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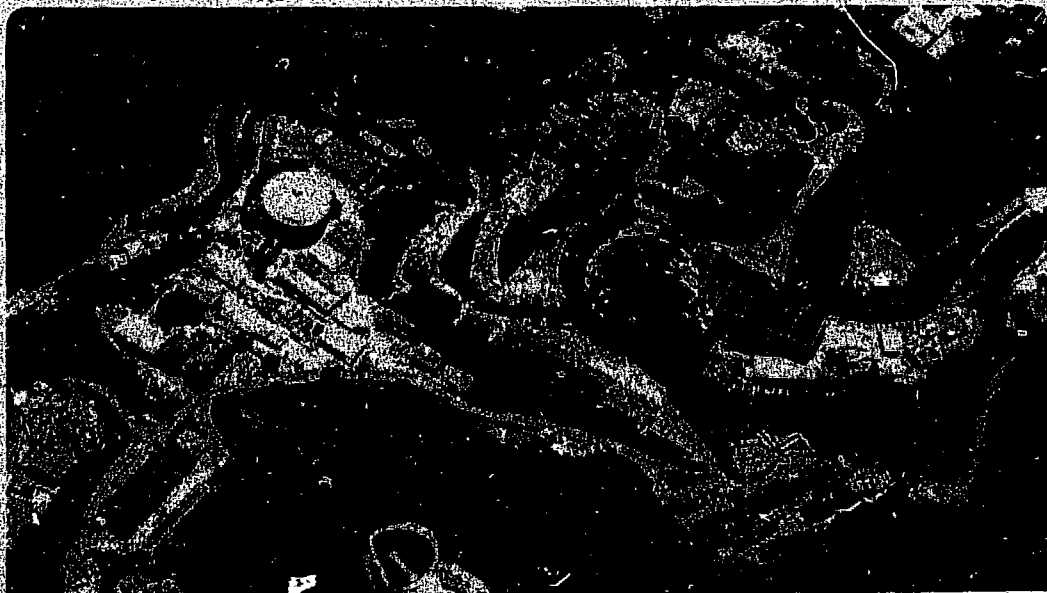
Physics, Computer Science & Mathematics Division

Invited talk presented at the International Symposium
on High Energy e^+e^- Interactions, Vanderbilt University,
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A REVIEW OF D AND B MESON DECAYS

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A Review of D and B Meson Decays

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I. INTRODUCTION

There is a class of down and out members of the legal profession, known as "ambulance chasers", who materialize at automobile accidents hoping to find clients for an injury suit. Glashow has aptly used this term in referring to a widely diffused style of doing theoretical physics. In theoretical physics, if not in the law, chasing ambulances is not necessarily an ignoble practice. It is part of what distinguishes us from the mathematicians who do not have experimental colleagues to stimulate and guide them. Some noble discoveries have been made trying to fit a curve, a notable example being Planck's fit to the black body radiation spectrum.

The emerging experimental picture of charmed meson decays has brought the ambulance chasers out in force. There is a sense of disaster in the air, but the actual magnitude of the accident is not yet clear. There is still the possibility that we are dealing with a mere "fender-bender". While the simplest picture of charm decays¹ led us to expect that D^+ and D^0 would have nearly equal lifetimes, $\tau_+ \approx \tau_0$, data from emulsion experiments and from SPEAR make it likely that τ_+ is appreciably greater than τ_0 . But we cannot tell yet whether the ratio is actually $\lesssim 3$ or $\gg 5$. In the former case an explanation can probably be found in the context of the generally accepted theoretical framework. But if τ_+/τ_0 is much larger than 5, I would say that the recent optimism about our understanding of all non-leptonic decays is called into question, including the basis of the $\Delta I = \frac{1}{2}$ rule for strange particle decays.

The plan of my talk is to review briefly the theoretical picture of K and D decays in order to explain why we did not expect large enhancements¹ in D decays. This expectation is contrasted with the available data on lifetimes and semileptonic branching ratios, which hint at large enhancements but are still ambiguous. I will then discuss some of the theoretical ideas proposed to explain the enhancement of D^0 , and possibly F^+ , nonleptonic decays. One of these, together with data presented earlier in this session, suggests a crazy way to try to detect the F^+ and a possible gluonium state in one fell swoop. Finally I will discuss what we learn about nonleptonic enhancements from decays into exclusive final states. In particular, the new data on $D \rightarrow \rho K/\pi K^*$ presented in this session have interesting implications for models which predict large enhancements.

In a briefer section, I will discuss two topics involving B decays which are related to the possibility of substantial

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nonleptonic enhancements. The first is the worrisome prospect that though we need to know the Kobayashi-Maskawa angles to determine whether B decays are enhanced, it may be a practical requirement to first understand the pattern of nonleptonic enhancements in order to extract the K-M angles from the data. Second I will discuss the decays of B mesons into $J/\psi(3095)$ from the perspective of what these decays might teach us about the dynamics of nonleptonic enhancement.

