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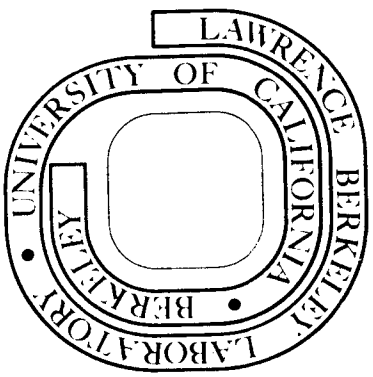

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HYPERFINE QUENCHING OF THE 2^3P_0 STATE IN HELIUMLIKE IONS*

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I. INTRODUCTION

In heliumlike ions with no nuclear spin, the 2^3P_0 state decays primarily to the 2^3S_1 state by an electric dipole radiative transition. In the case where the nucleus has spin, the interaction of the nuclear magnetic moment with the atomic electrons mixes electron states of different total angular momentum J, so that the one-photon dipole transition $2^3P_0 \rightarrow 1^1S_0$ is not forbidden. This additional decay mode has the effect of shortening the lifetime of the 2^3P_0 state and gives rise to emitted X-rays with the energy $E(2^3P_0) - E(1^1S_0)$.

A similar effect occurs for the 2^3P_2 state. If the nucleus has no spin, the 2^3P_2 state decays primarily by an electric dipole transition to the 2^3S_1 state or by a magnetic quadrupole transition to the 1^1S_0 state. If the nucleus has spin, certain hyperfine components of the 2^3P_2 state can, in addition, decay through hyperfine mixing by an electric dipole transition to the 1^1S_0 state. The lifetime of those components is thereby shortened. Evidence for this effect has been observed in the decays of heliumlike vanadium by Gould, Marrus, and Mohr.[1]

In this paper, we present an estimate of the lifetime of the 2^3P_0 state for odd-Z heliumlike ions in the range Z = 9 to 29. We employ an approximation scheme which utilizes the fact that both Z^{-1} and $(Z\alpha)^2$ are small parameters for the range of Z under consideration. This approach was used in the calculation in Ref. 1. The theory of hyperfine induced transitions in neutral atoms is discussed by Garstang.[2]

II. THEORY

To calculate the hyperfine induced decay rate, we take the hyperfine interaction into account in first order perturbation theory. The hyperfine interaction is diagonal in $F = J + I$, where I is the nuclear spin. In the present case, $J = 0$ for the initial and final states, so that only states of total angular momentum $F = I$ enter the calculation. We consider mixing of the 2^3P_0 state with only the nearby 2^3P_1 and 2^1P_1 states. Mixing of the 1^1S_0 state is neglected. The first order perturbed wave function for the 2^3P_0 state is then given by

$$|2^3P_0IM\rangle_1 = |2^3P_0IM\rangle + \frac{\langle 2^3P_1{*}IM|\delta H|2^3P_0IM\rangle}{E(2^3P_0) - E(2^3P_1{*})}|2^3P_1{*}IM\rangle + \frac{\langle 2^1P_1{*}IM|\delta H|2^3P_0IM\rangle}{E(2^3P_0) - E(2^1P_1{*})}|2^1P_1{*}IM\rangle \quad (1)$$

where I is the value of the total angular momentum F for the states, M is the z-component of F, and δH is the hyperfine interaction correction to the Hamiltonian. The zero-order states in (1) are sums of products of electron eigenstates and nuclear spin eigenstates, combined to give total angular momentum $F = I$. The asterisks (*) in (1) denote intermediate coupling electron states which are appropriate linear combinations of LS-coupling states

$$\begin{aligned} |2^1P_1{*}\rangle &= a|2^1P_1\rangle - b|2^3P_1\rangle \\ |2^3P_1{*}\rangle &= b|2^1P_1\rangle + a|2^3P_1\rangle . \end{aligned} \quad (2)$$

The LS-coupling states are eigenstates of the two-electron Schrödinger Hamiltonian H_S, which includes the Coulomb interaction energy of the electrons. The coefficients a and b, which we choose to be positive real, are determined by the condition that the sum $H_S + H_B$, where H_B represents the corrections to H_S which appear in the Pauli approximation to the Breit equation,[3] be diagonal in the basis formed by the intermediate coupling states, together with the condition $a^2 + b^2 = 1$.

