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Quark and Gluon Pair Production in SU(N) Covariant Constant Fields

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Abstract:

The neutralization rate of covariant constant SU(N) fields due to quark and gluon pair creation is calculated semiclassically including interactions between the produced pair. For SU(3) we find that this rate is remarkably independent to the color orientation in the Cartan subspace. Phenomenological consequences for quark-gluon plasma production in ultra-relativistic nuclear collsions are considered.

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Pair production in external Abelian fields is well understood[1,2,3,4]. The pair production rate of gluons in covariant constant non-Abelian SU(2) fields was computed in Ref.[5] in the one loop approximation neglecting interactions between the produced gluons. The same result was also obtained in Ref.[6] using canonical methods. In this note we generalize those results to SU(N) for both quark and gluon pair production, and consider semiclassical corrections to the rates due to the interactions between the pair. The competition between quark-antiquark and gluon pair production in the color neutralization process is illustrated. Possible phenomenological consequenses for ultra-relativistic nuclear collisions are considered.

Let $t_a, a = 1, \cdots, N^2-1$ be the generators of SU(N) in the fundamental representation. These traceless $N \times N$ Hermitian matricies satisfy $[t_a, t_b] = if_{abc}t_c$, $tr(t_a t_b) = \delta_{a,b}/2$. The conventional gluon field matrix is defined as $A_\mu \equiv A_\mu^a t_a$, the covariant derivative is $D_\mu = \partial_\mu + igA_\mu$, and the field tensor is $F_{\mu\nu} = [D_\mu, D_\nu]/(ig)$. In this notation the equations of motion for quarks and gluons are

$$(i\gamma^\mu D_\mu - m_f)\psi_f = 0 \;, \tag{1}$$

$$\partial^\mu F_{\mu\nu} + ig[A^\mu, F_{\mu\nu}] = gJ_\nu \;, \tag{2}$$

where $J_\nu = \sum_f \bar{\psi}_f \gamma_\nu t_a \psi_f t_a$ and f labels the quark flavors.

The covariant-constant field[5], which satisfies Eq.(2) in the source free region is of the form

$$F_{\mu\nu} = \langle F_{\mu\nu} \rangle n_a t_a \;, \tag{3}$$

where $\langle F_{\mu\nu} \rangle$ is independent of x_μ, and n_a is an N^2-1 dimensional color vector. The external A_μ field corresponding to Eq.(3) is $\langle A_\mu \rangle = -\frac{1}{2}\langle F_{\mu\nu} \rangle x^\nu n_a t_a$.

Since $\langle A_\mu \rangle$ is Hermitian, there exists a unitary matrix, U, that diagonalizes it. Since $N \times N$ traceless diagonal matricies can be expanded in terms of the $N-1$ traceless diagonal matricies, h_i, representing the Cartan subgroup of SU(N), it is convenient to expand A_μ in the Cartan-Weyl basis of SU(N). That basis consists[7] of $N-1$ Abelian generators, h_i, and the $N(N-1)$ non-Abelian generators, $\{e_{ij},\ i,j = 1, \cdots, N;\ i \neq j\}$, that satisfy

$$\begin{aligned} [h_i, h_j] &= 0 \\ [h_i, e_{jk}] &= (\vec{\eta}_{jk})_i e_{jk} \\ [e_{ij}, e_{jk}] &= \frac{1}{\sqrt{2}} e_{ik} \qquad \text{for } i \neq j \neq k \;, \end{aligned} \tag{4}$$

where the h_j are the Gell-Mann matricies

$$h_j = (2j(j+1))^{-\frac{1}{2}} diag(1, \cdots, 1, -j, 0, \cdots, 0) \;, \tag{5}$$

with $-j$ appearing in the $j+1$ column and where

$$\vec{\eta}_{ij} = \vec{\epsilon}_i - \vec{\epsilon}_j \tag{6}$$

are the root vectors of SU(N) as expressed in terms of the elementary weight vectors

$$\vec{\epsilon}_i = (\vec{h})_{ii} = ((h_1)_{ii}, \cdots, (h_{N-1})_{ii}) \ . \tag{7}$$

(Note that $e_{ij} = \hat{e}_i \hat{e}_j^\dagger / \sqrt{2}$ in terms of the N orthonormal unit vectors $\hat{e}_i$.)

