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# The photon polarization in $B \rightarrow X\gamma$ in the standard model

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The standard model prediction for the  $\bar{B}^0 \rightarrow X_{s,d}\gamma$  decay amplitude with a right-handed photon is believed to be tiny, suppressed by  $m_{s,d}/m_b$ , compared to the amplitude with a left-handed photon. We show that this suppression is fictitious: in inclusive decays, the ratio of these two amplitudes is only suppressed by  $g_s/(4\pi)$ , and in exclusive decays by  $\Lambda_{\text{QCD}}/m_b$ . The suppression is not stronger in  $\bar{B}^0 \rightarrow X_d\gamma$  decays than it is in  $\bar{B}^0 \rightarrow X_s\gamma$ . We estimate that the time dependent  $CP$  asymmetries in  $B \rightarrow K^*\gamma$ ,  $\rho\gamma$ ,  $K_S\pi^0\gamma$ , and  $\pi^+\pi^-\gamma$  are of order 0.1 and that they have significant uncertainties.

## I. INTRODUCTION

The standard model (SM) predicts that photons are mainly left-handed in  $b \rightarrow q\gamma$  ( $q = s, d$ ) decay (and right-handed in  $\bar{b} \rightarrow \bar{q}\gamma$ ). We define the ratio

$$r_q e^{i(\phi_q + \delta_q)} \equiv \frac{A_R}{A_L} \equiv \frac{A(\bar{B} \rightarrow f_q \gamma_R)}{A(\bar{B} \rightarrow f_q \gamma_L)}, \quad (1)$$

where  $\phi$  is a weak phase and  $\delta$  is a strong phase. It is usually stated that  $r_q = m_q/m_b \ll 1$  in the SM [1]. (Throughout this paper  $\bar{B}$  refers to  $\bar{B}^0$  or  $B^-$  that contain a  $b$  quark, and  $r$ ,  $\phi$  and  $\delta$  depend on the final state,  $f$ .) New physics can modify this prediction, and therefore several methods have been proposed to measure the photon helicity [1, 2].

In  $B \rightarrow f\gamma$ , where  $f$  is a  $CP$  eigenstate, since  $\gamma_L$  and  $\gamma_R$  cannot interfere, the time dependent  $CP$  asymmetry

$$\frac{\Gamma[\bar{B}^0(t) \rightarrow f\gamma] - \Gamma[B^0(t) \rightarrow f\gamma]}{\Gamma[\bar{B}^0(t) \rightarrow f\gamma] + \Gamma[B^0(t) \rightarrow f\gamma]} = S_{f\gamma} \sin(\Delta m t) - C_{f\gamma} \cos(\Delta m t), \quad (2)$$

is sensitive to  $r$ . In the SM,  $\phi_s$  and  $C_{f_s\gamma}$  are suppressed by  $|(V_{ub}V_{us})/(V_{tb}V_{ts})|$ , and to first order in  $r_s (\ll 1)$

$$S_{f_s\gamma} = -2r_s \cos \delta_s \sin 2\beta. \quad (3)$$

The first measurements of such  $CP$  asymmetries were carried out recently,

$$S_{K^*\gamma} = \begin{cases} +0.25 \pm 0.63 \pm 0.14 & \text{BABAR [3],} \\ -0.79_{-0.50}^{+0.63} \pm 0.10 & \text{BELLE [4],} \end{cases} \quad (4)$$

yielding a world average  $S_{K^*\gamma} = -0.28 \pm 0.45$ . At a super- $B$ -factory the statistical error with  $50 \text{ ab}^{-1}$  data is estimated to be  $\delta(S_{K^*\gamma}) = 0.04$  [5]. The Belle Collaboration also measured the  $CP$  asymmetry  $S_{K_S\pi^0\gamma} =$

$-0.58_{-0.38}^{+0.46} \pm 0.11$ , integrating over the invariant mass range  $0.6 \text{ GeV} < m_{K_S\pi^0} < 1.8 \text{ GeV}$  [6]. It will also be possible to measure this  $CP$  asymmetry in  $B \rightarrow \pi^+\pi^-\gamma$  [7], and maybe even with additional pions [8].

The purpose of this paper is to study the SM prediction for  $r$ . (For earlier attempts to go beyond the naive estimate, see Refs. [9, 10].) We find that  $r$  is only suppressed by  $g_s/(4\pi)$  in inclusive  $b \rightarrow X\gamma$  decay, and by  $\Lambda_{\text{QCD}}/m_b$  in exclusive  $\bar{B} \rightarrow \bar{K}^*\gamma$  and  $\rho\gamma$  decay.

