Review of *Oppositions and Paradoxes: Philosophical Perplexities in Science and Mathematics*, by John L. Bell. Peterborough, ON: Broadview Press (2016). Paper, 202p., $29.95.

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The distinguished logician and philosopher of mathematics John L. Bell is best known for his technical work on the foundations of mathematics, particularly in the fields of topos theory and model theory. The author of over a dozen books, he was elected a Fellow of the Royal Society of Canada in 2009. His *Toposes and Local Set Theories: An Introduction* (Oxford, UK: Oxford University Press, 1988) is a classic text on the subject—so much so that it was reprinted by Dover Books in 2008 for mass-market consumption.

Bell's recent (2016) book, *Oppositions and Paradoxes: Philosophical Perplexities in Science and Mathematics*, has a different character. Here Bell revisits some of the central puzzles in the history of philosophical thought – including themes, such as the status of infinitesimals, that Bell has returned to repeatedly throughout his career – from a non-technical perspective. Readers will not find new theorems here (though some of the analyses Bell offers were unfamiliar to me, and edifying); instead, the principal contributions come in the form of clear and accessible exposition and sometimes surprising links drawn between different areas of inquiry.

The book consists of seven chapters and four appendices. Its principal focus is on topics in late 19th and 20th century mathematics and physics, but it begins with much older puzzles. The first chapter, "The Continuous and the Discrete", treats of incommensurable magnitudes (as discovered by the Pythagoreans), Zeno's paradoxes, and problems facing any theory of infinitesimals, among other topics. I cannot speak to the historical accuracy of the discussions of Ancient philosophers, but the conceptual issues are treated very clearly. The next chapter jumps ahead a few millennia, to a subtle and engaging discussion of the classical set theoretic paradoxes, such as Russell's paradox, and other foundational issues in set theory, such as the transfinite hierarchy and the axiom of choice.

These two chapters are the strongest in the book, for several reasons. They provide novel and clear insights into truly deep conceptual problems, without introducing any technical apparatus. These chapters also make a compelling case for the relationship between the "Oppositions" – basically, opposing concepts, such as the Continuous and the Discrete or the Relative and the Absolute – and the "Paradoxes" appearing in the books title: briefly, paradoxes, Bell suggests, tend to arise when we are confronted with incompatible (or opposed) concepts that both seem essential to capturing some aspect of the world. For instance, Bell motivates the paradoxes of late 19th and early 20th century set theory as arising out of attempts to understand when "the many" may be treated as (forming) "one" entity. The opposition, here, is between the Many and the One; the paradoxes, of course, arise because not every collection (a "many") can be treated uniformly as a set ("one"). Experts in set theory are unlikely to find anything surprising in this discussion, but

they are not the audience, and I think the way of thinking that Bell offers will appeal to the interested novice.

Chapter 3 covers non-Euclidean geometry. The treatment is again perspicuous and accessible, but it is difficult to see how it relates to the principal themes of the book. After all, non-Euclidean geometry is a consistent theory, and unlike (for instance) early axiomatizations of set theory, it was not introduced to resolve contradictions in a naïve theory. This is not to say that the history of attempts to prove the Parallel Postulate is not philosophically rich and important; it is just less clear that that history is naturally conceived in terms of paradoxes or oppositions.

Chapter 4, meanwhile, discusses time travel, mostly through a careful analysis of science fiction scenarios. Here it is clear that paradoxes *do* arise, though Bell's focus is on how some apparent paradoxes can in fact be resolved if one allows time to "branch" as needed. For my own part, I would have liked to see more discussion of the constraints that self-consistency impose on time travel in a single "timeline" rather than so much attention paid to the physically (and metaphysically) ludicrous notion of time "splitting" merely to accommodate inconsistencies.

For roughly the first half of the book – in effect, these first four chapters – I felt that this would be the perfect introduction to more technical issues in philosophy (albeit presented at a conceptual level) for a bright high school student; it could also be highly appropriate for a small undergraduate seminar that could engage students with little background in philosophy. The exposition is clear and accurate, and the issues are discussed in a compelling way. The writing might not sustain the interest of a reader unwilling to engage seriously with the ideas, but it would deeply reward the focused novice. The final three chapters, however, struck me as much more problematic, and even misleading.

These chapters concern conceptual problems in 20th century physics. Chapter 5 covers special and general relativity; Chapter 6 is on quantum theory; and Chapter 7 is on cosmology. Each chapter offers a fairly standard exposition of the relevant physics, with a focus on thought experiments designed to highlight some of the ways in which these theories are surprising or counterintuitive. For instance, Chapter 5 discusses the fact that according to special relativity, were one of a pair of twins to travel away from earth at a substantial fraction of the speed of light, and then later return to earth, they would find themselves younger (in the sense of having experienced less elapsed time) than the twin who remained on earth. Likewise, Chapter 6 discusses, among many other things, Schrödinger's Cat, which, according to some interpretations of quantum theory, would find itself in a strange state of being neither alive nor dead.

The problem with these chapters is not exactly that the physics is incorrect. Rather, the issue is that these thought experiments are presented as *paradoxes*, or even putative paradoxes, whereas in fact they are simply novel and surprising predictions of contemporary physical theory. This is not to say that there are no paradoxes (or inconsistencies) in modern physics – quantum theory, in particular, is arguably inconsistent or incomplete as formulated in many textbooks, because it does not clearly distinguish between when two incompatible dynamics occur – but

rather that the mere fact that a theory defies classical expectations does not *ipso facto* render it paradoxical. At very least, there is a world of difference between, for instance, Russell's paradox, which reveals a true inconsistency in informal set theory, and the "twin paradox", which simply describes some consistent (and, apparently, true) facts about the physical world. Labeling both as "paradoxes" in a book on precisely that topic is a flaw that makes the book significantly less attractive as recommended reading its intended audience of non-specialist readers.