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Cognitive Underpinnings of Math Learning and Early Play Based Intervention

By

Chloe Teressa Green

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Education

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Frank C. Worrell, Chair

Professor Silvia Bunge

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Abstract

Cognitive Underpinnings of Math Learning and Early Play Based Intervention

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Chloe Teressa Green

Doctor of Philosophy in Education

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Professor Frank C. Worrell, Chair

For my graduate research presented in this dissertation, I employed cognitive development theory to evaluate key cognitive abilities that contribute to both typical and atypical mathematical learning in children and adolescence. I incorporated these findings into a novel play-based intervention for children at-risk for math learning disabilities (MLD). My dissertation work is represented in the following three papers.

In the first paper, I synthesized literature identifying the common cognitive precursors to math learning disabilities. I analyzed how core numerical processing weaknesses (e.g. number sense) in early childhood, restrict the developmental plasticity of mathematical learning. Furthermore, I identified how common weaknesses in other domain-general cognitive abilities (e.g. working memory and processing speed) serve to further exacerbate mathematical learning weaknesses in MLD. Taken together, these findings inform theoretically grounded approaches used to identify children with MLD, and identified promising approaches to early intervention.

In the second paper, I sought to characterize the cognitive factors that are most predictive of future math achievement in typically developing children and adolescents. I analyzed data from a longitudinal study of children between 6 and 21 years old who completed a battery of neuropsychological testing at 3 time points over the course of 5 years. I was specifically interested in the role of fluid reasoning (FR), or the ability to think logically to solve novel problems. Fluid reasoning has not been particularly well characterized in relation to math achievement. Structural equation modeling was employed to compare the relative contribution of spatial abilities, verbal reasoning, age, and FR in predicting future math achievement. This model accounted for nearly 90% of the variance in future math achievement. In this model, FR was the only significant predictor of future math achievement; age, vocabulary, and spatial skills were not significant predictors. The findings build on Cattell's conceptualization of FR as a scaffold for learning, showing that this domain-general ability supports the acquisition of rudimentary math skills as well as the ability to solve more complex mathematical problems.

In the third paper, I pilot-tested a novel game-play intervention for children at risk for math learning disabilities. The intervention involved playing numeracy and cognitive

speed games four days per week for 14 weeks. A single-case-study design was employed to evaluate response to intervention in 3 first- and second-grade students. The intervention took place during an after-school program. All three students demonstrated a significant improvement in weekly arithmetic fluency and marginal improvements in processing speed. However, there was variability during baseline testing in arithmetic fluency scores, limiting causal inference. This study provides preliminary evidence to suggest that game-based interventions that train basic numeracy and processing speed skills, may serve as an effective preventative approach that builds on children's intrinsic motivation to engage in playful learning.

Table of Contents

Chapter **Page**

Acknowledgements……..i

I. Cognitive Precursors to Math Learning Disabilities……1

II. Cognitive Underpinnings of Math Achievement in Typically Developing Children...16

III. Game Play Intervention for 1st and 2nd Grade Children at risk for Math Learning Disabilities…….43

Conclusion……..……84

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Chapter 1 - Cognitive Precursors to Math Learning Disabilities

Although large-scale research efforts on dyslexia have yielded effective remediation, research on mathematical learning disabilities (MLD), or dyscalculia, remains at a rudimentary stage, with little consensus regarding the underlying deficits that can be targeted by remediation. Using the traditional diagnostic model, estimates of the prevalence of math disabilities range from approximately 5% to 7% of all school-aged children. This figure is comparable to the prevalence of reading disabilities (Badian & Ghublikian, 1983; Gross-Tsur, Manor, & Shalev, 1996). The high prevalence of MLD is especially problematic due to the sequential and hierarchical nature of mathematics courses; children who have difficulty grasping the early mathematical concepts almost invariably fail to grasp later ones (Cawley & Miller, 1989), thereby increasing their risk of school failure (Badian & Ghublikian, 1983; Gross-Tsur, Auerbach, Manor, & Shalev, 1996).

Recognition that early failure to achieve proficiency in mathematics restricts a student's ability to compete for career opportunities has led national education organizations, such as the National Council of Teachers of Mathematics (2000) and the National Research Council (Kilpatrick, Swafford, & Findell, 2001), to advocate for mathematical proficiency in *all* children (Baroody, 1994). Yet, research efforts have remained limited, with inadequate research funding ($2.3 million by the National Institute of Health for MLD compared to $107 million for dyslexia) to support coordinated efforts to this end (Bishop, 2010).

There are several reasons for the lack of success in characterizing and remediating MLD. First, psychologists face many challenges in identifying children who meet the diagnostic criteria. Children with MLD constitute a heterogeneous group with common co-morbidities; they often have additional impairments in non-numerical domains of functioning, such as attention deficit hyperactivity disorder (ADHD), dyslexia, or difficulties with motor skills (Gross-Tsur, Manor, et al., 1996; Hanich, Jordan, Kaplan, & Dick, 2001). The lack of consensus among researchers on the core deficits that make up MLD leave psychologists challenged when it comes to differentiating children with poor math achievement from those with a learning disability or other disorders. Furthermore, treatment is often delayed because psychologists adhere to a traditional diagnostic model for learning disabilities, stipulating that to meet the criteria for a learning disability, children must demonstrate low math performance for a period of time and their performance must be significantly below their level of general intelligence (IQ). This traditional model is especially counterproductive for early remediation efforts because it too often requires that children demonstrate low performance in math for multiple semesters or even years before they are first identified as struggling with mathematics (Fletcher et al., 2002).

Today, there is an exigent need for rigorous research to hone in on commonalities in diverse populations of children with MLD to potentiate targeted remediation (Dowker, 2005). A unifying question guiding this work is whether deficiencies in any specific core cognitive process underlie impaired math performance. There is currently a theoretical dispute in the math learning disability literature concerning whether MLD is caused by an impairment in a domain-general cognitive mechanism such as *working memory* (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), or a more domain-specific deficit known as *number sense* (Butterworth & Reigosa, 2007; Butterworth, Sashank, & Laurillard, 2011), the ability to process small, exact and large, approximate representations of quantity.

In the literature review that follows, I begin by synthesizing the recent literature on the most common numeracy impairments found in children with MLD, in number sense. In the

subsequent sections, I review research on the complex intersection between common cognitive impairments in WM and MLD. I take the position that elucidating the relationship between WM impairments and MLD can offer a window into better understanding, early diagnosis, and treatment of MLD in diverse populations of children. Thus, in the last sections of this paper, I conclude with a discussion of the application of this line of research on the cognitive underpinnings of MLD to the development of early identification screening tools, and instructional techniques for teachers, that can be utilized in classrooms for children with MLD.

Numeracy Impairments Associated with MLD

A recent meta-analytic review of cognitive development and education research on MLD has revealed that a primary predictor of dyscalculia is a weakness in number sense (Geary, 2010). Number sense is the ability to use and understand numbers. Children with MLD do not intuitively grasp the value associated with a number and its relation to other numbers, a basic understanding that underpins all work with numbers (Geary, 2006). A key part of number sense is the ability to understand and manipulate sets of numbers to solve mathematical problems; children with MLD struggle to quantify a small set of objects without counting (estimate), as well as to break down large sets (or large numbers) into sets of smaller objects (numbers) that can be easily manipulated to facilitate efficient mathematical problem solving. In the classroom, there are four key indicators a child may have a number-sense deficit and may be at risk for MLD (Geary, 2010): (a) developmentally immature calculation procedures (i.e., use of fingers to count; Goldman, Pellegrino, & Merz, 1988); (b) delayed understanding of counting concepts such as frequent counting errors (Geary, Bow-Thomas, & Yao, 1992); difficulty remembering arithmetic facts (Geary, 1993; Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997); and poor conceptual knowledge of rational numbers (Mazzocco & Devlin, 2008).

Children with MLD demonstrate increasing deficits in math achievement over time. The Missouri Longitudinal Study of Mathematical Development and Disability compared math achievement trajectories in children with MLD (achievement scores below the 10^{th} percentile in math) to typically developing children (TA), low-achieving children (LA) who scored between the 11^{th} and 25^{th} percentiles, and a group of children with low intelligence (LIQ; IQ scores below the 10^{th} percentile). Longitudinal findings from 1^{st} to 5^{th} grade are illustrated in Figure 1. The authors concluded that low mathematic achievement in both the MLD and LA groups cannot be attributed to low intelligence (Geary, 2011a, 2011b; Geary, Hoard, & Bailey, 2012).

When clinicians consider diagnosing a child with MLD, it is critical that they explore potential causes of the child's unique deficiencies in number sense and definitively rule out the causal role of poor instructional techniques. Then, the pivotal questions in diagnosis are raised as follows: If deficiencies in numeracy development are not caused by poor instructional techniques, are they associated with impairments in basic cognitive functions? If so, which cognitive functions are impaired?

Neuroscience research on mathematical learning utilizes neuropsychological tests and neuroimaging techniques to localize and identify domain specific cognitive functions engaged during early math performance in the brain. Butterworth et al. (2011) reviewed brain-imaging studies in a recent meta-analysis and concluded that weakness in numerical processing corresponds to alterations in brain function and brain structure in children with MLD when they are compared to individuals without MLD. Almost all arithmetical and numerical processes involve the parietal lobes and learning new arithmetic facts primarily involves the frontal lobes and the intraparietal sulci. Children with dyscalculia exhibit weaker activation in the intraparietal sulci than normally developing children during performance of tasks that require

them to compare quantities of objects within different sets, or different number symbols, and during arithmetic tasks. Children with MLD additionally have alterations in gray matter in the intraparietal sulci (Butterworth et al., 2011).

Cognitive Impairments Associated with MLD in WM

When researchers have used neuropsychological tests to study children who perform in the lowest quartile in math across studies, they have found that in addition to number sense deficits, children with MLD often demonstrate impairments on tests that measure a core general cognitive ability that enables a person to keep information active in the mind and to manipulate this information to solve a problem (Swanson & Jerman, 2006). Children with MLD have difficulty bringing relevant information "on-line" (i.e., recalling math facts), manipulating the information (i.e., mental addition), and tracking each sequential operation required to solve a multistep problem. Children with MLD are likely to make frequent mistakes and often appear forgetful due to difficulty retrieving mathematical facts from long-term memory in an efficient way. We can describe this type of difficulty as low working memory capacity. A meta-analysis of studies involving children with MLD has suggested that the most common neuropsychological deficit among all children with MLD, regardless of co-occurring reading disability, is difficulty with working memory (Swanson & Jerman, 2006).

Conceptualizing WM. WM is a core cognitive function essential for all areas of academic performance (Alloway, Gathercole, & Elliott, 2010). WM is the ability to actively encode and hold information in one's mind in order to manipulate this information to perform a task (Baddeley, 1996). WM is considered to be a limited capacity system, meaning that an individual can only hold a limited number of items (i.e., information) in the mind at any given time to successfully solve a problem or to complete a task (e.g., remembering a phone number or solving an arithmetic problem). A child's WM capacity increases gradually throughout his or her development, with the typical adult being able to retain seven items (plus or minus one) on average (Miller, 1956). When children exceed their individual WM capacity (i.e., holding too many items in mind at once), they may experience a feeling of being overloaded during learning activities, which can potentially impair their ability to sustain attention and stay on task (Alloway, Gathercole, & Elliott, 2010).

To illustrate how WM works, consider the following task by Daneman and Carpenter (1980) for children. The examiner asked children to read several sentences, as follows:

1. Joe went to the store and bought fruit;
2. Lisa thinks that the cat is hungry;
3. Jemma said that she could climb the tree.

After children read the sentences, they are asked to answer questions about the content of the material. For example, the examiner could ask, "What did Joe buy at the store?" Following this, children are asked to recall sequentially the last words from each of the sentences they read. To complete this task successfully, a child would need to (a) read and process the content; (b) respond to the question by retrieving information about the relevant parts of the text; and (c) recall only the last word in each sentence, which requires inhibition of competing information from earlier words in the sentence and controlled attention (Baddeley, 2003). Thus, WM tasks engage children in actively maintaining information that is relevant and simultaneously processing the most pertinent information to solve the problem. Each time the child answered two questions correctly in a row, the difficulty level of the task was increased (i.e., more sentences were added to the sequence). The task was discontinued when the child answered two questions in a row incorrectly.

The most widely used conceptual model of WM is the multi-component model proposed by Baddeley and Hitch (1974) and updated by Baddeley (2000), where the concept of WM is broken down into the following four distinct parts: (a) a visuo-spatial sketchpad, (b) phonological loop, (c) central executive, and (d) episodic buffer. Baddeley and Hitch's conceptualization is the dominant model in research examining the role of WM in mathematical problem solving (Swanson & Beebe-Frankenberger, 2004). This model includes two storage systems, specifically (a) the visuo-spatial sketchpad, which enables short-term storage or maintenance of mathematical representations, and (b) the phonological loop, which stores and retrieves auditory information. These two storage subsystems operate under the control of the central executive, which serves as the director. In this model, the central executive enables the child to comprehend the goals of the task and then to appropriate attentional resources to the part of the WM system that can process the specific type of information required to solve the problem. Verbal information is sent to the phonological loop and visual or spatial information is sent to the visuo-spatial sketchpad (Alloway Gathercole, & Pickering, 2006; Bayliss, Jarrold, Gunn, & Baddeley, 2003).

Another key role of the central executive is to retrieve the key information from long-term memory that is pertinent to solve a math problem successfully. Selective retrieval requires children to resist distraction by related (but extraneous) information that is also stored in memory, but could hinder their ability to solve a particular problem (Baddeley, Emslie, Kolodny, & Duncan, 1998). Baddeley (2000) proposed that an additional component, the episodic buffer, should be added to the model. In Baddeley's (2000) updated model, the episodic buffer is under the control of the central executive and is conceptualized to be a temporary interface between the central executive and the storage systems (i.e., the phonological loop and visuo-spatial sketchpad.

WM and Math Achievement. Each component of WM in Baddeley and Hitch's (1974) conceptual model supports a different type of mathematical problem solving. A recent line of investigation with this more narrowly defined aim has distinguished how each of the WM components develops differentially with age and is employed independently according to the type of mathematical problem and strategy the child uses. Younger children tend to use the visuo-spatial sketchpad when they are acquiring new arithmetic skills to maintain visual representations of the mathematical operations and solution in mind (Hayes, 1972; Hitch, 1978). The visuo-spatial sketchpad is hypothesized to serve as a mental notepad for the visual representation of mental computations. For example, young children use a mental number line as a strategy to keep track of the magnitude of numerical information in an arithmetic problem.

When children get older, and gain experience working with arithmetic, they tend to rely more on their verbal WM to compute arithmetic problems mentally (Rourke, 1993). Finally, the central executive plays a more complex but still crucial role in tracking which parts of the problem have already been performed, allocating attentional resources to the appropriate subcomponents or storage systems, and maintaining each of the rules needed to solve each subsequent step of the math problem (Zheng, Swanson, & Marcourlides, 2011). Furthermore, the central executive is involved in carrying numbers during multi-step addition or subtraction problems (Imbo, Vandierendonck, & De Rammelaere, 2007; Seitz & Schumann-Hengsteler, 2002).

Components of WM and MLD A confluence of research findings converge on the link between WM deficits and difficulties that many children with MLD experience when they engage in mathematical problem solving, although researchers are still investigating the

complexities of the relationship among specific components of WM (Hitch & McAuley, 1991; Swanson & Jerman, 2006). This literature is complex because researchers have used heterogeneous tasks to measure WM based on different conceptualizations of the function of each component of WM, making it difficult to generalize findings across studies. In the following section, I review the literature regarding the specific type of impairments in components of WM and math learning disabilities in children. Table 1 summarizes the tasks employed in studies on WM and math learning difficulties (Raghubar, Barnes, & Hecht, 2010).

Verbal WM impairments. Research has shown that children with mathematical learning impairments typically have difficulty keeping verbal information that they read or hear in their verbal WM, or phonological loop (Geary, Hoard, & Hamson, 1999). This weakness in storing and manipulating verbal information creates several challenges that impede mathematical performance. For instance, this weakness limits children's ability to accurately monitor their steps during the counting process, often leading them to make frequent counting mistakes (Hitch & McAuley, 1991; Geary et al., 1999). Research studies on mental arithmetic in children have revealed that the phonological loop plays a critical role in enabling the child to verbally maintain the numbers and type of operation they need to carry out to complete each step of a problem and to track intermediate results in a multi-step equation or word problem (Heathcote, 1994). Swanson and Jerman's (2006) meta-analysis of 28 studies showed that the strongest predictor of math difficulties in children was weakness in verbal WM, after controlling for effects of several other variables, such as age, IQ, naming speed, and short-term memory for words and digits.

Raghubar et al.'s (2010) review of the literature elucidated the complexity of the relationship between various modes of measurement of verbal WM and math learning difficulties. They argued that many of the studies investigating the link between verbal WM and math learning difficulties employ domain-specific, numerical WM tasks to measure verbal WM, requiring the child to draw on the same domain as the child's impairment. The most common type of task used in these studies is the digit span backwards task, which requires the child to remember a string of digits read out loud by the examiner and then to repeat the digits out loud in the backwards order. Studies that use numerical measures of verbal WM (i.e., counting span and digit span backward) distinguish children with math difficulties more reliably than studies using non-numerical measures of WM (e.g., word span backward; Passolunghi & Cornoldi, 2008; Passolunghi & Siegel, 2001, 2004). At this point, the limited number of studies that have employed verbal WM tasks without numerical stimuli, such as the listening span task, have not consistently differentiated between children with and without math difficulties (D'Amico & Guarnera, 2005; Fuchs et al., 2008; Passolunghi & Cornoldi, 2008; Reukhala, 2001; Siegel & Ryan, 1989; Swanson & Beebe-Frankenberger, 2004; Van der Sluis, Van der Leij, & de Jong, 2005). Thus, the domain-specific versus domain-general nature of verbal WM impairments in MLD remains an open question for further research (Raghubar et al., 2010).

Visuo-spatial WM impairments. Children with MLD typically have difficulty representing numerical information spatially (i.e., comparing two quantities or accurately aligning number columns), which is linked to impairments in the visuo-spatial component of the WM system (Geary, 2004; Swanson & Jerman, 2006). Further biological support for a visuo-spatial deficit can be found in brain imaging studies that have suggested the parietal areas of the brain that are associated with number and magnitude processing are located near brain regions that support aspects of visuo-spatial processing in the intraparietal sulci Zorzi, Priftis, & Umilta,

2002). When parietal regions are damaged, there is a disruption in the ability to form spatial representations, particularly the ability to imagine a mental number line (Zorzi et al., 2002).

