.7\ \delta$) with values of the quantity V (customarily employed in perturbation treatments of beam stability) obtained from a program[25] based on general methods developed at CERN by Zotter.[26] To the extent that this is correct, the quantity $1/\tau_G$ for a growing collective oscillation of specific frequency, should (as mentioned before) be proportional to $1/R_s$ when the wall resistance is high but should be directly proportional to R_s for walls of rather low resistance [$\rho_V/\delta < R_s < \mu_o h\omega/2 = \mu_o h(M-\nu_r)\omega_\theta/2$]. We thus have been led to employ walls with a surface resistance considerably less than $1\Omega/\square$ -- for example $1/15\ \Omega/\square$ ($\tau_W \cong 300$ nsec) -- while providing one or more small windows for introduction of the inflector field. The adoption of low resistance walls, moreover, also should be favorable with respect to a longitudinal collective instability. Dynamical computations with respect to the transverse instability then indicate growth rates distinctly less than 1 μsec^{-1} for circulating beams of 10^{13} electrons in the absence of Landau damping and accordingly, as we have found, a very moderate energy spread may suffice for the suppression of this instability.

Longitudinal Coherent Stability -- The potential longitudinal (azimuthal) coherent instability of an intense beam has been seen to develop and is of considerable interest in connection with synchrotron accelerators,[27] storage rings, and electron-ring devices. The necessity of avoiding this particular potential instability indeed may present a major limitation to the attainable performance of electron-ring devices. Experimental work at Berkeley on the formation of rings of various intensities has shown that a greater amount of energy spread is necessarily present when stable rings of high intensity are achieved. The analytic result for the stability limit, expressing the maximum permissible number of particles in terms of a longitudinal coupling impedance Z_M for the mode M of interest,[28]

$$N = |\eta|\ \frac{\gamma R}{2\beta^3 r_e}\ \frac{Z_o}{|Z_M|/M}\ \left(\frac{\Delta E}{E}\right)^2$$

(in which $|\eta| = |(1-n)^{-1} - \gamma^{-2}| \cong (1-n)^{-1}$ for an electron-ring device), has been confirmed experimentally at CERN -- especially in a series of careful experiments with the I.S.R.[29]

The achievement of high-quality rings accordingly requires that the longitudinal coupling impedance Z_M -- or, strictly, $|Z_M|/M$ -- be kept low for a large number of modes. Careful design of an electron-ring device therefore will undertake to suppress the coupling impedance for low modes by the use of (longitudinally) conducting material close to the beam, will attempt to avoid (or to degrade) electro-magnetic resonances, and will aim for an impedance characteristic that for the higher mode numbers is close to that characteristic of a beam in free space ($|Z_M|/M \propto M^{-2/3}$). Calculations have been made of the values of Z_M/M for a wide variety of configurations potentially suitable for an electron-ring device, of which one[30] indicated the utility of situating the ring beam, at the time of its release from the magnetic well, between a pair of coaxial conducting tubes and preferably close to the inner member of the tube pair.

It will be recognized that the location of a conducting surface with little radial clearance to the beam, in the interests of reducing $|Z_M|/M$, precludes the presence of substantial energy spread, that will act to provide Landau damping but that also will lead to an increased radial spread of the beam. A good design would need to achieve, therefore, a suitable balance between these effects. Also, of course, a conducting surface of simple design, if situated at a radius only slightly smaller than that of the ring beam, necessarily must reduce the effectiveness of a pulsed (or RF) electric field intended to accelerate the ring in an electric acceleration column. In the successful magnetic-acceleration experiments reported from Dubna[5] it is difficult to see how values of $|Z_M|/M$ as low as would be required theoretically for stability could have been present throughout the acceleration. We may conclude, therefore, that the maintenance of azimuthal stability under such conditions should be no more difficult than is suggested by the present theory.

Stability with respect to azimuthal collective oscillation of course is essential throughout the compression stage of an electron-ring compressor. The situation immediately following injection may, in particular, be rather different than that just discussed. Specifically, the beam may have an initial density modulation of substantial magnitude as a result of the injection process. The beam also may be composed of several rather distinct turns, it may have a progressive energy variation along its length, and the amplitude of betatron oscillations may be significantly great. Simulation programs seem particularly attractive for the study of the de-bunching (or bunching) of circulating beams immediately following injection. We have begun to make simple computations of this nature at Berkeley, and the early results appear to be in general agreement with expectations -- it remains to be seen whether use of such programs can realistically be extended to give insight into the interpretation of experimental results obtained under various conditions with rings that have survived for many hundreds of turns.

Collective Electron-Ion Oscillations

The possibility of unstable transverse collective oscillations of ions vs. electrons in an electron-ring accelerator was considered in several early papers,[3] and recently has been examined in greater detail with respect to both dipole and quadrupole modes by Zenkevich and Koshkarev.[31] Ion-electron instabilities of the type described by this analysis indeed appear to have been observed in conventional accelerators such as the Bevatron.[32] The work of Zenkevich and Koshkarev certainly indicates that the achievement of stability with respect to this type of motion can restrict the performance of an electron-ring accelerator -- at least with respect to the number of ions that can safely be trapped by the electron ring beam. A further, somewhat detailed, examination[33] of the influence of Landau damping, of intra-species forces, and of image effects suggests, moreover, that these effects in practice will not prove to be significantly helpful.

