

UC Berkeley

Research Reports

Title

Some Theoretical Aspects Of The Benefits Of En-route Vehicle Guidance (ervg)

Permalink

<https://escholarship.org/uc/item/0c83x3gr>

Authors

Al-deek, Haitham M.
Kanafani, Adib K.

Publication Date

1989

Institute of Transportation Studies
University of California at Berkeley

PROGRAM ON ADVANCED TECHNOLOGY
FOR THE HIGHWAY

**Some Theoretical Aspects of the Benefits of
En-Route Vehicle Guidance (ERVG)**

**Haitham M. Al-Deek
Adib Kanafani**

PATH Research Report UCB-ITS-PRR-89-2

Prepared in cooperation with the State of California, Business
and Transportation Agency, Department of Transportation.

The contents of this report reflect the views of the authors who
are responsible for the facts and accuracy of the data presented
herein. The contents do not necessarily reflect the official views
or policies of the State of California. This report does not con-
stitute a standard, specification, or regulation.

August 1989
Revised March 1990

This paper has been mechanically scanned. Some errors may have been inadvertently introduced.

TABLE OF CONTENTS

ABSTRACT	vi
CHAPTER 1 - INTRODUCTION	1
A. Background.....	1
B. The Problem.....	3
C. Objective.....	3
CHAPTER 2 - STUDY APPROACH	5
A. Introduction.....	5
B. Continuum Analysis of Urban Transportation Networks	7
C. Basic Idea.....	8
CHAPTER 3 - IDEALIZED CORRIDORS FOR ANALYSIS	
CASE-I: LONG FREEWAY CORRIDOR	9
A. User Equilibrium Assignment Case-I	11
B. System Optimal Assignment Case-I	13
C. Sensitivity Analysis	14
D. Some Extensions	17
1. Travel Across the Freeway.....	17
2. Parallel Freeways.....	18

CHAPTER 4 - IDEALIZED CORRIDORS FOR ANALYSIS	
CASE-II: SINGLE DESTINATION CORRIDOR	21
A. User Equilibrium Assignment Case-II.....	23
1. Solution Algorithm for (UE) Assignment of Case-II.....	24
2. Numerical Example.....	26
B. System Optimal Assignment Case-II.....	28
CHAPTER 5 - CONCLUSIONS	33
REFERENCES	35
APPENDICES	37
Appendix -A- : Shed Boundary $y_1(x)$ for User Equilibrium Assignment Case-II.....	37
Appendix -B-: Derivation of $F_1(x)$ and $y_1(x)$ Formulas in the Numerical Example.....	39
Appendix -C-: Shed Boundary $y_2(x)$ for system Optimal Assignment Case-II.....	41

LIST OF FIGURES

Figure (3-1) Case-I : Traveller Path Choices	9
Figure (3-2) Case-II : Traveller Path Choices	10
Figure (3-3) User Optimal Shed Boundary y_1	11
Figure (3-4) Percent Time Savings Vs. Freeway Capacity.....	15
Figure (3-5) Percent Time Savings Vs. Trip Length.....	15
Figure (3-6) Percent Time Savings Vs. Speed on Surface Streets.....	16
Figure (3-7) Backtracking Trips (Type <i>I</i>) and Cross Freeway Trips (Type <i>II</i>)	17
Figure (3-8) System of Parallel Freeways and Optimal Spacing	19
Figure (4- 1) Cumulative Flow $F(x)$ and Shed Boundary $y(x)$ Functions (single Destination Corridor).....	22
Figure (4-2) Solution Algorithm for (UE) Assignment Case-II	25
Figure (4-3) Sequence of Algorithm Iterations	27
Figure (4-4) Algorithm Convergence.....	28
Figure (4-5) Derivation of Total Cost of Using the Corridor in Case-II.	29
Figure (4-6) Finding Optimal Shed Boundary $y_2(x)$ in Case-II.....	31
Figure (4-7) Benefits of (ERVG) as Related to Shed Boundaries $y_1(x)$ and $y_2(x)$ of Case-II.....	32

ABSTRACT*

This paper concerns the benefits from vehicle route guidance in urban networks. We suppose that vehicle routes can be altered by En-Route Vehicle Guidance (ERVG) in such a way as to achieve system optimal assignment. Benefits are measured by the savings in total travel time for a given demand when comparing this assignment with the user equilibrium, which is assumed to occur in the absence of route guidance. A continuum approach is used to analyze some idealized corridors in which a freeway is superimposed over a dense grid of surface streets. Two cases are considered: in the first, the freeway corridor is a distributor and is long compared to average trip length. In the second case the corridor is considered as a link to an end point such as a CBD. In the first case trips of length L are within the corridor and freeway flow along the corridor is constant. In the second, with all trips destined to the end point, the flow on the freeway accumulates as that point is approached. The main role of ERVG in both cases is to divert traffic from the freeway whenever its marginal cost exceeds that of the street system.

It is found that travel time savings of the order of 3-4% can be achieved from route guidance. Benefits are quite sensitive to city street speed. At low speed more users would choose the freeway resulting in congestion, and the potential benefits of route guidance are relatively high. But as street speed increases and approaches that of the freeway route guidance would be of less value as more of the motorists would be choosing the city street on their own. Route guidance benefits can be enhanced if information is customized to motorists on the basis of their origins and destinations. Finally, it is shown that route guidance benefits are reduced when the freeway network is dense. It is recommended that future research should focus on potential opportunities for using ERVG technology in managing networks under conditions of non-recurring congestion (accidents/incidents). This paper does not consider important aspects of the evaluation of route guidance, such as the equity issue stemming from increasing some trip times in order to achieve system optimum, or the local impact of diverted traffic.

* The authors greatly acknowledge the helpful discussions with Professor Pravin Varaiya (Department of Electrical Engineering and Computer Science at the University of California, Berkeley). The authors would also like to thank Steven Shladover, PATH Technical Director, for revising an earlier version of this report.

CHAPTER 1

INTRODUCTION

A. **Background**

The idea of using real time traffic information to guide vehicles through an urban network has gained a lot of attention in recent years. It has been thought that route guidance is capable of improving the performance of urban transportation networks. En-Route Vehicle Guidance (ERVG) provides information to the driver that suggests a route to be followed between the driver's origin and destination. The route given to the driver is based on minimizing some cost function. It can be based on the minimum distance path between the driver's origin and his destination or on the path of minimum travel time under existing network conditions. *

The concept of ERVG is that, with information transmitted to drivers, it might be possible to shift traffic assignment from user equilibrium (UE) to system optimal (SO) assignment such that available capacity in the network is utilized**. In a typical urban area in which the choice exists between the freeway system and the street network, it is often observed that congestion on the freeways rises while there is excess capacity on the street network. Under these conditions the marginal cost on the freeway will at some point exceed that on the city streets and it becomes beneficial, at least from the total system point of view, to divert traffic off the freeway.

This scenario, which is the subject of this study, is one of many where route guidance might be beneficial. Another, probably more important opportunity for the use of this technology is in managing networks under conditions of non-recurring congestions, i.e accidents that temporarily shut down segments of the network.

* In addition, it is possible to develop some generalized cost function that includes travel time as well as other cost components such as vehicle operating cost (wear and tear) and use it as a basis for selecting the cheapest path.

• * Both (UE) and (SO) assignments will be discussed later in chapter 2.

As the idea of route guidance gathers momentum and as experiments and demonstration projects proliferate, it has become clear that some sort of assessment of benefits from this technology is needed. Numerous attempts have been made to evaluate these benefits. In one of the earlier studies at the TRRL, Jeffries (1981a) estimated that between 8% and 10% of the vehicle miles can be saved if people do in fact take the shortest paths between origins and destinations. In later studies King (1987) estimated U.S losses in billions of dollars caused by excess travel time experienced by drivers who are unfamiliar with the network, wastage that can be recovered if drivers are guided properly to their destinations. However, estimates made by King did not emphasize benefits of real time traffic information systems. Jeffries (1987a) estimates of benefits of ERVG were based on drivers unfamiliar with the network under recurring and non-recurring congestion. Jeffries results, however, may not be valid for the U.S networks, simply because his results are network specific.

In earlier PATH studies of the potential benefits of real time traffic information systems Al-Deek, et. al. (1988 and 1989) simulated traffic conditions in the SMART Corridor in Los Angeles. In these studies it was demonstrated that the benefits of diverting traffic are limited in the case of recurring congestion when only a small percentage of the vehicles are equipped with information of route guidance.

This report documents a phase of an on-going research activity of which the main goal is to evaluate network level benefits of ERVG and its feasibility to various network conditions. It is recognized that there is a need for the assessment of system benefits. System benefits can be thought of in two ways. One is the total benefits to drivers equipped with information and benefits to drivers without information. The other is total benefits from the supplier or system manager perspective. This relates to the question of whether public money should be invested in the ERVG technology and if so how much public benefit this will produce. The customer decision to buy or not to buy an ERVG product depends on whether the customer perceives ERVG as being worthwhile or not. The decision to spend public dollars on ERVG depends in addition on whether ERVG is worthwhile from a system perspective.

B. The Problem

The problem addressed in this study seems to have more than one aspect. We have just recognized the need for a comprehensive evaluation of system benefits which includes technology users, nonusers and infrastructure suppliers (e.g public agencies).

Based on system and user benefits and costs, two important questions to answer are *where* and *when* is it most likely beneficial to use advanced driver information systems for route guidance? The “*where*” question refers to the configuration of the urban network and the sensitivity of route guidance to this configuration. The “*When*” question refers to traffic conditions (recurring and non-recurring congestion) under which ERVG is applied. The importance of the first question, for example, becomes obvious as one compares a network grid composed of a freeway with several parallel arterials or alternate routes (such as Los Angeles basin) with another network that has a single route (e.g a bridge or a tunnel) to reach destination (such as the Oakland-San Francisco Bay Bridge). Since driver options in the latter are limited there would be little or no benefits of ERVG.

C. Objective

The objective of this study is to explore some theoretical aspects of the benefits from route guidance. Sensitivity analysis is used to achieve an understanding of the relationships between potential benefits and network parameters used such as speed on the surface street network and trip length. In this research study analysis will be generic using an example of a rectangular shape network. The motivation is that if some general results can be found concerning this subject, then it would be possible to focus work on benefit estimation and to permit relaxing the many restrictive assumptions that have so far been made in this field.

CHAPTER 2

STUDY APPROACH**A. Introduction**

A quick review of the research on traffic assignment in urban networks is presented. This is followed by mathematical formulations of the user optimal and system optimal assignments as used in conventional analysis of assigning flows to network links.

Research on traffic assignment in highway networks is as old as highway networks themselves. The notion of traffic equilibrium assignment was described in a simple intuitive example by Knight (1924). In Beckmann et. al. (1976), Knight said (we quote):

“Suppose that between two points there are two highways, one of which is broad enough to accommodate without crowding all the traffic which may care to use it, but is poorly graded and surfaced, while the other is a much better road but narrow and quite limited in capacity. If a large number of trucks operate between the two termini and are free to choose either of the two routes, they will tend to distribute themselves between the roads in such proportions that the cost per unit of transportation, or effective return per unit of investment, will be the same for every truck on both routes. As more trucks use the narrower and better road, congestion develops until at a certain point it becomes equally profitable to use the broader but poorer highway”.

The above quoted example of one origin and one destination connected by two routes can be intuitively expanded to a network with many origins, many destinations, and many links, therefore resulting in a large number of possible routes.

The general principle of network equilibrium was introduced by Wardrop (1952), where he mentioned that under network equilibrium conditions all drivers on all used routes between a certain origin-destination pair in the network experience the same travel time and so no driver can improve his journey time by shifting to another used route. All unused routes, therefore, must have travel time greater than that of the used routes. This results from the assumptions that each driver seeks for the minimum travel time path

between his origin and his destination. Consequently used routes are actually the shortest paths between origins and destinations.

The flows resulted from the network equilibrium assignment are called “user optimized” flows and the network equilibrium assignment is also known as user-optimum traffic assignment. The (UE) traffic assignment was developed as a mathematical optimization problem by Beckmann et. al. (1956). In a glance we would like to formulate equations for the user optimal traffic assignment according to Beckmann.

Consider a transportation network of N nodes and K links. A node represents an origin, a destination or an intersection of streets. The user optimum (UE) assignment is formulated as following:

$$\text{Minimize } Z_1(f) = \sum_K \int_0^{f_k} C_k(w) \mathbf{d}w \quad (2-1)$$

Subject to:

1. Flow conservation constraints:

If there are q_{od} trips that need to travel from origin "o" to destination "d" then the summation of flows on all paths (P) from "o" to "d" must equal to q_{od} . This is expressed as :

$$\sum_P f_P^{od} = q^{od}$$

2. Non-negativity constraint:

$$f_k \geq 0 \quad , \text{ for all } k.$$

and

$C_k(w)$ is the travel time experienced by each driver on link (k) as a function of flow (w) on link (k).

f_k is the flow on link (k).