The rate for a dipole transition to the 1^1S_0 state is given by

$$A_{E1}^{HFS}(2^3P_0) = \frac{4\alpha\omega^3}{3c^2}\sum_{M'}|\langle 1^1S_0IM'|\vec{r}_1 + \vec{r}_2|2^3P_0IM\rangle_1|^2 . \quad (3)$$

In (3), ω is the transition frequency $\hbar\omega = E(2^3P_0) - E(1^1S_0)$. Substitution of (1) and (2) into (3) yields

$$A_{E1}^{HFS}(2^3P_0) = \frac{4\alpha\omega^3}{3c^2} |Q_I|^2 |\langle 1^1S| z_1 + z_2 | 2^1P \rangle|^2 \quad , \tag{4}$$

where

$$Q_I = \left[\frac{b^2}{E(2^3P_0) - E(2^3P_1^*)} + \frac{a^2}{E(2^3P_0) - E(2^1P_1^*)} \right] \langle 2^1P_1 IM| \delta H| 2^3P_0 IM \rangle$$

$$+ \left[\frac{ab}{E(2^3P_0) - E(2^3P_1^*)} - \frac{ab}{E(2^3P_0) - E(2^1P_1^*)} \right] \langle 2^3P_1 IM| \delta H| 2^3P_0 IM \rangle . \tag{5}$$

In obtaining (4), we have taken the relation

$$\sum_{M'} |\langle 1^1S_0 IM'| \vec{r}_1 + \vec{r}_2 | 2^1P_1 IM\rangle|^2 = |\langle 1^1S| z_1 + z_2 | 2^1P \rangle|^2 \tag{6}$$

into account. The vectors on the right-hand side of (4) and (6) are the normalized coordinate space factors of the electron wave functions. Evaluation of the constituents of (4) and (5) is described in the following section.

III. EVALUATION

The intermediate coupling energy levels and the ratio b/a are obtained by diagonalization of the operator $H_S + H_B$ in the subspace of the states in (2). In the LS-coupling basis, the matrix elements of H_S are given by

$$\langle n|H_S|n\rangle = (Z\alpha)^2 mc^2 \left[a_n + b_n Z^{-1} + c_n Z^{-2} + \ldots\right] \quad , \tag{7}$$

where the coefficients a_n, b_n, c_n,... have been calculated by Knight and Scherr,[4] by Sanders and Scherr,[5] and by Aashamar, Lyslo, and Midtdal.[6] The diagonal matrix elements of H_B are written in the form

$$\langle n|H_B|n\rangle = (Z\alpha)^4 mc^2 \left[\alpha_n + \beta_n Z^{-1} + \gamma_n Z^{-2} + \ldots\right] \quad , \tag{8}$$

where the coefficients α_n, β_n, and γ_n appear in Table I. The coefficients α_n correspond to the Pauli approximation for the hydrogenic fine structure.[3] The coefficients β_n have been calculated by Doyle in jj-coupling.[7] We have determined the coefficients

TABLE I. Values of α_n, β_n, and γ_n

n	α_n	β_n	γ_n
2^1P_1	-55/384	0.055403	-0.090
2^3P_1	-59/384	0.130429	-0.162
2^3P_0	-21/128	0.219768	-0.307
2^3S_1	-21/128	0.076935	-0.042

