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Deformed Pulsar Model of Subpulse Drift, Nulling, and Mode Switching

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Abstract

The motion of a rigid axi-symmetric deformed pulsar model yields many features of individual pulse structure, including drifting, nulling and mode switching that are qualitatively similar to features observed in real pulsars. As a corollary we examine the effects of a very small eccentricity for the pulsar discovered recently in SN1987A. Precession appears to be a plausible explanation of a small amplitude 2 hour frequency modulation of the millisecond pulses, or alternately, of periodic long term appearances and disappearances of the signal.

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1 Introduction

The well known gross characteristics of pulsar signals are that the pulse profile, averaged over many pulses, and the frequency are very stable. Otherwise the individual pulses generally are different one from the other [1,2]: (1) Some pulsars produce signals within the average profile with apparently random structure compared to each other. (2) Still others produce successive pulse timings that vary systematically around the mean, the pulses drifting across the average profile and after a few pulses beginning at the other side and repeating a similar sequence of pulses. (3) The drift can occur in either direction. (4) Successive bunches of pulses constituting a cycle in the drift are not precisely the same in number or structure. (5) In some pulsar signals there are nulls, meaning the absence of pulses when they are otherwise expected from the period. (6) In others the average pulse profile switches between several patterns.

We propose a simple mechanical model that yields pulse systematics that are similar to those described above. We accept the standard view that the pulse is the manifestation of beamed radiation along the magnetic axis which is fixed in the star and inclined at an angle with respect to the rotation axis. In addition we assume that the star is deformed axially and executes rigid-body motion and that the symmetry axis is not along the direction of the angular momentum. However unappealing the assumption of rigid-body motion may at first seem, we demonstrate that the model exhibits features that are similar to those observed in actual pulsars. We note that the structure introduced into pulses by the motion of a deformed pulsar are additional to and superimposed on any structure introduced by intrinsic radiation mechanisms such as discussed elsewhere[3,4]. Moreover, we certainly do not insist on precisely rigid-body motion. This is simply a solvable idealization which has many interesting features that may survive in more flexible motions under the stresses of gravitation, rotation, strong magnetic fields, the short-range tensor interaction which spatially aligns a succession of neutron-proton pairs with their spins, and the magnetic moments of the nucleons, which couples the preceding two.

2 Mechanics of the Model

In a torque free environment the model pulsar has two constants of motion, the kinetic energy and angular momentum, K and $\mathbf{L}$. The mechanical motion is determined fully by the two scalar constants K and L, all directions being simply referred to the direction of $\mathbf{L}$. Let the symmetry axis, $\mathbf{S}$ make an angle α with the instantaneous angular velocity vector $\boldsymbol{\omega}$ and an angle β with the vector $\mathbf{L}$, respectively. In a body with axial symmetry (say $I_1 = I_2 \neq I_3$), the magnitude ω is fixed and the vectors $\boldsymbol{\omega}$ and $\mathbf{S}$ lie in a plane that rotates about $\mathbf{L}$ with constant precession angular velocity Ω. From the Poinsot construction of mechanics, the above quantities are related by [5],

$$\frac{I_3}{I_1}\tan\beta = \tan\alpha, \quad \Omega\sin\beta = \omega\sin\alpha. \tag{1}$$

There is a third rotation, the body about its own symmetry axis, with angular velocity

$$n = (\frac{I_3}{I_1} - 1)\omega\cos\alpha \tag{2}$$

For chosen I_3, I_1 the unknowns are $\alpha, \beta, \omega, \Omega, n$. The two constants of the motion K and L together with the above three equations determine them. For our purpose it is more convenient to choose two of the above five and regard the other three and K and L as dependent. The two most relevant parameters of the problem are Ω and n as will become evident. Define

$$e \equiv \frac{I_3}{I_1} - 1, \quad r \equiv \frac{\Omega}{n} \tag{3}$$

From the first three equations we can also write,

$$n = \frac{e}{1+e}\Omega\cos\beta \tag{4}$$

We note that for $e < 0$ (prolate), the angular velocity, n about the symmetry axis is in the opposite sense to Ω and ω, while for $e > 0$ (oblate) it is in the same sense. Hence $re > 0$. Therefore without loss of generality we can take $0 < \beta < \pi/2$ and consequently

$$1 > \frac{1+e}{re} > 0 \tag{5}$$

For given eccentricity, e, this is a restriction on the ratio of frequencies, r!

$$|r| \equiv \left|\frac{\Omega}{n}\right| > \left|\frac{1+e}{e}\right| \tag{6}$$

For small eccentricities, which are likely in stars, this ratio of frequencies must be greater than one, indeed much greater.

Now we introduce the magnetic axis, **B**, and let it be fixed in the body making an angle γ with the symmetry axis. It rotates about this axis with angular velocity n. Define a reference system fixed in space with the z-axis defined by the constant direction of **L**. Let the origin lie at the center of mass of the star. At various instants **B**, **S** and the z-axis are coplanar. At one such instant when **B** lies below the **S** axis as measured from the z-axis, call the time $t = 0$. After some trigonometry we find that the time-dependent polar angles of **B** are,

$$\Theta(t) = \arctan\left(\frac{\sqrt{(\sin\beta + \cos\beta\tan\gamma\cos nt)^2 + \tan^2\gamma\sin^2 nt}}{\cos\beta - \sin\beta\tan\gamma\cos nt}\right) \tag{7}$$

$$\Phi(t) = \Omega(t + t_0) + \arctan\left(\frac{\tan\gamma\sin nt}{\sin\beta + \cos\beta\tan\gamma\cos nt}\right) \tag{8}$$

We explain the choice of t_0 below. With respect to the space-fixed axis, the second of these equations shows that **B** precesses around the z-axis with angular velocity Ω, modulated by the derivative of the second term.