An implicit assumption made in the above formulation is that traffic is homogeneous (i.e. there is no distinction made between cars, trucks,... etc). As the above is a convex minimization problem, there are algorithms which can solve it [see Sheffi (1985)].

A hypothetical situation is when a central controller assigns all drivers to routes from their origins to their destinations such that the total cost of travel (e.g total travel time) in the network is minimized. Equivalently this means that the average journey time is minimized. This is called Wardrop second principle (Wardrop 1952) and also known as “system optimal” (SO) traffic assignment. The system optimal link flows pattern can be known if the following mathematical program is solved :

$$\text{Minimize } Z_2(f_k) = \sum_K f_k C_k(f_k) \quad (2-2)$$

Subject to the same constraints as the (UE) assignment in Eq. (2-1) above, i.e

$$\sum_P f_P^{od} = q^{od}$$

and

$$f_k \geq 0 \quad , \text{ for all } k.$$

By definition of (SO) assignment it will be always true that if

T_1 is the total vehicle hours travelled per hour in the network under (UE) assignment,
and

T_2 is the total vehicle hours travelled per hour in the network under (SO) assignment,
then

$$T_1 \geq T_2$$

B. Continuum Analysis of Urban Transportation Networks

The formulation of both (UE) and (SO) assignments as described above uses conventional methods in representing and analyzing networks. In the conventional approach an urban area is represented by a network of streets and zones, and estimates

of the number of trips between each pair of zones are given. The problem is to determine traffic volumes on each link in the network according to either (LIE) or (SO) assignment. The conventional methods of representing the network with links and nodes as described above work well for rural and suburban networks where the number of nodes and links is not too large. But when it comes to dense urban networks then the level of complexity increases enormously. The very large number of links involved leads to a huge number of possible routes between origin-destination pairs in the network. This can be far beyond what any computer can handle. In the conventional network approach, a partial solution for this problem is that networks with large number of links are usually approximated by ones with a relatively small number of links by aggregating flows of minor roads and loading such flows to major roads or arterials. Minor roads are then eliminated and not coded into the computer.

Another approach of representing the highway network has been developed recently. The new approach uses “continuum” modeling of transportation networks, [see Dafermos (1980) and Newell (1980)]. Dense urban networks are modeled as a continuum in space where fast roads (e.g. freeways) are superimposed on fine grids with slower speeds (e.g. dense urban areas). One does not need to compute traffic flows on every road in the network but rather determine at every point the traffic density travelling in each direction.

C. Basic Idea

The basic idea in this study is that the maximum benefit that one can gain from route guidance is equal to the difference between system costs of both user and system optimal assignments (i.e. T_1 and T_2). We look at the sensitivity of benefits of route guidance to selected network parameters.

It is easy to verify, using Eq. (2-1) and (2-2), that under free flow conditions (UE) and (SO) assignments lead to the same link flow pattern. This results in that $T_1 = T_2$ whenever the network is not congested, i.e. ERVG has zero benefit in this case.

CHAPTER 3

IDEALIZED CORRIDORS FOR ANALYSIS
CASE-I: LONG FREEWAY CORRIDOR

We explore the difference between user and system optimal assignment in two types of corridors, both of which represent idealizations of real networks. In this chapter we discuss the first case where we consider a freeway corridor that is long in relation to the trip length L . Initially, we consider all trip trajectories to remain on one side of the freeway. We later consider the effect of adding trajectories that cross over from one side of the freeway to another. In this initial case a trip originating from a point $A(y)$ will have a destination at $B(x+L,y)$ as illustrated in Fig. (3-1). Since L is small compared to the corridor length, all trips remain within the corridor, and the flow on the freeway is constant and depends on the width of the freeway shed area.

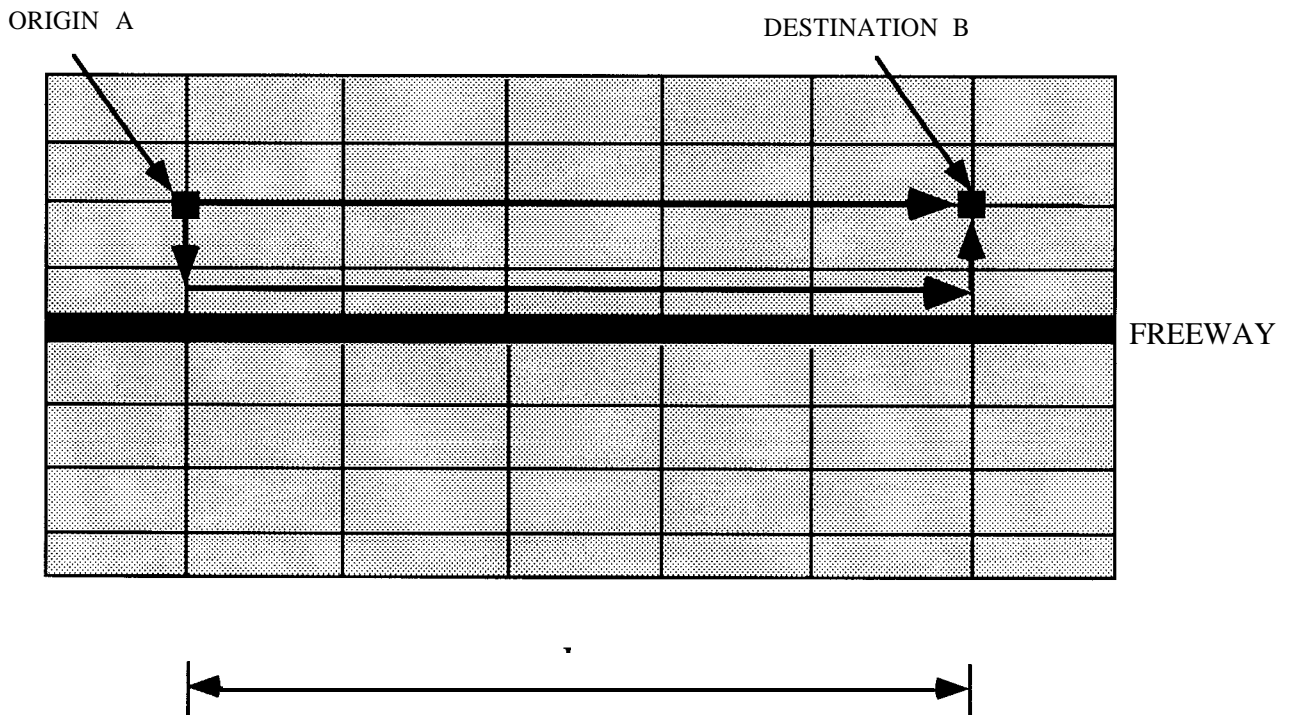


FIGURE (3-1) CASE-I: TRAVELLER PATH CHOICES

In Chapter 4 we discuss the second case where we consider a corridor leading to a destination, such as a CBD. All trips are destined to the same area, and have lengths varying depending on their origin points along the corridor. Flow on the freeway starts off at some distance, a , from the destination and increases up to a point, b , beyond which no additional motorists can enter the freeway. Trip trajectories are assumed as shown in Fig. (3-2), to go from $(a-x, y)$ to (a, y) .

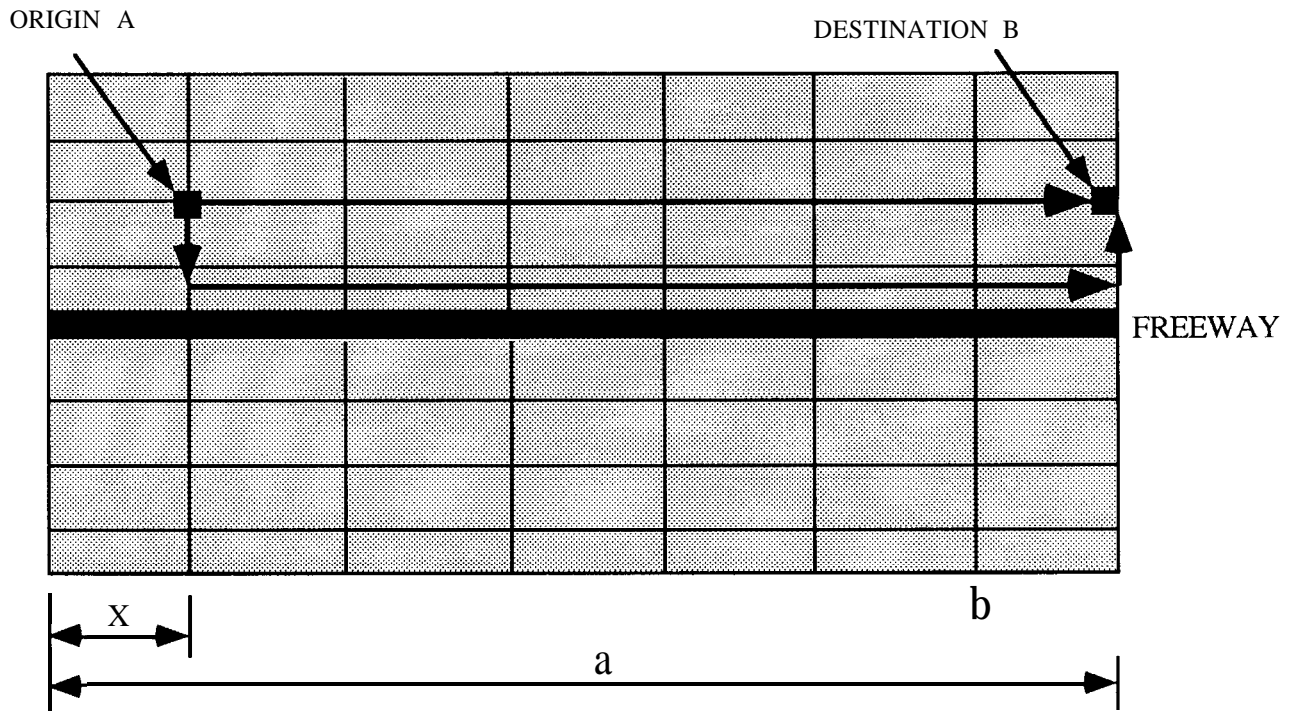


FIGURE (3-2) CASE-II: TRAVELLER PATH CHOICES

These are two of many cases that can be studied. They are chosen because they represent two conditions that are different in an important way: one in which freeway flow is constant, and the other in which freeway flow accumulates as a destination area is approached.

In the following analysis we compare user equilibrium with system optimal assignments.

A. User Equilibrium Assignment Case-I

Referring to Fig. (3-3) below, we consider a traveller originating at point **A(x,y)** and destined to point **B(x+L,y)**.

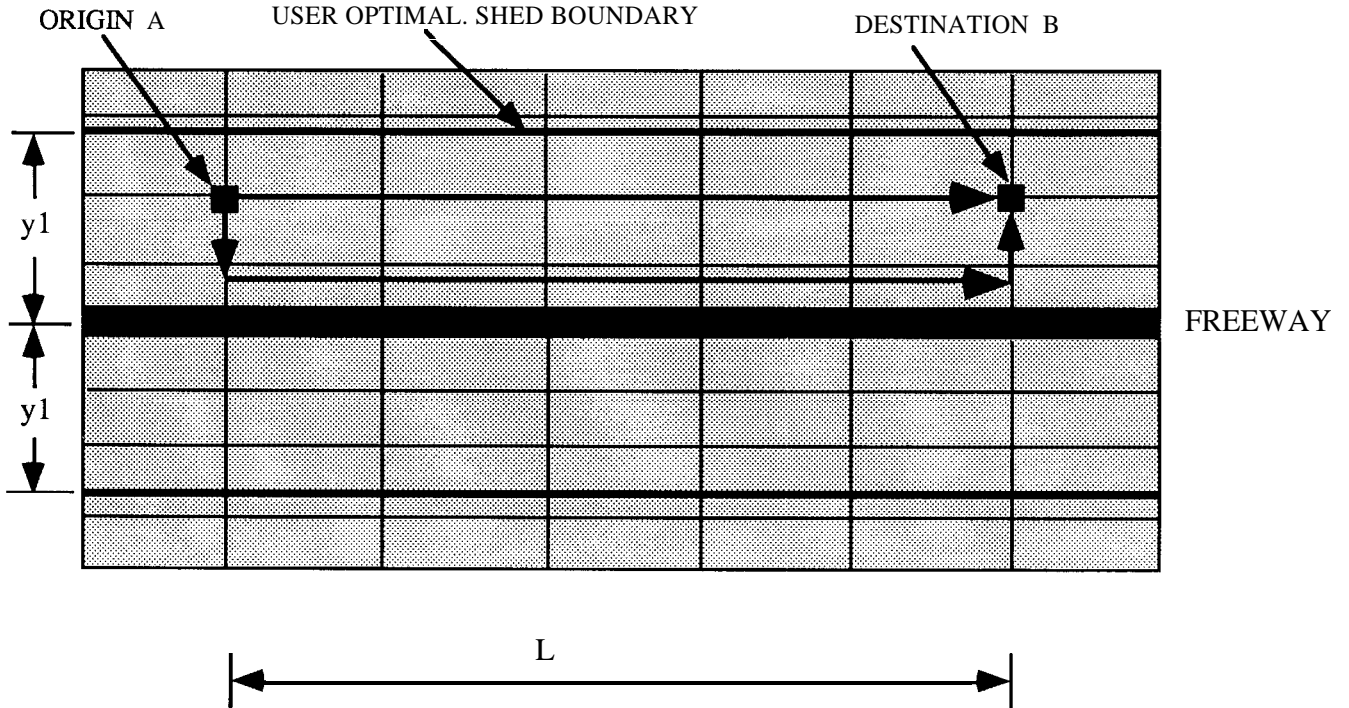


FIGURE (3-3) USER OPTIMAL SHED BOUNDARY y_1

The traveller has a choice between two routes: 1) using the surface street, or 2) driving down to the freeway and using it for the distance L and then returning back to point B. A traveller who minimizes travel cost will choose the freeway route if it costs less than driving straight through on the city streets. In other words, the freeway path is used when:

$$2 y_1 K + L c(f) < L K \tag{3-1}$$

where,

K is the constant average and marginal cost on the city street

L is the trip length

$c(f)$ is the freeway average travel cost function

f is the freeway flow

Equating the two sides of Eq. (1) gives the critical value y_1 which defines the user shed boundary of the freeway:

$$y_1 = \frac{L [K - c(f)]}{2 K} \quad (3-2)$$

The flow on the freeway can be calculated by considering that all users within the shed defined by y_1 on either side of the freeway, and between any two sections L apart use the freeway:

$$f = 2 \beta L y_1 \quad (3-3)$$

where β is the trip density, We now use a freeway cost function of the following standard type:

$$c(f) = t_0 \left[1 + \frac{f^2}{\mu^2} \right] \quad (3-4)$$

where,

t_0 is the free flow travel time on the freeway (eg. 1 minute per mile)

μ is the capacity of the freeway

Using this function we can calculate the flow f_1 and the total system cost T_1 resulting from the user equilibrium assignment:

$$f_1 = \frac{\mu^2}{2 t_0 \beta L^2} \left[-K + \frac{\sqrt{(A+B)}}{\mu} \right] \quad (3-5)$$

in which,

$$A = K^2 \mu^2$$

$$B = -4 \beta^2 L^4 t_0 (t_0 - K)$$

The total system cost T_1 resulting from the user equilibrium assignment can be calculated by adding the street flow cost to the total cost on the freeway as follows:

$$T_1 = K L (D - f_1) + 2 \beta L \int_0^{y_1} [2 y K + L c(f)] dy \quad (3-6)$$

in which D represents the total demand in the section of the corridor of length L , and depends on the overall width of the corridor.

B. System Optimal Assignment Case-I

System optimal assignment occurs when the marginal costs on the freeway and on the city streets are equal. The latter is constant and equal to the average cost K , and the former can be obtained by differentiating the total freeway cost functions with respect to freeway flow. By equating these two marginal costs, we obtain the system optimal freeway flow, f_2 , as follows:

$$f_2 = \mu \frac{[-\mu K + \sqrt{(A+3B)}}{6 t_0 \beta L^2} \quad (3-7)$$

in which A and B are as defined above. The total cost T_2 for the system optimal assignment is obtained in a manner similar to Eq. (3-6). Note that a new shed boundary y_2 applies in this case, and that $y_2 \leq y_1$. The difference, ϕ , between the two cost functions can be used as an indication of the benefits from route guidance:

$$\phi = T_1 - T_2 \quad \text{or}$$

$$\phi = \frac{t_0 L (f_1^3 - f_2^3)}{\mu^2} + \frac{K (f_1^2 - f_2^2)}{2 \beta L} + (t_0 - K) (f_1 - f_2) L \quad (3-g)$$

It should be noted that ϕ is considered as an upper limit to the potential benefits. It implies that it is possible to control the amount of traffic that enters the freeway.

C. Sensitivity Analysis

We use a numerical example to investigate the sensitivity of route guidance benefits ϕ with respect to system parameters. We consider a freeway with a lane capacity of 2000 vehicles per hour. The trip length L is 5 miles and the corridor is much longer than that. The trip density β varies from 10 to 100 vehicle trips per square mile per minute; the speed on the city streets varies from 10 to 40 mph corresponding to a value of K from 1.34 to 6.0. The width of the corridor is 4 miles. The number of lanes on the freeway ranges from 1 to 10 corresponding to a range for μ from 2000 to 20,000 vehicles per hour.

The sensitivity of benefits to capacity is shown in Fig. (3-4). When freeway capacity is small, user equilibrium route choice would not favor the freeway much and the benefits from diversion would be limited. On the other hand, it is evident that with unlimited freeway capacity there would be no point in traffic diversion since the marginal cost would be virtually constant. The quadratic relationship between benefits and capacity can in fact be seen from the manipulation of Eqs. 4-6. The freeway capacity at which maximum benefits are achieved also increases with trip density.

While the benefits from route guidance will rise with trip length L , the percentage savings will eventually decline as shown in Fig. (3-5). This is because as L increases the total time spent in the corridor increases more rapidly than the benefits.

As Fig. (3-6) shows, street speed appears to influence route guidance benefits significantly. As the speed approaches 40 mph on the city streets the freeway loses its advantage, and user equilibrium route choice will cease to favor it. The result is that route guidance will not have significant benefits. On the other hand, with street speeds down to 20 mph or less, benefits rise sharply and become quite sensitive to speed. The implication

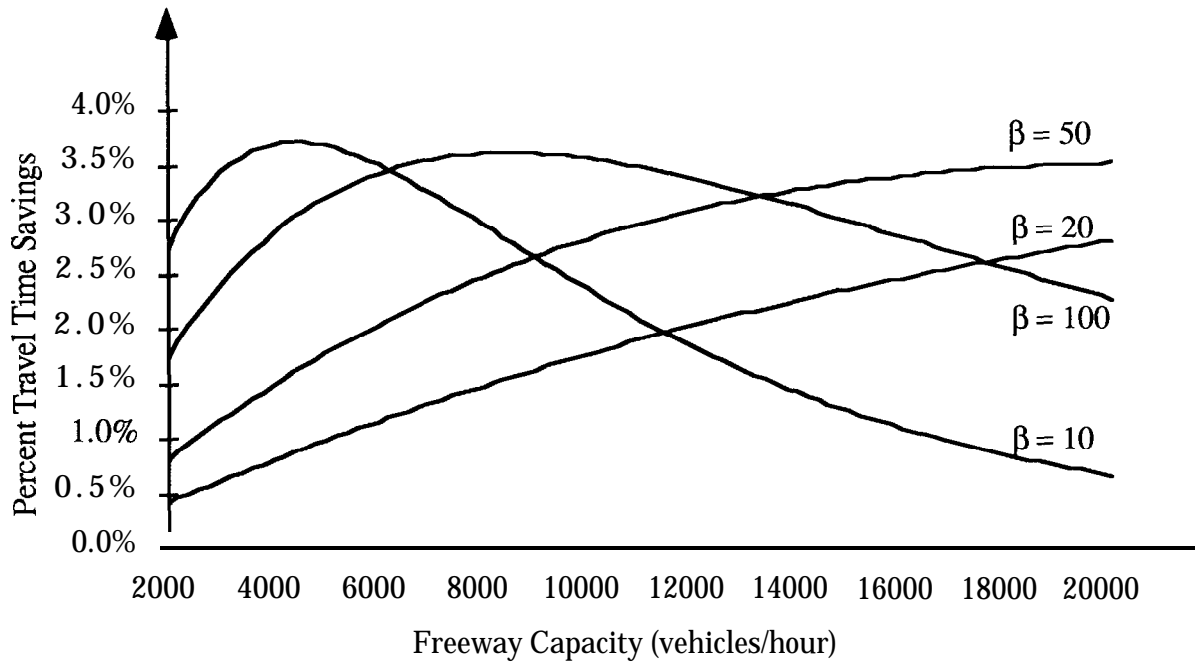


FIGURE (3-4) PERCENT TIME SAVINGS VS. FREEWAY CAPACITY

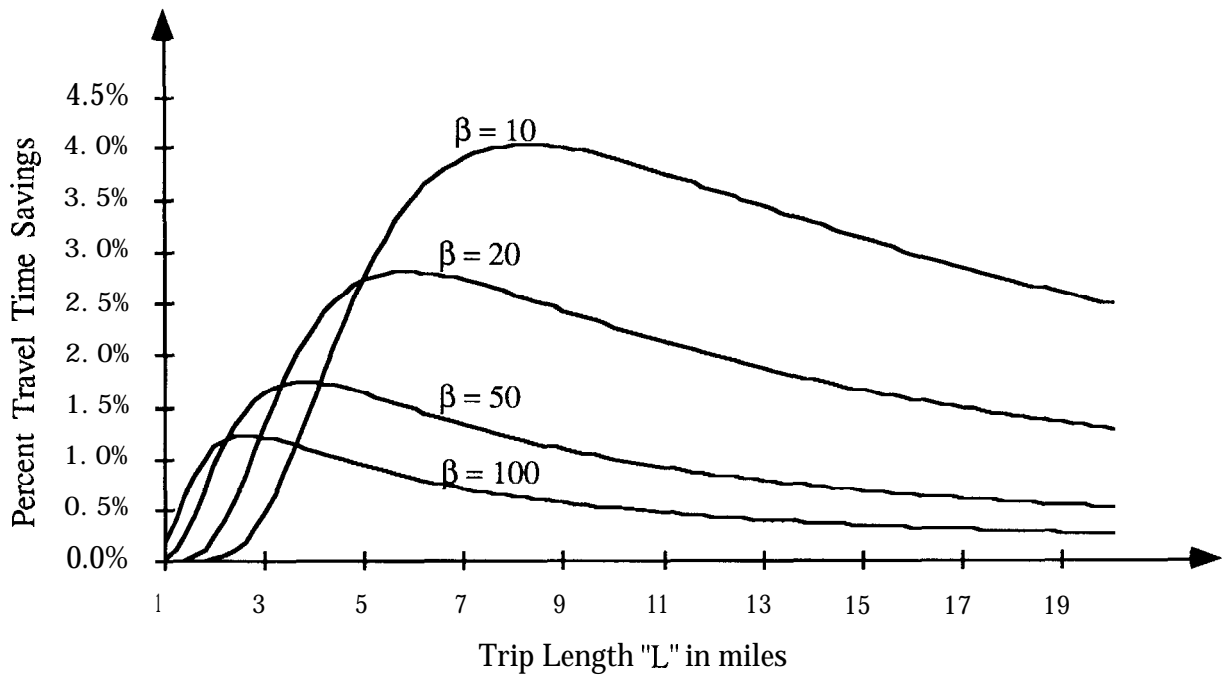


FIGURE (3-5) PERCENT TIME SAVINGS VS. TRIP LENGTH

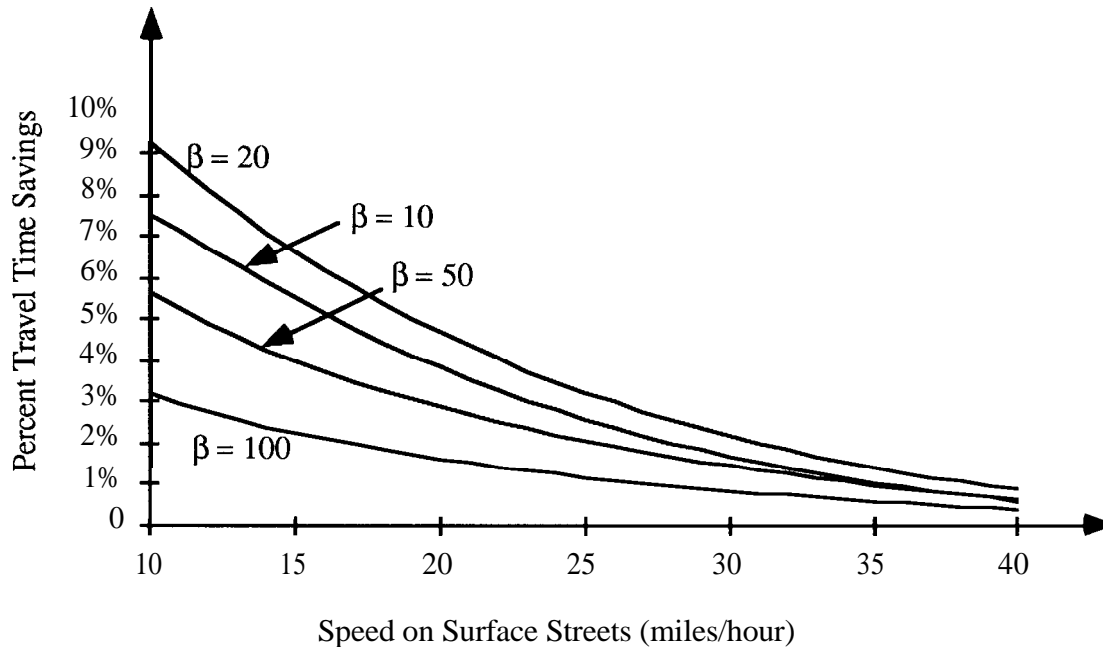


FIGURE (3-6) PERCENT TIME SAVINGS VS. SPEED ON SURFACE STREETS

of this is that in a simple corridor of the type studied effective street traffic management may be considered as an alternative to route guidance.