To understand the origin of such effects, recall that the effective Hamiltonian for  $b \rightarrow s\gamma$  is [11]

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^8 C_i(\mu) O_i(\mu). \quad (5)$$

For our discussion the operators directly relevant are

$$O_2 = (\bar{c}\gamma^\mu P_L b)(\bar{s}\gamma_\mu P_L c),$$

$$O_7 = \frac{e}{16\pi^2} \bar{s}\sigma^{\mu\nu} F_{\mu\nu}(m_b P_R + m_s P_L)b, \quad (6)$$

where  $P_{R,L} = (1 \pm \gamma_5)/2$ , and we neglect the  $m_s P_L$  part of  $O_7$  hereafter. At the parton level, as long as  $b \rightarrow s\gamma$  is a two-body decay (either from the leading contribution of  $O_7$  or subleading virtual contributions from  $O_{i \neq 7}$ ), the left-handed  $s$  quark is back-to-back to a photon. Then the two-body kinematics implies that only  $\gamma_L$  is allowed. This argument does not apply to multi-body final states, such as  $b \rightarrow s\gamma + \text{gluons}$ .

The  $m_b P_R$  part of the leading operator  $O_7$  contributes only to  $A_L$  to all orders in the strong interaction. To prove this, note that the electromagnetic tensor for  $\gamma_{L,R}$  is  $F_{\mu\nu}^{L,R} = \frac{1}{2}(F_{\mu\nu} \pm i\tilde{F}_{\mu\nu})$ , where  $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}$ , and  $O_7$  can be written in terms of  $m_b F_{\mu\nu}^L$ . Thus, independent of hadronic physics, the photon from  $O_7$  is left-handed. This argument only applies for  $O_7$ . Indeed, we find by explicit calculation that other operators produce right-handed photons once QCD corrections are included.

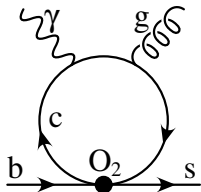


FIG. 1: Leading contribution to  $\bar{B} \rightarrow X_s \gamma_R$ . A second diagram with the gluon and the photon exchanged is not shown.

## II. INCLUSIVE $B \rightarrow X_s \gamma$ AND $B \rightarrow X_d \gamma$

In this section we estimate  $r$  from an inclusive calculation. The result can only be trusted if several hadronic final states are allowed to contribute, and  $r$  for specific final states cannot be obtained from this calculation model independently.

The leading contribution to the inclusive  $\bar{B} \rightarrow X_s \gamma_R$  rate is of order  $\alpha_s$ . It arises from bremsstrahlung contributions to the matrix elements of operators other than  $O_7$ . The numerically dominant contribution comes from  $O_2$  shown in Fig. 1. The corresponding amplitude was calculated in Ref. [12, 13]. We find that it yields equal rates for left- and right-handed photons at order  $\alpha_s$ , at any point in the  $b \rightarrow s \gamma$  Dalitz plot.

Because of the complicated  $m_c$ -dependence of the double differential rate,  $d\Gamma_{22}^{(\text{brem})}/dE_\gamma dE_g$ , we integrate over  $E_g$  and  $E_\gamma$  numerically. To reduce the large sensitivity to the scale of  $\alpha_s$ , we include the known order  $\alpha_s^2 \beta_0$  contribution to  $\Gamma_{22}^{(\text{brem})}$  [14]. (At this order the equality of the decay rates to left- and right-handed photons is violated by less than 1%, and can be neglected.)

Using the “effective” Wilson coefficients at leading order [15],  $C_2(m_b) = 1.1$  and  $C_7(m_b) = -0.31$ ,  $\alpha_s(m_b) = 0.22$  with  $m_b = 4.8$  GeV, and  $m_c = 1.4$  GeV, we obtain

$$\frac{\Gamma_{22}^{(\text{brem})}}{\Gamma_0} \simeq 0.025, \quad \Gamma_0 = \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha_{\text{em}} C_7^2 m_b^5}{32\pi^4}. \quad (7)$$

This result corresponds to integrating the numerator over  $x \equiv 2E_\gamma/m_b > 0.75$  (that is, roughly,  $E_\gamma > 1.8$  GeV). This result includes also the  $\mathcal{O}(\alpha_s^2 \beta_0)$  correction, which is sizable, indicating that the relevant scale of  $\alpha_s$  may be well below  $m_b$ ; without including it the result in Eq. (7) would be 0.015. Thus, we find at lowest order in  $g_s$

$$\langle r_s \rangle \Big|_{x>0.75} = \sqrt{\Gamma_{22}^{(\text{brem})}/(2\Gamma_0)} \simeq 0.11. \quad (8)$$

This value of  $\langle r_s \rangle$  decreases only slowly with a stiffer cut on  $x$ .