The relatively small number of studies on visuo-spatial WM and wide range of methodological differences among studies make it difficult to draw conclusions about the complex nature of visuo-spatial impairment in children with MLD. Researchers have employed different tasks to measure visuo-spatial WM based on different theoretical conceptualizations about the function of visuo-spatial WM. Additionally, these studies have targeted different ages and used different criteria or cut-offs on math tests to determine eligibility for the MLD groups. Despite these differences, most studies can be divided into those that employ static and those that use dynamic visuo-spatial tasks.

Static visuo-spatial tasks require the participant to hold information passively; for example, the matrices task requires participants to remember the location of black target blocks in a display in which half of the blocks are white and half are black. Dynamic measures of visuo-spatial WM require participants to engage in some type of manipulation of visuo-spatial information. For example, the dynamic matrices task requires participants to remember a sequence of flashing lights in a matrix. The results have produced mixed conclusions. Some researchers have found that both dynamic and static visuo-spatial WM tasks capture the differences between children with MLD and their typically developing peers (D'Amico & Guarnera, 2005; Reukhala, 2001), whereas others have reported that only dynamic visuo-spatial WM—and not static visuo-spatial WM—differentiates the groups (McLean & Hitch, 1999; Van der Sluis et al., 2005). Moreover, some have found that only specific types of dynamic visuo-spatial WM tasks differentiate the groups (e.g., backwards version; Passolunghi & Cornoldi, 2008).

Central executive WM impairments. In a meta-analysis of research on WM impairments in children with MLD, Geary (2003) found that one core marker of MLD may be a failure to access information from long-term memory due to poor inhibition of information that is irrelevant to solving a particular problem (Bull & Scerif, 2001). Researchers have suggested that this inability to retrieve facts from long-term memory may stem from an inability to shift from one numerical representation to another and simultaneously retain the number or math fact with which the child was previously working (Bull & Scerif, 2001; Miyake, Friedman, Emerson, Witzki, & Howerter, 2000). Weakness in the central executive component of WM is a primary candidate because it is theorized to take charge of the inhibition of irrelevant information, task switching, information updating, goal management, and strategic retrieval from long-term memory (Engle, Tuholski, Laughlin, & Conway, 1999).

Measures of the central executive require concurrent storage and processing of information and these measures are sometimes contrasted with performance on short-term storage memory tasks in which participants are required to hold small amounts of verbal or visuo-spatial information passively. The exact relationship between these cognitive factors—WM and inhibition—requires further investigation in regards to MLD (Herd, Banich, & O'Reilly, 2006). Finally, drawing conceptual connections between different mathematical concepts is especially challenging for children with WM impairments because each new math lesson is stored separately as its own fragmented memory instead of being integrated with previously learned information for efficient retrieval (Herd et al., 2006).

Early Identification of MLD

Currently, the modal diagnostic process used by schools to determine whether a child meets the eligibility criteria for MLD requires that the student demonstrate evidence of academic

failure in math (for multiple trimesters and often multiple years). According to some educators, this process can lead to a "wait to fail" model in which a child falls too far behind in school to catch up by the time he or she has been assessed and determined to have met the eligibility criteria for a learning disability (Reynolds, 2008). In recent years, most schools have begun to adopt a new special education model called response to intervention (RTI), in which children who experience difficulty learning in the early school years receive evidence-based instructional interventions as soon as the problem is identified. Their subsequent academic progress is then closely monitored to determine whether they should be referred for special education assessment. The aforementioned research on cognitive weaknesses that underlie MLD has applications for improving targeted and evidence-based RTI interventions in schools for children with MLD.

Currently, number sense weakness is an indicator for teachers that a child requires extra help (i.e., using number lines and learning math facts), but the ongoing effects of common WM weaknesses often go unaddressed by school interventions (Geary, Bailey, & Hoard, 2009). Early screening tools that measure children's WM in the classroom when MLD is a concern may potentiate the use of more effective intervention strategies that comprehensively and proactively address a child's underlying cognitive impairments as well as their number sense weakness. In addition, the availability of computerized tests makes administration of screening measures for learning disabilities more readily accessible to teachers in the classroom. Just as in reading learning disabilities, early detection is a vehicle that allows for early remediation. When WM deficits are identified, researchers suggest that instructional supports and new training interventions be enacted right away to address children's individual weaknesses (St. Clair-Thompson & Gathercole, 2006).

Instructional Strategies

Students with low WM are likely to experience academic setbacks (Gathercole & Alloway, 2008). Such students are at significant risk for developing anxiety and they are likely to have difficulty self-regulating as well as generating problem-solving strategies (Montague, Warger, & Morgan, 2000). Children with MLD are at risk for developing learned helplessness, or the loss of motivation to continue when they are faced with increasing challenges throughout each successive mathematics course (Diener & Dweck, 1978). To further complicate matters, when students struggle in math, too often teachers naturally make causal assumptions to explain their challenges, such as that the students simply lack the innate ability to do this type of math, or they are lazy (Diener & Dweck, 1978). When teachers believe that they cannot help students overcome their *innate* deficiencies in learning, it can limit the range of pedagogical techniques in which teachers engage in (Horn, 2007). However, such teachers' conceptualizations often fail to incorporate the learning trajectory research that reveals the important role that instructional experiences (with individual modifications/adaptations) play in supporting students as they progressively develop skills that enable them to meet the goals of school mathematics at their own rate (Daro, Mosher, & Corcoran, 2011).

To prevent students from giving up, the teacher's challenge is to provide targeted and strategic support that structures the learning environment for students to help them strengthen their areas of weakness. Research suggests that teachers can use simple instructional strategies such as memory aids to support students during structured learning activities (St. Clair & Gathercole, 2006). Thus, equipped with the right techniques, teachers can prevent children from experiencing excessive WM overload and help them to focus on the new goals of the current mathematics lesson (St. Clair & Gathercole, 2006). Meltzer (2007), an education researcher, suggested that teachers can support children with WM impairments by filling in the answers to

the initial steps of a multi-step problem that involve previously learned material to help students focus on the new content or material without being bogged down by previously learned operations that they have difficulty retrieving from long-term memory. For example, on a subtraction or division problem, when carrying numbers is required, the teacher could aid the student with an external reminder (a sign on his or her desk) of how to carry numbers, so that the child's attention is free to learn that day's specific math lesson with more advanced procedures. Additionally, teaching materials can restrict the amount of extraneous information presented or the number of task changes required.

Further Directions

Longitudinal research in populations of children with MLD is needed to advance our understanding of the causal contribution of WM impairments to dyscalculia. Future work should emphasize measures of WM that do not rely on numerical stimuli so as not to confound specific impairments in numeracy when aiming to measure more general impairments in WM. Though the efficacy of training interventions that aim to remediate the common cognitive weakness, WM, in children is still currently under investigation, the preliminary findings in this area of research have provided important insights about the plasticity of WM and these findings have wide application that advance the field of education. A key next step is to investigate the generalizability of WM training to improvements in math performance in the classroom. Researchers have only begun to scratch the surface of this hotly debated topic and early studies have suffered from methodological issues relating to small sample sizes, lack of control groups, and lack of blind randomization procedures (Melby-Lervåg & Hulme, 2013). A fruitful line of further investigation would be to study whether there is a synergistic effect of domain-general cognitive training in WM and training in more domain-specific number sense remediation programs. Computerized training programs with adaptive designs can flexibly meet the needs of diverse students with MLD.

Professional development for teachers. Professional development days can be a highly effective modality to provide information to teachers to increase awareness of the high prevalence rate of MLD in children as well as the most common cognitive impairments that teachers can screen for in their classroom to identify children at risk. For example, as part of an effective RTI model, kindergarten through third grade teachers can administer a brief universal screening tool for number sense and WM to all students as a means of diagnostic monitoring. Teachers should be made aware of the above mentioned instructional strategies that they can employ in their classrooms to support children at risk for MLD and help them combat common secondary consequences of persistent academic setbacks that result when students have a learning disability. To address the unique needs of students who meet diagnostic criteria for MLD, teachers should have access to evidence based computerized software that can be administered in their classroom, or a resource room, to treat common impairments in number sense and WM.

Conclusion

Given the high prevalence of MLD, it is evident that the current curriculum fails to support the needs of children with MLD. Despite heterogeneity in the population of children with MLD, the current research indicates that children with MLD have commonalities in weaknesses in two key areas, specifically (b) the domain-specific cognitive ability, number sense (b) the domain-general cognitive ability, WM. Despite ongoing debate in the literature regarding causality, it is clear that impairment in WM, a key cognitive process central to math, can significantly impede a child's mathematics achievement for years to come. The implication of

the research synthesized here is that early screening for the core impairments in number sense and WM can flag children at risk for MLD. Once identified, children at risk for MLD can receive progressive interventions and scaffolding techniques in their classroom that target and support their individual weakness and improve their access to the mathematics material.

Progressive targeted teaching practices that incorporate cognitive training interventions in schools are needed to support children to facilitate development in math learning. WM training may be an especially powerful remediation technique. If interventions are offered early, they can set a positive feedback loop into motion, empowering children to reap more reward per effort during mathematics lessons at the beginning of school before they begin to fail at math

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Chapter 2 - Cognitive Underpinnings of Math Achievement in Typically Developing Children

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American educators face the tall order of improving outcomes in science, technology, engineering, and mathematics (STEM) (Lee, 2012). To generate solutions to some of the globe's most pressing challenges, educators will need to teach children to become better problem-solvers who can apply to their work the information learned in their STEM courses. Courses in mathematics are especially challenging for many students, and these courses have become a gatekeeper to higher education and job opportunities in technological fields (Moses & Cobb, 2001). Because math instruction builds upon previously acquired knowledge and skills, it is difficult for children who fall behind early to catch up with their peers. In an effort to improve math and language outcomes across the nation, educators have recently released new national standards for math and language arts education called the Common Core State Standards (National Governors Association and Council of Chief State School Offices, 2014). The new standards lay out progressions of math skill building benchmarks that have opened up discussions about how teachers can provide better support for students in bolstering their math proficiency skills.

Complementary lines of research in psychology and education aim to identify which cognitive precursors lead to proficient acquisition of mathematics skills. A long-term aim of this line of research is to inform educators about the precursors to math development, so that they may create lesson plans that target not only specific math skills, but also underlying domain-general cognitive processes. The cognitive abilities required to solve math problems have been difficult to isolate because mathematics is a heterogeneous subject matter (e.g., arithmetic, fractions, geometry, statistics), and problems within the same topic area require several different operations and computations (e.g., adding, subtracting, multiplying, dividing). Nevertheless, researchers have begun to identify common key cognitive functions that are critically important for disparate types of mathematical computations (Bisanz, Sherman, Rasmussen, & Ho, 2005; Desoete & Gregoire, 2007; Krajewski & Schneider, 2009).

Relationships between math and cognitive abilities are often studied within the framework of the Cattell-Horn-Carroll (**CHC**) theory, arguably the most comprehensive and empirically supported theory of cognitive abilities derived from over 70 years of psychometric research using factor analytic theory (Keith & Reynolds, 2010). The utility of the theory is in clarifying the relations between cognitive and academic abilities to inform educational and psychological practices. The most recent revision of this model, by Schneider and McGrew (2012), includes 16 broad cognitive abilities that each contains more narrow cognitive abilities within them. This model does not include a general intelligence *g* factor; rather, it is based on accumulating evidence that broad and narrow CHC cognitive abilities explain more variance in specific academic abilities than *g* alone, and that these specific relationships are more informative to educational practice than

general intelligence (e.g. Floyd, McGrew & Evans, 2008; McGrew, Flanagan, Keith, & Vanderwood, 1997; Vanderwood, McGrew, Flanagan, & Keith, 2002).

In a recent synthesis of studies investigating the concurrent relationships between **CHC** cognitive abilities and achievement measures (CHC-ACH) by McGrew & Wendling (2010), fluid reasoning (FR) was one of three broad cognitive abilities that was consistently related to mathematical performance in calculation and problem solving at all age ranges throughout development (the other two were Verbal Comprehension and Processing Speed). FR was consistently related to future math achievement above and beyond the contribution of general intelligence. FR has been defined by contemporary CHC theory as the ability to flexibly and deliberately solve novel problems without using prior information (Schneider & McGrew; 2012). More specifically, it is the ability to analyze novel problems, identify patterns and relationships that underpin these problems, and apply logic. On FR tests, one or more of the following logic abilities is required: 1) *induction*, the ability to discover an underlying characteristic (e.g. rule, concept, or trend) that governs a set of materials, 2) *general sequential reasoning* (*deduction),* the ability to start with stated rules or premises and engage in one or more steps to reach a solution to a novel problem (Schneider & McGrew, 2012). FR tests are commonly administered as part of IQ batteries that are administered to children in schools or in clinical settings. While FR performance is strongly correlated to general intelligence (*g*), as is verbal comprehension*,* there is unique shared variance among tests of FR that cannot be accounted for by *g* alone (McGrew, Flanagan, Keith, & Vanderwood, 1997).

FR Development

In typically developing children, FR begins to emerge during the first two years of life, increases rapidly in early and middle childhood, continues to increase at a slower rate during adolescence, and reaches asymptotic values around age 25, after which it begins to decline (McArdle et al., 2002).

Analyses of longitudinal data from large samples that were used to create norms for the standardized Woodcock-Johnson Cognitive Abilities testing battery (Schrank & Wendling, 2009; Woodcock, Mather, & McGrew, 2001) reveal that both FR performance (as measured by Analysis Synthesis and Concept Formation tests) and Math Achievement increase rapidly during childhood, peaking in late adolescence to age 24 and beginning to decline in adulthood (Ferrer & McArdle, 2004). The trajectories of FR development and improvements in math abilities parallel one other throughout development – and more so from ages 5-10 as 11-24 (Ferrer & McArdle, 2004). This observation hints at the possibility that FR plays a bigger role in early math skill development in kindergarten and elementary school than in higher levels of education. However, additional data are needed to examine the relationships between these skills over development.

Fluid Reasoning and Math Achievement

Hypothesized link. One mechanism by which FR could support math skill acquisition is related to the fact that both FR and math problems engage a common underlying cognitive process called relational reasoning, or the ability to jointly consider multiple relationships between different components of a problem (Halford, Wilson & Phillips, 1998; Miller Singley & Bunge, 2014). According to this framework, understanding mathematics requires the ability to form abstract representations of quantitative and qualitative relations between variables (Halford, Wilson & Phillips, 1998). For instance, when children first learn fractions, they must keep several numerical relationships in mind:

whole unit integers have to be understood as subunits and they must learn to coordinate the value in the numerator and the value in the denominator (Saxe, Taylor, McIntosh, & Gearhart, 2005).

Furthermore, solving story word problems requires children to draw conceptual connections between real-world situations and analogous numerical symbols and operations to solve the problem (Clement, 1982). Another example of relational reasoning is evident in early algebra, when students are asked to solve for one or more unknown numbers, and must keep in mind the relationship between numbers on either side of the equal sign to determine which operand is required to solve for the missing variable. Empirical support for a link between FR and Math Achievement comes from both cross-sectional and longitudinal research.

While multiple longitudinal studies have elucidated the importance of spatial skills in math development (for a review see Mix & Cheng, 2012), only a few longitudinal studies have explored the unique developmental role of prior FR. Further research is needed to disentangle the role that each of these two cognitive abilities plays in math development because, although FR and spatial abilities are highly correlated (Fry & Hale, 1996), they rely on overlapping as well as separable cognitive processes and brain regions. FR tests (e.g. Matrix Reasoning) not only require spatial skills (including visualization), but additionally require relational reasoning, or the ability to consider relationships between multiple pieces of information to detect the underlying conceptual relationship among visual objects, and to use reasoning to identify and apply rules (Halford, Wilson & Phillips, 1998; Holyoak, 2012; Bunge & Vendetti, 2015). In regards to the neural correlates, a visuospatial skill that is commonly implicated in math achievement, visuospatial working memory, relies on the intraparietal and superior frontal regions (for a review see Klingberg, 2006), whereas the relational reasoning component of FR relies on the rostrolateral prefrontal coretex and lateral parietal regions (for reviews see Krawczyk, 2010; Bunge & Vendetti, 2014). Therefore, it is plausible that visuospatial abilities and FR make unique contributions math achievement.

Longitudinal Precursors to Math Achievement

Fluid Reasoning. As mentioned, there are only a limited numbers of longitudinal studies that have examined the extent to which FR skills uniquely contribute to the development of math proficiency in childhood, separately from general IQ, and other domain general cognitive abilities. In one such study (Fuchs et al., 2010), the authors compared the effect of basic numerical cognition and other domain-general cognitive abilities measured at the beginning of the school year on 280 1st-grade students' development of math problem solving over the course of that academic year. They found that FR (measured by Matrix Reasoning) in the fall semester was just as predictive of children's gains in word problem solving over the course of the year as their basic numerical cognition skills. Primi, Ferrão, and Almeida (2010) showed that 7th and 8th grade students' initial level of FR (measured by tests of numerical, verbal, spatial, and abstract reasoning) was positively related to their subsequent growth in quantitative abilities over the course of the next two academic years, such that children with higher FR ability at the start of the year demonstrated more growth in math over the course of two academic years than children with lower FR scores.

Spatial skills. Several longitudinal studies have examined the robust role of spatial skills in the development of math proficiency in childhood (*for a review see* Mix & Cheng).

However, studies in this literature rely on different operational definitions of spatial ability, such as visualization and spatial working memory.

Visualization. Visualization is the most commonly studied spatial ability related to mathematics (Mix & Cheng, 2012). Visualization is the ability to perceive visual patterns and mentally manipulate them to simulate how they might nook when transformed (e.g. rotated, changed in size, partially obscured) (Flanagan, Ortiz, & Alfonso, 2013). Visualization is frequently measured using tests such as Block Design, which measures the ability use two-color cubes to construct replicas of two-dimensional, geometric patterns under timed conditions. This test assesses the ability to mentally transform (or rotate) blocks. One such study by Zhang et al. (2014) found that spatial skills in kindergarteners (measured by spatial visualization), along with verbal skills, predicted level of arithmetic in the 1st grade as well as arithmetic growth through the 3rd grade. Another such study by Casey et al. (2015) examined the predictors in 1st grade of math problem solving in the 5th grade, comparing the predictive power of performance on Block Design with the predictive power of verbal and arithmetic skills. They found that Block Design performance in 1st grade were just as predictive of 5th grade math problem solving as early arithmetic skills.