It is interesting to note that the levels of benefits, in the range of 2-4% are lower than earlier assessments. Earlier results suggest that gains of the order of 8-10% in vehicle-miles of travel can be saved by improving the routing of motorists. While it may be possible to save that much in miles of travel, the savings in time are likely to be lower. The difference is probably due to congestion effects, which will occur even under the optimal assignment conditions. Minimum path routing, which is achievable with autonomous navigators on automobiles might save 8-10% of miles traveled. But such savings can be meaningless under conditions of congestion.

D. Some Extensions

We explore some extensions to the problem by modifying some of the characteristics of the idealized corridor considered above. We first relax the assumption that all trip trajectories begin and end on the same side of the freeway and consider travel across it. We then consider the case of a grid of parallel freeways.

1. Travel Across the Freeway

In relaxing the assumption that trips originating on one side of the freeway have their destination on the same side, we consider two types of trajectories *Z* and *ZZ* as illustrated in Figure (3-7).

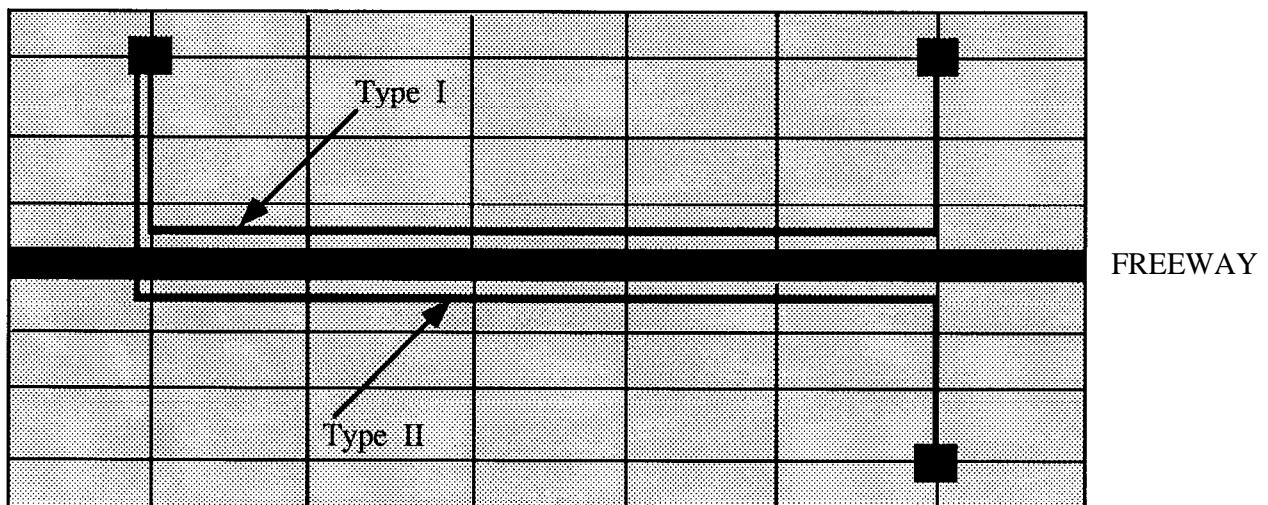


FIGURE (3-7) BACKTRACKING TRIPS (TYPE I) AND
CROSS FREEWAY TRIPS (TYPEII)

The fundamental difference between trip types *Z* and *ZZ* is that the first trajectory involves backtracking in order to use the freeway, whereas the second does not. For trips of type *Z* the shed boundary as defined by Eq. (3-2) results from equating the benefits of the faster freeway with the disbenefits of backtracking. For trips type *II* on the other hand, it will always be advantageous to use the freeway as long as $c(f) < K$. Thus for these trips there

is no shed boundary, and the selection of the freeway is an all-or-nothing decision depending on $c(f)$ in which the flow depends on the trip density, as shown in Eq.(3-3).

It can therefore be said that in user equilibrium route choice, cross-freeway trips retain a sort of priority on freeway use in the sense that they will more readily opt for the freeway and will only switch to city street routes when congestion causes $c(f)$ to exceed K , long after the trip ZZ makers have stopped using the freeway. It should be noted that while the average cost of a freeway trip is higher for type Z than for type ZZ trips due to the backtracking involved in the former, the freeway marginal cost function is the same for both. Thus from the system optimization point of view the route guidance strategy need not distinguish between trip makers. However, since they are more likely to be using the freeway, type ZZ motorists stand to lose more from diversion to city streets. Thus if route guidance is capable of providing guidance to specific vehicles on the basis of their origins and destinations, or at least if it can distinguish between type Z and type ZZ motorists and convey specific guidance to each, then it would be beneficial to favor type ZZ motorists for freeway use and divert type Z motorists first. It is noted that as trip length L increases, the relative effect of backtracking on the trajectory of trip types Z diminishes and the two types become similar.

2. Parallel Freeways

The case of a freeway network made of a set of parallel freeways represents an interesting extension to the problem of the benefits of route guidance. Suppose such a network has freeways spaced at a distance S . Using the assumption that a trip originating at $\mathbf{A}(x,y)$ is destined to $\mathbf{B}(x+L,y)$, it is easy to see that there is a freeway spacing S_o below which no traffic will use the city street network, regardless of whether user optimal or system optimal assignment is in force. This value is given by:

$$S_o = 2 y \quad (3-9)$$

where y represents the shed boundary. For equilibrium assignment:

$$S_o = L \left[1 - \frac{c(f)}{K} \right] \quad (3-10)$$

and for system optimal assignment:

$$S'_o = L \left[1 - \frac{c'(f)}{K} \right] \tag{3-11}$$

where $c'(f)$ is the freeway marginal cost function. It is easy to see that $S'_o \leq S$. As is shown in Fig. (3.8), if the spacing between parallel freeways is larger than the value given by Eq. (3-10) or (3-11), then the difference would represent the width of a strip generating non-freeway users. One can argue, then, that freeway grids should be spaced at distances less than S'_o . [See Newell (1980)]. At such spacings there is no difference between user and system optimal assignments for trips of length L or more. Such a spacing is often referred to as the optimal freeway spacing, since any increase in S would result in an increase in total cost. Ironically, route guidance will be of no use in this case.

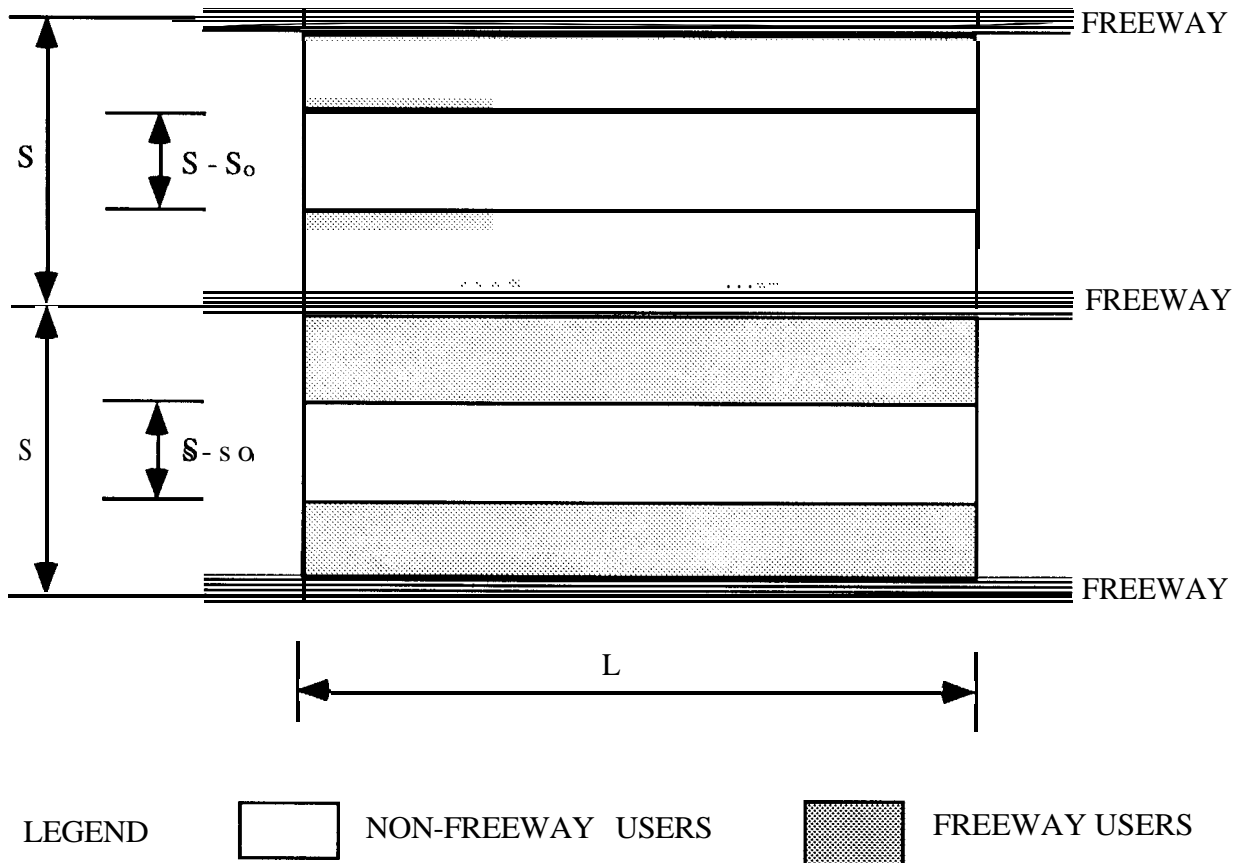


FIGURE (3-8) SYSTEM OF PARALLEL FREEWAYS AND OPTIMAL SPACING

CHAPTER 4

**IDEALIZED CORRIDORS FOR ANALYSIS
CASE-II SINGLE DESTINATION CORRIDOR**

Here we relax the assumption that the corridor is very long and consider one that leads to a given destination such as a CBD. As mentioned earlier in Chapter 3, the flow on the freeway in this case is not constant, but varies with distance. As shown on Fig. (4-1), the flow $F(x)$ starts off at zero some distance a from the destination and increases up to a point b at which no additional motorists would enter the freeway. The trip trajectory is assumed as shown in the Figure (4-1): a trip with origin $(a-x, y)$ will have a destination (a, y) . The shed boundary $y(x)$ for freeway users will look as shown.

This boundary can be calculated for the user equilibrium case by equating the average costs of freeway and non-freeway trajectories as in case-I, Chapter 3.