γ_n approximately by extrapolating to large Z the expectation values of H_B calculated for Z = 2 to 10 by Accad, Pekeris, and Schiff.[8] The off-diagonal matrix element of H_B between the 2^1P_1 and 2^3P_1 states is given by

$$\langle 2^1P_1|H_B|2^3P_1\rangle = (Z\alpha)^4mc^2\,[-\sqrt{Z}/96 + 0.028851\,Z^{-1} - 0.004\,Z^{-2} + \ldots\,], \qquad (9)$$

where the third coefficient has been determined by our extrapolation of the results of the variational calculation of Drake.[9] We note that the values of the matrix elements of H_B given by these formulas agree with the corresponding numerical values obtained by Ermolaev and Jones.[10]

The matrix elements of the hyperfine interaction correction are approximated by evaluating the contact interaction term[3]

$$\frac{4\pi\alpha}{3}\frac{\hbar^3}{mM_pc}\,g\vec{I}\cdot[\delta^3(\vec{r}_1)\vec{s}_1 + \delta^3(\vec{r}_2)\vec{s}_2] \qquad (10)$$

between symmetrized products of nonrelativistic hydrogenic wavefunctions. In (10), g is the g factor of the nucleus and M_p is the proton mass. Within this approximation

$$\begin{aligned}\langle 2^1P_1IM|\delta H|2^3P_0IM\rangle &= \frac{2\alpha}{3\sqrt{3}}\frac{m}{M_p}\,g[I(I+1)]^{\frac{1}{2}}(Z\alpha)^3mc^2\\ \langle 2^3P_1IM|\delta H|2^3P_0IM\rangle &= \frac{2\sqrt{2}\alpha}{3\sqrt{3}}\frac{m}{M_p}\,g[I(I+1)]^{\frac{1}{2}}(Z\alpha)^3mc^2\,.\end{aligned} \qquad (11)$$

The relative sign of the matrix elements in (11) has been made consistent with the sign of the matrix element in (9).

The matrix element in (4) is evaluated with the aid of the Z^{-1}-expansion calculation of Dalgarno and Parkinson[11]

$$\langle 1^1S|\, z_1 + z_2 \,|2^1P\rangle = \frac{2^8\hbar}{3^5 Z\alpha mc}\,(1 - 0.155\, Z^{-1} + \ \ldots\ ) \ . \qquad (12)$$

The transition frequency ω is approximated by

$$\hbar\omega = (Z\alpha)^2 mc^2\,[0.375 - 0.399\, Z^{-1} + 0.086(Z\alpha)^2\,] \ . \qquad (13)$$

The rate for the electric dipole transition $2^3P_0 \rightarrow 2^3S_1$ is evaluated by means of the expression[1]

$$A_{E1}(2^3P_0) = \frac{12\alpha\hbar^2\omega^3}{m^2c^4}\,(Z\alpha)^{-2}\,[1 + 0.759\, Z^{-1} + 0.685\, Z^{-2} - 0.417(Z\alpha)^2\,]^2 . \qquad (14)$$

Here, $\hbar\omega = E(2^3P_0) - E(2^3S_1)$. The hydrogenlike Lamb shift[12] $\Delta E(2P_{\frac{1}{2}}) - \Delta E(2S_{\frac{1}{2}})$ is included in the theoretical value for $\hbar\omega$ as an approximation for the actual Lamb shift in the separation. The coefficients of Z^{-1} and Z^{-2} in (14) are obtained from the results of the variational Z^{-1}-expansion calculation of Sanders and Scherr.[5]

IV. RESULTS

The results for the decay rates and lifetime of the 2^3P_0 state appear in Table II. Figure 1 shows the lifetimes, with and without the hyperfine induced decay, as a function of Z. Also shown is the result of the recent measurement of the lifetime of the 2^3P_0 state in heliumlike fluorine by Mowat et al.[13]

It is interesting to note that within the range of Z considered here, the hyperfine induced decay rate ranges from being a small fraction of the total decay rate of the 2^3P_0 state at small Z to being the dominant decay mode at large Z.