The value of  $\cos \delta_s$  is physical, as it enters  $S_{f_s \gamma}$  in Eq. (3). Yet it cannot be estimated from the inclusive calculation. The reason is that the dominant contribution to  $A_R$  comes from the  $b \rightarrow s \gamma g$  amplitude generated by  $O_2$ , while to  $A_L$  from the  $b \rightarrow s \gamma$  decay generated by  $O_7$ . These are different final states, for which one

can choose the phase conventions independently. These amplitudes can still contribute to the same hadronic final states and interfere once hadronization effects are included. Thus, the relevant phase for any final state is determined by the hadronization processes and cannot be extracted from the inclusive calculation. Comparing the absorptive and dispersive parts of the inclusive result, we find  $\cos^2 \delta_s \simeq 0.3$  with small variation over  $0.75 < x < 1$ . The only conclusion we can draw here is that we expect the strong phase to be generically large.

Next we discuss inclusive  $B \rightarrow X_d \gamma$ . In Eq. (5), operators multiplying the suppressed CKM factor  $V_{ub} V_{us}^*$  were neglected. The analogous terms are important in  $b \rightarrow d \gamma$ , since in this decay the  $u$  quark loop is not CKM suppressed compared to the  $c$  and  $t$  loops. The  $b \rightarrow d \gamma$  effective Hamiltonian is of the form

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{i=1}^8 C_i(\mu) O'_i(\mu) - \frac{4G_F}{\sqrt{2}} V_{ub} V_{ud}^* \sum_{i=1}^2 C_i(\mu) [O'_i(\mu) - O'_{i,u}(\mu)], \quad (9)$$

where prime denotes that in the standard operator basis in Eq. (6) the  $s$  quark is replaced by  $d$ , and the  $u$  subscript means that  $c$  and  $\bar{c}$  are replaced by  $u$  and  $\bar{u}$ ; for example,

$$O'_{2,u} = (\bar{u} \gamma^\mu P_L b) (\bar{d} \gamma_\mu P_L u). \quad (10)$$

It is useful to define

$$r_d e^{i(\phi_d + \delta_d)} = r_s e^{i\delta_s} + \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} r_d^u e^{i\delta_d^u}, \quad (11)$$

where the first [second] term comes from the contribution of the top [bottom] line in Eq. (9) to  $A_R$ . The first term is the same as in  $b \rightarrow s$  decay (recall that we neglected  $m_s$ ).

For the  $u$  quark loop, taking the limit of the quark mass to zero, the calculation of Ref. [14] simplifies considerably, and we can obtain analytic results

$$\frac{1}{\Gamma_0} \frac{d\Gamma_{22}^{(u, \text{brem})}}{dx} = \frac{C_2^2}{C_7^2} \left\{ \frac{\alpha_s}{27\pi} 2x(2-x) + \frac{\alpha_s^2 \beta_0}{27\pi^2} \left[ \frac{x(44 - 19x + 18x^2 - 24x^3)}{12} - \frac{x(2-x)}{2} \ln[x(1-x)] + \frac{2x^3}{3} \ln x \right] \right\}, \quad (12)$$

where  $\alpha_s$  is evaluated at the scale  $m_b$  in the  $\overline{\text{MS}}$  scheme. For the difference of the rates to  $\gamma_R$  and  $\gamma_L$  we obtain

$$\frac{1}{\Gamma_0} \frac{d[\Gamma_{22}^{(u,R)} - \Gamma_{22}^{(u,L)}]}{dx} = \frac{C_2^2}{C_7^2} \frac{\alpha_s^2 \beta_0}{27\pi^2} \frac{x^2(3 - 3x + 4x \ln x)}{6}. \quad (13)$$

This difference integrates to zero, and it gives a slight  $\gamma_L$  enhancement near  $x = 1$ . We obtain for  $x > 0.75$

$$\frac{\Gamma_{22}^{(u, \text{brem})}}{\Gamma_0} \simeq 0.030. \quad (14)$$

The absolute values of the amplitudes corresponding to the  $c$  and  $u$  loops [the squares of which yield Eqs. (7) and (14)] are comparable to each other. In the absence of strong phases this would result in a cancellation and lead to a very small  $r_d^u$ . Note that, as in the  $b \rightarrow s\gamma$  case, we cannot predict the sign of  $\cos\delta_d^u$ . Moreover, the values of  $r_d^u$  and  $\delta_d^u$  are sensitive to the difference between the strong phases of the  $c$  and  $u$  loops, for which we do not consider the perturbative result reliable. In particular, the matrix element of  $O'_{2,u}$  in  $b \rightarrow d\gamma$  may have sizable long distance contributions. In any event, the short distance calculation predicts that the strong phase vanishes for the  $u$  loop contribution, while it is sizable for the  $c$  loop. Therefore, the cancellation in  $r_d^u$  is unlikely to be effective. The important point is that we expect  $r_d \sim r_d^u \sim r_s$ .