In this basis, $\langle A_\mu \rangle$ can always experessed in terms of $N-1$ Abelian components, $\vec{H}^\mu = (H_1^\mu, \cdots, H_{N-1}^\mu)$, as

$$\langle A^\mu \rangle = \sum_{i=1}^{N} U H_i^\mu h_i \, U^\dagger \equiv U \vec{H}^\mu \cdot \vec{h} \, U^\dagger \ , \tag{8}$$

where $U \in \mathrm{SU}(N)$ and $\vec{h} \equiv (h_1, \cdots, h_{N-1})$. We can then expand the gluon field around the external field as

$$A_\mu = \langle A_\mu \rangle + U B_\mu U^\dagger = U(\vec{H}_\mu \cdot \vec{h} + B_\mu) U^\dagger \ , \tag{9}$$

where B_μ represents the quantum fluctuations around the external Abelian field, $\vec{H}_\mu$.

The physical significance of $\vec{\epsilon}_i$ can be seen from Eq.(1) by considering the equation of motion for the tranformed quark field, $\psi' \equiv U^\dagger \psi$. Eq.(1) then reduces to the set of equations

$$(\gamma_\mu (i\partial^\mu - g\vec{\epsilon}_c \cdot \vec{H}^\mu) - m_f)\psi'_c = O(B\psi') \ . \tag{10}$$

The approximation of neglecting higher order quantum fluctuations beyond the one loop order is equivalent to neglecting the $O(B\psi')$ terms on the right hand side of (10). We therefore see that in the one loop approximation, the equations for the N quarks (of each flavor) in the prime basis decouple and reduce to Abelian type equations where $\vec{H}^\mu$ plays the role of an effective electromagnetic field that couples to quarks with effective "charges" $g\vec{\epsilon}_c$. Since we know[1] the pair creation rate, $w_{\frac{1}{2}}(eF^{\mu\nu}; m)$, of fermions in an external Abelian field $F^{\mu\nu}$, we can immediately write down the pair creation rate per unit volume of ψ'_c quarks of flavor f as

$$w_{q_c,f} = w_{\frac{1}{2}}(g\vec{\epsilon}_c \cdot \vec{F}^{\mu\nu}; m_f) \ , \tag{11}$$

where $\vec{F}^{\mu\nu} = \partial^\mu \vec{H}^\nu - \partial^\nu \vec{H}^\mu$, and the elementary spin 1/2 rate is given by[1,8]

$$w_{\frac{1}{2}}(\sigma, m) = \theta(\sigma) \frac{\sigma}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int_{m^2}^{\infty} dE_\perp^2 \exp(-n\pi E_\perp^2/\sigma) \approx \theta(\sigma) \frac{\sigma^2}{4\pi^3} \varsigma(2) \ , \tag{12}$$

with $\theta(x) = 0(1)$ for $x < (>) 0$, $\varsigma(2) = \pi^2/6$, and where the approximation holds for $\pi m_f^2/\sigma \ll 1$.

Turning next to gluons, the equations of motion for B^μ in the one loop approximation are obtained by linearizing Eq.(2) in B^μ. Since $B^\mu \in \mathrm{SU}(N)$, we can expand it in the Cartan-Weyl basis as

$$B^\mu \equiv B_a^\mu t_a = \vec{C}^\mu \cdot \vec{h} + \sum_{i \neq j = 1}^{N} W_{ij}^\mu e_{ij} \ . \tag{13}$$

Inserting (9) into Eq.(2) and using the Cartan-Weyl expansion (13) for B^μ together with the algebra (4) leads to the following equations of motion for the fluctuations $\vec{C}^\mu$ and W^μ_{mn} in the (linearized) one loop approximation to

$$\partial_\mu(\partial^\mu \vec{C}^\nu - \partial^\nu \vec{C}^\mu) = 0 \quad , \tag{14}$$

and

$$(D_{mn})_\mu(D^\mu_{mn}W^\nu_{mn} - D^\nu_{mn}W^\mu_{mn}) - (W_{mn})_\mu[D^\mu_{mn}, D^\nu_{mn}] = 0 \quad , \tag{15}$$

where effective covariant derivative D^μ_{mn} is given by

$$D^\mu_{mn} = \partial^\mu + ig\vec{\eta}_{mn} \cdot \vec{H}^\mu \quad . \tag{16}$$