Define a variable $\tau = t/T$ with $T = 2\pi/\Omega$. As a consequence of eq.4 the polar angles of **B** are completely defined in terms of τ, β, γ and r satisfying eq.5, for any chosen eccentricity, e.

Knowing the direction of **B** as a function of the time variable τ, we can calculate the angle between the magnetic axis and an observer as a function of this variable. We have not chosen the x-axis within the constant, $\Phi = \Omega t_0$. Let it now be chosen so that the observer lies in the x,z-plane. This defines the constant t_0 introduced above. Let the angle between the observer and the z-axis (direction of **L**) be η. Then the time-dependant angle between observer and magnetic axis is

$$\delta(\tau) = \sin\eta\sin\Theta(\tau)\cos\Phi(\tau) + \cos\eta\cos\Theta(\tau) \tag{9}$$

Assuming the radiation is emitted with a gaussian distribution in angle measured from the magnetic axis **B**, the intensity of the signal seen by an observer will be proportional to

$$I(\tau) = \exp - \left(\frac{\delta(\tau)}{\Delta}\right)^2 + (\delta \to \delta - \pi) \tag{10}$$

The second term takes into account the radiation from both ends. Some authors believe that as well or instead of the above emission pattern, the radiation is emitted in a cone centered at the magnet axis, or both[6]. For the cone case we take the intensity proportional to

$$I(\tau) = \left(\frac{\delta(\tau)}{\Delta}\right)^2 \exp\left(-\left(\frac{\delta(\tau)}{\Delta}\right)^2 + 1\right) + (\delta \to \delta - \pi) \tag{11}$$

To have a chance of seeing the signal the observer's orientation has to approximately satisfy the following relation,

$$\beta + \gamma + \Delta \geq \eta \geq \beta - \gamma - \Delta \tag{12}$$

where the role of Δ is obvious, and the other parts of the limits follow from eq.7.

3 Qualitative Description of the Pulse Structure

Let us describe in words the motion that is executed by the magnetic axis. For a star with symmetry axis and rotation axis coincident and the magnetic axis inclined at an angle γ with the z-axis (fixed direction of $\mathbf{L}$), the magnetic axis traces out a cone. For the deformed star described above with symmetry axis offset from the angular momentum axis, it traces a surface that is bounded by two cones, at angles $\beta \pm \gamma$ with respect to the z-axis, completing a cycle in time $2\pi/n$. For small eccentricities, appropriate to stars, the period of the rotation of the body about its own axis is much longer than the precession of the body's symmetry axis about the z-axis. Consequently the surface traced by the magnetic axis will spiral between the above two cones, but the time to complete one cycle will be longer than the time for the magnetic axis to precess once around the z-axis by the ratio of frequencies $r = \Omega/n$. This surface will eventually retrace itself only if this ratio of frequencies is a rational number. Essentially that will never be the case for long, because of radiation damping. So in general the surface traced by the magnetic axis in the course of time will not retrace itself (not be periodic). The radiated signal has an angular width about the magnetic axis. Each pulse intensity and pattern seen by an observer fixed in space as the star rotates with precession frequency Ω will depend on the particular cut of the intrinsic radiation pattern his line of sight makes as the magnet axis sweeps by with the frequency Ω, modulated as in eq.8. *Therefore any sequence of pulses seen by an observer will never be repeated in intensity, structure and in timing.* However there will occur approximate repetition of sequences as we see below, a quasi-periodicity with period $2\pi/n$.

To gain further insight into the signals that an observer fixed in space would see from such a model pulsar, recall that the motion of the magnetic axis described in space fixed axis by eqs.7,8, is simply the superposition of the precession of $\mathbf{S}$ about the z-axis with angular velocity Ω and the rotation of $\mathbf{B}$ about $\mathbf{S}$ with angular velocity n, both making constant angles β and γ with their respective rotation axes. Let $\mathbf{S}$ define a z'-axis with x'-axis in the plane defined by the z and z'-axis and with the x'-axis below the xy-plane. Then the polar angle of $\mathbf{B}$ in this (uniformly precessing) system is $\phi' = nt$ and $\theta' = \gamma$. As already noted, the sign of n depends on the sign of e. Taking note that $\mathbf{S}$ is precessing in space say in counterclockwise sense, we then can say with respect to a fixed observer in space that if $0 < \phi' < \pi$ then the space-fixed polar angle Φ of the magnetic axis will be advanced compared to the symmetry axis, while if $\pi < \phi' < 2\pi$, it will be retarded. The position of greatest advancement or retardation occurs at $\phi' = \pi/2,\ 3\pi/2$. The positions when the magnetic axis is neither advanced or retarded is $\phi' = 0, \pi$. For each cycle of the pulsar (Ω), the magnetic axis will rotate by only a small angle, $\Delta\phi' = 2\pi(n/\Omega)$. What the observer sees will depend on his polar angle $\theta_{obs} = \eta$ in the space fixed axis. If $\theta_{obs} \approx \beta \pm \gamma$ he will see a succession of pulses near the angular frequency