The crucial difference between the time dependent  $CP$  asymmetries in  $B \rightarrow f_s\gamma$  and  $B \rightarrow f_d\gamma$  is that in the latter case, naively, there are two very strong suppression factors. First, considering  $O_7$  only,  $S_{f_d\gamma}$  is suppressed by  $m_d/m_b$ . Second, the phase of the dominant decay amplitude,  $V_{tb}V_{td}^*$ , cancels the phase of the  $B^0 - \bar{B}^0$  mixing amplitude, yielding another strong suppression of  $S_{f_d\gamma}$ . Both of these suppressions are fictitious, since the  $V_{ub}V_{ud}^*$  ( $O_2 - O'_{2,u}$ ) contributions lift both the  $m_d/m_b$  suppression, just like in the  $b \rightarrow s\gamma$  case discussed earlier, and also the suppression coming from the cancellation of the mixing and the decay phases. The leading contribution to  $S_{f_d\gamma}$  is proportional to  $r_d^u$ , which gives a contribution to  $A_R$  with weak phase  $\gamma$ . Using the fact that the phase of the mixing amplitude is  $2\beta$ , and that of  $A_L$  is  $\beta$ , we obtain at leading order

$$S_{f_d\gamma} = -2 \left| \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right| r_d^u \cos\delta_d^u \sin(\beta + \gamma). \quad (15)$$

This result is independent at first order in  $r_d^u$  of the small direct  $CP$  violation in  $b \rightarrow d\gamma$  in the SM.

A model dependent way to connect the inclusive calculations with exclusive  $B \rightarrow K^*\gamma$  or  $\rho\gamma$  is the Ali-Greub model [16] obtained by smearing the inclusive rate with a model shape function and integrating over  $E_\gamma > (m_B^2 - 1\text{ GeV}^2)/(2m_B)$  (i.e., attributing the rate to  $m_X < 1\text{ GeV}$  to the  $K^*$  or  $\rho$ ). Restricting the shape function parameters to reproduce  $\mathcal{B}(B \rightarrow K^*\gamma)$ , we obtain  $r_{K^*} \sim 0.025$ , with little sensitivity to the model parameters. This is comparable in magnitude but independent of the  $m_s/m_b$  contribution. This model also yields  $r_\rho \sim r_{K^*}$ , which is much larger than  $m_d/m_b$ . To compare with the ‘‘more inclusive’’ measurement [6], we computed  $\langle r_s \rangle_{m_{X_s} < 1.8\text{ GeV}} \sim 0.055$  using simple shape function models. Given the model uncertainties, this figure should be taken as a very rough estimate.

### III. EXCLUSIVE $B \rightarrow K^*\gamma$ AND $B \rightarrow \rho\gamma$

We consider next the photon polarization in exclusive  $\bar{B} \rightarrow \bar{K}^*\gamma$  and  $\rho\gamma$  decays using SCET [17]. We prove that

$O_{1-6}$  contribute only to  $A_L$  to all orders in  $\alpha_s$  at leading order in  $\Lambda_{\text{QCD}}/m_b$  (this is viewed as of the same order as  $m_{K^*}/E_{K^*}$ ). We identify several types of subleading SCET operators that give amplitudes to right-handed photons, and yield  $r_{K^*}$  and  $r_\rho$  of order  $\Lambda_{\text{QCD}}/m_b$ .

Using SCET, a factorization theorem for heavy-to-light form factors has been proven at leading order in  $\Lambda_{\text{QCD}}/m_b$  [18–20]. There are two contributions to the form factors, a nonfactorizable (or soft or form-factor) part and factorizable (or hard scattering) part, which are of the same order in  $\Lambda_{\text{QCD}}/m_b$ .

The exclusive radiative decays considered here were analyzed in an expansion in  $\Lambda_{\text{QCD}}/m_b$  in [9, 21–23], and recently in SCET [24]. The nonfactorizable part receives its dominant contribution from  $O_7$  and gives rise to  $A_L$  only. The operator  $O_2$  enters at leading order only via factorizable contributions.

When operators  $O_{i \neq 7}$  are included, the hierarchy of the relevant scales gets rather complicated, since  $m_b^2 > m_c^2 \sim \Lambda_{\text{QCD}} m_b > \Lambda_{\text{QCD}}^2$ . Ref. [24] assumed that the  $c\bar{c}$  loop is dominated by hard loop momenta and can be integrated out near the scale  $m_b$ . Here we adopt a simpler approach, by neglecting the charm mass and assuming that its effects can be included as a perturbation using the formalism of Ref. [25] without encountering singularities. This assumption is borne out by the explicit one- and two-loop calculations.