Visuospatial Working Memory. More recently, studies have also found that visuospatial working memory, or the ability to temporarily store and process visual information to complete a task, is a robust predictor of math achievement. For example, Li & Geary (2013) showed that developmental gains in visuospatial working memory between 1st and 5th grade was a strong predictor of math achievement at the end of 5th grade, as was general intelligence (measured by WASI Matrix Reasoning, Block Design, Vocabulary, and Similarities), and in-class attentive behavior. Similarly, LeFevre et al. (2010) advanced a developmental theory that suggests that three key pathways contribute differentially to early math development: quantitative, linguistic, and spatial pathways. They found that at age 4-5 years, early spatial attention (measured by spatial span) significantly predicted both number naming and processing of numerical magnitude two years later.

FR tests may require visualization or spatial working memory, but they are distinguished from these purely spatial ability tests because they require inductive or general sequential (deductive) reasoning (Schneider & McGrew, 2012). Factor analysis contributing to **CHC** theory has demonstrated that FR measures tap into a separable construct than spatial abilities (Schneider & McGrew, 2012).

Quantitative Skills. Though multiple studies have demonstrated the strong predictive power of early math skills on math achievement, over and above reading, attentive behavior, and domain general cognitive predictors, many of these studies have been conducted in populations of primary school-aged children and are consequently limited to more basic numerical competencies (e.g. magnitude comparisons, number naming, arithmetic, fractions etc.), and do not incorporate measures of FR as a unique factor in their models (e.g., Duncan et al., 2007; Fuchs et al., 2012, LeFevre et al., 2012; Watts et al., 2014). Therefore, more research is needed to better understand the relative predictive power of FR, and other domain-general cognitive skills, in relation to prior numerical skills in predicting math achievement across primary and secondary school grades.

Study Goals

In the present study, we sought to expand upon previous research to evaluate the

extent to which prior FR predicts later math outcomes in children between 6 and 21 years old, above and beyond other cognitive and numerical abilities that have previously been implicated in math development. Our aims were threefold: 1) to test a latent model of FR that combines three well-known psychometric tests, 2) to compare the contribution of prior FR to that of prior math reasoning in predicting future Math Achievement at T3, 3) to compare the relative contribution of prior FR to spatial skills, verbal skills, and age, in predicting future Math Achievement at T3. Each of these cognitive abilities has been shown to be a strong independent predictor of later Math Achievement (e.g. Primi, Ferrão, & Almeida, 2010).

To this end, we collected and analyzed data within the context of a larger longitudinal cohort sequential design study examining the neurodevelopment of FR. We administered a battery of age-normed neuropsychological tests to measure FR, as well as vocabulary, and spatial skills, at three timepoints (~1.5 years apart) in a group of 69 children who ranged in age from 6 to 21 at the first assessment. At the second assessment (T2) we assessed participants on a measure of math reasoning. At the final assessment (T3), we assessed participants on three different math achievement measures: math problem solving, arithmetic fluency, and math reasoning.

Methods

Participants

Participants were individuals in a longitudinal study designed to examine the cognitive and neural factors that underlie the development of FR. All participants and their parents gave their informed assent or consent to participate in the study approved by the Committee for Protection of Human Subjects. Additionally, all participants were screened for neurological impairment, psychiatric illness, and history of learning disabilities or developmental delays.

Understanding developmental processes requires longitudinal studies that focus on within-person changes over time. This study design involved a cohort-sequential design in which 201 participants, ranging from 5 to 15 at the time of recruitment, were assessed at one to three time points with an average delay of 1.5 years between time points. This cohort-sequential design enabled us to examine both between-person differences and within-person changes over a 5-year span – the five years of the funded research program – and with fewer participants than a traditional longitudinal design. This approach provides insight into the interplay of factors underlying such within-person changes over time i.e., improvements in cognitive abilities over development (Bell, 1953; McArdle, Ferrer-Caja, & Woodcock, 2002; Ferrer & McArdle, 2004).

Parents completed the Child Behavioral Check List (Achenbach, 1991) on behalf of their child. Participants who scored in the clinical range for either externalizing or internalizing behaviors were excluded from further analyses. Of the 172 children and adolescents enrolled in the study who scored in the normal range on the Child Behavior Check List, 69 participants successfully completed testing at three time points: T1, T2, and T3 – a substantive time commitment, involving six long testing sessions (one behavioral and one brain imaging session at each of the three time points). There was no statistical difference in performance between children who participated at all three time points as compared with those who did not.

The mean assessment ages for these 69 participants were 10.18 (*SD* = 3.32) at T1, 11.67 (*SD* = 3.35) at T2, and 13.45 (*SD* = 3.38) at T3. Across all of these participants and

timepoints, data were collected between ages 6 and 21 years. The ethnicity of the sample reflects the ethnic and racial diversity found in the local population (7.4% Hispanic/Latino, 56.21% White, 12.43% Asian, 10.45% Black or African American, 18.4% multiple ethnicities). Both genders were represented equally (48% males, 52% females). Most of these children came from middle-class homes, and the majority of families (85%) reported two adults living in the home. All mothers in the study had completed high school, and the majority (84%) had completed some post-secondary education, in the form of a Bachelor's or Associate's degree or a diploma from a vocational college. Most of the children spoke English at home.

Measures

The behavioral measures selected for our longitudinal study were standardized measures with very high internal consistency and test-retest reliability, ranging from .94 to .95 (McArdle et al., 2002; McGrew, Werder, & Woodcock, 1991).

Fluid Reasoning. FR ability was assessed using three standardized measures, including the Matrix Reasoning subtest of the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999), and the Analysis Synthesis and Concept Formation subtests of the Woodcock-Johnson Tests of Cognitive Abilities-Revised (Woodcock, Mather, & McGrew, 2001). Though these three tests are quite different from one another, they were all designed as measures of FR that rely one or more narrow FR abilities. As shown below, all three tests loaded onto a single factor "FR" in our sample, which is consistent with prior factor analytic work contributing to CHC theory (Schneider &McGrew; 2012). Thus, we used scores from this FR factor in all subsequent analyses (for example of this approach see: Primi et al., 2010).

Matrix Reasoning. This test was modeled after a traditional test of "fluid" or non-verbal reasoning—Raven's Progressive Matrix Reasoning (Raven, 1938)—and required the participants to examine an incomplete matrix, or geometric pattern, and then select the missing piece from five response options arranged according to one or more progression rules. The Matrix Reasoning subtest assesses FR induction skills, or the ability to identify an underlying characteristic (e.g. rule or trend) that governs the existing pattern, and then to choose a missing piece that contains this characteristic.

***Analysis Synthesis*.** On this test, participants are asked to analyze an incomplete logic puzzle made up of colored squares and to use a key to determine the missing color in the puzzle. To complete this task successfully, participants must use general sequential (or deductive) reasoning skills to draw correct conclusions from a color combination key, with more difficult items requiring a series of sequential steps.

Concept Formation. On this test, participants are asked to view a complete puzzle made up of colored squares, and to identify and state the "rules" (color and shape) when shown illustrations of both instances and non-instances of the concept (e.g. red square). The Concept Formation test requires frequent switching from one rule to another. To complete this task successfully, participants must use inductive reasoning skills to discover the rule that governs the puzzle.

Vocabulary. We used the Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary measure (Wechsler, 1999) to probe crystallized knowledge and, indirectly, semantic memory. This test is a norm-referenced measure of expressive vocabulary. On this test, the examiner presents stimulus words to participants and asks them to state each word's meaning.

Spatial Skills. We administered two tests of Spatial Skills. Spatial Span is considered a measure of visual memory, or the ability to remember visual images over short periods of time (less than 30 seconds) (Schneider & McGrew, 2012). Block Design is considered a measure of visualization, or the ability to mentally organize visual information by analyzing part-whole relationships when information is presented spatially (Schneider & McGrew, 2012). Based on prior factor analytic work contributing to CHC theory demonstrating that visual memory and visualization load onto a single factor, we created a factor score called Spatial Skills using multiple imputation in AMOS (Schneider &McGrew, 2012). We used scores from this Spatial Skills factor in all subsequent analyses.

Spatial Span. The Spatial Span test in the 4th edition of the Wechsler IntelligenceScale for Children (WISC-IV) is a norm-referenced measure that requires participants to remember a sequence of spatial locations on a grid in forward and reverse order. The Forward condition measures spatial attention and short-term visuospatial memory, whereas the Backwards condition additionally measures the ability to manipulate visuospatial representations in working memory. Participants' scores on each of the conditions are summed into a Spatial Span total score.

Block Design. The Block Design test in the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999) is a norm-referenced measure that requires participants to perceive patterns and mentally stimulate how they might look when transformed (e.g. rotated). On the Block Design test, participants are asked to arrange a set of red-and-white blocks in such a way as to reproduce a 2-dimensional visual pattern shown on a set of cards. The test is timed, and scoring is based on both efficiency and accuracy of the pattern reproduction.

Math Achievement. All math measures came from the Woodcock-Johnson III Tests of Achievement and Cognitive Abilities (WJ III ACH & COG), designed for use across the lifespan (Woodcock, Mather, & McGrew, 2001). A math reasoning test was administered at the second timepoint, and three math measures were administered at the final timepoint.

Number Series. We administered the Number Series test from the WJ-III Cognitive Abilities testing battery at the second and third timepoints as a measure of mathematical reasoning. On this test, the examiner presents the participant with a page of numerical sequences that contains a missing number. The participant is asked to complete each sequence by identifying and applying the rule that applies to the other numbers in the sequence. As the test advances, the underlying rules become more challenging (e.g., 2, 3, 4, ___? as compared with 15, 18, 21, ___?). Participants are awarded 1 point for each correct answer and 0 points for each incorrect answer. The examiner discontinues the test when the participant either finishes all items or misses six consecutive items.

Applied Problems. We administered this WJ-III subtest to measure participants' ability to solve practical math word problems using simple counting, addition, or subtraction operations at the third timepoint. On the Applied Problems test, a participant is presented with a picture, (e.g., a group of mixed coins) and asked to listen to a problem (e.g., "How much money is this?"). To solve a problem, the child must recognize the mathematical procedure to be followed and perform the appropriate calculations. As the test advances, the child must carry out more complex operations and have more advanced experience with each particular concept, such as telling time or solving word problems. Participants are awarded 1 point for each correct answer, and 0 for each incorrect answer.

The examiner discontinues the test when the child either finishes all items or missed six consecutive items by the completion of the test page.

Math Fluency. To measure participants' ability to complete basic arithmetic problems, we administered the Math Fluency test on the WJ-III Achievement testing battery at the third timepoint. This test measures participants' ability to solve simple addition, subtraction, and multiplication facts within a one-minute time limit. At the beginning of the test, the child is presented with a worksheet composed of simple arithmetic problems and asked to solve as many problems as he or she can in one minute.

Hypotheses

Latent Construct of Fluid Reasoning. To examine whether the three tests represent a common construct, we used confirmatory factor analyses (CFA) to create a latent variable 'Fluid Reasoning (FR)' from participants' scores on three different tests at each time point: Matrix Reasoning, Analysis Synthesis and Concept Formation. CFA procedures were conducted to test the fit of the data to the FR construct for each time point (Figure 1).

Comparing FR and Math Reasoning as predictors of later Math Achievement. Second, we tested a model including relations from FR skills and math reasoning skills to a diverse set of math skills at a future time points (see Figure 2). To this end, we created a math latent variable called 'Math Achievement' using three different math tests at T3, each measuring different math skills: Math Reasoning, Applied Problems, and Math Fluency. As shown in Figure 2, we hypothesized that FR at T1 and T2 would be the strongest predictors of Math Achievement at T3 after accounting for prior Math Reasoning at T2. In the next analyses, we added age to the model.

Comparing FR to verbal and spatial abilities as predictors of later Math Achievement. Third, we compared the relative contributions between FR and future Math Achievement in relation to other cognitive abilities that have previously been implicated in math development: Verbal Reasoning (Vocabulary) and Spatial Skills (Spatial Span and Block Design). We tested the model hypothesizing that FR skills would remain a strong predictor of future Math Achievement after accounting for verbal and spatial skills (see Figure 3).

Results

Missing Value Analysis

Since the percentage of missing values for four of the variables was above five (refer to Appendix A for the percentages of missing values per variable), the pattern of missingness was assessed via Little's MCAR (Missing Completely at Random) procedure (Tabachnick & Fidell, 2007). This procedure revealed that the data were missing at random (MCAR: χ^2 (336) = 360.32, p = .173). Because of this, we used the Expectation Maximization algorithm to estimate the model parameters (Tabachnick & Fidell, 2007).

Descriptive Statistics

Means and standard deviations for the study variables are shown in Table 1. Both the raw and T-scores are presented. The T-scores are standardized scores wherein the mean is 50 and the standard deviation is 10. Factor scores for FR and Spatial Skills were derived using multiple imputation in AMOS. As shown in Table 1, all mean raw and T-scores increased numerically over time, with the exception of the FR factor score, which

decreases from T2 to T3 because the standardized loadings for the FR factor score in T3 are smaller than the standardized loadings in T2. When composite (or average) FR scores are generated, it is evident that performance increases across time. Pearson correlations between study variables are shown in Table 2.

The Structure of Fluid Reasoning. To test whether Matrix Reasoning, Analysis Synthesis, and Concept Formation could be combined into a latent factor of FR, we conducted CFA for each of the time points using AMOS 23 software (Arbuckle, 2015). All factor loadings were statistically significant ($p < .001$), with standardized loadings above .65, thus, supporting a latent factor. Therefore, the FR construct was supported, and we computed one latent factor for each time period using the three psychometric tests.

Comparing FR and Math Reasoning as predictors of later Math Achievement. We employed structural equation modeling (SEM) to test our second hypothesis that prior FR at T1 and T2 would be stronger predictors of T3 Math Achievement than T2 Math Reasoning. This approach also allowed us to examine the effects of the predictor variables simultaneously on a latent dependent measure. As shown in Table 3, our hypothesis was supported: prior FR was the strongest predictor of Math Achievement at T3. Any model that did not involve FR as a predictor of T3 Math Achievement fit significantly worse and decreased the amount of explained variance. In contrast, removing the path from T2 Math Reasoning to T3 Math Achievement did not worsen the fit or decrease the amount of explained variance of T3 Math Achievement. Results are reported in Table 4**.** This pattern was similar when using T2 and T3 data only (i.e., eliminating FR T1 from the model). Including age at T1 and T2 in the model depicted in Figure 2 did not change the results. Indeed, removing all regression paths from age to the variables of interest (i.e., leaving age in the model but eliminating its effects) did not worsen the fit.

Results from these analyses showed that both of the hypothesized models in Figure 2 fit the data well (Tables 5 and 6). For the sake of simplicity, we feature here the results of the structural model with age having a direct effect on Math Achievement at T3 (Figure 3). Specifically, FR at T2 significantly predicted Math Achievement at T3, $\beta = .52$, $p < .001$. By contrast, Spatial Skills at T2 did not significantly predict Math Achievement at T3, $\beta = .19$, $p = .205$. Vocabulary at T2 also did not significantly predict Math Achievement at T3, $\beta = .15$, $p = .205$. Similarly, age at T2 did not significantly predict Math Achievement at T3, $\beta = .16$, $p = .150$. The T2 predictors (Age, Spatial Skills, Vocabulary, and FR) accounted for 90.2% of the variance in Math Achievement at T3. In summary, this analysis shows that FR at T2 was a strong, unique predictor of Math Achievement approximately 1.5 years later.

These analyses also enabled us to test the mediating effect of FR between Age and Math Achievement at T3. Age significantly predicted FR, and FR significantly predicted Math Achievement at T3. Therefore, the first two criteria of mediation were met. As shown in Table 7, the indirect effect was statistically significant, $p < .001$, but the direct effect was not, $p = .198$. Therefore, the third and fourth criteria for mediation were met. As such, FR significantly mediated the relationship between Age and Math Achievement at T3. By contrast, Vocabulary and Spatial Skills did not significantly predict Math Achievement; thus, these factors did not significantly mediate the relationship between age and Math Achievement at T3.

Discussion

Summary of results

In this paper, we sought to test whether FR, or the ability to analyze novel problems,

identify patterns and relationships, and apply logic, contributes to future math achievement throughout primary and secondary schooling. We were particularly interested in comparing FR to other cognitive precursors (verbal and spatial skills) that have been previously linked to math development (e.g. McGrew & Wendling, 2010). Most prior developmental math research studies have been conducted in populations of school-aged children in the primary grades, and did not incorporate measures of FR in their predictive models. Therefore, the current research provides a necessary extension to the existing developmental math literature by examining the role of FR and its relation to other pertinent cognitive precursors in predicting future math achievement across a wide age range of children, providing a more comprehensive model of math development.

To this end, we first created a latent factor score of FR from three psychometric tests designed to measure FR (Matrix Reasoning, Concept Formation, & Analysis Synthesis) using confirmatory factor analysis. Second, we compared the strength of the associations between prior FR and prior math reasoning on later math achievement at T3 (measured by Applied Problem Solving, Math Reasoning, and Math Fluency) using Structural Equation Modeling. Results showed that across all three-time points, prior FR was the strongest predictor of later math achievement at T3, after accounting for prior math reasoning and age. Once we had determined that FR was a better predictor of later math achievement than prior numerical reasoning, we compared the relative contribution of prior FR to other important cognitive abilities associated with math, indexed by verbal reasoning (measured by Vocabulary), and Spatial Skills (measured by Spatial Span and Block Design), to future Math Achievement at T3. This model accounted for over 90% of the variance in Math Achievement. In this model, FR was the strongest cognitive predictor of future Math Achievement measured approximately 1.5 years later. Notably, Spatial Skills, vocabulary, and age were not significant predictors in this model.

Though some studies have shown that spatial skills and verbal comprehension are also robust precursors to future math achievement (e.g. Li & Geary, 2013; LeFevre et al. 2010), many prior studies have not incorporated measures of FR in their predictive models. Thus, we interpret the current findings as support for the notion that FR is a foundational skill that influences future development of numerical reasoning and potentiates math problem solving skills. Thus, the findings indicate that FR should be incorporated into future developmental models. These results support and extend Cattell's (1971; 1987) notion that FR development is an important cognitive precursor for even the most basic math skill development, including timed arithmetic, as well as more complex equations and word problems.