For a given $y(w)$, the flow on the freeway increases with distance as follows:

$$F(x) = 2\beta \int_0^x y(w) dw \quad (4-1)$$

This flow function is transformed into a cost function by applying cost equation (3-4) to yield freeway average cost, $C(F)$ as a function of flow. Since the flow, and hence the cost, accumulates as we approach the CBD at point a , it is necessary in order to obtain the total cost, $g(x)$, from any point, x , along the corridor to integrate the cost function over the range from x to a :

$$g(x) = \int_x^a C(z) dz \quad (4-2)$$

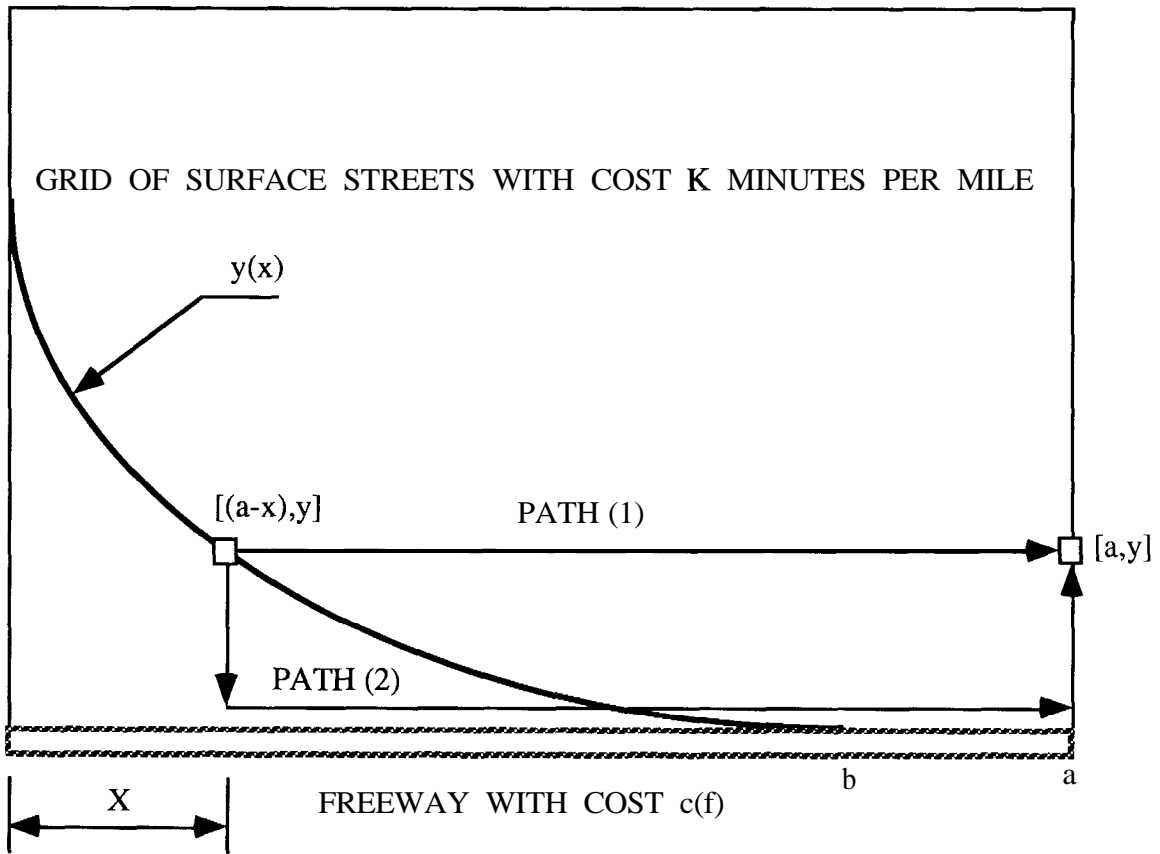
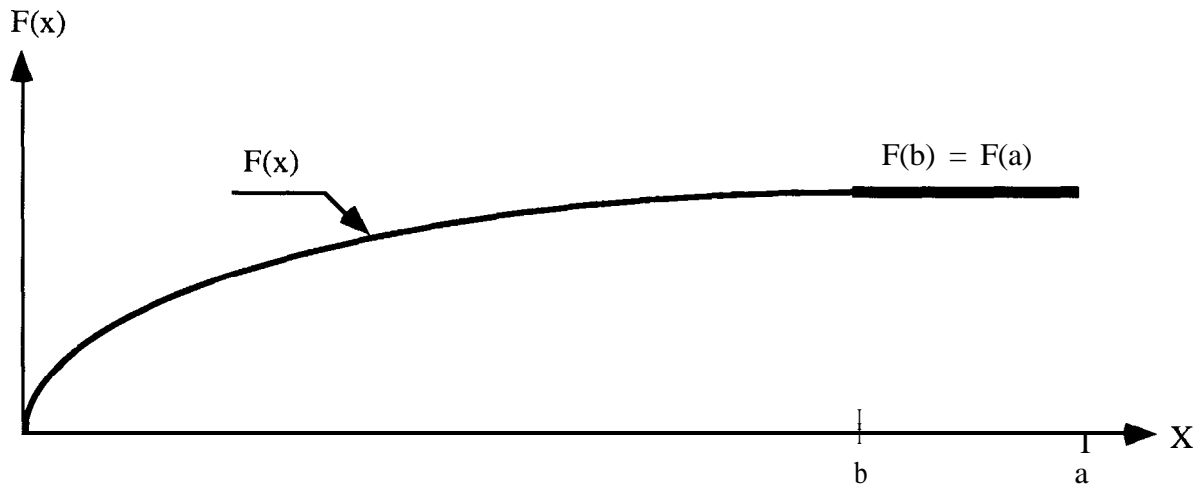


FIGURE (4-1) CUMULATIVE FLOW $F(x)$ AND SHED BOUNDARY $y(x)$ FUNCTIONS (SINGLE DESTINATION CORRIDOR)

A. User Equilibrium Assignment Case-II

To calculate the *user equilibrium* assignment, we equate the average costs of the freeway and non-freeway trajectories:

$$(a-x) K = 2K y_1(x) \int_x^a C(z) dz \quad (4-3)$$

which yields the boundary function $y_1(x)$ as:

$$y_1(x) = \frac{1}{2K} \left[(a-x) K - \int_x^a C(z) dz \right] \quad (4-4)$$

Equations (4-1) through (4-4) can be rewritten as a system of differential equations as follows:

$$2K y_1'(x) = -K + C(x) \quad (4-5)$$

$$F_1'(x) = 2\beta y_1(x) \quad (4-6)$$

where $y_1'(\cdot)$ and $F_1'(\cdot)$ are the first order differentials. It is possible [see Appendix -A-] to show that:

$$\left(\frac{dF_1}{dx} \right)^2 = A_1 F_1^3 + A_2 F_1 + A_3 \quad (4-7)$$

where the A's are known constants. As a first order nonlinear differential equation (4-7) requires one initial condition for its solution. Since as shown in Fig. (4-1) the flow on the freeway levels off at some point **b1** for user equilibrium assignment or point **b2** for system optimal assignment, it follows that

$$F_1(b_1) = F_1(a)$$

$$C(x) = C(b_1) = C[F_1(b_1)] \quad \text{for } b_1 \leq x \leq a$$

Substituting these conditions into a cost function such as Eq. (3-4) will yield the initial condition $F_1(b_1)$ as follows:

$$F_1(b_1) = \text{Min.} \left\{ \mu, \mu \sqrt{\frac{K}{t_0} - 1} \right\} \quad (4-8)$$

An iterative solution algorithm for Eq. (4-7) and a numerical example are presented in the next section.

1. Solution Algorithm for (UE) Assignment of Case-II

Since it may not be possible to get exact forms of $y_1(x)$ and $F_1(x)$ by solving Eq. (4-7) analytically, an algorithm was developed to solve it numerically.

A description of the algorithm developed as well as a graphical proof of its convergence are presented as follows:

We start with a given form of $F_1(x)$, e.g. $F_1^{(1)}(x)$ is linear in x as shown in the first quadrant of Fig. (4-2)^{***}. Regardless of how $F_1^{(1)}(x)$ is obtained the speed-flow curve is going to look as shown in the second quadrant where V is the difference between freeway speed and speed on the city streets. $F_1^{(1)}(V)$ is plotted only for $V \geq 0$. As V increases more drivers become attracted to use the freeway which means a larger $y_1(V)$. Once V is known from $F_1^{(1)}(V)$ curve, then $y_1(V)$ and $y_1(x)$ curves can be determined using Eq. (4-4). Iterative curves of $y_1(V)$ and $y_1(x)$ are shown in the third and fourth quadrants of Fig. (4-2).

To illustrate how one can determine final curves of $F_1(x)$ and $y_1(x)$ the following sequence of algorithm iterations is presented:

First Iteration

- (1) First we will consider as if the freeway has infinite capacity such that it can take all trips generated in the urban network. We start at any point $x=x_1$ and find $F_1^{(1)}(x)$.
- (2) Given $F_1^{(1)}(x)$ we can find $V[F_1^{(1)}(x_1)]$ from the F-V curve in the second quadrant.
- (3) Given $V[F_1^{(1)}(x_1)]$ we can find $y_1^{(1)}[V\{F_1^{(1)}(x_1)\}]$ in the third quadrant.

^{***} The superscript (1) in $F_1^{(1)}(x)$ refers to the sequential number of iteration.

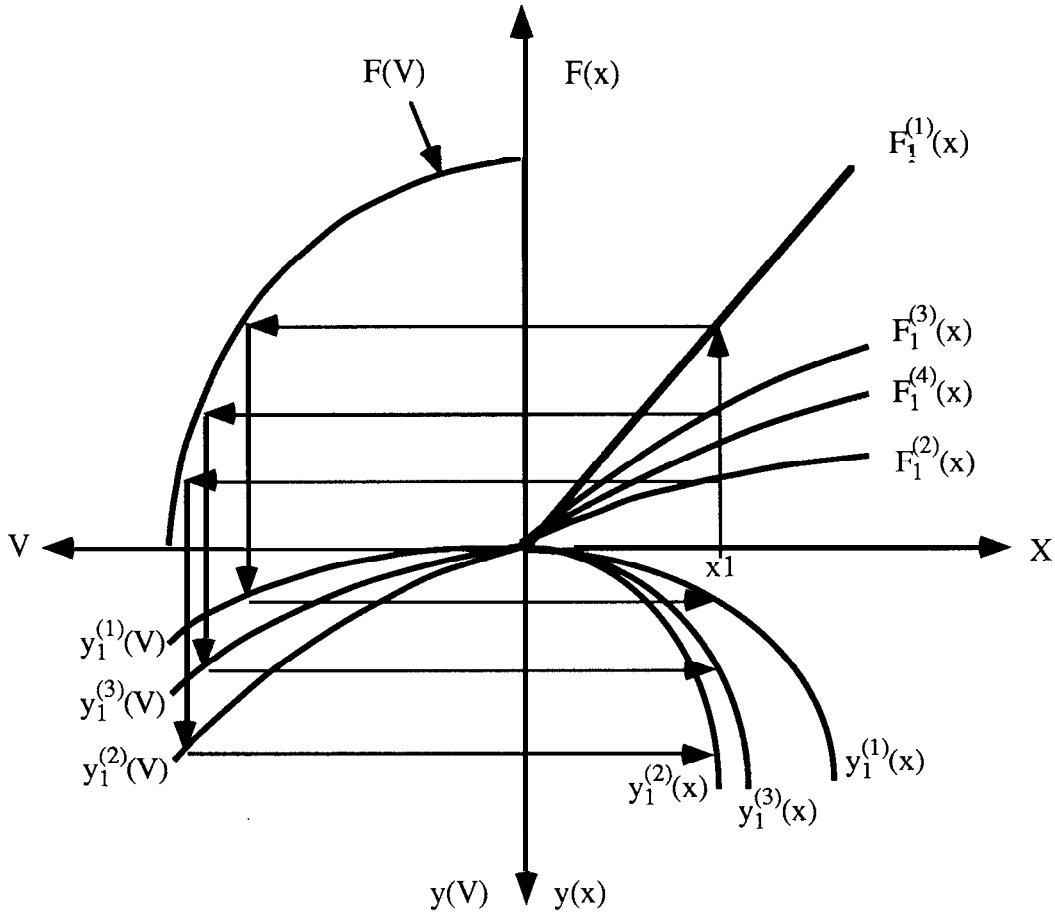


FIGURE (4-2) SOLUTION ALGORITHM FOR (UE) ASSIGNMENT CASE-II

- (4) Given $y_1^{(1)}[V\{F_1^{(1)}(x_1)\}]$ we can find $y_1^{(1)}(x_1)$ which is a point located on the $y_1^{(1)}(x)$ curve in the fourth quadrant. If one does this for all points of x one can determine $y_1^{(1)}(x)$ in the fourth quadrant.

Second Iteration

Since the shed boundary $y_1^{(1)}(x)$ is not infinite then there is some trips which use city streets and $F_1^{(2)}(x)$ curve should be lower than $F_1^{(1)}(x)$ curve (i.e $F_1^{(2)}(x) < F_1^{(1)}(x)$ for all x).

We repeat steps (1) through (4) above to get $y_1^{(2)}(x)$. It is noticed that $y_1^{(2)}(x) > y_1^{(1)}(x)$ which means that $y_1^{(2)}(x)$ curve would result more trips using the freeway than $y_1^{(1)}(x)$ curve. In other words $F_1^{(3)}(x) > F_1^{(2)}(x)$.