Ω but progressing from slightly advanced to retarded or vice versa, ie drifting in one sense or the other across the average pulse profile, with possible null pulses between one group of drifting pulses and the next. In Fig. 1 we show a sequence of individual pulses, plotted over the portion of the precession period for which the signal can be seen from the observer's position in space, and stacked one over the other. In this way we display both the sense of intensity variation and the frequency modulation. In contrast, the timing of the signal from a pulsar whose symmetry axis coincides with the rotation axis, would occur at constant frequency, if the source were a constant beam. Whether a sequences of drifting pulses is separated by nulls or by weaker pulses drifting in the opposite sense depends on the the beam width Δ, on γ, and on the observer's orientation. A sequence can also consist of pulses that drift from advanced to retarded positions with a frequency modulation that is approximately sinusoidal, interspersed with many nulls, as in Figs. 2 and 3. The cycles described are generally quasi-periodic, and not exactly so unless the ratio of frequencies $r = \Omega/n$ is a rational number. The sum of pulses and nulls in a cycle is approximately r. The parameters corresponding to the figures can be found in Table 1.

If the observer's orientation has about the same polar angle as the symmetry axis, $\eta \approx \beta$, then he will see the pulsar only when $\phi' \approx \pi/2, 3\pi/2$, if $\Delta << \gamma$. As ϕ' approaches either of these positions, the observer will see a sequence of pulses that occur very close in frequency (and near to the precession frequency) followed by a null period followed by another series of pulses that again occur near the precession frequency but which are displaced in timing from the first group, and correspond to the other position of ϕ'. One group is advanced and the other retarded, as in Fig. 4. There follows another null period and then a sequence at timings close to the first group and so on. The number of nulls will be equal if $\eta = \beta$, and otherwise there will be alternating number of nulls. The number of pulses seen near each of these positions of ϕ' will depend on the beam width and on the ratio of frequencies n/Ω, and will be approximately $(\Delta/2\pi)(\Omega/n)$. The average pulse profile of each group will be the reflection symmetric image of the other (but only approximately, because of the quasi-periodicity). This pattern resembles the phenomena exhibited

Table 1: Parameters that define the pulsar characteristics corresponding to the Figures.

Fig. no.	e	Ω/n	β/π	γ/π	η/π	Δ/π	$\|eg(\alpha)\|$
1	-.05	25	0.225	0.2	0.35	0.1	0.08
2	-.05	50	0.376	0.1	0.33	0.07	0.17
4	-.05	50	0.376	0.15	0.33	0.07	0.17
5	-.05	50	0.376	0.15	0.38	0.1	0.17
6	-.05	25	0.225	0.05	0.2	0.04	0.08

by certain pulsars that is referred to as mode switching. This is another example of the quasi-periodicity.

Under some circumstances pulses from both poles can be seen. One circumstance under which this can happen is when the angle between the symmetry axis and the magnetic axis is sufficiently large. An example of interpulse signals is shown in the intensity-pulse plot of Fig. 5.

As a final example, note that if the pulse emission pattern itself is more complicated than the gaussian pattern of eq.(11), say a cone emission as in eq.(12), then the individual pulses can vary not only in timing, but also in shape, as seen in Fig. 6.

4 Compatibility with Observed Frequency Damping

For many pulsars the period and its rate of change are measured. Periods range from milliseconds to seconds with most in the range 2/10 to 2 seconds, and the rate of change of periods lie in the range $10^{-18} < \dot{T} < 10^{-12}$. The damping is believed to be caused by magnetic dipole radiation or gravitational radiation for which $\dot{T} \propto 1/T$ and $1/T^3$ respectively. The gravitational damping will occur only if there is a time changing quadrupole mass transport, and so will occur in our model. The measured damping places an upper bound on the eccentricity, or more precisely on e times a function of α. Gravitational radiation yields a rate of change of the period according to,

$$\begin{aligned} \dot{T} &= (2\pi)^4 \frac{4}{25} \frac{1}{T^3} M R^2 e^2 \sin^2\alpha (16 \sin^2\alpha + \cos^2\alpha) \\ &= 1.6 \times 10^{-12} \Big(\frac{\mathrm{s}}{T}\Big)^3 \Big(\frac{R}{10\ \mathrm{km}}\Big)^2 \frac{M}{M_\odot} e^2 g^2(\alpha) \end{aligned} \tag{13}$$

where $g^2(\alpha)$ stands for the function of α in the first line and s stands for the unit second. For most of our examples, $eg(\alpha) \sim 0.1$. Hence for a nominal solar mass pulsar with radius 10 km, $\dot{T} \approx 1.6 \times 10^{-14}(\mathrm{s}/T)^3$. This defines a curve in the $\dot{T} - T$ plane above which many pulsars lie. Model parameters used in the figures are compatible with those pulsars that lie above this curve.