TABLE II. Decay rates and lifetime of the 2^3P_0 state

Z	A	I	gI	$A_{E1}(2^3P_0)$ $nsec^{-1}$	$A_{E1}^{HFS}(2^3P_0)$ $nsec^{-1}$	$\tau(2^3P_0)$ nsec
9	19	1/2	2.63	0.090	0.013[a]	9.73
11	23	3/2	2.22	0.114	0.012	7.96
13	27	5/2	3.64	0.139	0.074	4.71
15	31	1/2	1.13	0.165	0.041	4.85
17	35	3/2	0.82	0.194	0.030	4.47
19	39	3/2	0.39	0.224	0.016	4.17
21	45	7/2	4.76	0.257	4.15	0.227
23	51	7/2	5.15	0.292	10.5	0.093
25	55	5/2	3.44	0.330	10.7	0.091
27	59	7/2	4.62	0.372	36.0	0.027
29	63	3/2	2.22	0.416	21.7	0.045

[a] Calculated with $E(2^3P_1) - E(2^3P_0)$ from Ref. 14.

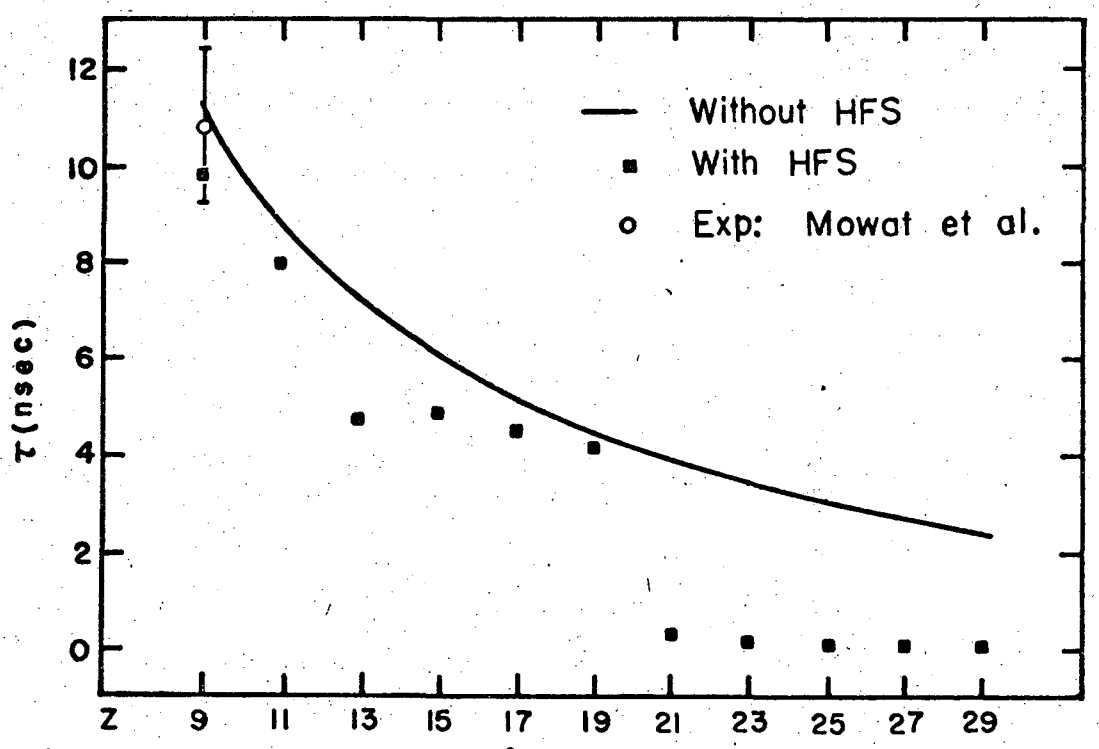

Figure 1. Lifetime of the 2^3P_0 state.

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