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DESIGN OF LONG STRAIGHT SECTIONS FOR SYNCHROTRONS*

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DESIGN OF LONG STRAIGHT SECTIONS FOR SYNCHROTRONS[*]

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ABSTRACT

A number of variations of long straight section insertions are available for synchrotron design. Collins-type (two quadrupole) insertions may be lengthened by altering the lengths of neighboring magnets and gaps. The momentum compaction increase of Collins-type insertions can be eliminated by pairing them. Another type of insertion using four quadrupoles and having a phase advance of π radians permits drift lengths several times greater than does the Collins-type insertion. The momentum compaction increase of these π insertions is reduced or eliminated by inclusion of bending magnets without gradient.

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[*] Work performed under the auspices of the U. S. Atomic Energy Commission.

In the course of the 200 BeV Design Study, we studied several variations of long straight sections. The type ultimately adopted is a modification of that originally proposed by Collins, but it may be useful to discuss some other possibilities as well.

The Collins-type straight section (Fig. 3) consisting of an F and a D quadrupole with a long gap between, permits matching of the betatron oscillations through the insertion with those in the normal structure.[1] However, the off-momentum closed orbits do not match, so that one suffers an increase in momentum compaction due to the insertion of

$$\hat{\alpha} = \frac{\Delta x}{\Delta p/p} = \hat{\alpha}_o[1 + (\sqrt{2}\sin \pi\nu/N_s)^{-1}], \qquad (1)$$

where $\hat{\alpha}_o$ is the value without insertions, N_s is the number of (evenly spaced) insertions, and ν is the number of betatron oscillations per revolution. We noticed that the increase in α can be avoided by pairing the insertions K with normal cells C : CKCCKC.[2] If the total phase advance through this assemblage is 2π, then 3 x 3 transfer matrix for the vector $(x, x', \Delta p/p)$ will be just the identity matrix, so that $\hat{\alpha} = \hat{\alpha}_o$, for parts of the machine away from the group ($\hat{\alpha} < \hat{\alpha}_o$ inside the group). This observation was also made independently at CERN.[3,4] We were not too interested in using such an arrangement, because it is more convenient to have the straight sections evenly spaced. However, the observation that any structure with identity matrix can be inserted in an AGS, led Lloyd Smith to propose extra long straight sections, with transfer matrix

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{or} \qquad M = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

(2π-phase advance); or (π-phase advance)

These insertions could have greater length than the Collins-insertion permits because there is now only one condition to fit ($\Delta\phi = \pi$ or 2π), rather than two for the Collins-insertion (where both the β and α betatron functions must match the normal structure), which limits the gap length. A. M. Sessler and the author investigated various quadrupole arrangements: the simplest one consists of two quadrupole pairs, with a gap between the two pairs twice as great as that in each end (between a pair and the adjacent magnet units).[5] This is the π-straight section (see Fig. 1). To obtain the π-phase advance, the lengths and gradient must satisfy

$$L = \frac{1}{K} \frac{1 - \frac{1}{2} Ka\,(\tan Km - \tanh Km)}{(\tan Km - \tanh Km) + Ka \tan Km \tanh Km} \tag{3}$$

where $K = \sqrt{\frac{1}{(B_o\rho)} \left|\frac{dB}{dR}\right|}$, and $B_o\rho$ is the momentum expressed as field x curvature. Two of them comprise a 2π straight section. Although the π insertion has about the same momentum compaction increase as the Collins, it can be made extremely long. The 2π insertion is inordinally long, over four times the longest field-free gap length. The π-straight section could have a field-free gap of 100 meters in a 200 GeV machine, with a betatron amplitude twice as great in the quadrupoles as in the normal cells.

By comparison, Collins-insertion gaps for 200 GeV machines are limited to about 30 meters (with amplitude factor about 1.2).

Internal targeting seems to be the only factor enhanced by such extra long straight sections. For this purpose, one very long insertion would suffice. However, freedom from resonances requires high periodicity, so preferably all the straight sections should be the same. Thus use of π-straight sections tend to increase circumference considerably.