Third and Fourth Iteration

It is noticed that

$$y_1^{(1)}(x) < y_1^{(3)}(x) < y_1^{(2)}(x)$$

and

$$F_1^{(2)}(x_1) < F_1^{(4)}(x_1) < F_1^{(3)}(x_1)$$

which means that both $F_1(x)$ and $y_1(x)$ are converging into some final forms of curves $F_1^*(x)$ and $y_1^*(x)$.

2. Numerical Example

We consider a freeway with capacity μ of 100 vehicles/minute, distance a of 10 miles, t_0 of 1 minute/mile, and K of 2.4 minutes/mile. The algorithm sequence is shown in Fig. (4-3) and is explained as follows:

(1) Start with a linear form of $F(x)$, call it $F_1^{(1)}(x)$, where

$$F_1^{(1)}(x) = \left(\frac{\mu}{a}\right) x$$

(2) Find $C_1(x)$ by substituting $F_1^{(1)}(x)$ into Eq. (3-4) and then find $y_1^{(1)}(x)$ by substituting $C_1(x)$ into Eq. (4-4).

(3) Find $F_1^{(2)}(x)$ by substituting $y_1^{(1)}(x)$ into Eq. (4-1). Notice that $F_1^{(2)}(x) < F_1^{(1)}(x)$.

(4) Repeat step (2) above to find $y_1^{(2)}(x)$. Notice that $y_1^{(2)}(x) > y_1^{(1)}(x)$.

- (5) Repeat step (3) to find $F_1^{(3)}(x)$ by substituting $y_1^{(2)}(x)$ into Eq. (4-1). Notice that $F_1^{(3)}(x) > F_1^{(2)}(x)$, hence, both $F_1(x)$ and $y_1(x)$ are converging into some final forms of curves $F_1^*(x)$ and $y_1^*(x)$. One more iteration shows that $F_1^{(4)}(x) < F_1^{(3)}(x)$.

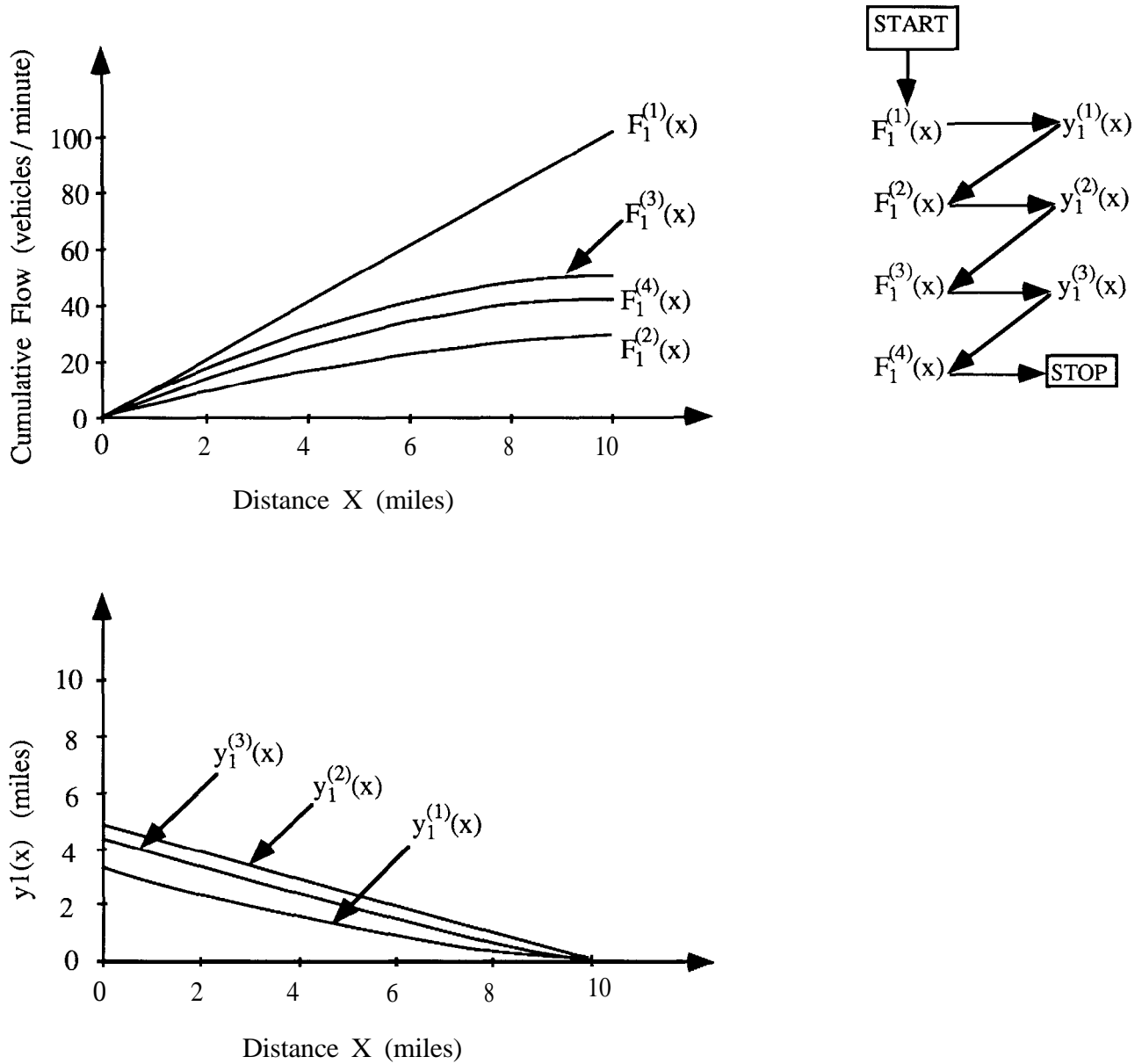


FIGURE (4-3) SEQUENCE OF ALGORITHM ITERATIONS

Derivation of equations of $F_1(x)$ and $y_1(x)$ are provided in Appendix -B-.

Figure (4-4) shows the algorithm convergence. The maximum difference between $F_1(x)$ cumulative values in the third and fourth iterations was less than 6% which may be considered close enough.

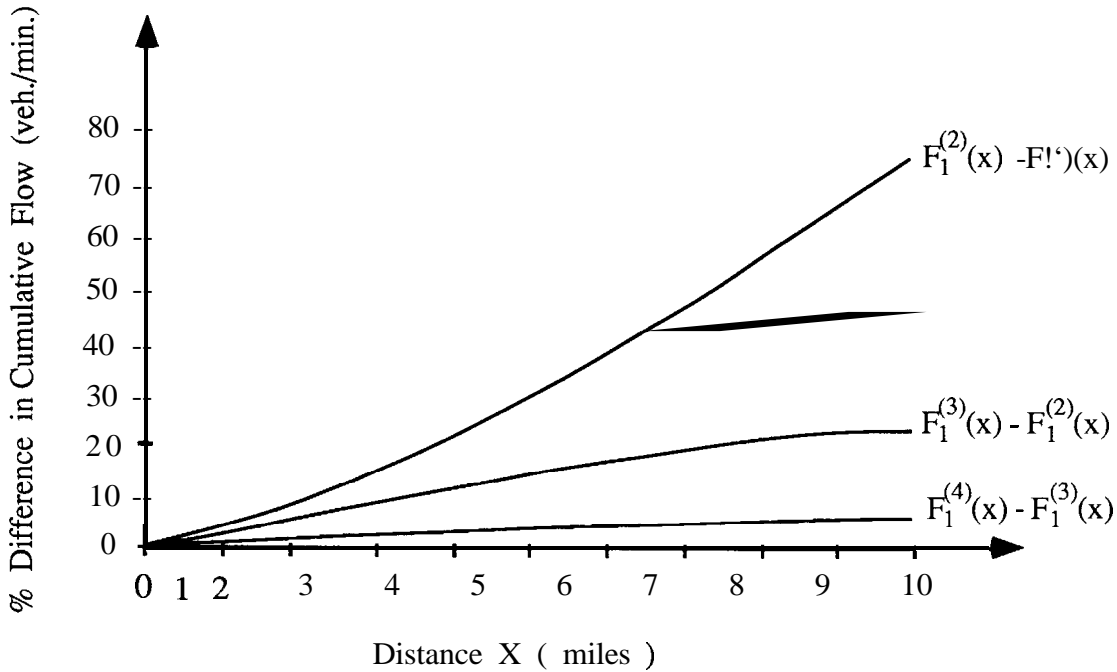


FIGURE (4-4) ALGORITHM CONVERGENCE

B. System Optimal Assignment Case-II

To obtain the system optimal boundary function y_2 we equate the marginal costs of the freeway and non-freeway trajectories, or equivalently by minimizing total system cost including all trajectories. The total cost for freeway trips originating at (a, x, z) and destined to (a, z) (as shown in Fig. (4-5)) is given by:

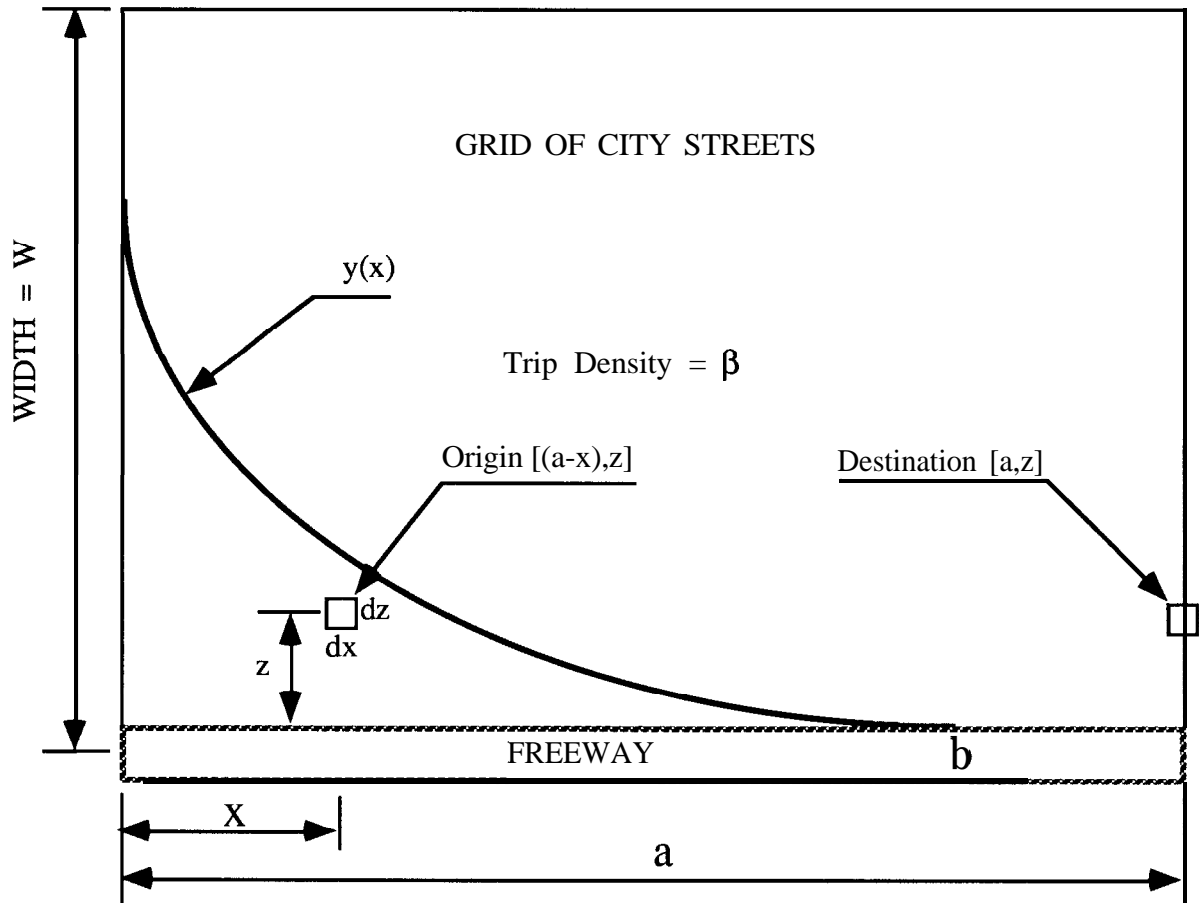


FIGURE (4-5) DERIVATION OF TOTAL COST OF USING THE CORRIDOR IN CASE-II

$$P(z,x) = 2zK + \int_x^a C[F(h)] dh \quad (4-9)$$

where $C[.]$ and $F(.)$ are the cost and the flow functions as defined earlier. If we let,

$$g(x) = \int_x^a C[F(z)] dz$$

and then integrate (4-9) for all trips originating within the freeway shed boundary as defined by $y(x)$ then we obtain the total cost T_f for freeway trajectories as follows:

$$T_f = 2\beta \int_0^b G[y(x)] dx \quad (4-10)$$

where

$$G[y(x)] = Ky^2(x) + g(x)y(x) \quad (4-11)$$

The total cost for non-freeway trajectories can also be calculated on the basis of the shed boundary given by $y(x)$ as follows:

$$T_{nf} = 2\beta K \int_0^a (a-x)[W-y(x)] dx \quad (4-12)$$

where W is the width of the corridor. The total system cost, which is optimized by the selection of the optimal boundary $y_2(x)$ is then given by the sum of the freeway and the non-freeway costs:

$$T = T_f + T_{nf}$$

or

$$T = 2\beta \int_0^b [Ky^2(x) + y(x)g(x) - Ky(x)(a-x)] dx + \beta K W a^2 \quad (4-13)$$

Eq. (4-13) applies for both (UE) and (SO) assignments; when $y(x)$, $g(x)$, and b are replaced by $y_1(x)$, $g_1(x)$, and b_1 respectively then T becomes T_1 of (UE) assignment. Similarly $y_2(x)$, $g_2(x)$, b_2 , and T_2 correspond to (SO) assignment. The last term in Eq. (4-13) is constant and does not depend on the type of assignment.