The effective Hamiltonian in Eq. (5) is matched in SCET<sub>I</sub> onto

$$H_{\text{eff}} = \frac{G_F V_{tb} V_{ts}^* e}{\sqrt{2} \pi^2} E_\gamma \left[ c(\omega) \bar{s}_{n,\omega} \mathcal{A}^\perp m_b P_L b_v \right. \quad (16) \\ \left. + b_{1L}(\omega_i) O^{(1L)}(\omega_i) + b_{1R}(\omega_i) O^{(1R)}(\omega_i) + \mathcal{O}(\lambda^2) \right],$$

where the  $\omega$ 's are the usual collinear label momenta. The relevant modes are soft quarks and gluons with momenta  $k_s \sim \Lambda$  and two types of collinear quarks and gluons along  $n$  and  $\bar{n}$  (including charm, which can be soft or collinear). Note that the approach of Ref. [24], treating  $m_c \sim m_b$ , will likely require additional modes. We take the photon momentum as  $q_\mu = E_\gamma \bar{n}_\mu$ , the collinear  $s$  quark to move along  $n_\mu$ , and  $\mathcal{A}_\perp^\perp$  denotes the transverse photon field. The operator in the first line in Eq. (16) occurs at leading order in the expansion parameter,  $\lambda = \sqrt{\Lambda/m_b}$ , and its Wilson coefficient is dominated by  $C_7$ ,  $c(\omega) = C_7 + \mathcal{O}[\alpha_s(m_b)]$ . The operators in the second line,

$$O^{(1L)}(\omega_1, \omega_2) = \bar{s}_{n,\omega_1} \mathcal{A}^\perp \left[ \frac{1}{\bar{n} \cdot \mathcal{P}} ig\mathcal{B}_n^\perp \right]_{\omega_2} P_R b_v, \\ O^{(1R)}(\omega_1, \omega_2) = \bar{s}_{n,\omega_1} \left[ \frac{1}{\bar{n} \cdot \mathcal{P}} ig\mathcal{B}_n^\perp \right]_{\omega_2} \mathcal{A}^\perp P_R b_v, \quad (17)$$

are the only SCET<sub>I</sub> operators suppressed by  $\lambda$  that couple to a transverse photon and are allowed by power counting and  $s$  chirality. Here  $ig\mathcal{B}_n^\nu \equiv [\bar{n} \cdot iD_c, iD_{c\perp}^\nu]$  is the collinear gluon field tensor; for the remaining notations see Ref. [26]. The operators  $O^{(1L)}$  and  $O^{(1R)}$  couple

only to  $\gamma_{L,R}$ , respectively. Their Wilson coefficients are

$$\begin{aligned} b_{1L}(\omega_1, \omega_2) &= C_7 + C_2/3 + \mathcal{O}[C_{3-6}, \alpha_s(m_b)], \\ b_{1R}(\omega_1, \omega_2) &= -C_2/3 + \mathcal{O}[C_{3-6}, \alpha_s(m_b)]. \end{aligned} \quad (18)$$

Although the operators in the first and second lines of Eq. (16) are of different orders in  $\lambda$ , after matching onto SCET<sub>II</sub>, they contribute at the same order in  $\Lambda_{\text{QCD}}/m_b$  to the  $B \rightarrow K^*\gamma$  amplitude. The leading order  $c(\omega)$  term gives the nonfactorizable contribution, which only contributes to  $A_L$ , while  $O^{(1L)}$  and  $O^{(1R)}$  give the factorizable contributions through time-ordered products with the ultrasoft-collinear Lagrangian,  $\mathcal{L}_{\xi q}^{(1)}$  [27]. After matching onto SCET<sub>II</sub> one finds, schematically

$$\begin{aligned} \int d^4x T\{O^{(1L)}(\omega_i), i\mathcal{L}_{\xi q}^{(1)}(x)\} &\rightarrow \int dk_+ \mathcal{J}_\perp(\omega_i^{(l)}, k_+) \\ &\otimes [(\bar{q}Y_n)_{k_+} \not{n} \mathcal{A}^\perp \gamma_\perp^\alpha P_R(Y_n^\dagger b_v)] (\bar{s}_{n,\omega'_1} \not{n} \gamma_\perp^\alpha q_{n,\omega'_2}), \quad (19) \\ \int d^4x T\{O^{(1R)}(\omega_i), i\mathcal{L}_{\xi q}^{(1)}(x)\} &\rightarrow \int dk_+ \mathcal{J}_\parallel(\omega_i^{(l)}, k_+) \\ &\otimes [(\bar{q}Y_n)_{k_+} \not{n} \mathcal{A}^\perp P_R(Y_n^\dagger b_v)] (\bar{s}_{n,\omega'_1} \not{n} P_L q_{n,\omega'_2}), \quad (20) \end{aligned}$$

where  $\mathcal{J}_{\perp,\parallel}$  are jet functions that have expansions in  $\alpha_s(\sqrt{\Lambda_{\text{QCD}}m_b})$ . The operator in Eq. (19) contributes to  $B \rightarrow K^*\gamma$  at leading order,

$$\begin{aligned} A_L &= \frac{G_F V_{tb} V_{ts}^* e}{\sqrt{2} \pi^2} \frac{m_B^3}{2} \left[ c(m_B) \zeta_\perp^{BK^*} + \frac{f_B f_{K^*}^\perp}{m_B} \int dx dt dk_+ \right. \\ &\times b_{1L}(m_B(1-t), m_B t) \mathcal{J}_\perp(x, t, k_+) \phi_{K^*}^\perp(x) \phi_B^\perp(k_+) \left. \right], \quad (21) \end{aligned}$$