Study limitations

A limitation of the study is that we did not administer math measures at the first assessment rendering us unable to control for the initial effect of these domain specific precursors on future math outcomes. However, we were able to include math reasoning at T2 in our model, which enabled us to compare the relative contribution of prior math reasoning to prior FR in predicting future math achievement. Prior FR emerged as a better predictor of future math achievement than prior math reasoning. This finding builds on prior studies showing that domain general FR is as good a predictor of later math skills as prior numerical reasoning skills (e.g. Fuchs et al., 2010). Another limitation of the study is the relatively small sample size. However, the results are statistically reliable, and the wide age range enables us to make a novel contribution to the literature.

Theoretical implications

This work demonstrates that FR and mathematics achievement are linked throughout development, and that FR supports mathematical thinking and reasoning throughout the school years. One account for the strong relation between FR and math assessments is that both engage a common underlying cognitive ability called relational reasoning, or the ability to jointly consider multiple relations between different components of a problem (Halford, Wilson & Phillips, 1998; Carpenter, Fennema, Franke, 2013; Miller Singley & Bunge, 2014; Richland, Holyoak, Stigler, 2004; White, Alexander, Daugherty, 1998). The emerging ability to reason relationally may form the foundation for mathematical conceptual development, from the time children learn to compare the value of one number to another, to the time they learn to extract the value of a fraction by comparing the value of the numerator to the value of the denominator, to when they learn algebra and have to solve for an unknown variable by keeping in mind the relationship between numbers on both sides of the equal sign, and so on (Miller Singley & Bunge, 2014).

Demonstrating that FR predicts future math achievement across ages, above and beyond the effects of age, math reasoning, and other cognitive factors correlated with math proficiency - vocabulary and spatial skills – advances existing developmental theories of mathematics. While many existing developmental theories were formed based on studies involving younger children (approximately 4-9 years of age), (e.g. LeFevre et al., 2010), the current sample spans a broader age range of 6-21 years. This work also replicates and extends the findings in previous longitudinal research conducted by Fuchs et al. (2010) and Primi et al. (2010), who found that FR was a robust cognitive predictor of future math achievement over the course of one to two academic years in children in grades 1, 7, and 8 (respectively). Our study included a wider age range of children and adolescents between 6 and 21 years old, and adopted an analytic approach that enabled us to look at sequential influences of FR and later math proficiency measured by three specific math achievement domains.

These findings expand upon an existing developmental framework proposed by LeFevre et al. 2010, who hypothesize that there are three different pathways that contribute to early math development in children 4 to 7 years old: prior linguistic (or verbal) skills, spatial skills, and quantitative skills. In the current analyses, we have included these same pathways as well as a fourth pathway, FR. Our findings indicate that FR is a robust pathway that may be even more influential to math development than linguistic and spatial skills, though the relative contribution of these predictors should be systematically compared in future research.

Broader Implications

More generally, this line of research has possible relevance to school classroom settings. Fluid reasoning is thought to support all forms of new learning for which an individual has to problem solve (by integrating new information) without relying solely on prior knowledge, and therefore fluid reasoning could be applicable to many subject areas. However, we posit that fluid reasoning is particularly helpful for learning math, which is hierarchical in nature and requires individuals to solve novel problems as each new level advances. Currently, educators often focus on building computational proficiency as a means to improving mathematical achievement, without much consideration of the cognitive precursors that underpin these skills or the students' strengths and weaknesses (Boaler, 1998). Theories such as CHC, as well as longitudinal studies such this one provide insights on the links between specific cognitive abilities and math achievement that can inform educational practices. Though some new math curricula do incorporate spatial

rotation or block construction exercises, FR has not typically been emphasized in current math curricula. However, even students with strong basic numerical skills and spatial skills may not be proficient in applying logical reasoning techniques to solve novel problems.

We argue that math curriculum should incorporate opportunities for students to practice core aspect of FR known as relational thinking, or the ability to jointly consider several relations among mental representations (Miller-Singley & Bunge, 2014). One example of a curriculum that incorporates relational thinking practice into math exercises is called Early Algebra (Carpenter, Franke & Levi, 2003). This curriculum involves teaching children as young as 6 years old to view the equal sign as a form of equivalency using non-traditional number sentences. For example, children solve equations such as "5+3 = 6 + _" and explain their thinking aloud. By solving these types of equations and having students explain their thinking, students come to understand the component relationship between numbers on opposite sides of the equation, and can often identify the correct answer without doing any calculations (Carpenter, Franke & Levi, 2003). Another effective approach involves practicing early abstract reasoning skills with kindergartners and preschoolers to improve early numeracy skills (Ciancio, Rojas, McMahon, & Pasnak, 2001; Kidd et al., 2008). Many other approaches can be used to incorporate FR skill building opportunities into math curriculum (e.g. Miller-Singley & Bunge, 2014).

Finally, the assessment of FR in elementary school could serve to identify students who are likely to have difficulty-learning math. This information could help guide teachers to better understand which interventions may be most fruitful for individual students at different developmental levels of FR and math achievement skills.

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Table 1

Descriptive Statistics for the Study Variables (N = 69)

	Time 1					Time 2					Time 3				
	Raw Score			T-Score		Raw Score			T-Score		Raw Score			T-Score	
Variables	Range	*M*	*SD*	*M*	*SD*	Range	*M*	*SD*	*M*	*SD*	Range	*M*	*SD*	*M*	*SD*
Assessment age	5 to 16	10.18	3.32	--	--	6 to 19	11.67	3.35	--	--	8 to 21	13.45	3.85	--	--
Vocabulary	11 to 66	41.62	14.49	56.02	11.44	23 to 71	48.46	12.21	59.61	10.48	30 to 74	54.36	10.38	54.36	8.21
Matrix Reasoning	3 to 32	22.05	7.47	54.96	9.61	5 to 35	26.38	5.56	59.13	7.81	8 to 34	28.70	4.06	59.98	5.89
Analysis synthesis	1 to 37	23.60	8.35	60.90	13.17	12 to 34	27.41	4.56	65.43	12.16	20 to 34	29.49	3.22	68.64	12.26
Concept formation	1 to 35	23.63	10.03	58.51	10.42	7 to 35	28.49	7.32	64.39	10.67	7 to 35	31.69	4.86	69.37	8.56
Fluid reasoning	4 to 31	21.64	7.17	--	--	11 to 39	32.31	5.91	--	--	14 to 30	26.45	2.88	--	--
Number series	--	--	--	--	--	7 to 22	15.77	3.42	--	--	11 to 23	17.43	2.81	--	--
Spatial Skills	--	--	--	--	--	21 to 83	56.28	17.81	--	--	30 to 90	68.60	13.57	--	--
Applied problems	--	--	--	--	--	--			--	--	29 to 60	46.24	7.60	65.90	12.74
Math fluency	--	--	--	--	--				--	--	29 to 160	95.66	30.73	--	--

Table 2

Pearson Correlations between the Study Variables

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
First assessment																				
1 Matrices																				
2 Analysis synthesis	.68																			
3 Concept formation	.71	.51																		
4 Vocabulary	.81	.63	.69																	
5 Fluid reasoning	.99	.72	.75	.84																
Second assessment																				
6 Matrices	.69	.77	.60	.77	.82															
7 Analysis synthesis	.60	.77	.59	.65	.75	.82														
8 Concept formation	.62	.60	.74	.73	.75	.76	.77													
9 Spatial Skills	.73	.68	.67	.80	.88	.77	.68	.78												
10 Number series	.68	.78	.55	.77	.80	.76	.77	.54	.75											
11 Vocabulary	.78	.66	.73	.87	.84	.77	.66	.64	.86	.74										
12 Fluid reasoning	.69	.79	.77	.77	.73	.94	.95	.88	.97	.82	.78									
Third assessment																				
13 Matrices	.60	.61	.50	.67	.76	.79	.71	.34	.64	.63	.58	.80								
14 Analysis synthesis	.55	.77	.48	.65	.70	.72	.75	.38	.65	.65	.62	.77	.65							
15 Concept formation	.57	.47	.63	.47	.62	.57	.54	.52	.53	.51	.49	.63	.52	.54						
16 Spatial Skills	.76	.75	.80	.88	.93	.89	.83	.87	.91	.83	.88	.96	.78	.78	.64					
17 Number series	.39	.53	.63	.55	.54	.57	.58	.48	.59	.59	.56	.65	.56	.71	.45	.73				
18 Vocabulary	.80	.65	.63	.82	.83	.73	.66	.56	.73	.64	.81	.75	.64	.56	.35	.81	.56			
19 Applied problems	.60	.66	.69	.79	.76	.72	.66	.54	.72	.74	.75	.77	.69	.74	.47	.86	.80	.71		
20 Math fluency	.67	.62	.65	.74	.77	.71	.67	.52	.86	.69	.81	.79	.61	.65	.37	.89	.60	.67	.71	
21 Fluid reasoning	.66	.76	.78	.72	.71	.83	.81	.78	.83	.72	.67	.87	.87	.91	.72	.90	.71	.72	.79	.70

Note. All correlations were statistically significant at .05.

Table 3

Fit statistics for the Structural Models Comparing FR to Math Reasoning in Predicting Future Math Achievement

Variables	χ^2	df	$\Delta\chi^2$/df	R^2
Model 1 Full (Figure 2 without Age)	61.02	19		.94
Model 1A ($FR_{t1} \rightarrow Math_{t3} = 0$)	62.63	20	1.61/1	.90
Model 1B ($FR_{t1} \rightarrow Math_{t3} = 0$) ($FR_{t2} \rightarrow Math_{t3} = 0$)	85.03	21	22.40/1 ***	.71
Model 1C ($FR_{t1} \rightarrow Math_{t3} = 0$) ($Math_{t2} \rightarrow Math_{t3} = 0$)	63.30	21	.67/1	.91
Model 2 Full (Figure 2 with Age)	139.71	31		.92
Model 2A ($Age_{t1} \rightarrow vars_{t2} = 0$) ($Age_{t2} \rightarrow vars_{t3} = 0$)	155.23	34	15.52/3	.90

* $p < .05$. ** $p < .01$. *** $p < .001$.

Table 4

Unstandardized and Standardized Path Coefficients for the Structural Models Comparing prior FR to Math Reasoning in Predicting Future Math Achievement

Variables	*B*	*SE*	β
Model 1C			
$FR_{t1} \rightarrow FR_{t2}$	.77	.09	.98 ***
$FR_{t1} \rightarrow Math_{t2}$	.39	.05	.90 ***
$FR_{t2} \rightarrow Math_{t3}$	.99	.12	.96 ***
Model 2			
$FR_{t1} \rightarrow FR_{2}$	.74	.09	.97 ***
$FR_{t1} \rightarrow Math_{t2}$	.34	.04	.86 ***
$FR_{t1} \rightarrow Math_{t3}$	-.10	.43	-.34
$FR_{t2} \rightarrow Math_{t3}$	.72	.49	.85

* $p < .05$. ** $p < .01$. *** $p < .001$.

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Table 5

Fit Indices for the Structural Model Predicting Math Achievement at T3 from Prior FR, Spatial Skills, Vocabulary and Age

Index	Indirect Effects	Direct Effect
Chi-square	67.68	65.69
Degrees of freedom	28	27
Probability level	.00	.00
Normed chi-square	2.42	2.43
Goodness of fit index (GFI)	.84	.85
Comparative fit index (CFI)	.94	.94
Root mean square error of approximation	.14	.15
(RMSEA)	.10	.10
Lower bound 90% confidence interval	.19	.19
Upper bound 90% confidence interval	.00	.00
P-close	.04	.04
Standardized root mean square residual (SRMR)		

Table 6

Unstandardized and Standardized Path Coefficients for the Structural Model Predicting Math Achievement at T3 (with Direct Effect from Age to Math Achievement)

Variables	*B*	*SE*	β
Age to:			
Vocabulary	2.96	.26	.81 ***
Spatial Skills	4.29	.45	.78 ***
Fluid Reasoning	1.37	.20	.71 ***
Math Achievement	.32	.22	.16
Vocabulary →Math Achievement	.08	.08	.15
Spatial Skills →Math Achievement	.07	.05	.19
Fluid Reasoning → Math Achievement	.52	.16	.52 ***

* $p < .05$. ** $p < .01$. *** $p < .001$.

Table 7

Standardized Direct and Indirect Effects of Age on Math Reasoning at Time 3

Effect	Model w/o Direct Effect	Model w/ Direct Effect
Total effect	.76 ***	.80 ***
Direct effect	--	.16
Indirect effect	.76 ***	.64 ***

* $p < .05$. ** $p < .01$. *** $p < .001$.

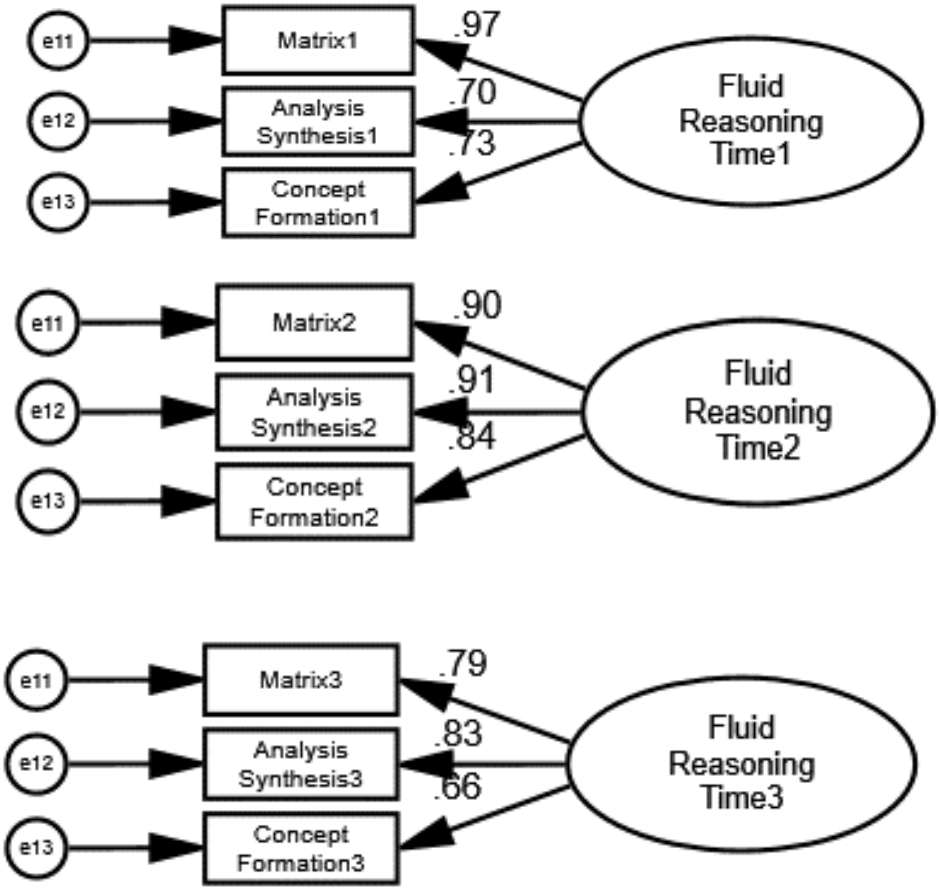


Figure 1. Standardized parameter estimates from the CFA of FR for each measurement occasion. All three indicators loaded on significantly to the FR constructs at each time point.

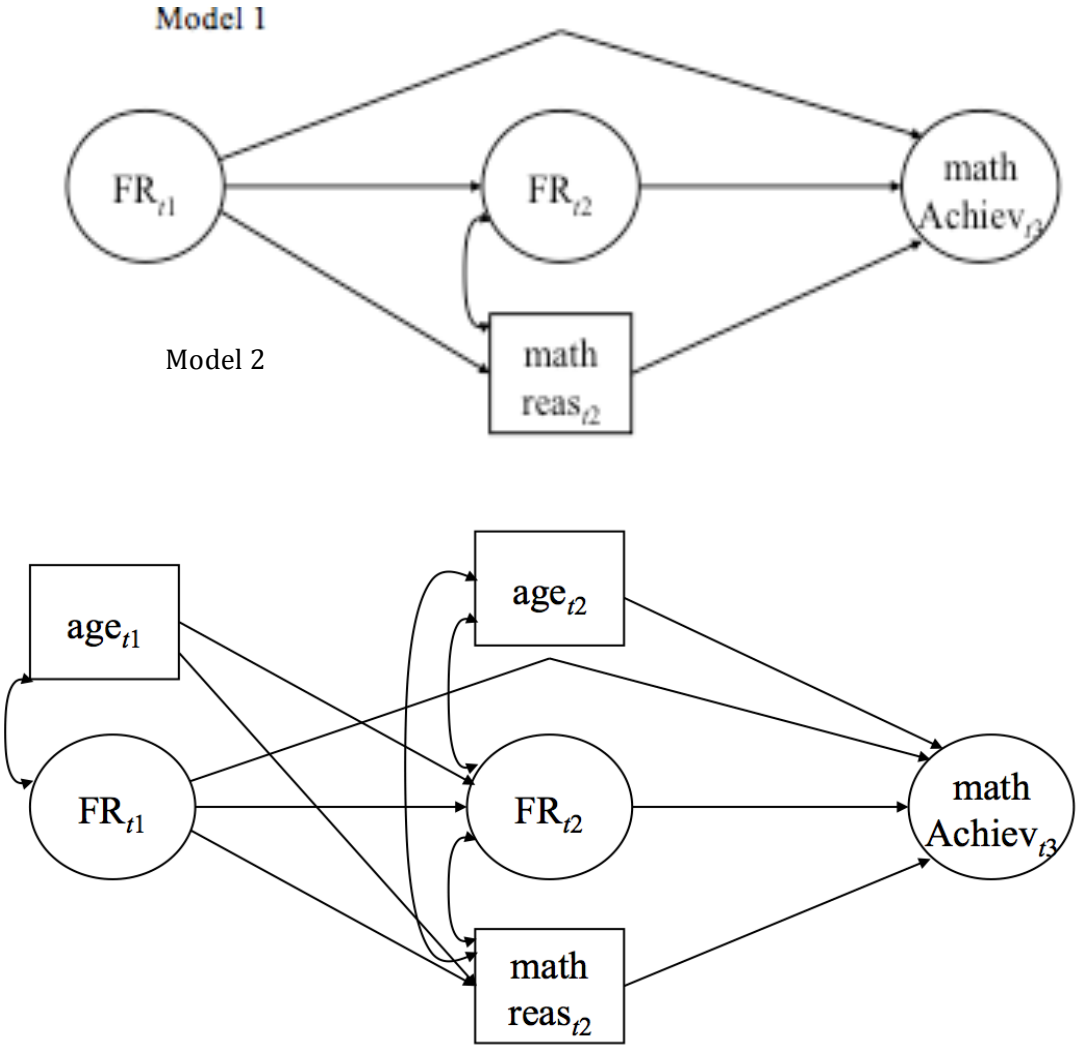


Figure 2. Longitudinal models predicting Math Achievement from Prior FR and Math Reasoning. In Model 1, Math achievement is predicted from previous assessments of FR and Math Reasoning. In Model 2 Age is included at the two previous occasions. Circles represent latent variables of FR comprised of three observed variables. One-headed arrows represent regressions and two-headed arrows represent covariance or correlations.