5 Sinusoidal Frequency Modulation

Under the condition $\tan\gamma << \tan\beta$ we have,

$$\begin{aligned} \Phi &\approx \Omega(t + t_0) + \arctan\Big(\frac{\tan\gamma}{\sin\beta} \sin nt\Big) \\ \dot{\Phi} &\approx \Omega + \Big\{1 + \Big(\frac{\tan\gamma \sin nt}{\sin\beta}\Big)^2\Big\}^{-1} n \frac{\tan\gamma}{\sin\beta} \cos nt \end{aligned} \tag{14}$$

We see that the frequency modulation can be sinusoidal, as for a pulsar whose signal is modulated by orbital motion. We check the constraints imposed on such an hypothesis for the pulsar discovered in the remnant of SN1987A. It has a period $T \approx 0.5 \times 10^{-3}$ seconds, the time rate of change of this period is $\dot{T} < 3 \times 10^{-14}$, and the pulsar period is modulated with an eight hour period. The modulation is sinusoidal and its amplitude is $\Delta\Omega/\Omega \approx 7.5 \times 10^{-7}$. The measurement of the time rate of change of the period is one constraint. Another is the period of the modulation compared to the pulsar period, eq.5, and the other is the amplitude of the frequency modulation, given by the second term of eq.14.

Gravitational radiation yields a rate of change of the period according to eq.13, and for this pulsar provides the constraint, $eg(\alpha) < 10^{-7}$. This is satisfied by $e \lesssim 10^{-6}$.

Identifying the long eight hour period with the rotation of the star about its own symmetry axis whose angular velocity we denoted by n, eq.4 provides the constraint $e \cos\beta = 10^{-7}/6$. For small e, eq.(1) tells us $\alpha \approx \beta$. Together the above constraints read,

$$\frac{1}{6} < \frac{e}{10^{-7}} < 10 \tag{15}$$

and are compatible. Thus the deformed pulsar model can account for the period of the pulsar and its modulation period with an eccentricity that satisfies the constraints imposed by the small rate of change of the pulsar period and with the ratio of the millisecond to the eight hour period. However referring to eq.4,14, we see that our model can yield,

$$\frac{\Delta\Omega}{\Omega} \approx \frac{n}{\Omega} \frac{\tan\gamma}{\sin\beta} << \left|\frac{e}{1+e}\right| < 10^{-6} \tag{16}$$

whereas the observed amplitude of the frequency modulation is of the order of the right side. Although the general form of the modulation is difficult to analyze analytically, it does seem unlikely that the observed eight hour modulation is due to precession. However in an improved analysis of the data [7], there is evidence of a second periodic frequency modulation of period ~ 2 hours and amplitude $|\Delta\Omega/\Omega| \sim 1.2 \times 10^{-8}$ which does satisfy the above inequality. So the two hour frequency modulation could be caused by precession due to a small non-axisymmetric deformation that is compatible with the measured rate of change of the millisecond period.

There is however another interesting possibility. One of our examples above showed a long period modulation in which the signal was very weak over about half of the long period. Now the pulsar in SN1987A was not found two weeks later at the next search. Perhaps the subsequent searches have occurred during the portions of the period when the magnetic axis has been rotated too far out of our line of sight by the small eccentricity. Identify the angular frequency with a time of the order of a week. This, according to eq.6 would require an eccentricity,$e < 8 \times 10^{-10}$, even smaller than those quoted above, relegating gravitational radiation to a very small role in the rate of change of the period. We tentatively propose the deformed pulsar model, in this case with minuscule eccentricity, as an explanation of the disappearance of the signal. If this explanation is correct, it should reappear and disappear at cyclic intervals. The duration of the appearance need not be the same as the duration of the disappearance. This depends on $\beta, \gamma, \Delta, \eta, e$. The smaller the eccentricity, the longer the period of the complete cycle, according to eq.4.

6 Summary

The motion of an axially deformed pulsar rotating about an axis that is not one of the principle axis, and which is assumed to emit beamed radiation in a cone centered on the magnetic axis fixed in the pulsar, produces signals that an appropriately situated observer fixed in space would observe. Depending on the observer's orientation with respect to the angular momentum axis, and the parameters defining the pulsars motion, the observer will see drifting pulses either in an advanced or retarded sense, with nulls or weaker pulses connecting the groups of drifting pulses, or he may see a group of pulses whose periods are nearly the same followed by nulls and then another group at a shifted timing, but again having nearly the same periods. The latter pattern resembles mode switching. Under certain circumstances an observer would see a sequence of pulses whose frequency is approximately sinusoidally modulated, and whose intensity varies, perhaps falling below the detection threshold so that a sequence of visible pulses is followed by nulls. In the absence of radiation damping, the above cycles would be periodic only if Ω/n is a rational number, say 50.45. Each cycle would consist of a total of about 50 pulses some of which may be nulls. Every 5045 pulses the same sequence of cycles would recur. The above cycles must in general be quasiperiodic, because precise periodicity can occur only if the ratio of precession frequencies is a rational number, and because of radiation damping this situation, though occasionally attained cannot be maintained.