II. D MESON DECAYS

A. Naive Expectations

The simplest imaginable picture of D meson decays is that the c quark decays into an s quark by bremsstrahlung of a virtual W^+ boson which materializes as a $u\bar{d}$ or $\nu\bar{l}$ pair. The light anti-quark- \bar{u} for D^0 , \bar{d} for D^+ , \bar{s} for F^+ - is a passive "spectator" to the decay. With this model we expect the D^0 , D^+ and F mesons all to have lifetimes equal to that of the c quark itself,

$$\tau_+ = \tau_0 = \tau_F = \tau_c . \quad (2.1)$$

The branching ratio for μ semileptonic decays is just the fraction of W^+ bosons which materialize as $\nu_\mu \bar{l}$ pairs,

$$B(D \rightarrow \nu_\mu \bar{l} X) = \frac{1}{3+1+1} = \frac{1}{5} \quad (2.2)$$

where the $u\bar{d}$ pair has a weight of three for color. The total width may be scaled from the rate for $\mu \rightarrow \nu_\mu e \bar{\nu}_e$:

$$\begin{aligned} \Gamma_{TOT} &= 5 \left(\frac{m_c}{m_\mu}\right)^5 \frac{1}{2} \Gamma(\mu \rightarrow \nu_\mu e \bar{\nu}_e) \\ &= [(1.5 \pm 0.1)] \cdot 10^{-12} \text{sec}]^{-1} \quad (2.3) \end{aligned}$$

where I assume $m_c = 1.5 \pm 0.15$ GeV and $m_s/m_c = 0.3$. The uncertainty in (2.3) is just that which reflects the assumed spread in m_c ; the factor $\frac{1}{2}$ is the reduction in the available phase space due to the strange quark mass. In the preceding I have discussed only Cabibbo allowed decays, which always give K's in the final state. The fraction of Cabibbo suppressed decays would be

$$B(D \rightarrow \text{no K}) \lesssim .08 , \quad (2.4)$$

slightly larger than $\tan^2 \theta_c$ because of the greater available phase space. In the four quark GIM model² we would expect the bound to be saturated while for six³ or more quarks the ratio could be smaller.

There are two other lowest order Feynman diagrams to consider. In these the light antiquark is not a passive spectator. The F^+ may decay through a virtual W^+ in the s-channel into a $u\bar{d}$ pair: $F^+ \sim c\bar{s} \rightarrow "W^+" \rightarrow u\bar{d}$. The $D^0 \sim c\bar{u}$ may decay by exchange of a W boson in the t-channel into an $s\bar{d}$ pair. No such mechanism is

possible for the D^+ . These diagrams were thought to be negligible for two reasons - a factor $(m_{u,d}/m_D)^2 \sim \frac{1}{25}$ due to helicity suppression (as in $\pi \rightarrow e\nu$) and a small factor $(F_D/m_D)^2 \sim \frac{1}{5} - \frac{1}{100}$ reflecting the probability for the initial quark pair to coincide in space as they must in these "annihilation" diagrams. Here F_D is the analogue of F_π for the pion, which in nonrelativistic models is proportional to the value of the wave function at the origin.

B. Nonleptonic Enhancement of K decays

Even if the annihilation diagrams are as small as presumed, it is not at all clear that the predictions (2.1) to (2.4) should be taken seriously. The point is of course that such a picture fails totally to account for the observed factor 400 enhancement in strange particle decays with $\Delta I = 1/2$. Our expectations for D decays are strongly coupled to our understanding of the K decays. In fact there was a wide-spread expectation among theorists that (2.1) - (2.4) would be a good zero'th order approximation. To explain this I have to make a slight detour to discuss the present understanding of the $\Delta I = 1/2$ rule.

If we consider the lowest order QCD corrections to the weak decay $s \rightarrow u\bar{u}$, the diagrams with loops containing both a W boson and a gluon give rise to large factor $\alpha_s(\mu) \ln \frac{M_W}{\mu} \gg 1$, where μ is the renormalization point typically taken to be of order 1 GeV. The leading logs in this parameter may be summed to all orders using the renormalization group. The result⁴ is that the effective four fermion interaction which in the absence of strong interactions is

$$\mathcal{H}_{\Delta S=1} \propto (\bar{s}u)_L (\bar{u}d)_L \quad (2.5)$$

becomes the sum of two operators

$$\mathcal{H}_{\Delta S=1} \propto f_- O_- + f_+ O_+ \quad (2.6)$$

where

$$O_\pm = \frac{1}{2} [(\bar{s}u)_L (\bar{u}d)_L \pm (\bar{s}d)_L (\bar{u}u)_L] \quad (2.7)$$

The abbreviated notation is that $(\bar{u}d)_L$ is the usual V-A weak current with an implied sum over color indices, $(\bar{u}d)_L \equiv \sum_{\alpha=1}^3 (\bar{u}_\alpha d_\alpha)_L$.

For $f_- = f_+ = 1$ (2.6) becomes the zero'th order interaction (2.5).