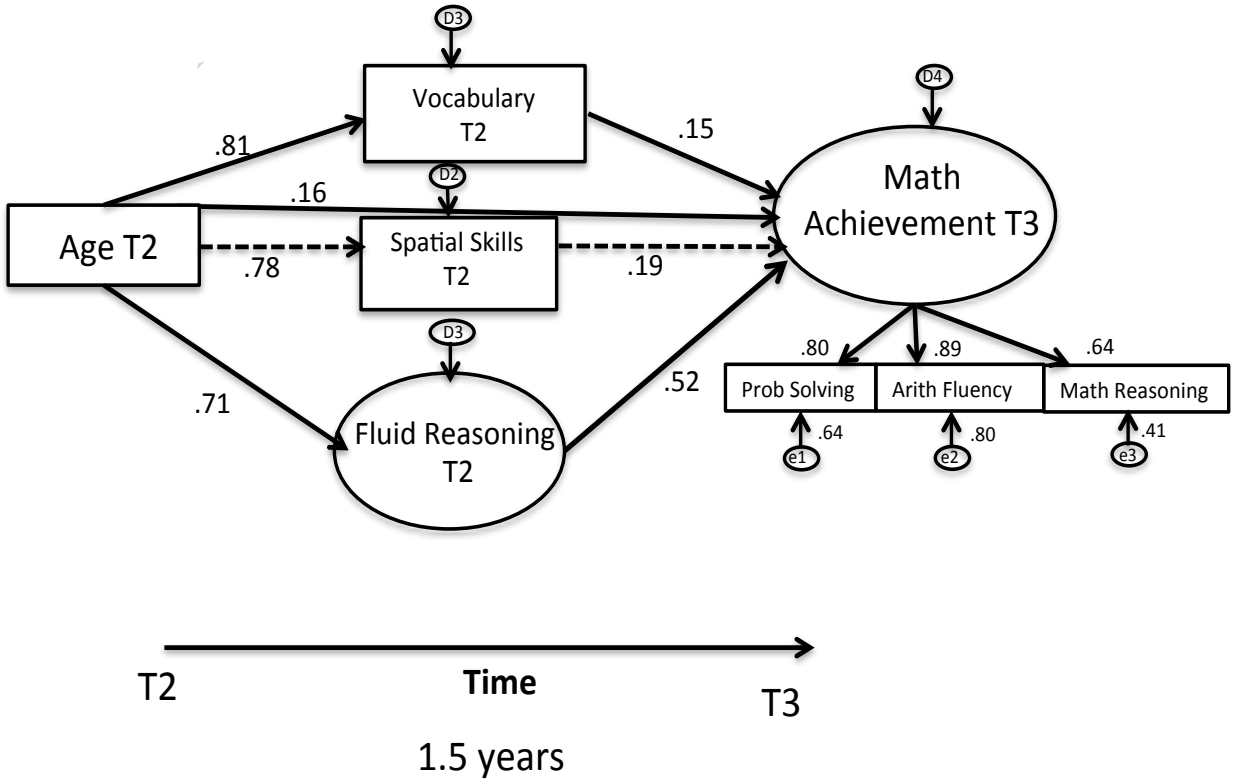


Figure 3. Longitudinal model predicting Math Achievement from Prior FR, Spatial Skills, Vocabulary and Age

Chapter 3 - Game Play Intervention for 1st and 2nd Grade Children at risk for Math Learning Disabilities

Math Learning Disability (MLD) interventions have been significantly under-researched, especially compared to interventions for reading disabilities, though the prevalence of these two disorders is equivalent, ranging from 5-7% of school aged children (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005). In the last 10 years, there have been a growing number of research studies published in this area. The underlying deficits that contribute to MLD and effective remediation techniques are becoming better understood. MLD is currently conceptualized as a disorder that is biologically based, but behaviorally defined, particularly in schools, by persistent challenges in learning even the most elementary mathematics (Mazzocco, 2007). Recent models further define math learning disabilities as a failure to benefit from standard instructional support in math (Burns & Vanderheyden, 2006).

Causes of MLD

The "core deficit" in MLD: number sense. Converging evidence shows that children with MLD have a common or *core* deficit in what is known as *number sense*, or the understanding of a number's magnitude and its relation to other numbers (Butterworth, Varma, & Laurillard, 2011). Number sense is a basic intuition about numbers that humans are born with (Jordan & Brannon, 2006). Number sense enables individuals to do tasks like estimate how many items are in a set without counting, compare two sets to determine which set is larger, or to break down a larger number into smaller numbers that can be more easily worked with to count or do arithmetic problems (Butterworth et al., 2011). Thus, number sense weaknesses can be observed on a range of basic numerical processing tasks, including magnitude comparison (Geary, 2000), subitizing, or the ability to identify how many objects are in a small set without counting (Koontz & Berch, 1996), and retrieval of arithmetical facts (Geary, 1993; Geary and Hoard, 2001; Ginsburg, 1997; Russell and Ginsburg, 1984; Shalev et al., 2001).

Domain general factors contributing to MLD. In addition to having number sense weaknesses, children with MLD typically demonstrate deficits in domain general cognitive abilities, including working memory, or the ability to keep information active in mind to solve a problem (Geary, Hoard, Byrd-Craven, Nugent & Numtee 2007). This weakness in working memory causes children with MLD to appear forgetful when they are learning math facts, and to lose track of their place while counting, or while completing a multi-step problem. Children with MLD also have weaknesses in processing speed, which cause them to process both numerical and non-numerical information more slowly than their typically developing peers (D'Amico and Passolunghi, 2009). Toll and Van Luit, (2013) showed that weaknesses in domain cognitive abilities further prevent children from benefitting from interventions aimed at improving early numeracy in children with math difficulties. Thus, it is common for accommodations to include extended time on assignments and tests (Bull & Scerif, 2001; Swanson & Sachse-Lee, 2001).

Biological underpinnings of MLD. Neuroscience research employing MRI shows that children with weaknesses in number sense have abnormalities in brain function and structure in a specific part of the parietal lobe, the intraparietal sulcus, which is responsible for quantity and magnitude processing (IPS; Butterworth et al., 2011). Additionally, children with MLD demonstrate abnormal brain activation in a network of brain areas related to numerical problem solving, including the prefrontal and ventral temporal–occipital cortices (Butterworth et al., 2011; Fias, Menon, & Szuc, 2013). These brain regions are responsible for symbol recognition

(ventral temporal–occipital cortex); as well as attention and working memory functions (frontoparietal network; Butterworth et al., 2011; Fias et al., 2013).

The Widening Achievement Gap

Longitudinal findings show that children with MLD experience accumulating deficits that cause them to fall further behind their peers every year (Geary, 2011a, 2011b; Geary, Hoard, & Baily 2012). Kindergartners' math achievement is a strong predictor of future math achievement across all grade levels (Duncan et al., 2007). The strength of this predictive relation between early and later math achievement is twice as large as the relation between early and later reading achievement (Duncan et al., 2007). Just as in the effective prevention of reading disabilities, early detection is a vehicle that allows for early remediation before children fall too far behind to catch up with the advancing curriculum (Griffin, 2007).

Early Screening to Identify At-Risk Students

There are early screening tools that teachers can administer to their class to identify children at risk for math learning disabilities in the early grades. These tests identify weaknesses in core numeracy skills (Number Sense Screener NSS; Glutting & Jordan, 2012), as well as the cognitive skills that are most predictive of MLD (e.g., Working Memory: Automated Working Memory Assessment [AWMA]; Alloway & Alloway, 2010). Tools such as these potentiate the use of more sensitive instructional strategies for students at risk for developing mathematical difficulties later on (Alloway, Gathercole, Kirkwood, & Elliott, 2009). Research on prevention of learning disabilities supports the use of early instructional interventions as well as frequent monitoring of subsequent academic progress to determine whether the interventions are effective (St. Claire-Thompson & Gathercole, 2006).

Components of Effective Interventions

Children with MLD are generally able to learn mathematics if they are provided with explicit, direct instruction of core numerical relations (Gersten et al., 2009; Clements, & Sarama, 2011). Principles for effective math instruction have been described by Ginsburg (2006), who stated that early math interventions should (a) follow natural developmental progressions of mathematical thinking, (b) provide hands-on games and activities that can encourage children to construct meaning, (c) encourage communication in spoken language and writing, and (d) ensure that activities capture children's emotions and imagination. Indeed, controlled studies (e.g., Starkey, Klein, & Wakeley, 2004) have shown that playing mathematical learning games can significantly enhance the informal math knowledge of both middle income and low-income groups.

Children with weaknesses in math are particularly at risk for developing learned helplessness, or the loss of motivation to continue when they are faced with increasing challenges throughout each successive mathematics lesson (Diener & Dweck, 1978). Motivation plays a crucial role in interventions for children with significant weaknesses in the areas being trained. A central component of effective intervention is the use of engaging, interactive games, with frequent monitoring and scaffolding by an adult (Klingberg, Forssberg, & Westerberg, 2002). Interventions are most effective when the supervising adult provides encouragement to persist in the face of challenge using motivational feedback (Diener & Dweck, 1978).

Prevention: Number Sense Training

A small number of intervention studies have employed number sense training for at-risk children in early grades (Gersten et al., 2009). A central component of number sense training involves practicing numeracy games using a number line with the aim of building familiarity with mental representations of numbers and magnitude. One such study by Siegler and Ramani

(2009) demonstrated that when 4-year-old children from low-income families played as little as one hour of a linear number line board game, they demonstrated considerably greater learning of arithmetic, magnitude comparison, and number line estimation. Subsequent iterations of this study showed that preschoolers from low-income backgrounds learned at least as much as, and on several measures more than, preschoolers with comparable initial knowledge from middle-income backgrounds (Ramani & Siegler, 2011).

A comprehensive intervention program developed by Griffin (1997) is the Number Worlds program. This curriculum, grounded in cognitive development, is a constructivist program that uses socially interactive games to encourage children to communicate mathematically. For example, some games involve rolling dice and moving pegs along a number line, or moving a teddy bear along a number line path and asking students to guess how many steps the bear has to take to reach the finish line. The Number Worlds program has been extensively evaluated with children from low-income populations, and has been demonstrated to enhance number sense, mathematical reasoning and communication, as well as enhancing performance on standardized mathematics achievement tests (Griffin, 2007). In one study of Number Worlds, Griffin (1997) showed that, at the end of first-grade, at-risk children in the number worlds group were performing at the same level as the normative group. By the end of second grade, the children in Number Worlds outperformed the normative group (Griffin, 1997). Transfer from the training was observed for real-world math measures involving telling time and using money. Long-term follow assessments showed that training improvements lasted one year and fewer referrals to special education were made (Griffin, 1997).

To target number sense weakness in children with dyscalculia, Wilson, Revkin, Cohen, Cohen, and Dehaene (2006) created a computerized game, The Number Race. The game was based on their work on parietal lobe dysfunction and number sense weakness in children with dyscalculia. The purpose of this game was to train children's ability to make numerical comparisons, by stimulating their conceptual connection between numbers and spatial representations (Wilson et al., 2006). In the first study, the program was administered individually to nine 7-9 year-old students, and the difficulty level was adapted to each student's level of performance. The training led to improvements in number comparisons, immediate number recognition, and subtraction in children with dyscalculia (Wilson et al., 2006). A follow-up study investigated the effects of the Number Race game using a crossover design in 53 kindergartners with low socioeconomic status in France (Wilson, Dehaene, Dubois, & Favol, 2009). Participating in a small number of short sessions of the Number Race game led to significant improvements on tasks that involved making symbolic numerical comparisons (Wilson et al., 2009). However, there were no improvements on non-symbolic magnitude comparison tasks, indicating that there was limited transfer to non-trained tasks. Further research is needed to investigate the generalizability of these effects to other broad domains of math achievement.

A limitation of domain-specific interventions that specifically target number sense is that they do not address the common weaknesses in cognitive processing (e.g., working memory and processing speed) that continue to hamper learning in children with math learning disabilities. It has been hypothesized that this omission can lead to more minimal transfer from the training to novel math concepts that require a broader range of cognitive skills beyond numerical proficiency (e.g., mathematical problem solving) (e.g., Ramani & Siegler 2011; Siegler & Ramani, 2009; Wilson et al., 2006).

Domain General Training of Cognitive Skills

An alternative intervention approach involves directly training the cognitive skills that are commonly impaired in children with math learning weaknesses (e.g., working memory, processing speed etc.). These interventions typically involve doing repeated cognitive games or exercises that engage the cognitive skill being trained. For example, studies have evaluated the effects of playing computerized working memory games that gradually increase in difficulty level (Peijnenborgh, Hurks, Aldenkamp, Vles, & Hendriksen, 2015). Studies that involving meta-analytic reviews of cognitive training studies have shown that this type of intervention is mostly effective in improving the specific cognitive skills trained by the intervention, as well as other closely related cognitive skills (e.g., inattention), though these improvements do not typically generalize to broader improvements in academic performance (e.g., Melby-Lervag & Hulme, 2013; Peijnenborgh et al., 2015).

Recent studies have compared these two intervention approaches (domain general cognitive training vs. number sense training) to evaluate which approach is more effective in improving math performance in children at-risk for MLD (Kuhn & Holling, 2014; Kyttala, Kanerva, & Kroesbergen, 2015; Passolunghi and Costa, 2016). Kuhn and Holling (2014) compared the effects of computerized number sense training to computerized working memory (WM) training on math performance in 59 nine-year-old children. Both training groups demonstrated significant improvements on dissociable domains of math performance. The number-sense training group demonstrated moderate gains in arithmetic skills compared to the control group (d = .54), whereas the WM training group demonstrated moderate gains in word problem solving compared to the control group (d = .57). Furthermore, in another recent study by Passolunghi and Costa (2016), working memory training was as effective at improving early numeracy skills in preschool children as counting training. Children who completed five weeks of adaptive working memory training improved on both working memory and early numeracy skills.

These lines of research provide encouraging preliminary evidence to suggest that numeracy and cognitive training may offer dissociable benefits for children at risk for math learning disabilities. Thus, a fruitful line of further investigation involves testing whether combining domain general cognitive training with early numeracy interventions produces more broad ranging effects on math skill development than either intervention approach alone.

The Present Study

In the current study, I investigate whether there is a synergistic effect of combining numeracy game training with speeded game training to enhance arithmetic fluency in children at-risk for math learning disabilities. To accomplish these goals, this research employed single-case design (SCD) methodology to evaluate the effects of 14 weeks of training on three children at risk for MLD. This research was conducted within the context of a larger study including typically developing children. Results from the first 13 weeks of training will be presented here.

Method

Participants

The case study participants were selected from a larger pool of participants taking part in a concurrent intervention study designed to improve cognitive and numeracy skills related to early math skill development in typically developing students. The intervention took place during an after-school program at a public elementary school in Berkeley, California. We recruited participants in first and second grade between the ages of 6 and 8. The after-school program from which participants were recruited is funded through the California Department of

Child Development. Though all students attending this elementary school are eligible to participate in the after-school program, the program is provided at no charge to families who meet income requirements (earning below 40% of the state median income) and demonstrate a need for care. Families who earn above the fee schedule limits are required to pay.

All participants and their parents provided their informed assent or consent to participate in the study approved by the Committee for Protection of Human Subjects at UC Berkeley. Additionally, all participants were screened for significant neurological impairment and severe developmental delays based on parent report (see Appendix A**).**

Parent information. Parents were asked to complete a brief survey (see Appendix A) with demographic questions about their child's birth date, gender, ethnicity, grade level, school history (i.e., repeated grades, special education services), and medical history.

Teacher information. Teachers were also asked to complete a brief survey (see Appendix B) that contained questions pertaining to the students' developmental math proficiency in comparison to grade level standards on arithmetic fluency and problem-solving activities.

Research Design Rationale

The SCD approach was employed for this study as this method is particularly useful for answering research questions in special education populations, which often focus on low-incidence or heterogeneous populations, for whom information about mean performance of large groups may be of less value for application to individuals (Horner et al., 2005). In SCD research, each participant serves as his own control. The dependent variable is measured repeatedly to establish the participant's pattern of performance prior to intervention, and then compared to his pattern of performance during and after the intervention. In this type of study design, the baseline condition is akin to a treatment-as-usual condition in study designs that compare effects between groups. Measurement of the dependent variable at baseline occurs until the pattern of performance is sufficiently consistent to enable prediction of future responding. The goal is to demonstrate that the intervention had a stable and robust effect, which is usually achieved after three demonstrations of the experimental effect at three points in time with a single participant (within-subject replication). Additionally, SCD research is also useful for conducting fine-grained analyses of non-responders. In SCD research, external validity is enhanced when results are replicated across subjects, and as such, three or more subjects are recommended in SCD research (Horner et al., 2005).

Case Study Participant Demographics

The selection criteria adopted by this study is consistent with prior research on at-risk children (Mazzocco, 2007). As such, students who met the following criteria were selected for case-study intervention: (a) below average performance (≤ 25th percentile) on tests of early numeracy (Woodcock Johnson Math Fluency Test and Digit Comparison Task); (b) low average or below grade level performance on classroom-based measures of math (according to teacher report; Appendix C); and (c) no prior diagnosis of intellectual or severe neurological impairments or severe ADHD (based on parent report, see form in Appendix B).