A simple way to find the exact value of the optimal shed boundary $y_2(x)$ is to calculate and then equate the marginal costs to freeway and to non-freeway traffic due to an incremental increase in trips generated at the boundary as shown in Figure (4-6). Even then the solution for $y_2(x)$ is fairly complex and is given by the implicit function [see Appendix -C- for derivations]:

$$y_2(x) = \frac{1}{2K} \left[K(a-x) - \int_x^a C[F_2(z)] dz - 2\beta \int_x^{b_2} y_2(w) s(w) dw \right] \quad (4-14)$$

where

$$s(x) = \int_x^a C'[F(z)] dz$$

$C'[\cdot]$ denote first derivative.

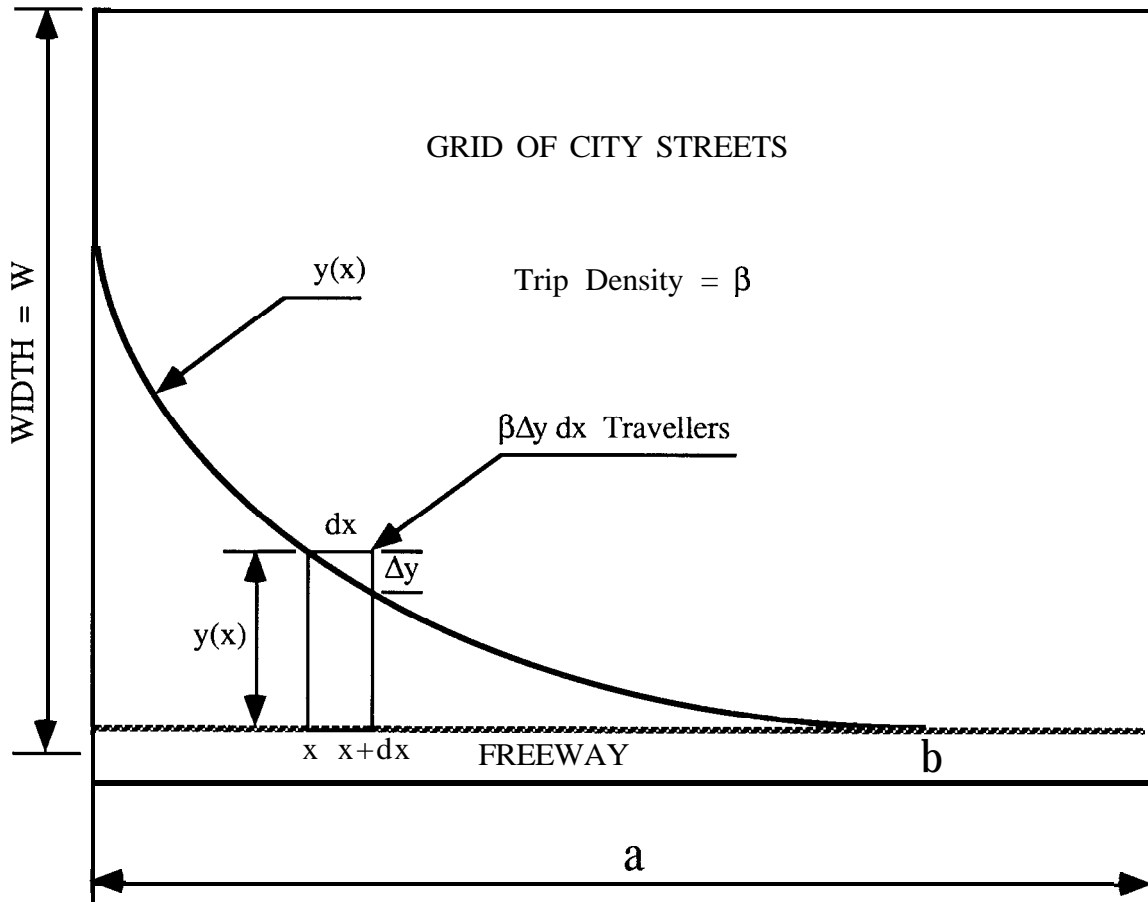


FIGURE (4-6) FINDING OPTIMAL SHED BOUNDARY $y_2(x)$ IN CASE-II

We can gain some insight onto the nature of the benefits from optimal assignment by comparing the shed boundaries for this and for the user equilibrium case. In Appendix -C- an argument is used to draw shed boundaries $y_1(x)$ and $y_2(x)$ as shown in Fig. (4-7).

This means that the system optimal assignment will always divert some traffic away from the freeway. The extent of benefits is shown by the magnitude of the shaded area on the Fig. (4-7). This area represents the origins of trips that use the freeway under no guidance, but would divert from the freeway if they were to follow optimal route guidance instructions from ERVG.

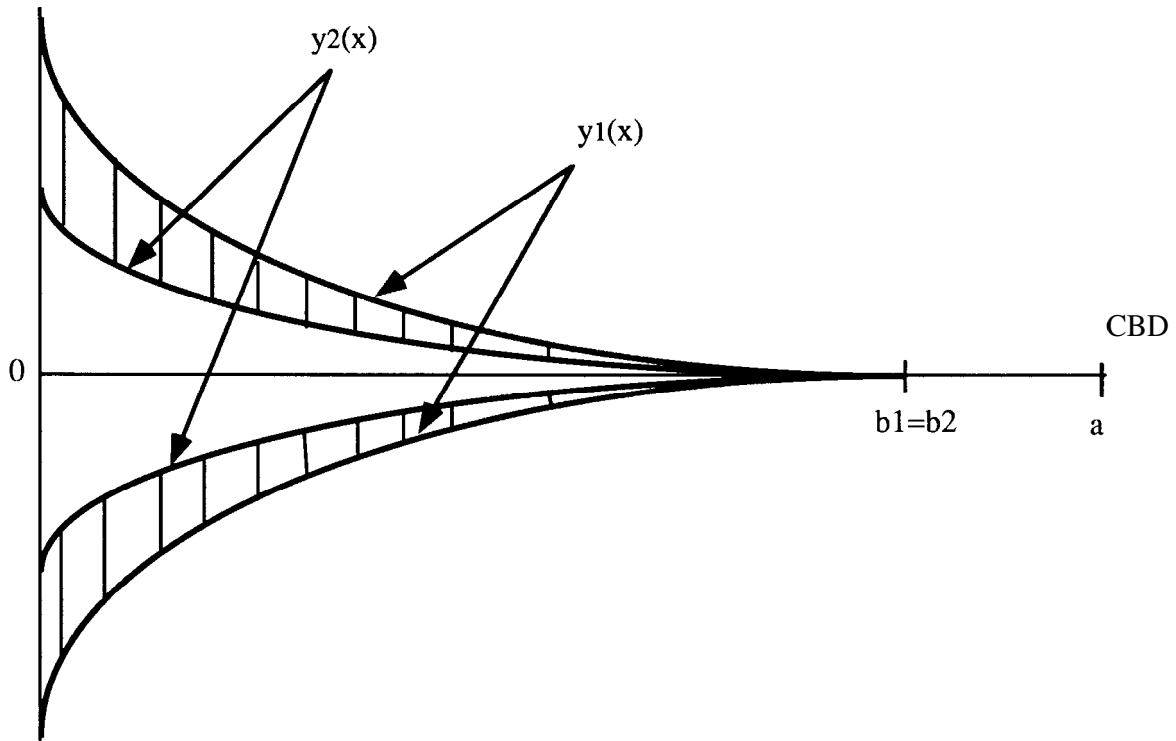


FIGURE (4-7) BENEFITS OF (ERVG) AS RELATED TO SHED BOUNDARIES $y_1(x)$ AND $y_2(x)$ OF CASE-II

CHAPTER 5

CONCLUSIONS

The goal of en-route vehicle route guidance is to influence individual motorist trajectories in such a way as to move toward system optimal assignment. If it is assumed that user equilibrium assignment will prevail in the absence of route guidance, and that motorists will follow guidance instructions then the benefits from route guidance technology can be measured by the difference in total travel time, for a given demand, between the system optimal and the user equilibrium assignments.

In this study some idealized freeway corridors are considered for an assessment of these benefits. The main effect of route guidance in these corridors is to divert traffic from the freeway to the street network when the marginal cost on the former exceeds that on the city streets. We find total time savings to be typically of the order of 3-4%, but to be also quite sensitive to some corridor parameters. The benefits of route guidance are quite sensitive to city street speed. At low speed more users would choose the freeway resulting in congestion, and the potential benefits of route guidance are relatively high. But as street speed increases and approaches that of the freeway route guidance would be of less value as more of the motorists would be choosing the city street on their own.

For the corridor studied in the first case maximum route guidance savings from 3% to 9% depending on trip density can be achieved when the average speed on the street network is 10 mph. These savings drop to 1.5%-3.5% with a speed of 25 mph and virtually disappear at a speed of 40 mph. The implication is that in a corridor of the first type studied, local street traffic management may be an alternative to route guidance. In other words excessive traffic on the freeway can be avoided either by diverting motorists with route guidance, or by improving conditions on the street network.

Benefits are also quite sensitive to freeway capacity. There is little gain from route guidance if there is a severe shortage of freeway capacity since user equilibrium assignment would not send many motorists to the freeway anyway. On the other hand if there is ample freeway capacity then, depending on trip density, freeway congestion may not arise and there may be little need for route guidance.

There are opportunities to enhance the benefits from route guidance if it is capable of providing information tailored to specific vehicles on the basis of their origins and destinations. It was seen earlier that trip trajectories across the freeway will more readily use the freeway than trajectories that involve backtracking. Route guidance that would divert backtrackers first would achieve the optimal assignment with less individual “sacrifice” than a system that diverts traffic indiscriminantly.

In the second case corridor it was found that the level of complexity of this analysis increases rapidly as one begins to relax simplifying assumptions.

Since the scenario analyzed in this study considered only corridors under normal traffic conditions then, in order to have an overall assessment of benefits, it would be necessary for future research to investigate corridors under incident conditions,

In any case, it appears inevitable that route guidance in congested freeway corridors will result in diverting traffic off the freeway and onto local city streets. Many issues are related to such a strategy. Some of them are perhaps more important than savings in total travel time. These include the noise and air quality impacts on affected neighborhoods, and the potential disruption of local mobility due to the diversion of large flows of through traffic onto local streets. There are also the equity issues stemming from the possibility of diverting individual vehicles onto longer routes in order to achieve overall system optimum. Such effects could seriously affect the manner by which motorists respond to route guidance, or whether they would acquire the technology in the first place. Thus, while certainly part of the overall picture, time savings alone are not going to tell us whether route guidance is worthwhile or not.

REFERENCES

- Jeffrey, D. J. (1981a) *The Potential Benefits of Route Guidance*. TRRL Report LR 997, Transport and Road Research Laboratory, U.K.
- Jeffrey, D. J. (1981 b) *Ways and Means for Improving Driver Route Guidance*. TRRL Report LR 1016, Transport and Road Research Laboratory, U.K.
- King, G. F. and T. M. Mast. (1987) *Excess Travel: Causes, Extents and Consequences*. KLD Associates.
- Jeffery, D. J; K. Russam and D.I. Robertson. (1987) *Electronic Route Guidance by Autoguide: The Research Background*. Traffic Engineering + Control, vol.28 no. 10.
- Jeffery, D. J. (1987a) *Route Guidance and In-Vehicle Information Systems*. Information Technology Applications in Transport, Edited by Peter Bonsall and Michael Bell, VNU science Press BV.
- Al-Deek, H. ; M. Martello; A. May; and W. Sanders. (1988) *Potential Benefits of In-Vehicle Information Systems in A Real Freeway Corridor Under Recurring and Incident Induced Congestion*. PATH Research Report, Institute of Transportation Studies, University of California, Berkeley. UCB-ITS-PRR-88-2.
- Al-Deek, H. M. and A. D. May. (1989) *Potential Benefits of In-Vehicle Information Systems (IVIS): Demand and Incident Sensitivity Analysis*. PATH Research Report UCB-ITS-PRR-89- 1.
- Beckmann, M. and H. P. Kunzi. (1976) *Traffic Equilibrium Methods*. Proceedings of the International Symposium held at the Universite de Montreal, Edited by M. A. Florian, Springer-Verlag.
- Wardrop, J. G. (1952) *Some Theoretical Aspects of Road Traffic Research*. Proceedings of the Institute of Civil Engineers, Part II.