One may improve the circumference, and at the same time reduce the momentum compaction increase, by adding a suitable number of bending magnets, each with bending angle ϕ, located symmetrically at distances d_i from the ends of the insertion.[6] The 3 x 3 matrix of the insertion is

$$M_\pi \cong \begin{pmatrix} -1 & 0 & 2\phi\Sigma d_i \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad (4)$$

Suppose that the insertion is at a symmetric point of the structure, where the closed orbit without insertions would be $(\hat{\alpha}\Delta p/p, 0, \Delta p/p)$. The condition that the insertion have no effect on this orbit is easily seen to be

$$\phi\Sigma d_i \cong \hat{\alpha}\,\frac{\Delta p}{p} . \qquad (5)$$

The factor $\hat{\alpha}$ is smaller at DD than at FF points, so compensation can be achieved with least bending if the π-insertion is at a DD point. If the π-insertion is too short, one may not be able to satisfy Eq. (5). However inclusion of the bending magnets will still improve $\hat{\alpha}$. It is also possible in this case to invade the region between quadrupole pairs, though this increases the bending aperture required.

An accelerator with twelve π-straight sections with bending magnets was considered for the 200 GeV synchrotron, incorporating free drift regions 46 meters long. The circumference is 6% greater than that of the adopted design with twelve Collins straight sections, the betatron amplitude and momentum compaction are respectively 8 and 20% smaller in the normal cells for the π-straight section machine. However, the total quadrupole length needed is over three times as great for the π machine, and in addition the bending magnets are an added complication. These drawbacks seemed too high a price to pay for a more commodious internal target area.

Having determined that the Collins-type insertion was best for the particular accelerator being considered, we sought possible improvements in its design.[7] Modifications in the structure of neighboring magnets had been investigated by H. S. Synder, though we did not have his calculations at hand. Therefore, a study was made of the effect of altering the length and placement of the two gradient magnets, one on each end of the Collins straight section (see Fig. 4). One thinks of the region between dotted lines in Fig. 4, which includes the two gradient magnets of altered length, as well as the quadrupoles. However this as assemblage is <u>not</u> inserted at an FD point of the normal structure, rather it <u>replaces</u> the first half of an FDDF cell. Hence the transfer matrix of the assemblage, in order to preserve matching of betatron oscillations, must have a transfer matrix of the form

$$M = \begin{pmatrix} \sqrt{\frac{\beta_D}{\beta_F}} \cos \psi & \sqrt{\beta_F \beta_D} \sin \psi \\ -\frac{\sin \psi}{\sqrt{\beta_F \beta_D}} & \sqrt{\frac{\beta_F}{\beta_D}} \cos \psi \end{pmatrix} \qquad (6)$$

where β_F and β_D are the values of the betatron function β at FF and DD points of the normal cell and ψ is arbitrary. Formulas for the gap lengths and quadrupole strength to satisfy this matching requirement are given in the appendix. A computer has been given to evaluate the parameters, as functions of ℓ, m_o, (KM), and ψ, where K is the quadrupole parameter, Eq. (3). It is found that as the magnet length m_o increases, so does the drift length L, while simultaneously the short gaps of length a decrease, as does the value of the function β in the quadrupoles.

This scheme was employed directly in the adopted injector synchrotron, with modified gradient magnets 2.34 m long, compared to 1.95 m for the normal units. The drift length L so obtained is 9.3 m, compared to about 7 m without the modification.

For the main ring, the corresponding procedure led to modified units too long for fabrication and support. Hence we instead added additional short units 1.84 m long next to the normal 5.73 m units adjoining the straight section (see Fig. 5). By this means the drift length was increased from 31 to 34 meters, and the total radius was decreased from 702 to 690 meters.

These examples illustrate the fact that numerous variations of long straight sections for strong focusing synchrotrons are available. The selection made will vary with the requirements of the particular machine in question.

The author wishes to acknowledge the invaluable contribution of Mr. James Eusebio to the SYNCH computer program, by means of which these studies were implemented.[8]

FOOTNOTES AND REFERENCES

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3. K. Johnsen, CERN Report AR Int/SG/62-4 (1962).
4. E. Keil, Comparison of Two Arrangements of Matched Long Straight Sections, CERN Report AR/Int. SG/64-16 (1964). This report shows details of the closed orbit behavior for an example of paired straight sections. The displacement is almost normal outside the structure CKCCKC (see text) and smaller than normal inside it.
5. A. Garren, Construction of a π-Phase Advance Straight Section, Lawrence Radiation Laboratory Report UCID-10155 (1965).
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APPENDIX

Formulas for Satisfying the Matching Requirements in Modified Collins Straight Sections

An assemblage of the form shown in Fig. 4 or 5 between dotted lines is required to have transfer matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad .$$