We therefore see that the Abelian fluctuations, C^μ, obey free field equations whereas the non-Abelian fluctuations, W^μ_{mn} obey Abelian vector field equations in the external field, $\vec{H}^\mu$, with an anomalous magnetic moment coupling[6]. Note that $[D^\mu_{mn}, D^\nu_{mn}] = ig\vec{\eta}_{mn} \cdot \vec{F}^{\mu\nu}$. Obviously these equations are decoupled in this approximation. The effective "charge" of the W^μ_{mn} gluon is given by $g\vec{\eta}_{mn}$. Pair production in SU(N) covariant constant fields is thus equivalent to $N(N-1)/2$ different SU(2) problems. Therefore, the pair creation rate per unit volume of $W_{mn}W_{nm}$ gluon pairs can be calculated from the known[5,6] rate, $w_1(gF^{\mu\nu})$, of vector mesons for SU(2) covariant constant fields as

$$w_{g_{mn}} = w_1(g\vec{\eta}_{mn} \cdot \vec{F}^{\mu\nu}) \quad , \tag{17}$$

where $\vec{F}^{\mu\nu}$ is the same external covariant constant SU(N) field as in Eq.(11) and the spin 1 rate is given by[4]

$$w_1(\sigma) = \theta(\sigma)\frac{\sigma}{4\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^\infty dp_\perp^2 \exp(-n\pi p_\perp^2/\sigma) = \frac{1}{2}w_{\frac{1}{2}}(\sigma;0) \quad . \tag{18}$$

For a discussion on the physical origin of the $(-1)^n$ factors in the above formulas see ref.[2]. .

The case of particular interest in phenomenological applications[8,9] corresponds to constant color electric fields created between interacting partons in high energy collisions. For that case $\vec{F}^{30} = -\vec{F}^{03} = \vec{E} = \vec{Q}E_0$, where $E_0 = g/A_\perp$ for a flux tube of transverse area $A_\perp$ and $\pm\vec{Q}$ are the effective color charges of the projectile and target. We can think of that external field as being generated by a charge density $J_0 = \rho_Q + \rho_{-Q}$, where

$$\int d^3x \rho_Q \equiv U\vec{Q} \cdot \vec{h}\, U^\dagger \quad . \tag{19}$$

The elementary $q\bar{q}$ string corresponds in this picture to $\vec{Q} = \vec{\epsilon}_c$. We identify the string tension with $\sigma = \frac{1}{2}\langle\vec{E}\rangle \cdot \langle\vec{E}\rangle A_\perp = \frac{1}{2}\vec{Q} \cdot \vec{Q} g E_0$. Strictly speaking the string tension is $\frac{1}{2}\langle\vec{E} \cdot \vec{E}\rangle A_\perp$, which is proportional to the second order Casimir operator, C_2. Inherent in our semiclassical approximation is the assumption that $\langle\vec{E} \cdot \vec{E}\rangle \approx \langle\vec{E}\rangle \cdot \langle\vec{E}\rangle$. This approximation is valid for large external fields, but even for strings in the fundamental and adjoint representations it is a good approximation since $\sigma_A/\sigma_F = C_2(A)/C_2(F) = 2N^2/(N^2-1)$ whereas $\vec{\eta}\cdot\vec{\eta}/(\vec{\epsilon}\cdot\vec{\epsilon}) = 2N/(N-1)$. For $N \gg 1$, the ratio of the string tensions in the adjoint and fundamental representation in both cases is thus $\sigma_A/\sigma_F \approx 2$. For SU(3) $\sigma_A/\sigma_F = 3$ in our approximation as compared to the exact value 9/4.