Beckmann, M.; C. B. McGuire; and C. B. Winsten. (1956) *Studies in The Economics of Transportation*. Yale University Press, New Haven.

Sheffi, Y. (1985) *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice-Hall, Inc.

Dafermos, S. C. (1980) *Continuum Modelling of Transportation Networks*. Transportation Research, Vol. 14B.

Newell, G. F. (1980) *Flow on Transportation Networks*. The MIT Press.

Kanafani, A. (1987) *Towards a Technology Assessment of Automated Highway Navigation and Route Guidance*. PATH Working Paper, UCB-ITS-PWP-87-6.

Yumoto, N.; H. Ihara; T. Tabe; and M. Naniwada. (1979) *Outline of the CACS Pilot Test System*. 58th Annual Meeting, Transportation Board, Washington, D.C.

APPENDIX -A-

**SHED BOUNDARY $y_1(x)$ FOR USER EQUILIBRIUM ASSIGNMENT
CASE-II**

Derivation of Equation (4-7)

Recalling Eq. (3-4) one can express $C(x)$ in terms of $F_1(x)$ in Eq. (4-5) and then one can solve for $y_1(x)$ as follows:

$$y_1(x) = \frac{dy_1}{dx} = DF_1^2 + H \quad (A-1)$$

where,

$$D = \frac{t_0}{2K\mu}$$

and

$$H = \frac{t_0 - K}{2K}$$

Eq. (4-6) can be rewritten as

$$\frac{dF_1}{dx} = 2\beta y_1(x) \quad (A-2)$$

Dividing Eq. (A-1) by Eq. (A-2) yields

$$\frac{dy_1}{dF_1} = \frac{D F_1^2 + H}{2\beta y_1} \quad (A-3)$$

by separation of y_1 and F_1

$$\int 2\beta y_1 dy_1 = \int [D F_1^2 + H] dF_1 \quad (A-4)$$

which gives:

$$\beta y_1^2(x) - \left(\frac{D}{3}\right) F_1^3(x) - H F_1(x) = I \quad (A-3)$$

where I is the constant of integration. It is possible to solve for $y_1(x)$ as a function of $F_1(x)$ and then substitute for $F_1(x)$ into Eq. (4-6) to get the final Eq. (4-7):

$$\left(\frac{dF_1}{dx}\right)^2 = A_1 F^3 + A_2 F + A_3 \quad (4-7)$$

where,

$$A_1 = \frac{4}{3} \beta D, \quad A_2 = 4\beta H, \quad \text{and} \quad A_3 = 4\beta^2 I$$

APPENDIX -B-

**DERIVATION OF $F_1(x)$ AND $y_1(x)$ FORMULAS IN THE NUMERICAL
EXAMPLE**

First Iteration

(1) Start with a linear form of $F_1(x)$ as follows

$$F_1^{(1)}(x) = \left(\frac{\mu}{a}\right) x \quad (\text{B-1})$$

where $\mu=100$ vehicle/minute, $a=10$ miles, and x is measured in miles.

(2) Find $C_1^{(1)}(x)$

$$C[F_1^{(1)}(x)] = t_0 \left[1 + \left(\frac{F_1^{(1)}(x)}{\mu} \right)^2 \right] \quad (\text{B-2})$$

Substitute for $F_1(x)$ as in Eq. (B- 1) into Eq. (B-2) which gives

$$C_1^{(1)}(x) = t_0 \left[1 + \left(\frac{x}{a} \right)^2 \right] \quad (\text{B-3})$$

and

$$g(x) = \int_x^a c(w) dw$$

hence,

$$g_1^{(1)}(x) = t_0 \left[(a - x) + \frac{(a^3 - x^3)}{3a^2} \right] \quad (\text{B-4})$$

(3) Find $y_1^{(1)}(x)$ by substituting for $g_1^{(1)}(x)$ into Eq. (4-4) therefore

$$y_1^{(1)}(x) = \frac{1}{2K} \left[(a - x) (K - t_0) - t_0 \frac{(a^3 - x^3)}{3a^2} \right] \quad (\text{B-5})$$

$y_1^{(1)}(x)$ is plotted in Fig. (4-3).

Second Iteration

(1) Find $F_1^{(2)}(x)$

To find $F_1^{(2)}(x)$ we substitute for $y_1^{(1)}(x)$ into Eq. (4-1) to get

$$F_1^{(2)}(x) = \frac{\beta}{K} \left[\left(ax - \frac{x^2}{2} \right) (K - t_0) - t_0 \left(\frac{a^3 x}{3} \right) + \frac{t_0 x^4}{12 a^2} \right] \quad (\text{B-6})$$

with $\beta = 1$ trip/minute $F_1^{(2)}(x)$ is plotted as shown in Fig. (4-3).

(2) To find $y_1^{(2)}(x)$ we repeat steps (2) and (3) above.

Third and Fourth Iteration

Similarly one can proceed to find equations for the third and fourth iterations. However, equations for $y_1(x)$ and $F_1(x)$ would become substantially longer and complicated as more iterations are done.

APPENDIX -C-

SHED BOUNDARY $y_2(\mathbf{X})$ FOR SYSTEM OPTIMAL ASSIGNMENT
CASE-II

Derivation of Equation (4- 14)

Suppose that there is an incremental increase in traffic generated at some shed boundary $y(x)$ between $(x, x+dx)$ and $(y(x), y(x)+\Delta y)$ as shown in Fig. (4-6). This incremental increase is equal to $(2\beta Ay dx)$. For the optimal shed boundary $y_2(x)$ increase in total cost due to incremental increase in traffic is supposed to be minimum.

Let At be the increase in total cost due to the incremental increase in traffic generated at $y(x)$ boundary. Hence,

$$At = (T + \Delta T) - T$$

and

$$T + \Delta T = 2\beta \int_0^b [K (y + \Delta y)^2 + (y + \Delta y) g(x) - K (y + \Delta y) (a - x)] dx + \beta W K a^2 \quad (C-1)$$

The integral \int_0^b can be partitioned into three integrals as follows:

$$\int_0^b \dots = \int_0^x \dots + \int_x^{x+dx} \dots + \int_{x+dx}^b \dots \quad (C-2)$$

Consequently one can rewrite T and $T+\Delta T$ as:

$$T = \int_0^x \dots + \int_x^{x+dx} \dots + \int_{x+dx}^b \dots \quad (C-3)$$

$$T + \Delta t = \int_0^x \dots + \int_x^{x+dx} \dots + \int_{x+dx}^b \dots \quad (C-4)$$

the first integral \int_0^x is common between T and $T + \Delta t$ because the incremental increase in traffic is generated between x and $x + dx$ and therefore it affects time of travellers in the spacing from $(x$ to $a)$ but not from $(0$ to $x)$. Therefore when finding Δt the first integral \int_0^x cancels out. Also if we neglect higher order terms of Δy^2 in Eq. (C-1) and any term that includes Δy in the integral \int_{x+dx}^b (because Δy exists only in the interval $(x, x + dx)$) then one can write Δt as follows:

$$\Delta t = 2\beta \Delta y \int_x^{x+dx} [2K y(w) + g(w)] dw - 2\beta \Delta y \int_x^{x+dx} K(a-w) dw + 2\beta \int_{x+dx}^b y(w) [g^*(w) - g(w)] dw \quad (C-5)$$

where

$$g^*(w) = \int_w^a C[F^*(z)] dz, \text{ for } w > x \quad (C-6)$$

and

$$F^*(z) = F(z) + 2\beta \Delta y dx$$

$F^*(z)$ is being the flow after the incremental increase in traffic. Using series expansion and neglecting higher order terms:

$$C[F(z) + 2\beta \Delta y dx] = C[F(z)] + 2\beta \Delta y dx C'[F(z)] + \dots$$

hence,

$$g^*(w) = \int_w^a C[F(z)] dz + 2\beta \Delta y dx \int_w^a C' [F(z)] dz$$

and consequently

$$g^*(w) = g(w) + 2\beta \Delta y dx \int_w^a C' [F(z)] dz \quad (C-7)$$

We use Eq. (C-7) to substitute for $g^*(w)$ in Eq. (C-5) and simplify to get Δt :

$$\Delta t = (2\beta \Delta y dx) 2K y(w) + (2\beta \Delta y dx) g(w) + (2\beta \Delta y dx) 2\beta \int_x^b y(w) s(w) dw - (2\beta \Delta y dx) K(a - w) \quad (C-8)$$

where

$$s(w) = \int_w^a C' [F(z)] dz$$

The sum of the first and second terms of the right hand side of Eq. (C-8) represent average cost of using the freeway by the incremental flow $(2\beta \Delta y dx)$. The third term is an added congestion cost to total system cost due the incremental flow. The last term is the decrease in total system cost due to change in routes from city streets to the freeway by the flow $(2\beta \Delta y dx)$. Eq. (C-8) can be rewritten as follows:

$$\frac{\Delta t}{2\beta \Delta y dx} = 2K y(w) + g(w) + 2\beta \int_x^b y(w) s(w) dw - K(a - w) \quad (C-9)$$

We take the limit of Eq. (C-9) when the incremental flow $(2\beta \Delta y dx)$ is small. But when $(2\beta \Delta y dx)$ is small, or close to zero, Δt diminishes. Also Δt is equal to zero when we have a system optimal shed boundary $y_2(x)$. Therefore at the limit $y(w)$, b , and $g(w)$ are replaced by $y_2(x)$, b_2 and $g_2(w)$ in Eq. (C-9). After this replacement, the sum of the first three terms of the right hand side of Eq. (C-9) becomes equal to the marginal cost on the freeway while the absolute value of the last term is equal to the marginal cost on city

freeway while the absolute value of the last term is equal to the marginal cost on city streets. Obviously at system optimal shed boundary $y_2(x)$ the two marginal costs are equal which means that the limit of Eq. (C-9) is equal to zero, or

$$2K y_2(w) + g_2(w) + 2\beta \int_x^{b_2} y_2(w) s(w) dw - K(a - w) = 0 \quad (\text{C-10})$$

where

$$g_2(w) = \int_x^a C[F_2(z)] dz$$

Eq. (C-10) can be rewritten as:

$$y_2(x) = \frac{1}{2K} \left[K(a - x) - \int_x^a C[F_2(z)] dz - 2\beta \int_x^{b_2} y_2(w) s(w) dw \right] \quad (\text{C-11}) \text{ or } (4-14)$$

We compare Eq. (4-14) with Eq. (4-4). Note that Eq. (4-4) can be written as

$$y_1(x) = \frac{1}{2K} \left[K(a - x) - \int_x^a C[F_1(z)] dz \right] \quad (4-4)'$$

where $F_1(\cdot)$ and $F_2(\cdot)$ are user equilibrium and system optimal flows on the freeway. When $x=b_2$ the last term in Eq. (4-14) becomes equal to zero. This means that there is no more congestion effect because no more traffic uses the freeway for $x>b_2$. Since $F_2(x) \leq F_1(x)$ then $g_2(w) < g_1(w)$ for all w , where

$$g_1(w) = \int_x^a C[F_1(z)] dz$$

hence,

$$g_2(b_2) < g_1(b_2)$$

From Eq. (4-14) and (4-4) one can conclude that

$$y_2(b_2) > y_1(b_2)$$

but since we know that

$$y_2(x) \leq y_1(x) \text{ for all } x$$

consequently

$$y_2(b_2) = y_1(b_2)$$

This can be true only if user optimal and system optimal freeway flows diminish at the same point $x = b_2 = b_1$.

At the other end where $x=0$

$$y_2(0) < y_1(0) \quad (\text{i.e } y_2(0) \text{ is strictly less than } y_1(0))$$

because at $x=0$ the congestion term

$$2\beta \int_0^{b_2} y_2(w) s(w) dw$$

(which has a negative sign in Eq. (4-14)) is likely to have large effect on reducing the value of $y_2(0)$ such that $y_2(0)$ becomes strictly less than $y_1(0)$. Therefore shapes of the two curves $y_2(x)$ and $y_1(x)$ are expected to look as shown in Fig.