where all nonperturbative matrix elements are defined as in [26]. The operator in Eq. (20) only gives rise to longitudinal  $K^*$  and therefore does not contribute to  $B \rightarrow K^*\gamma$ . This proves that  $O_2$  contributes at leading order in  $\Lambda_{\text{QCD}}/m_b$  to  $\bar{B} \rightarrow \bar{K}^*\gamma_L$  via Eq. (19), but its contribution to  $\bar{B} \rightarrow \bar{K}^*\gamma_R$  vanishes at this order. (Interestingly, it would contribute to  $\bar{B}^* \rightarrow \bar{K}^{(*)}\gamma_R$  at leading order.) The same proof also holds for the other four-quark operators,  $O_{1-6}$ .

This result agrees with the  $\mathcal{O}(\alpha_s)$  computation [21, 22], and extends it to all orders in  $\alpha_s$ . This is important, since  $\alpha_s(\mu)$  at the hard-collinear scale  $\mu^2 \sim \Lambda m_b$  may or may not be perturbative [18, 28, 29]. The suppression of  $A_R$  can be understood from a simple helicity argument. Since the  $s$  quark is left-handed, the  $K^*$  cannot be right-handed, unless additional right-handed gluons end up in the final meson. However, the contributions of higher Fock states are power suppressed.

The  $\bar{B} \rightarrow \bar{K}^*\gamma_R$  amplitude does arise at subleading order in  $\Lambda_{\text{QCD}}/m_b$ . There are several sources of such corrections. For example, (i) time ordered products of  $O^{(1R)}$  with the subleading collinear Lagrangian  $\mathcal{L}_{\xi\xi}^{(n \geq 1)}$ , which lead to factorizable contributions similar to Eq. (19) containing an explicit factor  $\alpha_s(\sqrt{\Lambda_{\text{QCD}}m_b})$ ; (ii) higher order terms in the SCET<sub>I</sub> effective Lagrangian in Eq. (16),

such as the  $\mathcal{O}(\lambda^2)$  operator

$$\begin{aligned} O^{(2R)}(\omega) &= \int \frac{dx_+}{2\pi} dk_- e^{-ix_+k_-/2} \kappa \left( \frac{2E_\gamma k_-}{m_c^2} \right) \\ &\times \bar{s}_{n,\omega} [Y_n^\dagger i\mathcal{D}^{\text{us}} Y_n] \left( \frac{\bar{n}x_+}{2} \right) \mathcal{A}_\perp P_R b_v, \quad (22) \end{aligned}$$

where the gauge invariant operator  $[Y_n^\dagger i\mathcal{D}^{\text{us}} Y_n](x)$  contains Wilson lines of the ultrasoft fields in the  $\bar{n}$  direction.  $O^{(2R)}$  is obtained by matching the graph in Fig. 1 with one ultrasoft gluon, and its Wilson coefficient is related to  $b_{1R}$  in Eq. (18) by reparameterization invariance [30]. The  $C_2/3$  terms in Eq. (18) correspond to  $m_c = 0$ , when  $\kappa(z) = 1/2$ . In this case the  $k_-$  and  $x_+$  integrals in Eq. (22) can be performed trivially. However, in Eq. (22) we kept  $m_c \neq 0$  to exhibit the analogy with the nonperturbative corrections to inclusive  $b \rightarrow s\gamma$  decay [31, 32]. For  $m_c \neq 0$ , additional nonlocality is introduced by  $\kappa(z) = 1/2 - 2 \arctan[\sqrt{z/(4-z)}]^2/z$  with  $z = 2E_\gamma k_-/m_c^2$  [12]. [For  $m_c \neq 0$ , the  $C_2/3$  terms in Eq. (18) should be multiplied by  $2\kappa(2E_\gamma\omega_2/m_c^2)$ .] Expanding  $O^{(2R)}$  in  $x_+$  reproduces the series of operators proportional to  $(m_b\Lambda_{\text{QCD}}/m_c^2)^n$  in Eq. (5) in [32]. Time-ordered products of  $O^{(2R)}$  with the ultrasoft-collinear Lagrangian,  $\mathcal{L}_{\xi q}^{(2)}$ , give  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$  soft contributions to  $A_R$ , with no  $\alpha_s$  suppression compared to  $A_L$ .