In high energy nuclear collisions[9] or very high energy hadronic collisions, multiple soft gluon exchange may lead to a large effective charges $Q \gg 1$. If $\mathcal{N}$ gluons are exchanged with random charges, then $\langle Q^2\rangle = \mathcal{N}(1+N^{-1})^{-1}$ since only $N(N-1)$ of the N^2-1 gluons are "charged" with $\vec{\eta}_{ij}\cdot\vec{\eta}_{ij} = 1$. Of course such color electric fields are unstable againt pair production. The pair production rate per unit volume of massless quark-antiquark and gluon pairs is given from Eqs.(11,17) by

$$w_q = \frac{N_f g^2}{24\pi}\sum_{c=1}^{N}(\vec{\epsilon}_c\cdot\vec{E})^2 = \frac{Q^2}{2}N_f w_0$$

$$w_g = \frac{g^2}{48\pi}\sum_{i>j=1}^{N}(\vec{\eta}_{ij}\cdot\vec{E})^2 = \frac{Q^2}{4}N w_0 \ , \qquad (20)$$

where $w_0 = (gE_0)^2/(24\pi)$ and N_f is the number of quark flavors such that $\pi m_f^2/\sigma \ll 1$. We thus see that in the large color limit gluons are produced at a rate $N/2N_f$ faster than quarks in the color neutralization process.

The above rates of course neglect the interactions between the produced pair. In QED they are difficult to include because the electric field, e/r^2, falls off rapidly with separation. However, in non-Abelian theories it is assumed that color electric flux can only propagate in narrow flux tubes. For a flux tube of transverse area, $A_\perp$, the color electric field felt by a particle of charge $g\vec{q}$ due to its partner is $-g\vec{q}/2A_\perp$, independent of separation. Because that field stregth is constant, it possible to include its effect on the pair production rate in a simple way[10,11]. The external covariant constant color electric field in Eq.(20) needs only to be replaced by the total field felt by the particle:

$$\vec{E} = \frac{g\vec{Q}}{2A_\perp} + \frac{g\vec{Q}}{2A_\perp} - \frac{g\vec{q}}{2A_\perp} = (\vec{Q} - \frac{1}{2}\vec{q})\frac{g}{A_\perp} \ . \qquad (21)$$

The effective screened string tension that controls the pair production rate in (11,17) is therefore

$$\sigma(\vec{q},\vec{Q}) = g\vec{q}\cdot\vec{E} = \sigma_A\left(\vec{q}\cdot(2\vec{Q}-\vec{q})\right) \ , \qquad (22)$$

where $\sigma_A = g^2/(2A_\perp)$ is the tension of the adjoint string, as produced, e.g.,in pp collisions. Noting that $\vec{q} = \vec{\epsilon}_i, \vec{\eta}_{ij}$ for quarks and gluons respectively, the rate per unit volume for pair production including interactions between the pair in the semiclassical approximation is thus given by

$$w_q = \sum_f \sum_{\pm\vec{\epsilon}_i} w_{\frac{1}{2}}(\sigma(\vec{\epsilon}_i,\vec{Q}); m_f) \ , \qquad (23)$$

$$w_g = \sum_{\vec{\eta}_{ij}} w_1(\sigma(\vec{\eta}_{ij}, \vec{Q})) \ , \tag{24}$$

where the sum over the $\pm\vec{\epsilon}_i$ includes both the $N-1$ quark weight vectors and the $N-1$ antiquark weight vectors, the sum over $\vec{\eta}_{ij}$ includes the $N(N-1)$ root vectors.

As pairs are produced the local color electric field becomes partially neutralized. For large external fields ($Q^2 \gg \eta^2 = 1$), the average local color electric field decreases at a rate

$$\frac{d\langle\vec{E}\rangle}{dt} \approx -\sum_f \sum_{\vec{\epsilon}_i} \frac{g\vec{\epsilon}_i}{A_\perp} w_{\vec{\epsilon}_i} \delta V_{\vec{\epsilon}_i} - \sum_{\vec{\eta}_{ij}} \frac{g\vec{\eta}_{ij}}{A_\perp} w_{\vec{\eta}_{ij}} \delta V_{\vec{\eta}_{ij}} \ , \tag{25}$$

where the pair production rates per unit volume and effective volume elements, $\delta V_{\vec{q}}$, depend on the particular charges, $\vec{q}$, as well as on the mean external charge, $\langle\vec{Q}(t)\rangle = \langle\vec{E}(t)\rangle A_\perp/g$. The minimum volume elements for pair production are constrained by energy conservation[12]. For particles with charges, $\pm g\vec{q}$, the energy gained by separating them a distance, r, is $\Delta E(r) = \sigma(\vec{q}, \vec{Q})r$ in terms of the effective string tension (22). For a pair produced with transverse momenta, $\pm\vec{p}_\perp$, energy conservation implies that the particles can come on shell only after a separation, $r_c = 2m_\perp/\sigma(\vec{q}, \vec{Q})$, where $m_\perp^2 = m^2 + p_\perp^2$. The average volume required for pair production of such pairs is therefore

$$\delta V_{\vec{q}} \approx r_c A_\perp = \frac{2\langle m_\perp\rangle}{\sigma(\vec{q}, \vec{Q})} A_\perp \ . \tag{26}$$

