Lawrence Berkeley National Laboratory

Recent Work

Title

NONLINEAR PENETRATION OF A LANGMUIR WAVE INTO A PLASMA DENSITY GRADIENT

Permalink

<https://escholarship.org/uc/item/09x2q8vv>

Author

Kaufman, Allan N.

Publication Date

1974-08-02

NONLINEAR PENETRATION OF A LANGMUIR WAVE INTO
A PLASMA DENSITY GRADIENT

Allan N. Kaufman

August 2, 1974

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

For Reference

Not to be taken from this room

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

NONLINEAR PENETRATION OF A LANGMUIR WAVE
 INTO A PLASMA DENSITY GRADIENT

Allan N. Kaufman

Physics Department and Lawrence Berkeley Laboratory
 University of California, Berkeley, California 94720

August 2, 1974

ABSTRACT

The nonlinear Schrödinger equation, descriptive of an intense Langmuir wave near critical density, is solved for the steady state of reflection from a linear unperturbed density profile.

Recent studies¹ of the behavior of nonlinear electron plasma waves have shown that the "nonlinear Schrödinger equation", with suitable modifications when necessary, can describe the mutual effect of density perturbations and amplitude modulations. The terms used to describe the phenomena vary: solitons, oscillating two-stream instability, modulational instability, self-trapping, four-wave interaction; but the fundamental physics is the same.

Since one characteristic of a soliton is its density depression, allowing propagation through a slightly overdense plasma, the question arises as to its behavior in a density gradient. In this paper, we present a particularly simple solution of the nonlinear Schrödinger equation in a one-dimensional density gradient, showing that the density depression associated with the wave penetrates into the overdense region a distance proportional to the wave energy density.

In the field of the Langmuir wave $\text{Re } E(x,t)e^{-i\omega_0 t}$, the oscillation-center of an electron experiences an effective low-frequency ponderomotive potential energy $\psi_e(x,t) \equiv \langle \frac{1}{2} m_e v^2 \rangle = e^2 |E|^2 / 4m_e \omega_0^2$.

In (low-frequency) thermal equilibrium, the electron density is $n_e(x,t) = n_0 \exp[-\beta_e(-e\phi + \psi_e)]$, where ϕ is the low-frequency potential. For the special case of a stationary density profile, the ion density may also be taken as a Boltzmann distribution: $n_i(x) = n_0 \exp[-\beta_i(e\phi + \psi_i)]$, where ψ_i is a pseudopotential responsible for the unperturbed ($\psi_e = 0$) density profile. If the densities vary little over a Debye length, we may equate $n_i(x) = n_e(x)$, and solve for $e\phi(x) = (\beta_e\psi_e - \beta_i\psi_i)(\beta_e + \beta_i)^{-1}$, finally obtaining for the common density

$$n(x) = n_0(x) \exp(-\psi_e/T), \quad (1)$$

where $T \equiv \beta_e^{-1} + \beta_i^{-1}$, and $n_0(x) \equiv n_0 \exp(-\psi_i/T)$ is the unperturbed profile.

The nonlinear equation for the (stationary) Langmuir amplitude $E(x)$ is most expeditiously obtained from the linear dispersion relation

$$\omega^2 = \omega_p^2 + 3k^2 v_e^2, \quad (2)$$

by setting $\omega \equiv \omega_0$, $\omega_p^2 = 4\pi n(x)e^2/m$ (using (1)), $k^2 \equiv -d^2/dx^2$, and treating (2) as an operator on $E(x)$. Then (2) becomes

$$3v_e^2 \frac{d^2 E}{dx^2} + \omega_0^2 E(x) = \left[4\pi n_0(x)e^2/m \right] E(x) \exp(-e^2|E|^2/4m_e\omega_0^2 T). \quad (3)$$

We choose a linear unperturbed profile: $n_0(x) = n_0(1 + x/L)$, with $\omega_0^2 \equiv 4\pi n_0 e^2/m$, so that $x = 0$ is the critical surface. If the exponent is small (as discussed below), it can be expanded to first order. Setting $\lambda_0^2 \equiv 3v_e^2/\omega_0^2$, we then obtain

$$\lambda_0^2 d^2 E / dx^2 = \left[(x/L) - (e^2 |E|^2 / 4 m_e \omega_0^2 T) \right] E . \quad (4)$$