Of the 24 students tested at baseline, 3 students met all selection criteria for the case study. All three students performed in the below average range on both numeracy and processing speed tasks (see Table 1). In each case, the participants' teachers noted concerns in math on the questionnaire. Finally, none of the participants had pre-existing neurological or developmental disorders that could explain their low math achievement scores. The following descriptions of the case study students were formed based on teacher and parent reports.

Jesse. Jesse is a second-grade Hispanic boy, whose teacher and mother describe him as a "contextualized learner," who is clever and highly engaged in many subjects, but who dislikes math and reading. After having attended a non-structured Montessori play-based preschool, it was relatively difficult for him to adapt to the structured school environment in Kindergarten. Initially, he had some minor difficulty learning to read, which his mother attributes to him entering school with less formal reading experience than other students who attended traditional pre-schools. He has received after-school tutoring in reading since first grade, which has propelled his reading skill development, though his teacher notes that he continues to have some difficulty with comprehension.

His teacher reports that Jesse's greatest area of academic difficulty is in math. In both addition and subtraction domains, he performs below grade expectations. Because addition and subtraction skills represent a core component of second grade level standards, his teacher has expressed concern about his basic math proficiency. On pre-testing, he scored in the average range on a test of fluid-reasoning and working memory, though his processing speed, and numeracy skills fell to the below average range. His cognitive profile indicates that he is at risk for a math learning disability.

Jennifer. Jennifer is a first-grade Caucasian girl, whose teacher describes her as a shy student with shaky confidence in her academic skills. Her timid nature makes it difficult for her teacher to get a sense for what she knows, though her teacher does recognize that her math skills are underdeveloped for her age. When a task becomes too challenging, Jennifer attempts to cover up what she doesn't know by making up answers and turning in her work. Her teacher notes that she avoids asking for help, in an attempt to blend in with her peers. Her mother reported that at the age of two, Jennifer had difficulty articulating S's and was diagnosed with a speech disorder, for which she received speech therapy.

At pre-testing, Jennifer performed in the low average range across tests of fluid reasoning, processing speed, and working memory. Her numeracy skills fell in the mildly impaired range on both the math fluency and digit comparison tasks. Her cognitive and academic profile indicates that she is at risk for a math learning disability.

José. José is a bilingual Hispanic second-grade boy whose parents emigrated from Spain before he was born. Spanish is his first language and he began speaking English at the age of three. His mother reports that when he started school, he had some minor difficulty keeping up with his peers in writing, reading, and math. He has received after-school tutoring in all subject areas since first grade. His academic skills have improved substantially in reading and writing, though his math skills remained relatively behind his same age peers. He is a socially oriented who gets along well with many peers.

His pre-testing results indicated that he performed in the low average range across fluid reasoning, processing speed, and numeracy domains. His cognitive and academic profile renders him at risk for a math learning disability.

Dependent Measures

Pre/post tests. All participants in the larger study underwent a battery of neuropsychological tests including age-normed standardized measures of cognition and math abilities. Two standardized numeracy measures were administered to identify students who performed in the below average range: WJ-IV Math Fluency Test and the Digit Comparison Task.

Math Fluency Woodcock Johnson Achievement Battery, Fourth Edition (WJ-IV). To measure participants' ability to complete basic arithmetic problems under timed conditions, participants are asked to complete a worksheet containing simple addition, subtraction, and multiplication problems. At the beginning of the test, the child is presented with the worksheet and asked to solve as many problems as he or she can in three minutes. The score is the number of items correct. Test-retest reliability for scores on this measure is .95 (McGrew, LaForte & Schrank, 2014). We used three alternative forms of this test to reduce practice effects that can occur when this test is repeatedly administered within a short period of time.

Digit comparison task. To measure children's explicit number processing abilities, a numerical comparison task was used, in which participants were presented with two single digit numbers (ranging from 1 to 9) on a computer screen, and were asked to choose the numerically larger number as fast as they could without making any errors. Both numbers had a font size of 72 and appeared on a 14-inch computer screen on either side of a centrally located fixation dot until the participants made a response. There were 80 trials in which the ratio between the two numbers (lower number divided by higher number) was manipulated and fell between 0.11 and 0.89, for example the ratio between two and three is 0.67 (see Appendix C for a list of pairs and ratios). There were 27 levels of ratio for the numerical comparison task. Each ratio was repeated four times in random order, and each number was counterbalanced for the side of presentation. Each participant received a break halfway through the task. Both accuracy and response times were recorded during each trial.

Cognitive ability tests. The following subtests were administered at pre and post testing.

Letter-pair matching (*WJ-IV).* This is a timed cognitive processing speed test from the Woodcock Johnson Cognitive Battery Fourth Edition (WJ-IV, Schrank, McGrew, & Mather, 2014). The test requires participants to determine which letter or letter groups within a row are the same. For example, participants are asked to "circle the identical letters or letter groups:" bl va dl bl na. Participants are instructed to complete as many items as possible within three minutes. Test-retest reliability for scores on this measure is .91 for ages 7-11 (McGrew, LaForte, Schrank, 2014). This test was administered at T1 and T2.

Cross-out (WJ-IV). This is a timed cognitive processing speed test from the Woodcock Johnson Cognitive Battery Fourth Edition (WJ-IV, Schrank, McGrew, & Mather, 2014). Participants were told to cross-out drawings that are identical to the first drawing in each row. They were instructed to work quickly to complete as many rows as possible within three minutes, while maintaining accuracy. Test-retest reliability for scores on this measure is .91 for ages 7-11 (McGrew, LaForte, Schrank, 2014). This test was administered at T3.

Matrix Reasoning (WIPPSI-IV). This test was modeled after a traditional test of fluid, or non-verbal reasoning; Raven's Progressive Matrix Reasoning (Raven, 1938). This test required the participants to examine an incomplete matrix, or geometric pattern, and then select the missing piece from five response options arranged according to one or more progression rules. The Matrix Reasoning subtest assesses fluid reasoning (FR) induction skills, or the ability to identify an underlying characteristic (e.g., rule or pattern) that governs the existing pattern, and then to choose a missing piece. The test is discontinued when a participant misses three problems in a row. Test-retest reliability for scores on this measure is .82 (Wechsler, 2012).

Weekly Dependent Math Measure: Arithmetic Fluency

Arithmetic fluency was measured with timed math probes. Each probe included 30 addition problems administered using a paper and pencil assessment within a time limit of one minute. 10 different probes were administered over the course of the intervention. Each probe each contained thirty addition facts with sums under 10. The researchers timed and scored the number of correct responses. This is a brief, repeatable method of assessment that is sensitive to student improvement over time. The test can be used to examine the functional relationship between performance and the intervention (Shinn, 1989). Test-retest reliability for scores on this measure is .82 (Marston, 1989).

Intervention Procedure

Overview. During the intervention phase, participants met with the researchers approximately 3-4 times per week for 30-minute sessions for 13 weeks during their after-school program (Figure 1). Over the course of the intervention in the fall and spring, each participant received approximately 21 hours of intervention. Two out of four days per week were spent playing processing speed games, and the other two days were spent playing numeracy games. All games were played in small groups of 2-3 children. The decision to use a game-based intervention stems from the importance of motivation in learning math, and principles for effective math instruction described by Ginsburg (2006). Students played new games each week, which ensured that children remained engaged and motivated to play the games throughout the program.

Staffing. A senior research assistant and graduate student oversaw implementation of both the case study and larger intervention study. The intervention team consisted of 11 undergraduate research assistants and two graduate students. On a given day, six research assistants worked with small groups of two or three students at a time during each game-play session.

Numeracy training. Half of the training consisted of students playing a series of numeracy games that advanced in a manner consistent with developmental math progressions over the course of the intervention. Each numeracy game was intended to build a unique component of basic numeracy skills. At the beginning of each session, a researcher explained the game instructions to ensure comprehension, and monitored the participants closely throughout the remainder of the game to ensure that participants adhered to the rules of the game. Each game was played two days per week to facilitate sufficient practice and improvement while avoiding fatigue or boredom. Furthermore, if a participant stopped being engaged or willing to continue playing a particular game on a given day, that game was discontinued for the remainder of the session and another game was introduced. All games are commercially available. See Appendix E for a complete list of numeracy games included in the intervention. The games were played in a pre-planned order intended to scaffold the natural progression of math skills by building upon numeracy skills involving number recognition, number comparisons, and numerical equivalency skills.

Importantly, none of the math games included in the intervention explicitly trained arithmetic skills. An example of a number recognition game was Zingo 1-2-3, which builds number recognition skills. In this game, players matched their numbered tiles to their corresponding challenge card. A more advanced game, Olympian Number Line, targeting numerical comparison skills, involved rolling dice and moving pieces along a pre-planned path and then determining which of two numbers written in the space was bigger using a number line.

Processing speed games. The other half of the training consisted of students playing processing speed games. The speed game training design was modeled after prior research by Mackey et al. (2011). All games involved some form of rapid visual processing and rapid motor responding based on simple task rules. Most games involved working quickly under a time limit or racing against another player. All games are commercially available. The complete list of speed games is listed in Appendix F. Each game was played approximately 3-4 times to enable time for sufficient practice and improvement while avoiding fatigue or boredom. For example, in the game Spot It, two or more players are given a card with six colorful animals. The animals may vary in size and position, but there is always one, and only one, animal match between any two cards. The aim of the games is to be the first to spot a matching animal between the target card and your own card, before another player spots one.

Statistical Analyses

Group level changes from pre to post testing. To determine whether the intervention led to significant improvements in trained abilities at the group level on numeracy and processing speed measures, statistical analysis was performed on pre- and post-test data using the non-parametric tests. The Related-Samples Wilcoxon Signed Rank procedure was conducted for tests measured at two time points and the Related-Samples Friedman's Two-Way ANOVA by Ranks was conducted for tests measured at three time points.

Individual changes in weekly arithmetic fluency. To evaluate each participant's performance on timed arithmetic fluency throughout the course of the intervention, weekly arithmetic fluency scores were plotted on a time series line graph.

Percentage of non-overlapping data (PND), was calculated by first determining the number of data points in the intervention phase that exceeds the highest data point in the baseline phase. This value was divided by the total number of data points in the intervention phase and multiplied by 100, yielding a percentage score. PND scores are indicative of effect size in SCD research. Higher percentage scores reflect more effective interventions: values of 90% or higher reflect "highly effective" interventions; values of 70% to under 90% reflect "moderately effective" interventions; values from 50% to under 70% reflect "mildly effective" interventions; and values below 50% reflect an "ineffective" intervention (Ma, 2006).

Slope coefficients were calculated using ordinary least squares regression for each student on the number of digits computed correctly per week. This regression indicated the average increase in the number of correctly computed digits per minute for each week over the course of the intervention.

Next, *visual analysis* was performed to examine the slope over multiple testing sessions, using published guidelines outlined by Horner et al. (2005). This approach involved calculating student's *level* (or mean performance score) of arithmetic fluency during each phase of the study, as well as their rate of increase or decrease (e.g., *slope*) in performance. Visual analysis enabled evaluation of (a) variability within each subject's performance within each phase of the study, (b) the immediacy of effects following the onset and/or withdrawal of the intervention, (c) the magnitude of changes in the dependent variable, and (d) the consistency of data patterns across multiple presentations of intervention and nonintervention conditions (Parsonson & Baer, 1978). The integration of information from these multiple assessments and comparisons was used to determine whether a functional relationship existed between the intervention and dependent

variables. In order to identify the intervention as effective, the data across all phases of the study was analyzed to determine if there were at least three demonstrations of an effect at a minimum of three different points in time (Kratochwill et al., 2010).

Ratio effect. A more fine-grained approach was taken to evaluate changes in performance on the Digit Comparison Task with regard to the well-known ratio effect. The ratio effect is observed on digit comparison tasks when children are asked to compare the relative magnitudes of two digits. Participants are typically faster and more accurate as the ratio between the two digits increases (e.g., though number pairs 2 and 3 and 8 and 9 both have a numerical distance of 1, their ratio is significantly different, and it takes longer to discriminate the relative magnitude of 8 and 9 than it does for 2 and 3; Holloway & Ansari, 2009). The ratio effect is strongly correlated with individual differences in performance on standardized math measures in children; participants who show a large ratio effect typically demonstrated low math scores (Holloway & Ansari, 2009). Prior research shows that the presence of a large ratio effect is associated with an immature representation and processing of numerical magnitude. Participants who show a large ratio effect are predicted to have low standardized math scores (Holloway & Ansari, 2009). Specifically, in order to enumerate the relationship between ratio and reaction time (RT), the slope and intercept of the regression line that relates ratio and RT was calculated using reaction times for each correct trial for each participant.

To determine whether the ratio effect on the digit comparison task was related to individual differences in math achievement scores, partial correlations were conducted using math fluency subtest raw scores, and the intercept and slope of the ratio effect. If there is a significant negative correlation between math fluency raw scores and the intercept and with slope, then it can be concluded that the ratio effect is related to individual differences in math achievement scores.

Results

Qualitative Observations

Jesse. Overall, Jesse was engaged during games that he enjoyed (e.g., Stormy Seas), but had difficulty maintaining attention during games that he did not prefer to play (e.g., Tangoes, Jr.). It was particularly important to pair him with peers who demonstrated strong behavioral skills and could model good behavior. Whenever possible, the researcher gave Jesse a choice between two games. Over the course of the intervention, he demonstrated observably faster reaction times on the speed games and his inclination to play numeracy games increased. Researchers observed that he demonstrated notably improved numerosity skills during game-play.

Jennifer. From the start of the intervention, it was evident that Jennifer had difficulty with numerosity, particularly when she had to make comparisons between two numbers to determine which one was smaller or larger. She used immature counting strategies, such as counting on her fingers. On weekly arithmetic fluency tests, she often rushed through the test and wrote incorrect answers. She worked best individually or with one or two close female peers, and sought attention from adults. Her reaction time was notably faster on games that did not involve numbers. Her numerosity skills accelerated when she played number-line board games on a one-on-one basis with a researcher. She became notably faster at recognizing numbers on a number line and making numerical comparisons on the board games. She continued to have difficulty on more abstract number games that required her to integrate information (e.g., Number Chase).

José. Generally, José was well behaved and engaged in the games throughout the intervention. He worked well with many peers. He routinely asked for help whenever he was not sure about a rule or a strategy. His reaction time was notably slower than other students, though he improved throughout the course of the intervention. His numeracy skills improved with practice, though his performance was variable.

Group Changes across Time

To determine whether performance changed across time on standardized tests of numeracy (Digit Comparison and Math Fluency) and processing speed (Letter Span), non-parametric procedures were conducted. For tests that were measured at two time points (e.g., Letter Pattern), Related-Samples Wilcoxon Signed Rank tests were conducted. For tests that were measured at three time points (e.g., Digit Comparison and Math Fluency), Related-Samples Friedman's Two-Way ANOVA by Ranks were conducted.

Processing speed raw score. The findings in Table 2 reveal that Letter Pattern raw scores did not differ significantly from baseline (T1) to the end of the first intervention (T2), $W = 6.00$, $p = .109$. Note, however, that there was a trend towards improvement: the mean baseline (T1) score ($M = 22.67$, $SD = 2.52$) was lower than mean T2 score ($M = 29.67$, $SD = 5.03$).

Processing speed standard score. The findings in Table 2 reveal that Letter Pattern standard scores did not differ significantly from baseline (T1) to the end of the first intervention (T2), $W = 6.00$, $p = .109$. But, as shown in the table, the mean baseline (T1) score ($M = 83.33$, $SD = 2.08$) was marginally lower than the mean (T3) score ($M = 94.67$, $SD = 8.33$). Notably, processing speed scores improved from the below average range to the average range.

Math Fluency. The findings in Table 2 reveal that Math Fluency scores changed significantly across time, $\chi^2 (2) = 6.00$, $p = .050$. Pairwise comparisons indicated that baseline scores (T1) were significantly lower than T3 scores, $p = .043$.

Digit comparison. The findings in Table 2 reveal that performance on the digit comparison task improved across time, as measured by accuracy and reaction time, though the results did not reach statistical significance. Similarly, the ratio effect (slope and intercept) improved across time, though the results did not reach statistical significance.

Correlations between Math Fluency and ratio intercept and slope. The findings in Table 3 show that Math Fluency scores were negatively associated with *ratio intercept* scores throughout all testing occasions. Baseline Math Fluency scores were also negatively associated with baseline *ratio slope* scores ($\tau = -1.00$, $p < .01$). These findings can be interpreted to indicate that the students demonstrated a large ratio effect at the start of the intervention that was associated with lower Math Fluency scores. Over the course of the intervention, they demonstrated a relatively smaller ratio effect that was associated with higher Math Fluency scores.

Changes in Weekly Arithmetic Fluency Scores

Percent of non-overlapping data. Examination of the PND scores was carried out to evaluate the overall efficacy of the intervention. PND analyses indicate that the intervention ranged from mildly to highly effective for the three students (Table 4). Jesse experienced the most growth; in fact, during the second phase of the intervention, 100% of the intervention points fell above baseline points, suggesting that the intervention was "highly effective" for this student. Jennifer experienced mild growth during both phases of the intervention, with approximately 67% of the intervention points exceeding that of baseline data during both phases. José, initially the highest performing student in the

group, experienced the least amount of growth in the first seven weeks of the intervention, with only 50% of the intervention points in the first phase exceeding baseline points. However, in the second phase of the intervention, José demonstrated significantly more growth, with 100% of the intervention points falling above baseline points.

A post-hoc analysis was carried out to analyze whether a specific component of the intervention, involving playing number line board games (Olympian Number Line game), had a more substantial effect on arithmetic fluency scores, than other games. Because number line board games were introduced during the tenth week of the intervention and played for three consecutive weeks, PND analyses were conducted to analyze the percentage of math fluency scores that exceeded baseline during the three weeks that these games were played (weeks 11-13). The results indicated that arithmetic fluency scores were indeed higher after number line games were introduced for Jesse and José (Jesse: 100% and José: 100%), than before (Jesse: 86% and José: 67%). Whereas, Jennifer's arithmetic fluency scores after number line game play remained commensurate with her scores prior to number line game play (67% exceeded baseline before and after).

Jesse: Slope. To determine the slope of Arithmetic Fluency, weekly Arithmetic Fluency scores were regressed on the week numbers. The slope for Jesse was statistically significant, $\beta = .75$, $p = .005$. Jesse's Arithmetic Fluency scores increased across time.