If we choose $\delta V > \delta V_{\vec{q}}$, then we must take into account of the fact that the probability that the pair will screen the field at a particular point is only $\delta V_{\vec{q}}/\delta V$.

The neutralization rate is dominated by production of pairs with small mass ($\pi m^2/\sigma \ll 1$), for which

$$\langle m_\perp\rangle_{\frac{1}{2}} = \sigma^{1/2}\varsigma(5/2)/(2\varsigma(2)) = (2 - 2^{-1/2})^{-1}\langle m_\perp\rangle_1 \ , \tag{27}$$

where $\varsigma(5/2) \approx 1.3415$. Therefore, we estimate that

$$\begin{aligned} w_{\vec{\epsilon}_i}\delta V_{\vec{\epsilon}_i} &\approx \frac{1}{\tau_1}\left(\vec{\epsilon}_i \cdot (2\vec{Q} - \vec{\epsilon}_i)\right)^{3/2} \\ w_{\vec{\eta}_{ij}}\delta V_{\vec{\eta}_{ij}} &\approx \frac{(1 - 2^{-3/2})}{\tau_1}\left(\vec{\eta}_{ij} \cdot (2\vec{Q} - \vec{\eta}_{ij})\right)^{3/2} \ , \end{aligned} \tag{28}$$

where $\tau_1 \propto (g^2\sqrt{\sigma_A})^{-1}$ is independent of Q. Obviously for large Q, the interactions between the pair can be neglected, and thus Eq.(25) reduces to

$$\frac{d\langle\vec{Q}(t)\rangle}{dt} \approx -\frac{2^{3/2}}{\tau_1}\left[N_f {\sum_{\pm\vec{\epsilon}_i}}' \vec{\epsilon}_i(\vec{\epsilon}_i \cdot \langle\vec{Q}\rangle)^{3/2} + (1 - 2^{-3/2}) {\sum_{\vec{\eta}_{ij}}}' \vec{\eta}_{ij}(\vec{\eta}_{ij} \cdot \langle\vec{Q}\rangle)^{3/2}\right] \ , \tag{29}$$

where the sums are restricted to charges with $\vec{q} \cdot \vec{Q} > 0$.

Eq.(29) controls the rate of color neutralization of large covariant constant SU(N) fields and is the natural generalization of the equation derived for the Abelian case in Ref.[12]. The SU(N $\geq$ 3) case is only complicated by the fact that Eq.(29) is a vector equation. The power 3/2 follows from dimensional considerations under the assumption that the only dimensional quantity in the problem is $\vec{E}$.

For $\vec{Q}(0)$ pointing along one of the weight or root vectors Eq.(29) reduces to an Abelian equation

$$dQ/dt = -Q^{3/2}/\tau(\hat{Q}) \ , \tag{30}$$

where the relaxation time is given in the two special cases by

$$\begin{aligned}\tau(\hat{\epsilon}) \ &= \tau_1((N-1)/2N)^{1/4}\left[N_f((1-N^{-1})^{3/2}+N^{-3/2})+(1-2^{-3/2})N\right]^{-1} \\ \tau(\hat{\eta}) \ &= \tau_1\left[N_f+(1-2^{-3/2})(N+2^{3/2}-2)\right]^{-1}\end{aligned} \tag{31}$$

The terms proportional to N_f are those due to $q\bar{q}$ pair production. For SU(2) the two times are of course identical. Amazingly, for SU(3) they only differ in the fourth decimal place.