We suppose the quantity $\phi = Km$, where m is magnet length and K is defined in Eq. (3), is given for both the modified gradient magnets and the quadrupoles. Let these be called, respectively

$$\phi_o = K_o m_o \ ,$$

$$\phi = Km$$

and abbreviate

$$\begin{aligned} s_o &= \sin \phi_o & s &= \sin \phi \\ c_o &= \cos \phi_o & c &= \cos \phi \\ \bar{s}_o &= \sinh \phi_o & \bar{s} &= \sinh \phi \\ \bar{c}_o &= \cosh \phi_o & \bar{c} &= \cosh \phi \quad . \end{aligned}$$

We suppose the gradient magnets have a value of K_o identical to that of the normal F and D units. Further suppose that the outer gap length, ℓ , is unknown. Then the remaining parameters K , a , L , m are obtained by solving the following equations. Let

$$P_1 = s_o\bar{c} - A\,\bar{s}_o c \qquad Q_1 = \frac{C}{K_o}(\bar{c}_o + K_o \ell \bar{s}_o)c \qquad R_1 = c_o\bar{c} - D\bar{c}_o c \qquad T_1 = BK_o\bar{s}_o c$$

$$P_2 = s_o\bar{s} + A\,\bar{s}_o c \qquad R_2 = s_o\bar{c} + D\bar{s}_o c \qquad T_2 = BK_o\bar{s}_o s$$

$$P_3 = c_o\bar{s} - A\,\bar{c}_o s \qquad Q_2 = \frac{C}{K_o}(\bar{c}_o + K_o \ell \bar{s}_o)s \qquad R_3 = c_o\bar{s} + D\bar{c}_o s \qquad T_3 = BK_o\bar{c}_o s$$

$$P_4 = c_o\bar{c} - A\,\bar{c}_o c \qquad Q_3 = \frac{C}{K_o}(\bar{s}_o + K_o \ell \bar{c}_o)s \qquad R_4 = s_o\bar{s} - D\bar{s}_o s$$

$$P_5 = s_o\bar{s} - A\,s_o s \qquad R_5 = s_o\bar{s} + D\bar{s}_o s$$

$$P_6 = s_o\bar{c} + A\,\bar{s}_o c$$

$$P_7 = c_o\bar{s} + A\,\bar{c}_o s$$

Then

$$K = K_o \frac{\dfrac{P_1 + Q_1}{P_2 - Q_2} + \dfrac{R_1 - K_o\ell R_2 + T_1}{-R_3 + K_o\ell R_4 + T_2}}{\dfrac{P_3 + Q_3}{P_2 - Q_2} + \dfrac{-R_5 - K_o\ell R_3 + T_3}{-R_3 + K_o\ell R_4 + T_2}}$$

$$m = 0/K$$

$$a = \frac{1}{K}\,\frac{KP_3 - K_oP_1 - C(\bar{c}_o c - \frac{K}{K_o}\bar{s}_o s - K\ell\bar{c}_o s + K_o\ell\bar{s}_o c}{K_o(P_2 - Q_2)}$$

$$L = \frac{2}{K}\,\frac{\left[P_4 - \frac{K_o}{K}P_2 - K_o a P_6 + \frac{C}{K}\left(\frac{K}{K_o}\bar{s}_o c + \bar{c}_o s + K(\ell + a)\bar{c}_o c + K_o\ell\bar{s}_o s + KK_o\ell a\bar{s}_o c\right)\right]}{\left[-P_7 + \frac{K_o}{K}P_6 + K_o\,a\,P_5 + \frac{C}{K}\left(\frac{K}{K_o}\bar{s}_o s - \bar{c}_o c + K(\ell + a)\bar{c}_o s - K_o\ell\bar{s}_o c + KK_o\ell a\bar{s}_o s\right)\right]}$$

For the modified half-cell, Fig. 4, the matrix M is given by Eq. (3). For the scheme of Fig. 5, one uses

$$M = \begin{pmatrix} \cos\psi + \alpha \sin\psi & \beta \sin\psi \\ -\dfrac{1+\alpha^2}{\beta} \sin\psi & \cos\psi - \alpha \sin\psi \end{pmatrix}$$

where α and β are evaluated at the FD point of the normal cell.