Now in O_- the u and d quark fields are arranged antisymmetrically, $I = 0$, so the net isospin of O_- is the $I = (1/2)$ of the \bar{u} . In O_+ the ud pair has $I = 1$ so the net isospin of O_+ may be $1/2$ or $3/2$, $1 \otimes (1/2) = (1/2) \oplus (3/2)$. Therefore if f_- is much larger than f_+ we will have found an explanation for the $\Delta I = 1/2$ rule. The actual result of the calculation⁴ is

$$f_- = \frac{1}{f_+} = \frac{\alpha_s(\mu)}{\alpha_s(M_W)} \Big|^{0.48} \cong 2.4 ,$$

$$f_+ \cong 0,65, \quad (2.8)$$

and

$$\left(\frac{f_-}{f_+}\right)^2 \cong 20. \quad (2.9)$$

Equation (2.9) is a generous estimate, but it falls far short of the needed 400.

A second possible source of $\Delta I = 1/2$ enhancement has been much discussed in recent years - the so-called penguin diagrams.⁵ These occur by virtue of the strangeness - changing neutral currents which the GIM mechanism² was invented to suppress. In the GIM model they do still occur at a level given by the mass differences among the quarks, such as $m_c - m_u$. In the penguin diagrams, the loops contain the $Q = + (2/3)$ quarks and the W-boson, and the expansion parameter summed to leading-log order is $\alpha_s(\mu) \ln \frac{m_c}{\mu}$. Because

these logs are appreciably smaller than the $\log M_W$ effects discussed above, the leading log approximation must be taken with an even larger grain of salt in this case. The penguin diagrams are pure $\Delta I = 1/2$ because their net effect is to change an s quark into a d quark. The \bar{u} or \bar{d} quark in the K meson initial state interact in these diagrams only by gluon exchange which is flavor-preserving.

This unlikely mechanism has two large factors in its favor. First, there are large color factors of order 10. Second, because the u or \bar{d} quark interact by the purely vectorial gluon interaction, the penguin diagrams give rise to four quark operators with left-right helicity structure rather than the usual Fermi left-left structure of Eqs. (2.5)-(2.7). The L-R structure is not susceptible to the suppression that occurs in Fermi decays of a pseudoscalar meson, such as $\pi \rightarrow e\nu$. For instance, a penguin induced operator is

$$\mathcal{K}_{\Delta S=1}^{(\text{Penguin})} \propto (\bar{s}d)_L (\bar{u}u + \bar{d}d + \dots)_R \quad (2.10)$$

A crude estimate of the helicity effect yields

$$\frac{\langle \pi\pi | (\bar{s}d)_L (\bar{u}u)_R | K \rangle}{\langle \pi\pi | (\bar{s}d)_L (\bar{u}u)_L | K \rangle} \sim \frac{m_\pi^2}{m_u m_s} \sim 30 \quad (2.11)$$

where $m_{u,s}$ are in this case the bare or current quark masses (because Eq. (2.11) is derived from the equations of motion and therefore uses the masses which appear in the Lagrangian rather than the effective constituent masses), $m_u \sim 5$ MeV and $m_s \sim 120$ MeV. A more careful estimate using the M.I.T. bag model gives a similar result.⁶ The conclusion is that despite the considerable uncertainties, which mean these estimates are somewhere between qualitative and (semi)ⁿ-quantitative with $n \geq 2$, it is plausible that the penguin mechanism may be the origin of the factor 400 enhancement in K decays. If the M.I.T. bag calculation is reliable, the factor 400 is not a synergistic combination of the

penguin and the $\ln M_W$ effects due to the operator O_- : the two contributions interfere destructively so the enhancement must result from the overwhelming effect of the penguin mechanism alone.

C. D Decays with QCD Corrections

If we accept the penguin diagrams as the basis for the $\Delta I = (1/2)$ rule in K decays, it is easy to see why the free quark model was expected to be a reasonable guide to D decays. The effect of penguin diagrams should be vastly smaller in D decays than in K decays⁷. First, the penguin mechanism is Cabibbo-suppressed. This is no handicap in the K system but for D decays it costs a factor $\sim (1/20)$ in the rate. Second, the helicity enhancement of Eq. (2.11), which was a factor $m_\pi^2/m_u m_s \sim 30$ in the amplitude, becomes in the D case $m_\pi^2/m_u m_c \sim 2.5$. So relative to their importance in K decays, the penguins are demoted by roughly $\frac{1}{20} \cdot \left(\frac{2.5}{30}\right)^2 \sim \frac{1}{3000}$ in the D decays. It seems very unlikely that they could contribute a sizeable enhancement to the width of D mesons.