In our idealization we took two of the principle moments of inertia to be equal. Freed from this restriction, the pulse patterns would have greater complexity. We also assumed that the beamed radiation was *constant* in intensity and distributed around the magnetic axis with a simple pattern. In real pulsars whatever effects discussed here that are caused by rotation about an axis that is not the symmetry axis, are *superimposed* on the radiation which is produced in the complex and possibly time-varying environment of the magnetosphere. The source itself may vary in intensity and frequency and its distribution about the magnetic axis may be more complex than assumed in our examples.

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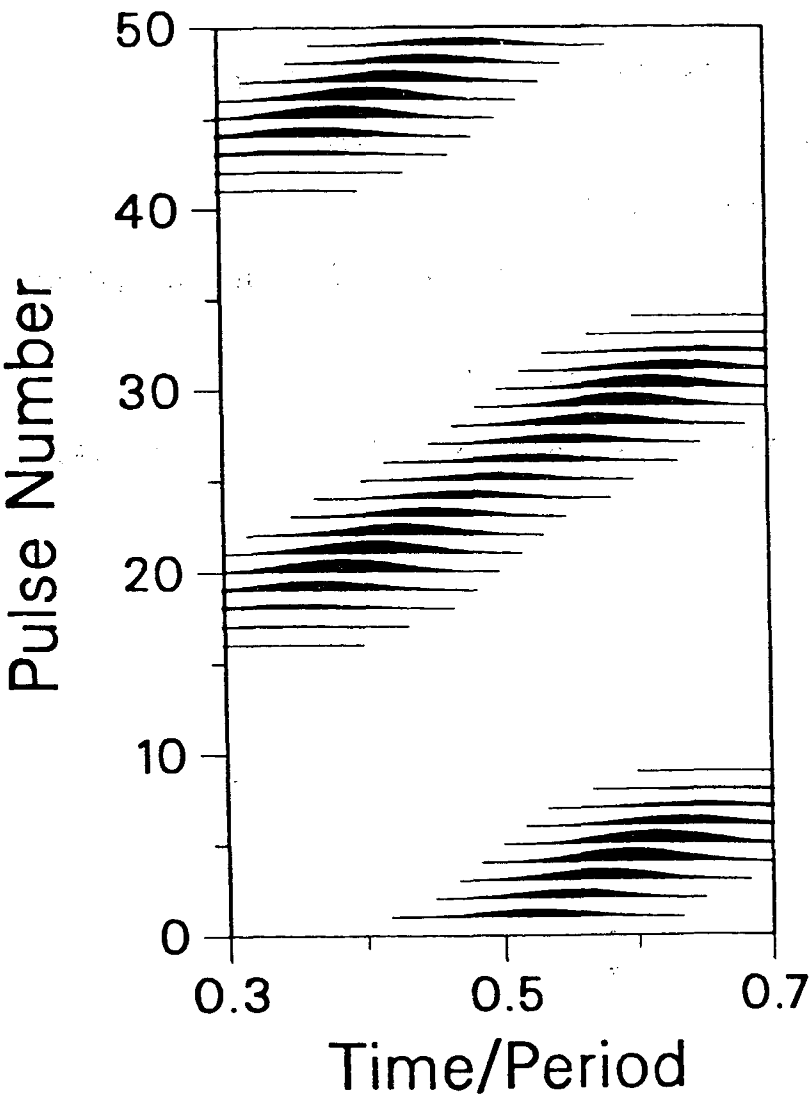

Fig. 1 Pulses that would be seen from the model deformed pulsar defined by the parameters of Table 1, for a continuous source of radiation. Pulses are stacked modulo the precession period $T = 2\pi/\Omega$. This example illustrates drifting pulses separated by nulls. If the observer were positioned near $\eta = \beta - \gamma$ instead of $\beta + \gamma$ he would see a drift in the opposite direction, as would be the case if the sign of the eccentricity were reversed.