In the normal Collins insertion, the corresponding formulae are

$$KL = 2\,\frac{\alpha \sin\psi - s\bar{s}}{\bar{s}c + \bar{c}s} \qquad \ell = \frac{(KL)(\bar{s}c - \bar{c}s) + 2(c\bar{c} - \cos\psi)}{2\,\dfrac{1+\alpha^2}{\beta}\sin\psi}$$

$$K = \frac{\dfrac{1+\alpha^2}{\beta}\sin\psi}{(KL)\,s\bar{s} - \bar{s}c + \bar{c}s} \qquad L = (KL)/K,\ m = \phi/K \quad .$$

The maximum length L is obtained when $\psi = \pi/2$. The maximum shifts when the modifications are introduced.

FIGURE CAPTIONS

Fig. 1 Schematic Diagram of π-Straight Section

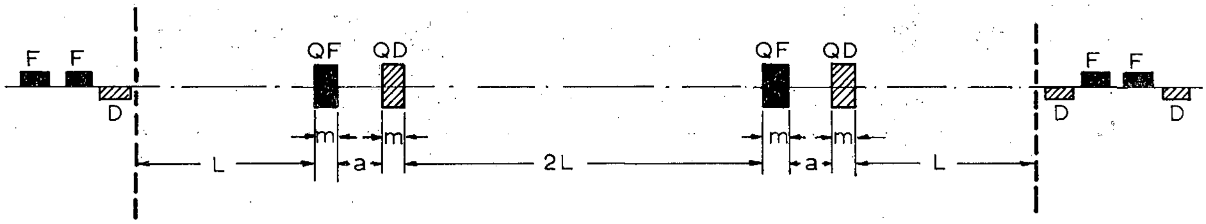
Fig. 2 π-Straight Section with Compensating Bending Magnets

Fig. 3 Envelope of Betatron Oscillations in a π-Straight Section at Full Energy

Fig. 4 Collins-Type Long Straight Section

Fig. 5 Collins Straight Section with Short Gradient Mangets in End Gaps

Fig. 6 Insertion with Quadrupole Pair and Gradient Magnets Replacing an FD Half-Cell



Schematic Diagram of π Straight Section

fig.1

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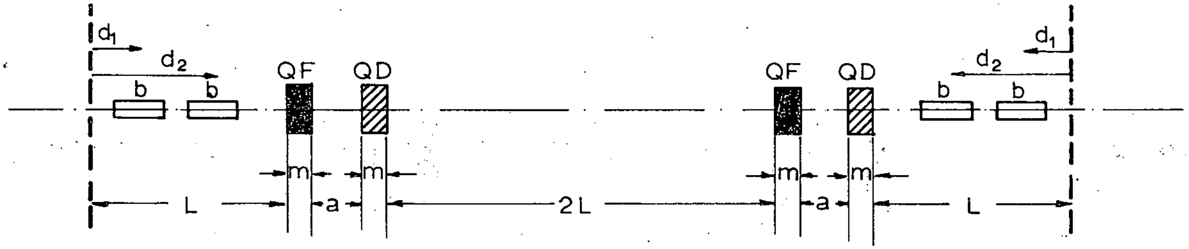
Fig. 1



π Straight Section with Compensating Bending Magnets

fig. 2

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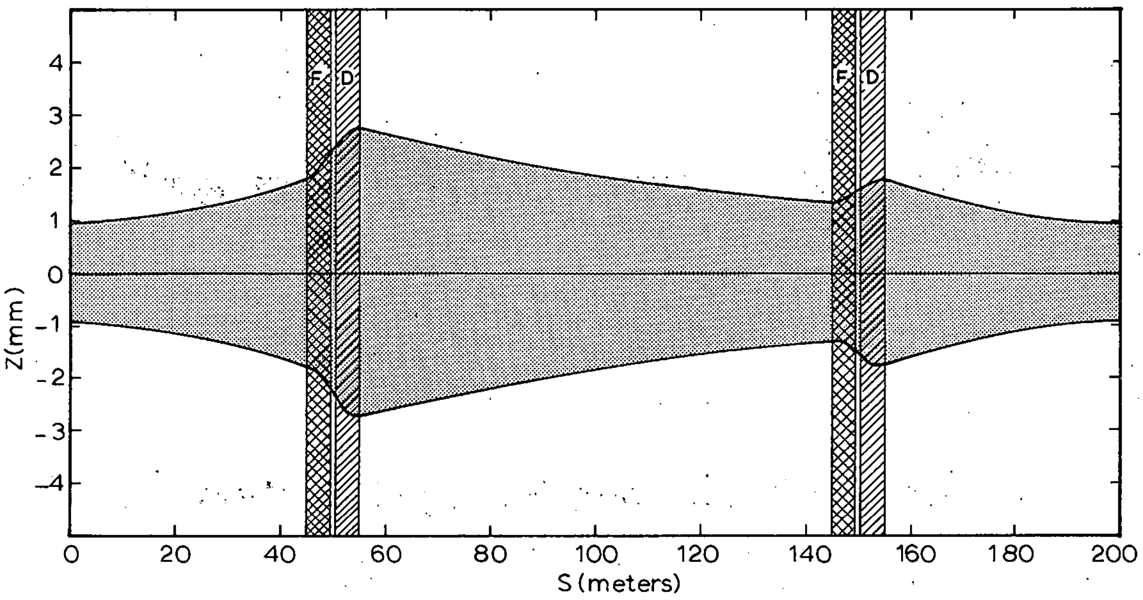
Fig. 2