The analogue of the $\ln M_W$ effects summarized in Eqs. (2.6)-(2.9) are less important for D than K decays because $\alpha_s(m_c) \ln \frac{M_W}{m_c}$ is smaller than the analogous parameter in the K system. The calculation proceeds just as before.^{8,9} The lowest order four quark interaction

$$\mathcal{H}_{\Delta C=1} \propto (\bar{c}s)_L (\bar{u}d)_L \quad (2.12)$$

becomes in leading log approximation

$$\mathcal{H}_{\Delta C=1} \propto f_- O_- (\Delta C=1) + f_+ O_+ (\Delta C=1) \quad (2.13)$$

where

$$O_\pm = \frac{1}{2} \left[(\bar{c}s)_L (\bar{d}u)_L \pm (\bar{c}u)_L (\bar{d}s)_L \right] \quad (2.14)$$

f_\pm are nearer the free quark values $f_+ = f_- = 1$ than in Eq. (2.8):

$$f_- = \left(\frac{\alpha_s(m_c)}{\alpha_s(M_W)} \right)^{0.48} \sim 2$$

$$f_+ = \frac{1}{\sqrt{f_-}} \sim 0.7 \quad (2.15)$$

The operator $O_- (\Delta C=1)$ is in the 6^* of $SU(3)$ while $O_+ (\Delta C=1)$ is in the 15 of $SU(3)$. So we expect a moderate "6-dominance." But no sizeable enhancement is expected in the rate of nonleptonic decays. The spectator ansatz for D decays together with Eqs. (2.13)-(2.15) implies that the total nonleptonic width is¹⁰

$$\Gamma_{NL}(D) = \frac{2f_+^2 + f_-^2}{3} \Gamma_{NL} \text{ (free quark)}$$

$$\sim \frac{5}{3} \Gamma_{NL} \text{ (free quark)}. \quad (2.16)$$

That is, the color factor 3 is replaced by a factor 5.

QCD corrections to the width of the semi-leptonic decays are also expected to be moderate. No $\ln M_W$ effects occur because there are no loops containing both W-bosons and gluons. The $O(\alpha_s)$ correction to $c \rightarrow s \nu \bar{\ell}$, computed from the sum of virtual corrections and real gluon emission, is

$$\Gamma_{SL} = \left(1 - \frac{1}{2} \alpha_s(m_c)\right) \Gamma_{SL} \text{ (free quark)}$$

$$\sim \frac{2}{3} \Gamma_{SL} \text{ (free quark)} \quad (2.17)$$

using $\alpha_s(m_c) \sim 0.7$ for $\Lambda \sim 0.5$ GeV. The perturbation expansion is not quantitatively believable here but may be a reliable qualitative guide.

In this framework of QCD corrections, the free quark model predictions Eqs. (2.1) - (2.4) are very little modified. We are still committed to the spectator ansatz

$$\tau_+ = \tau_0 = \tau_F. \quad (2.18)$$

The estimate for the semi-leptonic branching ratio decreases by a factor 2, as a consequence of (2.16) and (2.17):

$$B(D \rightarrow \nu_\mu \bar{\mu} X) \sim \frac{2/3}{5 + \frac{2}{3} + \frac{2}{3}} \sim \frac{1}{10} \quad (2.19)$$

The estimate of the life-time decreases by only $\sim 30\%$,

$$\Gamma_{TOT} \sim \frac{5 + 2 \cdot 2/3}{5} \Gamma_{TOT} \text{ (free quark)}$$

$$\sim \left[(1.2_{-0.6}^{+0.9}) 10^{-12} \text{ sec} \right]^{-1}. \quad (2.20)$$

The estimate of the rate for Cabibbo-suppressed decays is unchanged from (2.4):

$$B(D \rightarrow \text{no } K) \lesssim .08 \quad (2.21)$$

D. Comparison with the Data

The experimental situation is different from these predictions but it is not yet clear how different. By SU(2) symmetry the semi-leptonic widths of D^+ and D^0 must be equal, so that

$$\frac{B(D^+ \rightarrow \nu_\mu \bar{\mu} X)}{B(D^0 \rightarrow \nu_\mu \bar{\mu} X)} = \frac{\tau_+}{\tau_0}. \quad (2.22)$$

One measurement¹² of the semileptonic branching ratios then gives $\tau_+/\tau_0 \gtrsim 4$ (95% CL) and another¹³ gives $\sim 3^{+7}_{-2}$. Early results¹⁴ from the E-531 emulsion experiment at Fermilab with still sparse statistics, give $\tau_+/\tau_0 \sim 10^{+20}_{-7}$. $B_{SL}(D^+)$ is measured at $\sim 0.16 \pm 0.05$ (Mark II¹³) or $\sim 0.24 \pm 0.04$ (DELCO)¹² while $B_{SL}(D^0)$ is $\sim 0.052 \pm 0.033$ (Mark II)¹³ and ≤ 0.045 (DELCO).¹² The emulsion data¹⁴ give $\tau_+ \sim (1.00^{+0.89}_{-0.46}) \cdot 10^{-12}$ sec and $\tau_0 \sim (0.93^{+0.52}_{-0.29}) \cdot 10^{-13}$ sec.

The branching ratios for Cabibbo suppressed decays are reported¹³ to be $B(D^0 \rightarrow \text{no K}) \sim 0.25 \pm 0.11$ and $B(D^+ \rightarrow \text{no K}) \sim 0.41 \pm 0.16$.