A complete study of these subleading contributions is rather involved and we leave it for future work. It seems unlikely to us that a cancellation could result in a suppression of  $r_{K^*}$  and  $r_\rho$  to order  $\Lambda_{\text{QCD}}^2/m_b^2$ . This leads to the dimensional estimate

$$r_{K^*} \sim \frac{1}{3} \frac{C_2}{C_7} \frac{\Lambda_{\text{QCD}}}{m_b} \sim 0.1. \quad (23)$$

This effect dominates over the  $m_s$  piece of  $O_7$  for  $r_{K^*}$ . The estimate for  $r_\rho$  is more involved because of the contributions with different weak phases,

$$r_\rho \sim r_{K^*} \left[ 1 + \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \left( C_{\text{loop}} \frac{m_c^2}{m_b^2} + C_{\text{WA}} 4\pi \frac{\Lambda_{\text{QCD}}}{m_b} \right) \right], \quad (24)$$

The  $C_{\text{loop}}$  term comes from the non-cancellation of the  $c$  and  $u$  loops, and we expect it to have a numerically large coefficient. The  $C_{\text{WA}}$  term arises from weak annihilation, whose contribution to  $A_R$  at order  $\Lambda_{\text{QCD}}/m_b$  vanishes [9]. The latter contributes significantly only in  $B^\pm$ , while in  $B^0$  it is color suppressed. Thus, we expect that the SM prediction for  $S_{\rho\gamma}$  is not much smaller than it is for  $S_{K^*\gamma}$ .

For higher mass one-body hadronic final states,  $A_R$  still vanishes at leading order, but the suppression by  $m_X/E_X$  is expected to be less effective as  $m_X$  increases (although there is no evidence for this in the  $B \rightarrow DX$  data [33]). Thus, the SM value of  $r$  is expected to depend on the final state. For high-mass and multi-body final states  $A_R$  may arise, formally, at leading order in  $\Lambda_{\text{QCD}}/m_b$ . For example,  $\bar{B} \rightarrow \bar{B}^*\pi_{(\text{soft})}$  followed by  $\bar{B}^* \rightarrow \bar{K}^{(*)}\gamma_R$  can give rise to  $\bar{B}^0 \rightarrow K_S\pi^0\gamma_R$  with  $m_{K_S\pi^0} \sim \sqrt{\Lambda_{\text{QCD}}m_b}$ , without a  $\Lambda_{\text{QCD}}/m_b$  suppression.

Therefore, averaging the results of  $B \rightarrow K^*\gamma$  [3, 4] with  $B \rightarrow K_S\pi^0\gamma$  [6] is not free from theoretical uncertainties.

#### IV. DISCUSSION AND SUMMARY

We studied the standard model prediction for the  $\bar{B} \rightarrow X_{s,d}\gamma_R$  decay amplitude with a right-handed photon,  $A_R$ , compared to the amplitude with a left-handed photon. Considering only  $O_7$ , their ratio is  $r_{s,d} = m_{s,d}/m_b$ , which reproduces the often-quoted prediction independent of the hadronic final state. However, including the other operators,  $O_{i \neq 7}$ ,  $A_R$  becomes much larger than this naive estimate, and hadronic physics gives rise to sizable uncertainties. The time dependent  $CP$  asymmetries also become sensitive to the strong phase.

In inclusive  $B \rightarrow X_S\gamma$  and  $X_d\gamma$  decays,  $A_R$  is only suppressed by  $g_s/(4\pi)$ . We calculated  $r$  inclusively, and found it to be of order 0.1 depending on the cut on the photon energy [see Eq. (8)]. While this calculation is reliable, it cannot be easily compared to data. If one restricts the hadronic final state, such as in the measurement of the time dependent  $CP$  asymmetry  $S_{K_S\pi^0\gamma}$  with a range of  $K_S\pi^0$  invariant masses, then that can no longer be calculated using inclusive methods. Still, our results indicate that it would be hard to argue that a measurement of  $S_{K_S\pi^0\gamma} \sim 0.1$  cannot be due to SM physics.

In exclusive  $\bar{B} \rightarrow \bar{K}^*\gamma$  and  $\rho\gamma$  decays, we proved using SCET that  $A_R$  vanishes at lowest order in the  $\Lambda_{\text{QCD}}/m_b$  expansion to all orders in  $\alpha_s$ . The leading contribution to  $A_R$  is formally of order  $\Lambda_{\text{QCD}}/m_b$ , but numerically it is enhanced by  $C_2/C_7$  [see Eqs. (23) and (24)]. The result depends on unknown hadronic matrix elements, which

give rise to a sizable uncertainty in the SM prediction.

Both the inclusive and the exclusive calculations predict that  $r_s$  and  $r_d$  are comparable to each other in the SM. Thus, we also expect the time dependent  $CP$  asymmetries in  $\bar{B}^0 \rightarrow f_d\gamma$  to be comparable to those in  $\bar{B}^0 \rightarrow f_s\gamma$  (except for a modest suppression by  $|V_{ub}/V_{td}|$ ), in contrast to what has been widely believed. We estimate these asymmetries in decays such as  $B \rightarrow K^*\gamma$ ,  $\rho\gamma$ ,  $K_S\pi^0\gamma$ , and  $\pi^+\pi^-\gamma$  to be of order 0.1, with large uncertainties.

