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Abstract

Analytic heat diffusion solutions for compressible gas in cylindrical cells for Gaussian and arbitrary beam profiles are compared to standard (neglecting compressibility) heat diffusion equation solutions.

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Most theoretical treatments of the nonresonant spectrophone have used as their starting point the standard inhomogeneous heat diffusion equation:

$$\rho_o C_v \frac{\partial T'}{\partial t} = \kappa \nabla^2 T' + H \tag{1}$$

where T' is the temperature disturbance, H is the heat released due to collisional deactivation of the optically excited molecules, ρ_o is the density, C_v is the specific heat per unit mass at constant volume, and κ is the thermal conductivity. This equation neglects the work done on or by the gas in compression or expansion, respectively, and treats it as if it were incompressible. A more proper equation for an ideal gas, taking into account expansion and compression, is given by:

$$\rho_o C_p \frac{\partial T'}{\partial t} = \kappa \nabla^2 T' + H + \frac{\partial p'}{\partial t} \tag{2}$$

where p' is the pressure disturbance. When the cell dimensions are small compared to the acoustical wavelength, p' is constant throughout the cell, and if one uses the ideal gas equation of state and the fact that the total mass of gas is conserved, one finds that p' is proportional to the volume integral of the temperature disturbance T'; if one assumes an $e^{-i\omega t}$ time dependence, one obtains an equation like that obtained by Sall,[1]:

$$\nabla^2 T' + \frac{i\omega\rho_o C_p}{\kappa} T' - \frac{i\omega(\gamma-1)\rho_o C_p}{\gamma\kappa} \int_V T'(\vec{r}')dV' = -\frac{H}{\kappa} \tag{3}$$

where we have neglected the microphone impedance correction included by Sall. Sall solved this equation for the special case of a radially- and longitudinally-uniform source function H in an infinitely long cylindrical cell. In this paper we present a more general solution by a Green's function method, still assuming longitudinal uniformity and infinite length, and in addition, radially symmetric excitation, but allowing an arbitrary radial dependence of H. We show that the pressure amplitude p' obtained by integrating

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the temperature solution over volume is

$$p' = \frac{i(\gamma-1)}{\pi R^2 \omega} \frac{\int_0^R H(r')\left[1 - \frac{I_0(kr')}{I_0(kR)}\right] 2\pi r'\, dr'}{1 + \frac{2(\gamma-1)}{kR}\frac{I_1(kR)}{I_0(kR)}} \tag{4}$$

where $k = (-i\omega\rho_0 C_p/\kappa)^{1/2}$ and I_0 and I_1 are modified Bessel functions. In the special case of uniform H(r) this reduces to Sall's solution. We have also solved the heat diffusion equation (1) in the frequency domain for arbitrary H(r) and obtained for the pressure:

$$p' = \frac{i(\gamma-1)}{\pi R^2 \omega} \int_0^R H(r')\left[1 - \frac{I_0(k'r')}{I_0(k'R)}\right] 2\pi r'\, dr' \tag{5}$$

where $k' = (-i\omega\rho_0 C_v/\kappa)^{1/2} = k/\sqrt{\gamma}$. The heat diffusion equation (1) tends to overestimate the temperature disturbance near the center of the cell and underestimate it near the walls due to the respective expansion and compression of the gas in these regions. While these two effects tend to cancel, the former produces a greater effect on the pressure, since the additional heating of the gas near the walls is limited by the ease of loss of heat to the walls. In the high-frequency and low-frequency limits, both equations (4) and (5) have identical limiting behavior. At low frequencies, both give:

$$p' = \frac{(\gamma-1)\rho_0 C_v}{4\pi\kappa} \int_0^R H(r')\left[1 - \frac{r'^2}{R^2}\right] 2\pi r'\, dr' \tag{6}$$

while at high frequency (but still well below the first radial resonance) both give:

$$p' = \frac{i(\gamma-1)}{\pi R^2 \omega} \int_0^R H(r')\, 2\pi r'\, dr' \tag{7}$$

At intermediate frequency, Equations (4) and (5) differ by an amount that is proportional to (γ-1) and is greatest when the beam diameter is very small. The worst case occurs when $\gamma = 5/3$ and $\omega\rho_0 C_p R^2/\kappa \simeq 34$; in this instance the heat diffusion equation solution (5) overestimates the pressure amplitude by 30%. In spite of this, the heat diffusion equation provides a reasonably good description of the cell pressure response at most frequencies, particularly for smaller values of γ. However, where the temperature and/or density solutions are needed, as in thermal lensing, photothermal deflection, etc., in finite radius cells, the solution of Equation (3) may be more appropriate.

We will also present a solution of Equation (1) for arbitrary H(r) in terms of an eigenfunction expansion, and show that this solution is equivalent to Equation (5). We will discuss the reasons why a similar approach to Equation (3) does not yield very useful results.

As an example of the use of Equations (4) and (5), we consider the important case of a Gaussian beam,

$$H(r) = \alpha I(r) = \alpha \frac{2W}{\pi w^2} \exp(-2r^2/w^2) \tag{8}$$

where α is the absorption coefficient, I the intensity, W the Fourier component of the beam power at frequency ω, and w is the beam radius at $1/e^2$ intensity. When $w \ll R$, the integrals from 0 to R in Equations (4) and (5) are well approximated by integrals from 0 to infinity, which can be done analytically. Then Equation (4) gives:

$$p' = \frac{i(\gamma-1)\alpha W}{\pi R^2 \omega} \frac{I_0(kR) - \exp(k^2 w^2/8)}{I_0(kR) + \frac{2(\gamma-1)}{kR} I_1(kR)} \tag{9}$$

The magnitude of p' obtained from this equation is accurate to 1% as long as $w/R < 0.65$, and to 5% as long as $w/R < 0.81$. Beyond this, numerical integration of Equation (4) must be performed. The corresponding solution to the standard heat diffusion equation obtained for the Gaussian beam using Equation (5) is:

$$p' = \frac{i(\gamma-1)\alpha W}{\pi R^2 \omega} \left(1 - \frac{\exp(k'^2 w^2/8)}{I_0(k'R)}\right) \tag{10}$$

with the same limitations on w/R.

References:

1. A.O. Sall, Sov. Phys. Tech. Phys. *1*, 152 (1956) (English translation of J. Tech. Phys. U.S.S.R. *26*, 157 (1956)).

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