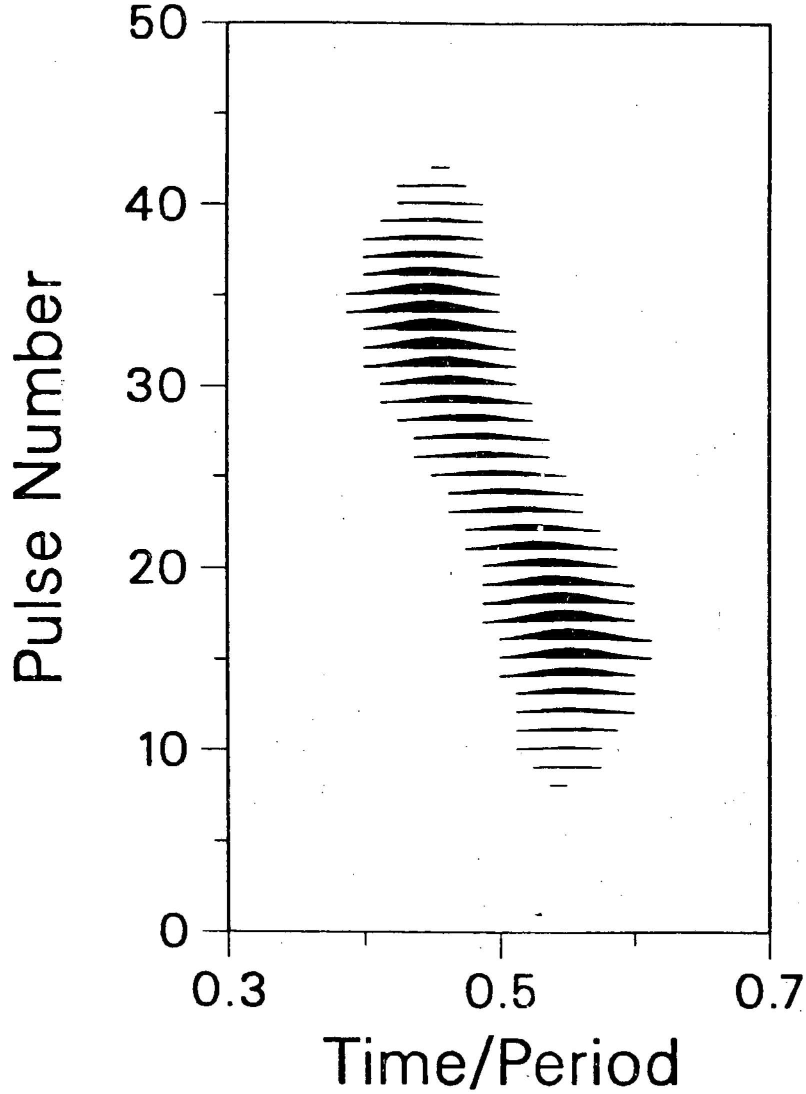

Fig. 2 Frequency modulation is approximately sinusoidal in this case, with intensity of observed signal varying as in Fig. 3. As in all cases the signal source is constant.

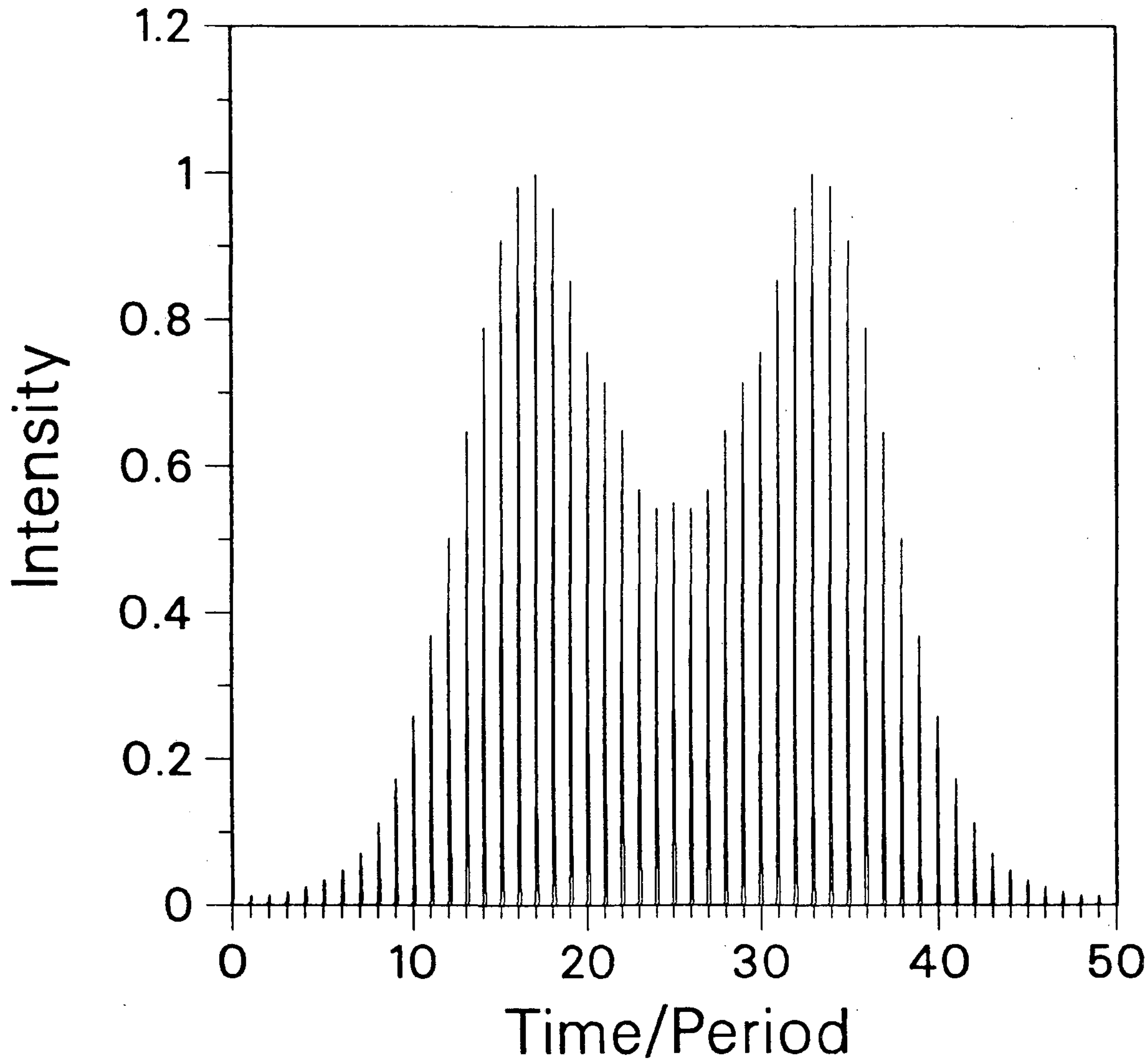

Fig. 3 Intensity of individual observed pulses corresponding to Fig. 2, for one complete cycle of rotation of the pulsar about its own symmetry axis.