To conclude, our main result is that the standard model prediction for  $A_R/A_L$  is of order 0.1, with sizable uncertainty. A measurement of it much above this level would indicate new physics. To draw conclusions from a smaller value would require a more complete analysis and knowledge of hadronic matrix elements including strong phases. More effort in this direction would be welcome.

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- [1] D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. **79**, 185 (1997) [hep-ph/9704272].
  - [2] T. Mannel and S. Recksiegel, Acta Phys. Polon. B **28**, 2489 (1997) [hep-ph/9710287]; Y. Grossman and D. Pirjol, JHEP **0006** (2000) 029 [hep-ph/0005069]; M. Gronau, Y. Grossman, D. Pirjol and A. Ryd, Phys. Rev. Lett. **88** (2002) 051802 [hep-ph/0107254]; G. Hiller and A. Kagan, Phys. Rev. D **65**, 074038 (2002) [hep-ph/0108074]; M. Gronau and D. Pirjol, Phys. Rev. D **66** (2002) 054008 [hep-ph/0205065].
  - [3] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93** (2004) 201801 [hep-ex/0405082].
  - [4] K. Abe *et al.* [Belle Collaboration], hep-ex/0409049.
  - [5] A. G. Akeroyd *et al.* [SuperKEKB Physics Working Group Collaboration], hep-ex/0406071.
  - [6] K. Abe *et al.* [Belle Collaboration], hep-ex/0411056.
  - [7] D. Atwood, T. Gershon, M. Hazumi and A. Soni, hep-ph/0410036.
  - [8] O. Long, private communications.
  - [9] B. Grinstein and D. Pirjol, Phys. Rev. D **62** (2000) 093002. [hep-ph/0002216].
  - [10] L. M. Sehgal and J. van Leusen, Phys. Lett. B **591** (2004) 235 [hep-ph/0403215].
  - [11] B. Grinstein, R. P. Springer and M. B. Wise, Phys. Lett. B **202** (1988) 138; Nucl. Phys. B **339** (1990) 269.
  - [12] A. Ali and C. Greub, Z. Phys. C **49** (1991) 431.
  - [13] N. Pott, Phys. Rev. D **54** (1996) 938 [hep-ph/9512252].
  - [14] Z. Ligeti, M. E. Luke, A. V. Manohar and M. B. Wise, Phys. Rev. D **60** (1999) 034019 [hep-ph/9903305].
  - [15] A. J. Buras, M. Misiak, M. Munz and S. Pokorski, Nucl. Phys. B **424** (1994) 374 [hep-ph/9311345].
  - [16] A. Ali and C. Greub, Phys. Lett. B **259** (1991) 182.
  - [17] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001) [hep-ph/0005275]; C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001) [hep-ph/0011336]; C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001) [hep-ph/0107001]; C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65**, 054022 (2002) [hep-ph/0109045].
  - [18] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003) [hep-ph/0211069].
  - [19] M. Beneke and T. Feldmann, Nucl. Phys. B **685**, 249 (2004) [hep-ph/0311335].
  - [20] B. O. Lange and M. Neubert, Nucl. Phys. B **690**, 249 (2004) [hep-ph/0311345].
  - [21] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B

- 612** (2001) 25 [hep-ph/0106067].
- [22] S. W. Bosch and G. Buchalla, Nucl. Phys. B **621** (2002) 459 [hep-ph/0106081].
- [23] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C **23** (2002) 89 [hep-ph/0105302].
- [24] J. g. Chay and C. Kim, Phys. Rev. D **68**, 034013 (2003) [hep-ph/0305033].
- [25] A. K. Leibovich, Z. Ligeti and M. B. Wise, Phys. Lett. B **564**, 231 (2003) [hep-ph/0303099].
- [26] D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003) [Erratum-ibid. D **69**, 019903 (2004)] [hep-ph/0211251].
- [27] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643**, 431 (2002) [hep-ph/0206152].
- [28] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70** (2004) 054015 [hep-ph/0401188].
- [29] R. J. Hill, T. Becher, S. J. Lee and M. Neubert, JHEP **0407** (2004) 081 [hep-ph/0404217]; T. Becher and R. J. Hill, JHEP **0410** (2004) 055 [hep-ph/0408344].
- [30] A. V. Manohar, T. Mehen, D. Pirjol and I. W. Stewart, Phys. Lett. B **539** (2002) 59 [hep-ph/0204229].
- [31] M. B. Voloshin, Phys. Lett. B **397** (1997) 275 [hep-ph/9612483].
- [32] Z. Ligeti, L. Randall and M. B. Wise, Phys. Lett. B **402** (1997) 178 [hep-ph/9702322].
- [33] Z. Ligeti, M. E. Luke and M. B. Wise, Phys. Lett. B **507** (2001) 142 [hep-ph/0103020]; C. W. Bauer, B. Grinstein, D. Pirjol and I. W. Stewart, Phys. Rev. D **67** (2003) 014010 [hep-ph/0208034].