To make all three terms of the same order, we choose dimensionless variables of order unity:

$$\xi \equiv x/x_0 , \quad F \equiv E/E_0 , \quad (5)$$

where $x_0 \equiv \lambda_0^{2/3} L^{1/3}$,

$$E_0^2 \equiv 16\pi n_0 T (\lambda_0/L)^{2/3} . \quad (6)$$

Equation (4) then reads

$$d^2 F / d\xi^2 = (\xi - F^2) F , \quad (7)$$

with F assumed real, and the boundary condition $F(+\infty) \rightarrow 0$. With the change of variables (5), the exponent in Eq. (3) is $(\lambda_0/L)^{2/3} F^2$. Thus the assumption $\lambda_0 \ll L$, which is necessary for quasi-neutrality, as well as for the use of (2) as an operator equation, assures us that the exponent is indeed small for F of order unity.

The family of solutions of (7) is shown in Fig. 1. The small amplitude limit ($|F| \ll 1$) is the Airy function, as seen from (7) and the Figure. As the amplitude increases (from one solution to another), the ponderomotive force depresses the density, allowing penetration of the wave into the overdense region of the unperturbed profile. The depth of penetration can be characterized by the last inflection point: $\xi = F^2$, or in dimensional terms: $x = (|E|^2 / 16\pi n_0 T) L$.

The density $n(x) = n_0 [1 + (\xi - F^2)(\lambda_0/L)^{2/3}]$ is shown in Fig. 2 for the family of solutions of Eq. (7). Similar density depressions have been observed in simulation of resonance absorption

of obliquely incident electromagnetic radiation^{2,3}, in laboratory experiments⁴, and in cold-plasma two-species analytic theory⁵.

While the discussion here referred to a Langmuir wave, the same analysis applies to a normally incident electromagnetic wave, with the new definition $\lambda_0 = c/\omega_0$.

ACKNOWLEDGMENTS

This work was begun at the Göteborg Workshop on Electrodynamics on Nonequilibrium Plasmas, June 1974. I thank H. Wilhelmsson for his gracious hospitality, V. Karpman for stimulating discussions, and H. Gustafsson and G. Smith for numerical solutions. The work was partially supported by the U. S. Atomic Energy Commission.

REFERENCES

1. Because the literature is so voluminous, the papers cited here are selective of recent ones:
 - (a) V. Zakharov and A. Shabat, Sov. Phys. JETP 34, 62 (1972);
 - (b) P. Kaw, G. Schmidt, and T. Wilcox, Phys. Fluids 16, 1522 (1973);
 - (c) Y. Ichikawa, T. Suzuki, and B. Fried, J. Plasma Phys. 10, 219 (1973);
 - (d) A. Kingsep, L. Rudakov and R. Sudan, Phys. Rev. Lett. 31, 1482 (1973);
 - (e) G. Morales Y. C. Lee and R. White, Phys. Rev. Lett. 32, 457 (1974);
 - (f) K. Nishikawa, H. Hojo, K. Mima, and H. Ikezi, Phys. Rev. Lett. 33, 148 (1974).
2. J. Kindel, K. Lee, and E. Lindman, Surface Wave Absorption, submitted to Phys. Rev. Lett.; D. Forslund, J. Kindel, K. Lee, E. Lindman, and R. Morse, Theory and Simulation of Resonant Absorption in a Hot Plasma, submitted to Phys. Rev. Lett.
3. K. Estabrook, E. Valeo, W. Kruer, J. De Groot, A. B. Langdon, and B. Lasinski, 4th Annual Anomalous Absorption Conference, Livermore, April 1974.
4. H. Kim, R. Stenzel, and A. Wong, to be published.
5. P. Koch and J. Albritton, Phys. Rev. Lett. 32, 1420 (1974).

FIGURE CAPTIONS

- Fig. 1. The family of solutions of Eq. (7), representing a standing wave, i.e., an incident wave and its reflection from the self-consistent density profile.
- Fig. 2. The family of density profiles, perturbed by the ponderomotive force of the radiation reflected from it.

0 0 0 0 4 1 0 8 1 7 8

-7-

XBL 748-3788

Fig. 1.

XBL 748-3789

Fig. 2.

LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720