The data suggest that Cabibbo allowed D^+ decays are not enhanced but that D^0 nonleptonic decays are enhanced, perhaps substantially. For instance, Eq. (2.20) agrees remarkably with the emulsion measurement of τ_+ (even the reported experimental uncertainty is correctly predicted in (2.20)!). $B_{SL}(D^+)$ appears to agree with the free quark model prediction of $1/5$; given the theoretical uncertainties in (2.16) and (2.17) the discrepancy with the expected $1/10$, Eq. (2.19), is not unsettling. The branching ratios for $B(D \rightarrow \text{no K})$ hint at the possibility that Cabibbo suppressed nonleptonic decays of the D^+ may share the enhancement of the Cabibbo allowed D^0 decays. But a hard look at the quoted experimental uncertainties shows that the predictions (2.18) - (2.21) may yet survive at the level of a factor of two or better. Equations (2.19) and (2.20) should not be trusted at more than the factor two level in any case. Equations (2.18) and (2.21) are more reliable consequences of the assumed theoretical framework, but it is not hard to imagine effects which could also cause them to be modified by a factor two or three. So it is not yet clear whether theory and experiment are in a full scale collision or if the theorists will be able to walk away with only a dented fender and a few scratches to show.

E. Other Sources of Nonleptonic Enhancements

Regardless of the scale of the accident, it is clearly interesting to think now about additional sources of non-leptonic enhancements. One idea, which goes back to before the discovery of the J/ψ , is that decays into nonexotic channels are enhanced¹⁵ or, equivalently, that the net quantum numbers of the final state of enhanced decays may be represented by the same number of quarks as the initial state.^{16,17} D^0 and F^+ decay into states with the net quantum numbers of $s\bar{d}$ and $u\bar{d}$ respectively so they are enhanced, but the D^+ decays into a final state with exotic quantum numbers

$\bar{u}d\bar{d}$. Cabibbo suppressed decays of D^+ lead to a final state with the net quantum numbers of $\bar{u}d$, hence they may be a larger fraction of all D^+ decays. Had our understanding of K decays not "progressed" as described in Section B, this would probably have been the prevalent line of theoretical reasoning and $\tau_0 \ll \tau_+$ would have seemed a likely prospect.

But this is only a rule or mnemonic. Even if it turns out to be correct we will still want to know its dynamical origin. One possibility is that hadronic final state interactions are the basis of the rule. Broad s -channel resonances could enhance the D^0 and F^+ decay amplitudes. Hadronic final state interactions may have a tremendous effect on decays into particular final states (see Section F), but their effect on the total width should be less dramatic. Even if there were an s -channel $K\pi$ resonance around 1.86 GeV, its effect would be diminished by its probably large total width. We can get a feeling for how large the effect may be by looking at the structure in the e^+e^- total cross section between 1.5 and 2 GeV. My conclusion is that hadronic final state interactions are unlikely to contribute more than a factor two to τ_+/τ_0 . If in the end we learn that $\tau_+/\tau_0 \sim 2$, then hadronic f.s.i. could be part of the reason.

The "annihilation" diagrams are another possible dynamical explanation of the quark number conservation rule. It was argued in Section A that these are suppressed by a helicity factor $(m_u/m_c)^2$ and by the small probability for annihilation $(F_D/m_D)^2$. But in a first order calculation of the QCD corrections which assumes a nonrelativistic bound state model of the D , it is found that the annihilation diagrams might make a substantial contribution.¹⁸ After bremsstrahlung of a gluon the initial state is no longer in an s -wave so that there is no helicity suppression of the subsequent weak decay. Furthermore the scale m_D in the factor $(F_D/m_D)^2$ is replaced by a light constituent quark mass m_u . The conclusion is that

$$\frac{\tau_+}{\tau_0} \sim 1 + 0.4\alpha_s(m_c) \left(\frac{m_D}{m_c}\right)^5 \frac{F_D^2}{m_u^2} . \quad (2.23)$$

A reasonable guess for F_D based on the e^+e^- decay widths of old and new vector mesons gives³⁷ $F_D \sim 150$ MeV. and then $\tau_+/\tau_0 \sim 1.2$. Of course even if we know F_D precisely, (2.23) could be no more than a rough guide given the unreliability of the perturbation expansion for $\alpha_s(m_c) \cong 0.7$.

Another attitude¹⁹ is more realistic but has less predictive

power. As a relativistic bound state the D surely has a component of its wave function with one, two or many gluons. For this component the quark and antiquark need not be in an s -wave and can annihilate with no helicity suppression. Predictions are made by treating the magnitude of the gluonic component of the wave function as a free parameter. For reasonable values of this parameter the D^0 and F annihilation amplitudes might be substantially larger than the spectator quark amplitude that yielded $\tau \sim 10^{-12}$ sec.