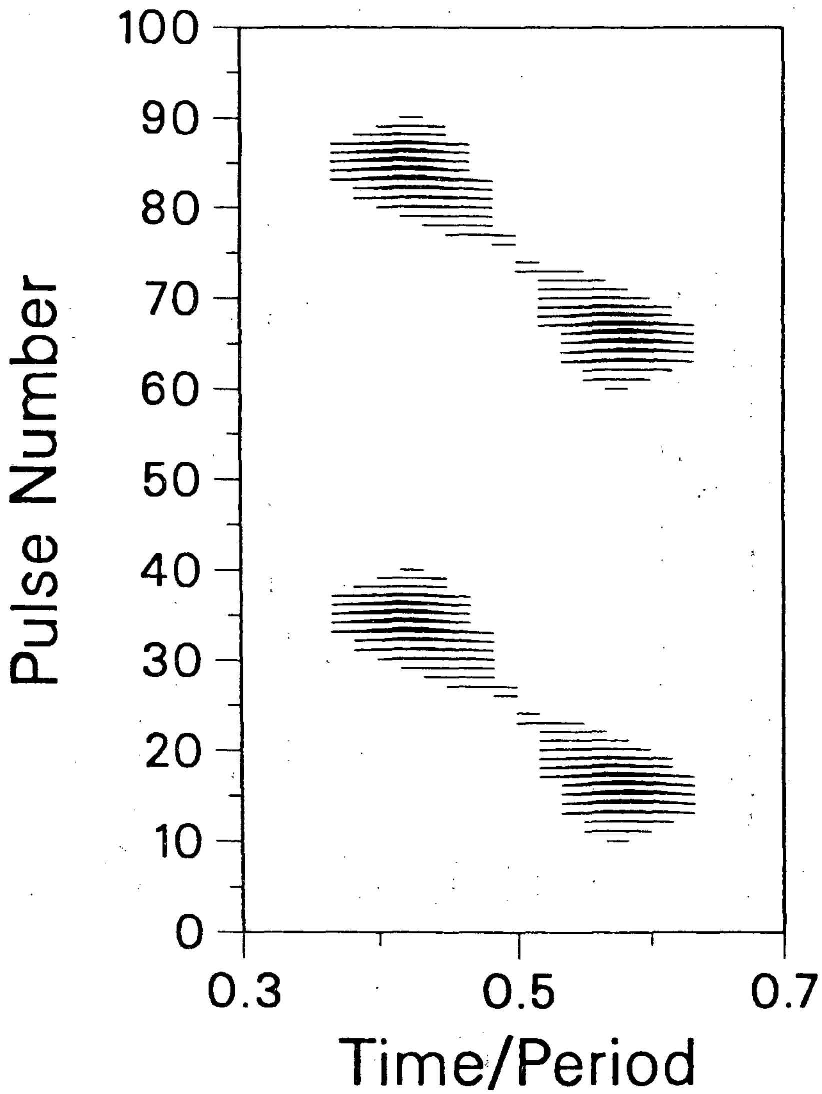

Fig. 4 Illustrating pulses that alternate in timing with approximately the same frequency in each bunch. The plot covers two complete cycles.