Radiation Reaction

The radiation reaction experienced by an intense electron ring in passing through a periodic structure (as in an electric-acceleration column) was of considerable concern during the past years,[3] especially because it was uncertain whether the energy loss from this mechanism would increase with γ_z. The situation seems now to have become clarified, as a result of experimental work at SLAC,[34] computations by Keil,[35] and analytic calculations.[36] The conclusion, as summarized in a recent panel discussion,[37] is that the radiation loss per cell fortunately exhibits no γ dependence over a wide range of energies, the electron-ring dimensions do not affect the loss in a critical way, but the reaction force is substantial unless the bore of the acceleration column is large.

Expected Performance Characteristics

The selection of "optimum" parameters for a complete electron-ring compressor and acceleration system would be an extensive task and, moreover, would be influenced by the figure-of-merit considered appropriate to the application of interest. Information to stimulate discussion can be gained, however, by employing[14] current concepts to up-date an earlier analysis[38] -- with particular emphasis on the holding field for protons that could be produced by a stable electron ring at the end of compression. To present a highly simplified summary, one may express the useful holding field (in the absence of safety factors) as[39]

$$e\,\mathcal{E} \cong \frac{Ne^2}{\pi R(\sigma_a+\sigma_b)}\;\frac{1}{\alpha}$$

for a ring of N electrons, of major radius R, and with radial and axial minor dimensions of standard deviation σ_a, σ_b, while α is a factor that should be at least as great as 2 for magnetic acceleration.[14]

If we now confine our attention solely to the important requirement of azimuthal collective stability and write[30]

$$|Z_M|/M = 300\,\frac{h}{R-h} \sim 1200\,\sigma_a/R \text{ ohms}$$

for a ring situated a distance $h = 4\sigma_a$ from a conducting tube, the stability limit occurs for

$$N = \frac{377}{2400}\;\frac{\gamma R^2}{r_e\sigma_a}\;\left(\frac{\Delta E}{E}\right)^2$$

and

$$e\,\mathcal{E} = \frac{377}{2400\pi\alpha}\;\frac{\gamma R}{\sigma_a(\sigma_a+\sigma_b)}\;\left(\frac{\Delta E}{E}\right)^2\;(m_oc^2).$$

With $\Delta E/E$ at most given by $2.36\sigma_a/R$ (betatron-oscillation amplitude small) and $\sigma_a \gg \sigma_b$, then

$$e\,\mathcal{E} = \frac{377\,(2.36)^2}{2400\pi\alpha}\;\frac{\gamma}{R}\;m_oc^2.$$

For highly relativistic particles circulating in a guide field of strength B gauss, $\gamma/R = (e/m_oc^2)B$, and, for electrons, $\gamma/R = 5.867 \times 10^{-4}$ B. We thus obtain, with $\alpha = 2$,

$$e\,\mathcal{E} = \frac{377\,(2.36)^2}{(2400\pi)\,2}\;(5.867\times10^{-4})(0.511\times10^6)B$$

$$= 41.7 \text{ B ev/cm for B in gauss}$$

$$= 4.17 \text{ B MeV/m for B in kilogauss.}$$

Thus for $B = 20$ kilogauss and magnetic acceleration of the electron ring, the maximum effective acceleration field for trapped ions becomes approximately 83 MV/m, which could be attractive for some applications, although recognition of additional constraints may lead to smaller estimates for this field.[14] Higher values of the guide field B of course could lead to larger values of the useful acceleration field [as has been noted by Möhl (ERAN-178)], but such an increase would not necessarily be advantageous economically.

Conclusion

In conclusion, one can state that understanding of beam stability has been, and continues to be, particularly important to development of the ERA. The situation in this regard appears to be reasonably well in hand for single-particle resonances and for transverse collective motion. Knowledge concerning characteristics of the longitudinal instability is being acquired experimentally. There is available a considerable volume of theory pertaining to this phenomenon and to electron-ion collective motion also. Work at present is actively continuing with the object of establishing the connection between experimental observations and the theory. Simulation programs are beginning to aid in extending the theoretical work, and most importantly, it is gratifying that the experimental programs of several ERA groups have arrived at the point where quantitative experiments can be designed and conducted to test our present understanding and to stimulate its further advance.

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21. The quantity $1/\tau_G$ will be recognized as corresponding to the quantity V of earlier papers (cf. Ref. 17a), or to one term of a complex "coupling impedance" [see, for example, D. Möhl and A.M. Sessler, Proc. VIII Internat. Conf. on High Energy Accelerators (CERN, Geneva, Switzerland; 1971), pp. 334-337 and references cited therein]. A precise statement of the stability condition involves both the real and imaginary parts of this coupling impedance and of course depends on the form of the distribution function N(E). The present discussion concerning suppression of the collective radial instability is believed, however, to give adequate estimates of the minimum magnitude of $(E\,\partial S/\partial E)(\Delta E/E)$ required for this purpose [where $S = (M-\nu_r)\omega_\odot$].

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TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720