D^0 annihilation proceeds by the t -channel exchange of a W -boson so the $c\bar{u}$ pair may be in a color octet and a single gluon is sufficient to make up the initial color singlet state. But F^+ annihilation proceeds through an s -channel W -boson so two gluons in a color singlet are needed to balance the color. Since the second gluon can be soft, this does not mean that F^+ annihilation is necessarily suppressed relative to D^0 annihilation. F^+ semileptonic decays may also proceed by this mechanism; hence these decays may be a source of gluon rich hadrons such as η' and possibly even of gluonium states.²⁰

A good test of the quark number rule is given by the isospin structure of the final states.²¹ The final states of D^0 decays must be dominantly $I = \frac{1}{2}$ since they are formed from an $s\bar{d}$ pair plus $I = 0$ gluons. Therefore we expect

$$\frac{\Gamma(D^0 \rightarrow \pi^+ K^-)}{\Gamma(D^0 \rightarrow \pi^0 K^0)} = \frac{\Gamma(D^0 \rightarrow \rho^+ K^-)}{\Gamma(D^0 \rightarrow \rho^0 K^0)} = 2 \quad (2.24)$$

and so on. For the $K\pi\pi$ mode we have²¹

$$\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0) = \frac{1}{2} \Gamma(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) - \frac{1}{4} \Gamma(D^0 \rightarrow \bar{K} \pi^+ \pi^0). \quad (2.25)$$

There are also constraints on the decays into K plus n pions for any value of n .

Before leaving the subject of annihilation diagrams I want to make a crazy suggestion which is motivated by the experimental presentations of Coyne and Scharre in this session. We have heard that the anticipated F -associated rise in η production is not yet seen at the Crystal Ball, and we have also heard of the very large signal for $\psi \rightarrow \gamma + E(1420)$ seen at both the Mark II and the Crystal Ball. Since the $E(1420)$ is not generally a prominent state in hadronic reactions and since $\psi \rightarrow \gamma X$ may be a copious source of gluon production, it is natural to speculate that the E may be a gluon-rich state, perhaps even a gluonium state. Unless the chain of reasoning is checked by a strong cup of coffee, it leads to the notion that the glueball cum E may be a good tag for F production. $F^+ \rightarrow E\pi^+$ should be the dominant mode, since $F^+ \rightarrow E\rho^+$ is only permitted on the ρ tail. So by looking for $F^+ \rightarrow E\pi^+ \rightarrow (K\bar{K}\pi)\pi^+$ it might be possible to detect the F and a

glueball in one fell swoop. If the annihilation amplitude is large this mode could be a very substantial fraction of F decays.

Another proposal^{22,23} to explain $\tau_+ \gg \tau_0$ requires assuming that f_-/f_+ is much larger than the leading log value, Eq. (2.15), and that gluon final state interactions do not change the color structure of the two $q\bar{q}$ color singlet clusters created according to the spectator quark ansatz. Recall that in leading log approximation

$$\begin{aligned} \mathcal{H}_{\Delta C=1} &= f_- O_- + f_+ O_+ \\ O_{\pm} &= \frac{1}{2} (O_1 \pm O_2) \\ O_1 &= (\bar{c}s)_L (\bar{d}u)_L \\ O_2 &= (\bar{c}u)_L (\bar{d}s)_L \end{aligned} \quad (2.26)$$

where for instance $(\bar{d}u)_L$ denotes the usual V-A color singlet current responsible for beta decay. Now it is easy to see that when O_1 acts on $D^+ \sim c\bar{d}$ it creates two color singlet quark-antiquark pairs, $(\bar{d}s)$ and $(\bar{d}u)$. O_2 acts on D^+ to create the same two pairs. But O_1 and O_2 create different color singlet pairs when they act on $D^0 \sim c\bar{u}$: O_1 creates $(\bar{u}s)(\bar{d}u)$ while O_2 creates $(\bar{u}u)(\bar{d}s)$. Therefore in D^+ decays the contribution of $O_- = \frac{1}{2} (O_1 - O_2)$ may cancel coherently in the amplitude whereas in D^0 decays no such cancellation occurs because O_1 and O_2 give rise to different final states which add incoherently. The result is that

$$\Gamma_{NL}(D^+) = \frac{4}{3} f_+^2 \Gamma_{NL}(\text{free quark}) \quad (2.27)$$

$$\Gamma_{NL}(D^0) = \frac{1}{3} (2f_+^2 + f_-^2) \Gamma_{NL}(\text{free quark}) \quad (2.28)$$

$$\frac{\tau_+}{\tau_0} = \frac{f_-^2 + 2f_+^2 + 4/3}{4f_+^2 + 4/3} \quad (2.29)$$

With the values obtained in leading log approximation, (2.15), $f_- = (f_+)^{-2} \sim 2$, the enhancement is $\tau_+/\tau_0 \sim 2$. For the value $f_- = (f_+)^{-2} \sim 5$, chosen²² to fit the observed ratio of $D^0 \rightarrow K\pi$ decays as discussed in the next Section, the result is $\tau_+/\tau_0 \sim 10$.

According to this approach, which I shall refer to as "enhanced 6 dominance", the effective $\Delta C = 1$ Hamiltonian is overwhelmingly dominated by the operator which is in the 6^* of $SU(3)$.