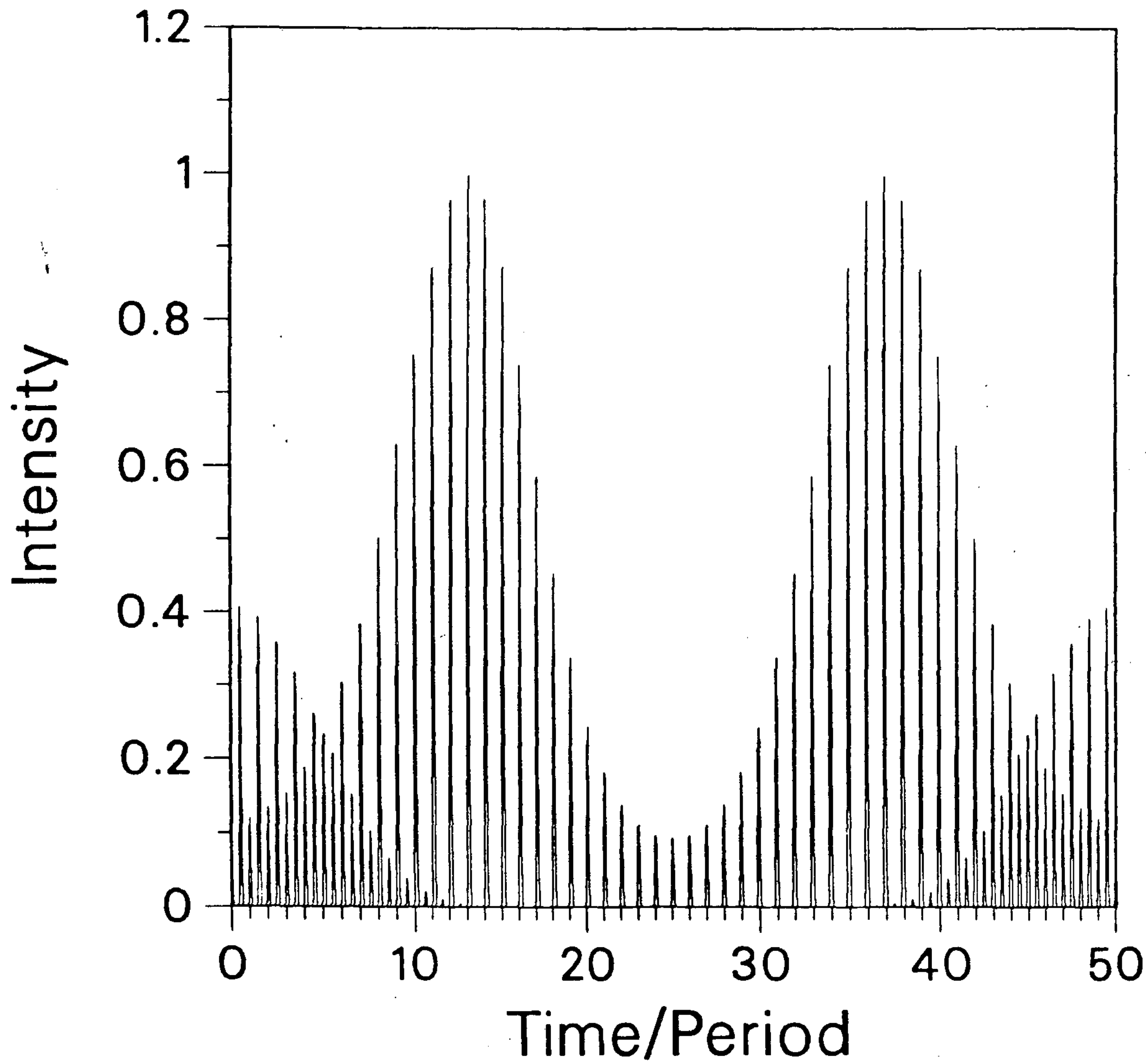

Fig. 5 Observed intensity of pulses from one end of pulsar occur near the integer units of t/T and weaker pulses seen from the other end occur near 1/2 integer units. The source at each end is constant and equal in intensity.

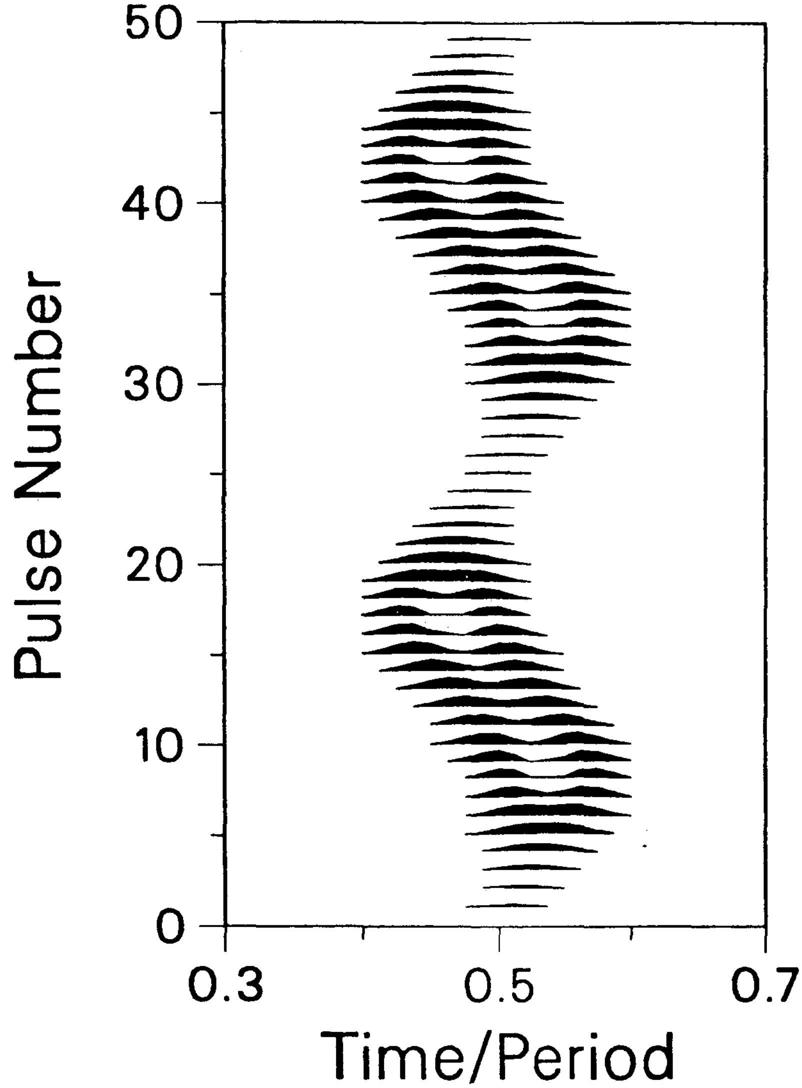

Fig. 6 Drifting pulses for the cone shaper emission pattern of eq.11.

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