Jesse: Visual analysis. Data varied within the baseline and two interventions (see Table 5). Evaluation of level of change during all three conditions indicates improvement within the intervention condition (see Table 5). Split-middle method of trend estimation revealed that there was an increasing and positive trend across all conditions (see Table 6) but data were considered variable following application of a stability envelope to trend lines (see Figure 2).

Jennifer: Slope. To determine the slope of Arithmetic Fluency, Arithmetic Fluency scores were regressed on the week numbers. The slope for Jennifer was statistically significant, $\beta = .64$, $p = .035$. Thus, Jennifer's Arithmetic Fluency scores increased across time.

Jennifer: Visual analysis. Data varied within the baseline and two interventions (see Table 7). Evaluation of level of change during all three conditions indicates improvement within; improvement was minimal during the baseline and steepest during the second intervention (see Table 7). Split-middle method of trend estimation revealed that there was an increasing and positive trend across all conditions (see Table 8) but data were considered variable following application of a stability envelope to trend lines (see Figure 3).

José: Slope. To determine the slope of Arithmetic Fluency, Arithmetic Fluency scores were regressed on the week numbers. The slope for José was statistically significant, $\beta = .64$, $p = .035$. Thus, his Arithmetic Fluency scores increased across time.

José: Visual analysis. Data varied within the baseline and two interventions (see Table 9). Evaluation of level of change during all three conditions indicates improvement within the baseline but slight deterioration during the two intervention conditions (see Tables 9 and 10). Split-middle method of trend estimation revealed that there was an increasing and positive trend across the baseline only (see Table 10) but data were considered variable following application of a stability envelope to trend lines (see Figure 4).

Discussion

The purpose of this study was to evaluate whether combining numeracy and speed game training using a play-based intervention would generalize to improvements in weekly arithmetic fluency scores in children at-risk for math learning disabilities in first and second grade. A 13-week intervention was carried out using a single-case-study design to analyze individual

differences in performance over the course of the intervention in three at-risk students. The case study students were selected from a larger pool of 24 participants who were pre-screened at the outset of the study. The three case study students were classified as at-risk for a MLD according to criteria used in prior research (Mazzocco, 2007). These criteria included normative weaknesses (< 25th percentile) on two tests of numeracy (math fluency and digit comparison), and below grade level skills in math (according to parent or teacher report), which could not be explained by other factors (e.g., other neurodevelopmental disorders).

Based on observational data, all three case study students demonstrated high levels of engagement with games throughout the intervention and made notable advancements in game performance, as tracked by research personnel. Individual differences in attention were noted. Furthermore, evaluation of change in performance from pre- to post-tests during the fall, winter and spring indicated that the game-play intervention was associated with improvements on standardized tests of skills targeted by the intervention. There was marginal improvement in processing speed and significant improvement in math fluency over the course of the intervention. Though, improvement was observed on the digit comparison task, in accuracy and reaction time, the pattern was not significant. Similarly, the ratio effect, an indicator of numerical representation on the digit comparison task, appeared to improve (decrease) over the course of the intervention, though this pattern was not significant.

To examine transfer of trained skills to more general arithmetic fluency skills, weekly arithmetic fluency scores were analyzed. Timed arithmetic fluency is a particularly important outcome in this population because prior research shows that children with MLD perform worse on timed calculation measures because they tend to use developmentally immature calculation procedures (e.g., counting on fingers), resulting in more errors and slower response times (e.g., Geary, Bow-Thomas, & Yao, 1992). Therefore, timed tests are used as a proxy for measuring progress in both accuracy and efficient calculation strategy use. Results indicated that all three participants demonstrated a significant increase in the number of correctly computed digits per minute on weekly arithmetic fluency measures. Furthermore, effect size analyses indicated that the intervention was modestly or highly effective for Jesse and José, whereas the intervention was only mildly effective for Jennifer. It is plausible that Jennifer did not experience as much gain as the other two students because she has a more severe weakness in number sense than the other two students, and thus, she may respond better to a more individualized intervention that is more structured, and comprehensive than informal game play.

Furthermore, I evaluated whether there was a dissociable effect of playing number line games on math fluency performance during the weeks these games were played. Results indicated that math fluency scores were indeed higher for Jesse and José after the number line games were introduced, but not for Jennifer. Thus, this finding generally aligns with the literature demonstrating that playing number line games has a robust impact on early math development in young children with numeracy weaknesses (e.g., Ramani & Siegler 2011; Siegler & Ramani, 2009). In future iterations of this study, number line games should be introduced earlier in the year, to facilitate better number representation skills that set the foundation for higher-level math skills (e.g. arithmetic).

The weekly rate of math fluency improvement (or slope) for all three students exceeded 0.6, which is above the expected rate of improvement, 0.3, for other first and second grade math curriculum interventions (Fuchs & Fuchs, 1993). Visual analysis of the weekly arithmetic fluency data yielded more variability in the findings. For Jesse (who was observed to have mild attention weaknesses) there was an increasing and positive trend across both phases of the

intervention. For Jennifer (who was observed to have low numeracy) there was also an increasing and positive trend across both phases of the intervention. For José (who was observed to have a particularly slower reaction time) the intervention effects were more variable, though substantial improvement was observed overall. All three participants demonstrated variability in arithmetic fluency during baseline testing, which places limits on the causal inferences that can be drawn from the study (Horner et al., 2005). Nevertheless, the fact that improvement was observed on timed arithmetic fluency tests, at a rate that exceeded most other curricula, suggests that the students not only achieved more accurate responses over the course of the intervention, but also were more efficient in their computational strategy use (employing more advanced strategies such as counting on from the higher addend). In future iterations of this study, strategy use should be explicitly measured.

The final two weeks of the intervention are still currently underway. At the end of the study, the efficacy of the intervention will be re-evaluated. It is predicted that the students will continue to demonstrate growth on arithmetic fluency measures. However, if any of the participants remain in the "below-average" range on standardized tests of math fluency at the end of the study, the intervention would be classified as ineffective for that particular student. In that case, the student's teacher and parent would be notified that more a more comprehensive and structured numeracy intervention may be warranted (e.g., Griffin, 2007).

These preliminary findings contribute to the existing math learning disability prevention literature (Griffin, 2007) by demonstrating that children at risk for math learning disabilities can benefit from playing a combination of socially engaging numeracy and speed games. To the best of our knowledge, this is the first study to show that combining numeracy training with cognitive speed training leads to general improvements in math skills that were not explicitly trained by the intervention. The fact that all three students maintained high levels of engagement and interest throughout the study is important because low-performing children are at risk for developing a negative attitude towards math that can cause them to fall further and further behind (Diener & Dweck, 1978). Notably, participants have commented that they would like their parents to buy their favorite games so that they can play at home with their families. This is a particularly promising outcome because extending game play to the home may potentially contribute to longer lasting and stable improvements over time.

Study Limitations

The current case study was carried out within the context of a larger intervention study necessitating methodological consistency between the two study designs. As a result, we were not able to implement one element of practice recommendations for single case study design research (Kennedy, 2004), which stipulates that it is best to introduce one variable at a time and then measure the resulting effect on the targeted outcome before introducing another variable, as doing so would have been incompatible with the larger study. Without such a design, it is difficult to pinpoint precisely which component of the training is responsible for the effects found. In a future study, numeracy games should be implemented first, and then speed games should be introduced once an effect is observed (or vice versa), to determine whether there are any synergistic effects. Furthermore, due to space limitations at the school, the study was carried out in the cafeteria, where other students were participating in their regular after-school program activities (e.g., homework, reading, or small group activities). As a result, there was frequently extraneous noise in the room that posed as a distraction to study participants.

Future Directions

Future studies should evaluate the extent of transfer to other mathematical tasks (both in the short- and long-term), and the duration of improvement with follow-up evaluations. Furthermore, future studies should also integrate cognitive training games that build other cognitive skills related to math learning disabilities (e.g. working memory, spatial reasoning etc.). For example, integrating games that target working memory skills, a cognitive function that is commonly impaired in children at-risk for mathematical learning disabilities, would be an important next step for this training (Peijnenborgh et al., 2015). Furthermore, an important future direction for research on prevention is to investigate the most effective method for training teachers to incorporate early identification tools and evidence based interventions for children at-risk for math learning disabilities (Griffin, 2004).

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Table 1

Descriptive Statistics for the Study Participants (N = 3)

						Fluid Reasoning	Processing Speed			Calculation Fluency		
Subject ID	Sex	Age	Grade	Prior Diagnosis	Academic Concerns	T1	T1	T2	T3	T1	T2	T3
3	M	7	2	None	Math (Teacher)	100	85	104	106	80	84	89
11	F	6	1	Speech Disorder	Math/Reading	80	81	92	92	78	94	98
22	M	8	1	None	Math	85	84	88	97	88	88	89

Table 2

Group Level Changes Across Time and Friedman's Two-way ANOVA Results for Study Measures (N = 3)

Measure	Fall (Time 1) *M (SD)*	Winter (Time 2) *M (SD)*	Spring (Time 3) *M (SD)*	χ^2
Arithmetic Fluency	8.00 (3.18)	11.90 (3.83)	15.17 (5.53)	6.00*
Math fluency WJ-IV	13.67 (8.15)	18.33 (5.86)	23.67 (5.69)	6.00*
Letter Pattern[a]				
Raw score	22.67 (2.52)	29.67 (5.03)	-- --	6.00
Standard score	83.33 (2.08)	94.67 (8.33)	-- --	6.00
Letter Span	9.00 (4.00)	9.33 (2.89)	9.00 (3.47)	.67
Matrix Reasoning				
Raw score	15.00 (4.00)	19.33 (4.04)	13.00 (8.08)	2.67
Standard score	8.00 (2.83)	10.50 (2.12)	17.67 (7.07)	3.00
Digit Comparison				
Accuracy	.71 (.07)	.74 (.06)	.74 (.08)	.67
Reaction time	.99 (.41)	.86 (.25)	.78 (.26)	2.00
Ratio				
Intercept	.91 (.29)	.89 (.35)	.77 (.34)	2.67
Slope	.15 (.36)	-.07 (.24)	.00 (.17)	.67

[a]Wilcoxon Signed Rank values are shown.
*$p < .05$.

Table 3

Kendall Tau Correlations between Math Fluency and Ratio Measures (N = 3)

Ratio Measures	MF 1	MF 2	MF 3
Ratio intercept			
Baseline	-.33	-.33	-.33
End of First Intervention Phase	-1.00 **	-1.00 **	-1.00
Start of Second Intervention Phase	-1.00 **	-1.00 **	-1.00
Ratio slope			
Baseline	-1.00 **	-1.00 **	-1.00
End of First Intervention Phase	.33	.33	.33
Start of Second Intervention Phase	1.00 **	1.00 **	1.00

Note. MF 1 = Math Fluency during baseline. MF 2 = Math Fluency during end of the first intervention phase. MF 3 = Math Fluency after winter break.
$^{**}p < .01$.

Table 4.

Percent of Non-Overlapping Data for Weekly Arithmetic Fluency Data

Student	PND Phase 1	PND Phase 2	Effect of Intervention
Jesse	71	100	Moderate-Highly Effective
Jennifer	67	67	Mildly Effective
José	50	100	Mild-Highly Effective

Table 5

Descriptive Statistics for Weekly Arithmetic Fluency Jesse (Step 3)

	Mean	Median	Mode	Range	Stability Envelope
Baseline	7.67	9.00	3	3 to 11	66.67
Intervention 1	13.71	14.00	17	7 to 18	42.86
Intervention 2	17.00	16.00	13	13 to 22	100.00

Note. Envelope = 2.25.

Table 6

Visual Analysis - Level Change in Weekly Arithmetic Fluency for Jesse

	Baseline	First Intervention	Second Intervention
Relative Level Change (Step 4A)			
Median of first half	3.00	12.00	13.00
Median of second half	11.00	17.00	16.00
Relative level change	8.00	5.00	3.00
Absolute Level Change (Step 4B)			
First value	3.00	12.00	13.00
Last value	11.00	17.00	22.00
Absolute level	8.00	5.00	9.00
Mid-dates and Mid-rates (Step 5)			
Mid-date			
First half	1	5.5	11
Second half	3	9.5	12
Mid-rate			
First half	3	12	13
Second half	11	17	16

Table 7

Descriptive Statistics for Jennifer on Weekly Arithmetic Fluency (Step 3)

	Mean	Median	Mode	Range	Stability Envelope
Baseline	5	4.00	3	3 to 8	67.00
Intervention 1	7.50	8.00	8	3 to 10	67.00
Intervention 2	9.67	9.00	7	7 to 13	50.00

Note. Envelope = 1.00.

Table 8

Visual Analysis - Level Change in Weekly Arithmetic Fluency for Jennifer

	Baseline	First Intervention	Second Intervention
Relative Level Change (Step 4A)			
Median of first half	3.00	6.00	7.00
Median of second half	4.00	10.00	13.00
Relative level change	1.00	4.00	6.00
Absolute Level Change (Step 4B)			
First value	8.00	16.00	7.00
Last value	11.00	15.00	13.00
Absolute level	3.00	1.00	6.00
Mid-dates and Mid-rates (Step 5)			
Mid-date			
First half	1	5	11
Second half	3	9	12
Mid-rate			
First half	3	6	7
Second half	4	10	13

Table 9

Descriptive Statistics for José

	Mean	Median	Mode	Range	Stability Envelope
Baseline	11.33	11.00	8	8 to 15	67.00
Intervention 1	14.50	14.50	10	10 to 20	67.00
Intervention 2	20.00	20.00	20	18 to 222	100.00

Note. Envelope = 2.75.

Table 10

Visual Analysis - Level Change in Weekly Arithmetic Fluency for José

	Baseline	First Intervention	Second Intervention
Relative Level Change (Step 4A)			
Median of first half	8.00	16.00	22.00
Median of second half	11.00	14.00	20.00
Relative level change	3.00	2.00	2.00
Absolute Level Change (Step 4B)			
First value	8.00	16.00	22.00
Last value	11.00	15.00	20.00
Absolute level	3.00	1.00	2.00
Mid-dates and Mid-rates (Step 5)			
Mid-date			
First half	1	5	11
Second half	3	8	12
Mid-rate			
First half	8	16	22
Second half	11	14	20

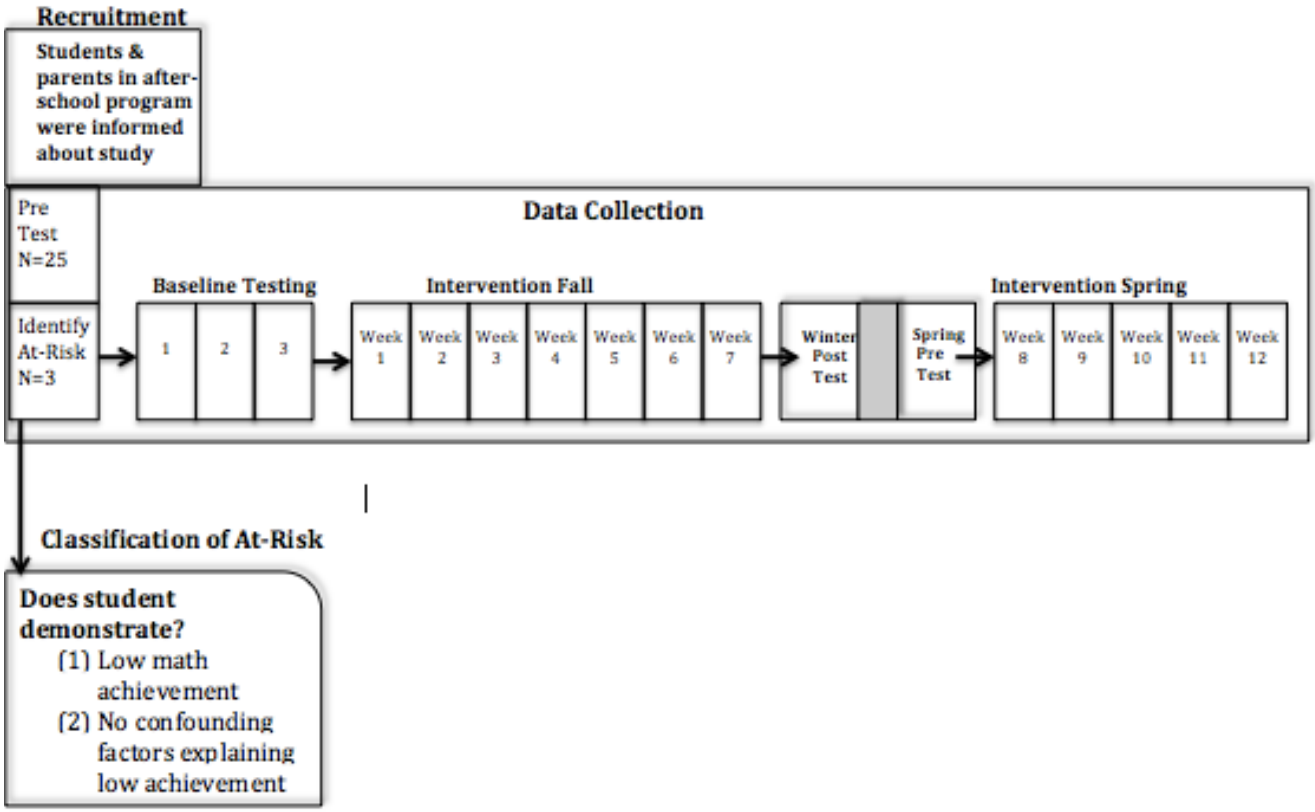


Figure 1. Study timeline and participant selection diagram

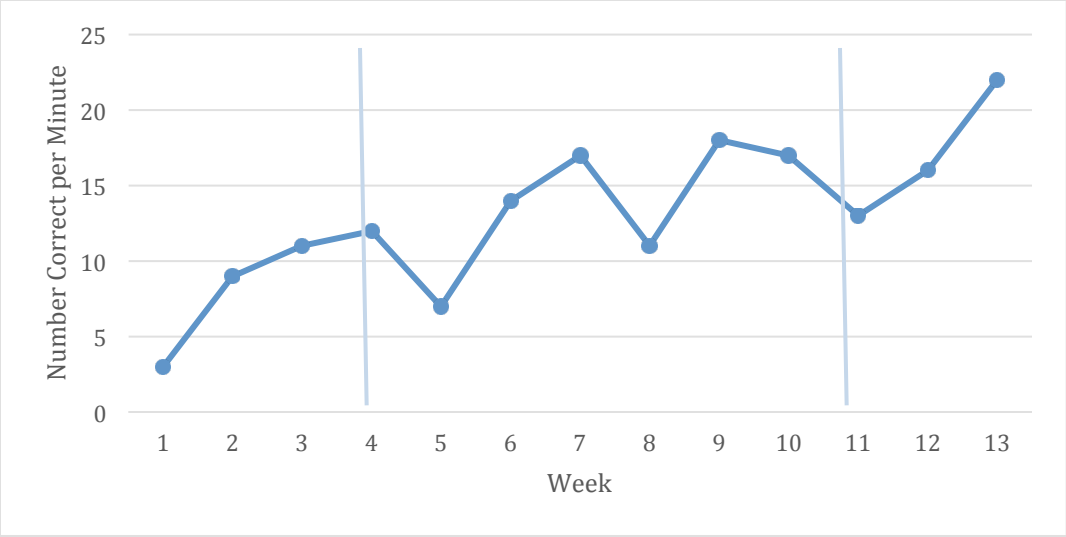


Figure 2. Weekly Arithmetic Fluency across time for Jesse.