The numerical coincidence of $\tau(\hat{\epsilon})$ and $\tau(\hat{\eta})$ for SU(3) has the pleasant consequence that $\tau(\hat{n}) \approx 0.18\tau_1$ is independent of the orientation of the color charge in the Cartan subspace to a very high accuracy. Therefore for SU(3), the vector nature of Eq.(29) is irrelevant, and the solution is accurately given by the power law

$$\vec{Q}(t) = \vec{Q}_0/(1+t/\tau_{1/4}(Q_0))^2 \ , \tag{32}$$

as in the Abelian case[12], but with the characteristic time required to neutralize 3/4 of the initial color field given by

$$\tau_{1/4}(Q_0) = 0.36\ \tau_1\ Q_0^{-1/2} \ , \tag{33}$$

For an initial color field generated by a random walk[9] in SU(3) with $\mathcal{N}$ steps, $\sqrt{Q_0} \approx (3\mathcal{N}/4)^{1/4}$. In collisions of heavy nuclei of mass A, Glauber theory gives $\mathcal{N} \propto A^{2/3}$ for the number of binary collisions per unit area, and therefore the characteristic neutralization time decreases for heavy nuclei as $\tau_{1/4} \sim 0.4\tau_1 A^{-1/6}$. This formula is however only valid as long as $Q_0 \sim A^{1/3} \gg 1$. For $Q_0 \sim 1$, Eq.(29) strickly speaking does not apply, and the interactions between the pairs must be included. A rough estimate for the neutralization rate in that case can be obtained by including pair interactions via (28). For $\vec{Q} = \vec{\eta}$ $(Q^2 = 1)$ this gives $d\vec{Q}/dt \approx -1.7/\tau_1$ for SU(3) with $N_f = 2$. In contrast, Eqs.(30,31) ignoring those interactions give $d\vec{Q}/dt \approx -5.5/\tau_1$. Therefore, pair interactions delay the neutralization time for $Q^2 = 1$ by a factor ~ 3 relative to Eq.(33). In this way we estimate the neutralization time of adjoint (gg) strings to be $\tau_{1/4}(\vec{\eta}) \sim \tau_1$. The reduction of the characteristic neutralization time by an extra factor ~ 3 for large A is due to the lesser importance of the interactions between the produced pairs.

These results imply that most of the quarks and gluons produced in the neutralization of large color fields created in ultrarelativistic nuclear collisions may appear at proper times an order of magnitude smaller than in elementary pp or e^+e^- collisions. Shorter neutralization times are encouraging in connection with the hope that a quark-gluon plasma can be generated and studied through ultra-relativistic nuclear collisions. Obviously shorter times imply that plasmas with higher initial energy densities can be produced[12,13]. However, it is not clear whether local thermal equilibrium can be achieved on such small time scales[14]. Fortunately, though, the color neutralization mechanism leads to initial conditions that are not far from local equilibrium. (1) First, in high fields the neutralization mechanism involves production of quarks and gluons at comparable rates (for SU(3)). Furthermore, u,d,s quarks are produced with nearly the same abundance since their masses become irrelavant. Thus the chemical composition of the non-equilibrium plasma produced through neutralization is close to that it would be in local equilibrium. Even charmed pairs ($c\bar{c}$) could be produced with many orders of magnitude greater probability than in pp collisions ($w_c/w_u \sim \exp(-\pi m_c^2/\sigma(\vec{\epsilon},\vec{Q})) \sim \exp(-10/A^{1/3})$). (2) Second, the distributions of initial transverse momenta are nearly exponential as in local equilibrium, although gluons have initially about 30% larger transverse momentum than quarks via (27). (3) Finally, the produced quarks and gluons are subject to accelerations in the external color field. This leads to Joule heating of the plasma at a rate $\propto \sigma_c \vec{E}\cdot\vec{E}$, where $\sigma_c \propto T/(\alpha \ln \alpha^{-1})$ is the color conductivity[15]. Such Joule heating helps to drive the plasma toward local equilibrium more rapidly. Thus the non-equilibrium quantum tunneling dynamics and external field effects may play an important role in creating plasma initial conditions approximating local equilibrium at very early times in the collision.

We close by pointing out that the rates derived here can be used to extend Monte-Carlo models such as the Lund model[16] to study quantitatively the production and evolution of quark-gluon plasmas in large color fields. Such work is in progress and will be reported elsewhere.

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