For the decays of D^0 and F^+ into two pseudoscalars, this has the same consequences for the isospin of the final state as the quark number conservation rule. The symmetric product of two octets projects into the 27 and 8 of SU(3). The D^0 is in the SU(3) 3^* so the final state created by action of O_- is given by $3^* \otimes 6^* = 8 \oplus 10^*$. Therefore 6 dominance requires the D^0 final state to be in an SU(3) octet and therefore in the $S = -1$ iscdoublet. However 6 dominance does not require an $I = 1/2$ final state for decays of D^0 into a pseudoscalar plus a vector, since in this case the antisymmetric 10^* is a permissible final state.

F. Exclusive Channels

In this section I want to give some examples of what can be learned from exclusive final states about the dynamical issues discussed in the preceding sections.²⁴ Consider first the measured rates¹³ for $D^0 \rightarrow K\pi$:

$$\begin{aligned} B(D^0 \rightarrow K^- \pi^+) &= 0.028 \pm 0.005, \\ B(D^0 \rightarrow \bar{K}^0 \pi^0) &= 0.021 \pm 0.009. \end{aligned} \quad (2.30)$$

The $K\pi$ final state is a sum of $I = 1/2$ and $I = 3/2$ components

$$\begin{aligned} m(D^0 \rightarrow K^- \pi^+) &= \sqrt{\frac{2}{3}} a_1 + \sqrt{\frac{1}{3}} a_3 \\ m(D^0 \rightarrow \bar{K}^0 \pi^0) &= \sqrt{\frac{1}{3}} a_1 - \sqrt{\frac{2}{3}} a_3 \end{aligned} \quad (2.31)$$

The quark number conservation rule requires the final state to be dominantly $I = 1/2$, so we expect

$$\frac{B(D^0 \rightarrow K^- \pi^+)}{B(D^0 \rightarrow \bar{K}^0 \pi^0)} = 2 \quad (2.32)$$

which is consistent with the data (2.30).

The same model which led to Eqs. (2.27) and (2.23) was first applied^{8,10} to the $D \rightarrow K\pi$ exclusive decays. The $q\bar{q}$ clusters are identified with the appropriate K and π mesons and again color rearrangement due to gluon final state interactions is neglected. The result is

$$\frac{B(D^0 \rightarrow K^- \pi^+)}{B(D^0 \rightarrow \bar{K}^0 \pi^0)} = 2 \cdot \frac{(2f_+ + f_-)^2}{(2f_+ - f_-)^2} \quad (2.33)$$

Using the leading log estimate for f_{\pm} the ratio is very large; for instance, substituting (2.15) I find 60 for the right hand side of (2.33). The choice²² $f_{-} = f_{+}^{-2} \sim 5$, which gives $\tau_{+}/\tau_{0} = 10$ in (2.29), gives a ratio of 4 in (2.33), compatible with the data at the $1 \frac{1}{2} \sigma$ level.

The prediction using the leading log estimate for f_{\pm} appears to be dramatically excluded by the data. In fact the situation is not so simple, because of the effect of hadronic final state interactions^{25,26} which shift the phases of the amplitudes in (2.31):

$$\begin{aligned} m(D^0 \rightarrow K^- \pi^+) &= \sqrt{\frac{2}{3}} e^{i\delta_1} a_1 + \sqrt{\frac{1}{3}} e^{i\delta_3} a_3 \\ m(D^0 \rightarrow \bar{K}^0 \pi^0) &= \sqrt{\frac{1}{3}} e^{i\delta_1} a_1 - \sqrt{\frac{2}{3}} e^{i\delta_3} a_3 \end{aligned} \quad (2.34)$$

The leading log prediction that the $K^- \pi^+$ mode is much more frequent than the $\bar{K}^0 \pi^0$ mode means that in (2.31) we expect a cancellation, $a_1 \sim \sqrt{2} a_3$. But this delicate cancellation is easily undone if, as is not unlikely, $|\delta_1 - \delta_3|$ is large. If we assume for illustration that $a_1 = \sqrt{2} a_3$ exactly, then²⁵

$$\frac{\Gamma(D^0 \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)} = \frac{1}{8} \left[9 \cot^2 \left(\frac{\delta_1 - \delta_3}{2} \right) + 1 \right] \quad (2.35)$$

which can vary from infinity for $\delta_1 = \delta_3$ to $1/8$ for $\delta_1 - \delta_3 = \frac{\pi}{2}$. The situation is further complicated by the inelasticity in the $K\pi$ channel, which is not included in (2.34). The conclusion is that we probably cannot learn much about the leading log approximation from the decays $D^0 \rightarrow K\pi$. On the other hand, the predictions of the quark number rule and of enhanced 6 dominance are far less sensitive to these final state phases since they imply $|a_1| \gg |a_3|$.