Envelope of Betatron Oscillations in a π Straight Section at Full Energy
fig. 3

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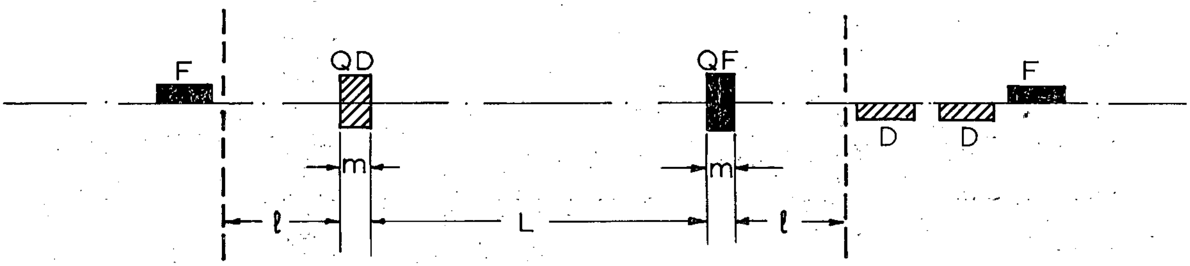
Fig. 3



Collins-Type Long Straight Section

fig. 4

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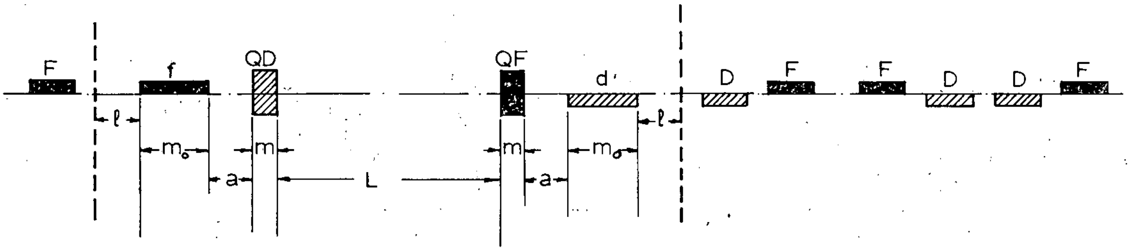
Fig. 4



Insertion with Quadrupole Pair and Gradient Magnets
Replacing an FD Half Cell

fig. 5

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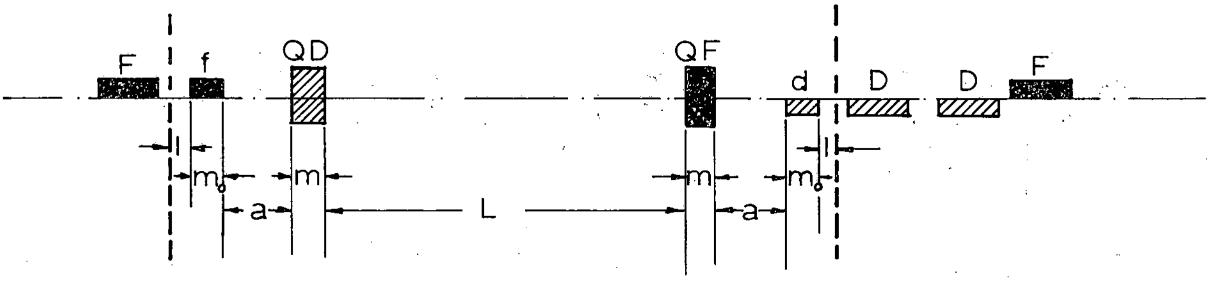
Fig. 5



Collins Straight Section
with Short Gradient Magnets in End Gaps

fig. 6

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Fig. 6