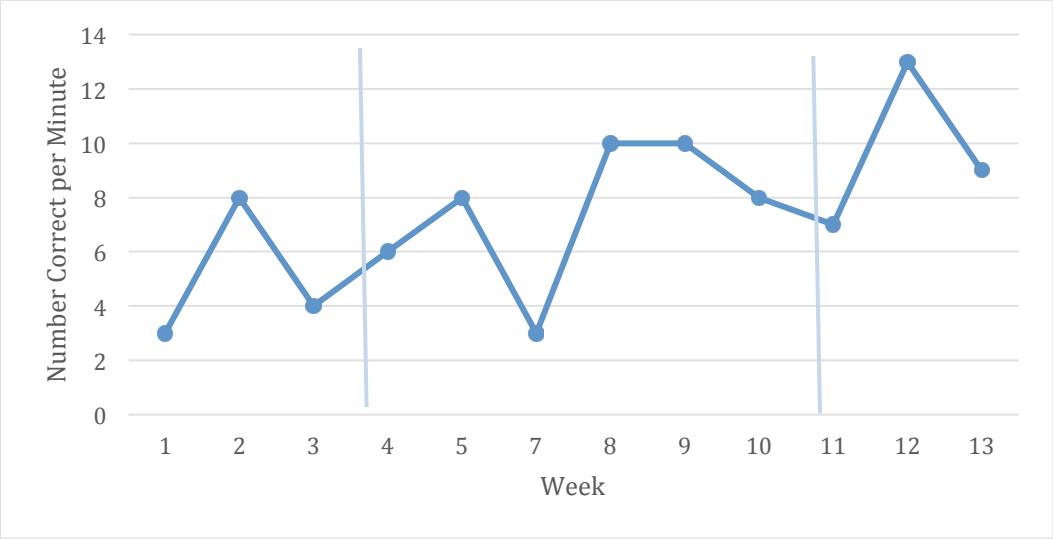


Figure 3. Arithmetic Fluency across time for Jennifer.

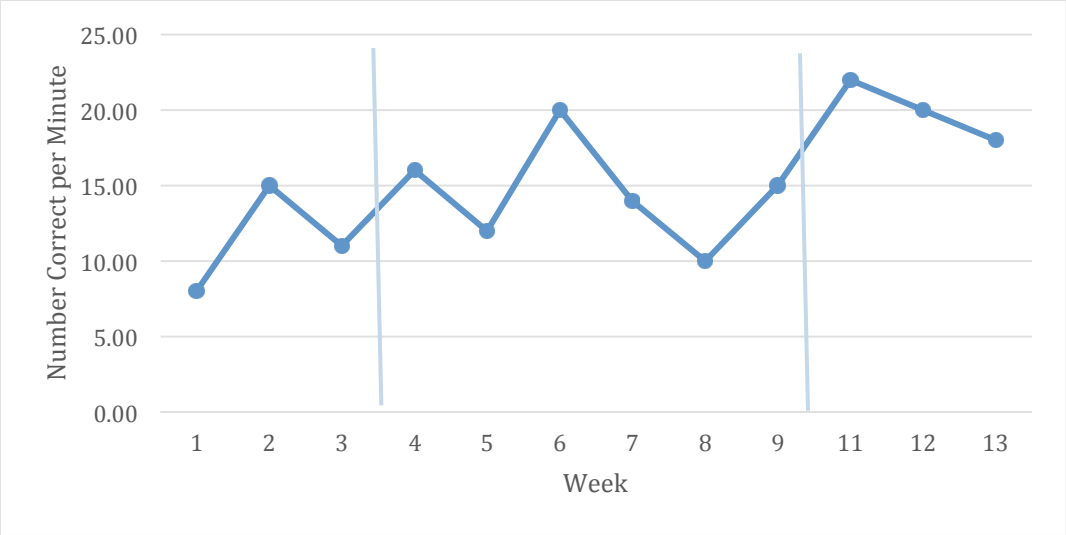


Figure 4. Arithmetic Fluency across time for José.

Appendix A

Parent Questionnaire: Demographics and Educational History

In research, we need to understand various factors that may affect our results. Your honest answers
these questions will help us learn more about your child. Please print all answers as clearly as possibl

1. Child's Full Name (first name, lastname):_____

2. Child's Date of Birth (mm/dd/yy):_____

3. Child's Grade:_____4. Child's Gender: ☐ F ☐ M

5. Child's Race/Ethnic group (check all that apply):
   - ☐ American Indian/Alaskan Native
   - ☐ Asian
   - ☐ Native Hawaiian or Pacific Island
   - ☐ Black or African American
   - ☐ White
   - ☐ More than one race
   - ☐ Decline to report

   5a. Is your child Hispanic or Latino? ☐ Yes ☐ No

6. Is English your child's first language? ☐ Yes ☐ No
   6a. If not, what language does your child speak in the home? _____

   6b. If not, at what age did your child start learning English? _____

Please check the box that best represents your answer.

7. How would you describe your child's academic progress in school? (check one box)

Unsatisfactory	Marginally satisfactory	Average	Satisfactory	Very satisfactory
☐	☐	☐	☐	☐

8. Have his/her teachers conveyed any concerns about your child's performance or behavior?

Yes, academic	Yes, behavior	Yes, both	None
☐	☐	☐	☐

   8a. If so, is this concern expressed regularly? ☐ ☐

   ✓ 8b. **If yes,** please indicate which subject areas your child's teacher has expressed concerns in

   _____Math_____Reading_____Writing

   _____Speech_____Other (if other, please specify_____)

9. How high can your child count without making a mistake? _____

10. Circle all that apply:

Is your child:

a) Not ***reading*** yet
b) Can ***read*** names of letters
c) Can ***read*** their own name
d) Can ***read*** some words
e) Can ***read*** words easily
f) Can ***read*** sentences

Is your child:

a) Not ***writing yet***
b) Can ***write*** names of letters
c) Can ***write*** their own name
d) Can ***write*** some words
e) Can ***write*** words easily
f) Can ***write*** sentences

11. Does your child know the names of these digits (i.e. if he/she saw these numbers on paper, would he/she be able to name them)? Circle ALL that apply.

1-10 10-20 20-30 30-40 40-100 100+

12. Is your child in special education or remedial classes for any subject? ☐ Yes ☐ No

12. a. If so, which?_____

13. Is your child in an advanced class for any subject? ☐ Yes ☐ No

13.a. If so, which?_____

14. Have your child ever been diagnosed as having (if so, please state when diagnosis wasmade):

☐ Speech impairment
☐ Hyperactivity
☐ Learning disability:
☐ Attention Deficit Disorder (ADD or ADHD)

If so, what type of learning disability?_____

14a. If they have been diagnosed before, at what age was the diagnosis made?_____

14b. If yes to any of the above, is your child currently undergoing treatment? ☐ Yes ☐ No

If yes, please explain._____

15. Does your child receive resource room help or have an IEP? ☐ Yes ☐ No

16. Does your child get any tutoring required by the school? ☐ Yes ☐ No

13a. If yes, for what subjects? _____

17. Is there anything else about your child's learning or school experiences that you think is significant and would like to share with us?

Appendix B

Teacher Questions:

- How would you describe the student's math skills?
- How would you rate his math skills in relation to his-same-age peers?
  1) Below Average (please specify) 2) Average 3) Above Average
- How would you rate his math skills in relation to 1st/2nd grade level standards?
  1) Below Average (please specify) 2) Average 3) Above Average
- Do you have any concerns about his math progress?
- Are there are any particular math skills that he is having difficulty mastering?
- Does this student generally complete his math work in class and his homework?
  1) yes 2) no (please specify)
- How would you rate his overall academic skills?
  1) Below Average (please specify) 2) Average 3) Above Average
- Has he ever received any math interventions in addition to the regular curriculum?

Is there anything else you would like to add about your observations related to this student and his academic progress in your class?

Appendix D

Table 1

Digit Comparison Task

The following digits pairs were presented over the course of 80 trials.

2	7
3	8
2	5
3	7
4	8
5	9
4	7
3	5
2	3
5	7
6	8
7	9
4	5
7	8
8	9

Appendix E

Table 2

Numeracy Intervention Games. Games are listed in order played by difficulty level

Gaame	**Numeracy Skill Trained**	**Game Description**
Zingo123 (Thinkfun)	Number Recognition	Players race to match their numbered tiles to the corresponding challenge card.
Spot It Numbers (Dobbel)	Number and Shape Recognition	Players race to identify (or spot) the matching number or shape between two cards
Tangoes Jr. (Smart Toys)	Spatial and Pattern Identification	Players use the seven magnetic puzzle pieces to recreate designs
Zip Zap (Gamewrite)	Numerical Sequencing	Players race to place their cards down in numerical order to get rid of their hand
Stormy Seas (Thinkfun)	Equivalency	Players take turns placing weighted cargo pieces on a wooden ship, while trying to maintain equal weight on both sides so that it doesn't tip
Rat-A-Tat-Cat (Gamewright)	Number Comparisons	Players take turns trading in higher cards for lower cards. The player with the lowest total at the end wins.
Number Chase (Thinkfun)	Number Comparisons	One player draws a number and each of the players try to guess the number by asking numerical order questions on their cards
Olympian Number Line (Didax)	Numeracy	5 number line board games that each build a different numeracy skill • Compare numbers between 50 and 100 • Round numbers to the nearest 10 • Count by 2s, 5s and 10s • Practice number bonds

Appendix F

Table 2

Processing Speed Intervention Games. Games are listed in order played.

Game	Game Description
Zingo (Thinkfun)	Dealer exposes two tiles. If a player spots a tile that matches his or her card, the player calls out the name of the image on his card. Then the player takes the tile and covers the matching image on the card. The first player to cover his board with tiles wins the game
Spot It (Asmodee)	Players are given a card with six colorful animals. The animals may vary in size and position, but there is always one, and only one, animal match between any two cards. The aim of the games is to be the first to spot a matching animal between the target card and your own card, before another player spots one.
Spy Tag (Ravensburger)	Each player is given 3-4 characters cards to display in front of them. Players take turns drawing spy cards and slapping them on the matching agent cards laid out in front of your teammates. Get caught with a match when the timer goes off and draw a card from the secret briefcase pile.
Ugly Dolls (Gamewright	Players take turns turning over cards until someone spots three matching ugly dolls. Then players race to grab a match before they all get snatched. The player to claim the most cards wins.
Slamwich (Gamewright)	One by one, each player takes the top card of her deck and flips it onto a central pile. Players race to slap the pile when the following conditions are met: • the flipped card is identical to the card directly underneath it (a "double decker") • If two identical cards have exactly one card in between them (a "slamwich") The player who slaps the decks collects all the cards in the pile. The player with the most cards at the end wins.
Chomp (Gamewright)	In unison, all players turn over the top card of their pile into the center of the playing area. The first player to slap the card with

	the lowest creature on the food chain and shout “Chomp!” wins the round.
Pictureka (Hasbro Games)	Players race to find target pictures on a very detailed board. The first player to find the target picture wins the card. The player with the most cards at the end wins.
Blink (Mattel)	Players try to match the shape, count, or color on their cards to either one of two target cards..
Robot Face Race (Educational Insights)	Players roll the die to find out which robot head features they should search for and then they scan the board for the robot head with the correct colored face, nose, eyes, and mouth. The first player to find the matching robot head wins the round.
Quick Cups (Spin Master)	Players are given five colored cups. During each round, players race to stack up or line up their five colored cups to match a picture on a card. The player to finish first and ring the bell wins the card.
Fast Flip (Blue Orange)	Players compare two cards in order to find a match. One of the cards will display a bunch of fruit while the other either displays a fruit or a number. If a fruit is displayed the players need to count up how many of that type of fruit are on the table. The first player to say the correct number gets to take the card that shows one fruit. If one of the cards display a number, players need to find the fruit that has exactly that many pictured on the other card. The player that yells out the correct answer takes the card that displays the number
Nada (Blue Orange)	In each round, players use six white dice and six orange dice, with each die having six different symbols on it. Someone rolls all the dice in the center of the table, then players race to be the first to yell out a symbol that is on at least one white and one orange die. If correct, the player collects *all* the dice showing this symbol. Once a player has claimed dice, this player rerolls all the remaining dice. If no match can be found, the first player to yell "Nada!" claims all the remaining dice. Players tally their scores – one point for each die collected – then play another round. Whoever has the highest score after three rounds wins!

Conclusion

Despite the serious negative life consequences associated with low numeracy, and the high proportion of students failing to meet grade level standards in the US, math learning weaknesses and interventions remain poorly understood. There is a large gap between research and practice in this domain.

Math learning disabilities are widespread, affecting 5% to 7% of children (Butterworth, 2005; Shalev, Auerbach, Manor, & Gross-Tsur, 2000). Though math learning disabilities (MLD) are understood to be a congenital disorder, children with MLD frequently remain unidentified throughout the early school years. Teachers hesitate to flag children as learning disabled prior to third grade for numerous reasons. The reasons include (a) students have differing levels of exposure to number concepts prior to starting school, (b) teachers are not well informed about the identifying features of MLD, (c) poor instruction needs to be ruled out, (d) diagnostic criteria indicate there should be pattern of poor performance over an extended period of time (Berch & Mazzocco, 2007). Unfortunately, this all-too-common delay in identification leads to students falling too far behind to catch up. Early identification is therefore a critical component of effective intervention.

In the first paper presented in my dissertation, my aim was to contribute to narrowing the gap between research and practice by synthesizing literature that pinpoints the key identifying features of math learning disabilities, including early number sense weaknesses. Number sense is the understanding of a number's magnitude and its relation to other numbers, and is considered the *core deficit* in math learning disabilities (Brian Butterworth, Varma, & Laurillard, 2011). Number sense capacity can easily be measured in preschool or kindergarten using simple tests of dot counting and number comparisons. The implication of this research for practice is that teachers should routinely screen low performing students for number sense weaknesses in the early grades so that they can ensure the appropriate referrals are made for interventions to take place. Professional development for educators on MLD is critically needed.

In the second paper presented in my dissertation, I sought to characterize the cognitive abilities that are most predictive of future math achievement in typically developing children. I hope that this research can be used to develop skill-building techniques that can help low-performing students catch up. I was particularly interested in the role of fluid reasoning (FR) in the acquisition of math skills. FR is the ability to analyze novel problems, identify patterns and relationships, and apply logic. Researchers posit that FR may be particularly important to math because math is hierarchical in nature, requiring students to consistently learn and apply new problem-solving techniques. In previous research, however, FR has not been well characterized in relation to math development, and spatial reasoning has been emphasized. In this paper, I analyzed data from a longitudinal study with a cohort sequential design, to investigate whether prior measures of FR predicted future math outcomes for a group of 69 participants between ages 6 and 21 years old. I used structural equation modeling (SEM) to examine the direct and indirect relations between children's previous cognitive abilities and their future math achievement. I found that FR was the only significant predictor of future math achievement, after accounting for spatial skills, verbal reasoning, and age.

These results underscore the unique role that FR plays in influencing development of more complex math problem-solving skills. Based on these findings, I assert that Math curricula should incorporate opportunities for students to practice FR skills.

In the third paper, I carried out a novel game-play intervention designed to train numeracy and processing speed skills in children at risk for MLD. This study extended previous research involving number sense training by including numeracy games and games that aimed to improve more general cognitive skills that are commonly weak in children with MLD (processing speed). The intervention was carried out over the course of 14 weeks, 4 days per week, during an after-school program at a local elementary school. I used the single-case-study design to investigate response to intervention in 3 children in first and second grade. The participants were highly engaged and interested in the games throughout the course of the intervention. They demonstrated significant improvement in math fluency skills and marginal improvements in processing speed. These findings provide promising preliminary evidence to indicate that early game play intervention may facilitate math skill development in at-risk children. Thus, incorporating games that train general cognitive skills may lead to broad effects on higher-level math achievement.

Future Directions

The pilot intervention research presented in this dissertation is based on cognitive psychology and neuroscience research that has identified the types of tasks that train early number sense and cognitive abilities in at-risk children. To extend this research into practice, expert teacher guidance would be needed to inform the presentation and sequencing of these tasks to align with grade level curricular progressions. This would involve teacher consultation, and an iterative design process using successive evaluations in classrooms (e.g., Butterworth & Laurillard, 2010). It may be optimal to create a computerized version of the games that can adapt in real-time to the learner's level of understanding, provide immediate feedback, and become progressively more challenging, thus personalizing the intervention to the individual student (Butterworth & Laurillard, 2010; Healy & Kynigos, 2010). Computerized interventions enhance feasibility and reliability of intervention research by reducing the number of staff needed to carry out the intervention, allowing for consistent implementation, and enhancing data tracking and analytics. Additionally, software potentiates design modifications to be made on an iterative basis through teacher consultation.

Additionally, larger studies are needed because most prior studies evaluating early numeracy interventions lack sufficient statistical power to analyze differences in responsiveness related to student characteristics. This is important because children at-risk for math learning disabilities are heterogeneous, and some students appear to be more responsive than others to particular types of interventions (e.g. game play versus structured practice). Future studies may therefore benefit from drawing on larger sample sizes that will enable comparison of intervention techniques as well as the mediational effects of differing learning styles. Such studies would enable teachers to select interventions that are better tailored to individual students.

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