Decays into a pseudoscalar plus a vector meson are of interest because, as discussed in the previous section, they offer more possibilities for distinguishing between the quark number rule and the enhanced 6 dominance hypothesis. The quark number rule always requires the D^0 to decay to an $I = 1/2$ final state, so that

$$\frac{\Gamma(D^0 \rightarrow \rho^+ K^-)}{\Gamma(D^0 \rightarrow \rho^0 \bar{K}^0)} = \frac{\Gamma(D^0 \rightarrow \pi^+ K^{*-})}{\Gamma(D^0 \rightarrow \pi^0 \bar{K}^{*0})} = 2 \quad (2.36)$$

Equation (2.36) need not hold for the enhanced 6 dominance hypothesis which does, together with SU(3) symmetry, imply that²⁷

$$\Gamma(D^+ \rightarrow \rho^+ \bar{K}^0) = \Gamma(D^+ \rightarrow \pi^+ \bar{K}^{*0}) \quad (2.37)$$

Together with the SU(2) symmetry relations

$$m(D^0 \rightarrow \rho^+ K^-) + \sqrt{2} m(D^0 \rightarrow \rho^0 \bar{K}^0) = m(D^+ \rightarrow \rho^+ \bar{K}^0)$$

$$m(D^0 \rightarrow \pi^+ K^{*-}) + \sqrt{2} m(D^0 \rightarrow \pi^0 \bar{K}^{*0}) = m(D^+ \rightarrow \pi^+ \bar{K}^{*0})$$

(2.37) implies the inequality²⁸

$$\frac{B(D^+ \rightarrow \pi^+ \bar{K}^{*0})}{B(D^0 \rightarrow \rho^+ K^-)} \gtrsim \frac{\Gamma_+}{\Gamma_0} \left(1 - \sqrt{\frac{2B(D^0 \rightarrow \rho^0 \bar{K}^0)}{B(D^0 \rightarrow \rho^+ K^-)}} \right)^2 \quad (2.38)$$

Scharre in this session has reported measurements of these branching ratios. Two D^0 decay modes are measured at

$$B(D^0 \rightarrow \rho^+ K^-) \gtrsim \frac{2}{3} B(D^0 \rightarrow \pi^+ \pi^0 K^-), \quad (2.39)$$

$$B(D^0 \rightarrow \pi^+ K^{*-}) \cdot B(K^{*-} \rightarrow \pi^- \bar{K}^0) \gtrsim \frac{2}{3} B(D^0 \rightarrow \pi^+ \pi^- \bar{K}^0), \quad (2.40)$$

from which we can deduce that

$$B(D^0 \rightarrow \rho^0 \bar{K}^0) \lesssim \frac{1}{3} B(D^0 \rightarrow \pi^+ \pi^- \bar{K}^0), \quad (2.41)$$

$$B(D^0 \rightarrow \pi^+ K^{*-}) \cdot B(K^{*-} \rightarrow \pi^0 K^-) \lesssim \frac{1}{3} B(D^0 \rightarrow K^- \pi^+ \pi^0). \quad (2.42)$$

Using the three body branching ratios

$$B(D^0 \rightarrow K^- \pi^+ \pi^0) = 0.085 \pm 0.032, \quad (2.43)$$

$$B(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) = 0.038 \pm 0.012, \quad (2.44)$$

we have the lower bound

$$\frac{B(D^0 \rightarrow \rho^+ \bar{K}^-)}{B(D^0 \rightarrow \rho^0 \bar{K}^0)} \gtrsim 4.4 \pm 2.1, \quad (2.45)$$

where I have combined the uncertainties in (2.43) and (2.44) in quadrature. At the moment (2.45) is still just compatible at the 1 σ level with the quark number conservation rule, Eq. (2.36), but a contradiction will develop if the bounds (2.39) - (2.42) are improved significantly. In this case the failure to observe

$D^+ \rightarrow \pi^+ \bar{K}^{*0}$ could also lead to a contradiction with the 6-dominance inequality, Eq. (2.38).

III. B Meson Decays

There is (fortunately) only enough time remaining to discuss two aspects of B meson decays, both of which are related to the issues discussed in the previous section. I will simply assume the b quark assignment of the standard Kobayashi-Maskawa model. I will discuss the problem of determining the K-M angles if there are significant enhancements in nonleptonic B decays and what we can learn about nonleptonic dynamics from the decays $B \rightarrow \psi X$.

A. Determining the K-M Angles

In the standard six quark model³ the charged weak current is given by

$$J = (\bar{u} \ \bar{c} \ \bar{t})_L U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \quad (3.1)$$

where

$$U = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}. \quad (3.2)$$

The b quark decays by its coupling to the u quark

$$V_{ub} = s_1 s_3, \quad (3.3)$$

and to the c quark

$$V_{cb} = c_1 c_2 s_3 + s_2 c_3 e^{i\delta}. \quad (3.4)$$

Measurement of the K-M angles will be one of the most profound results of the study of B meson decays. The K-M angles are intimately connected with the origin of the quark masses. For instance, in the standard Higgs model the diagonalization of the Higgs-fermion coupling matrix yields both the fermion masses and the K-M angles. In general, models which offer an explanation of the quark masses will also predict or constrain the K-M angles.

Knowledge of the values of the angles is also an essential prerequisite to the study of weak interaction dynamics. For instance, it is necessary to know the K-M angles in order to extract dynamical enhancement factors from B lifetime measurements. Lepton yields which are not separated by B^0 or B^- cannot necessarily tell us whether there are enhancements - as we are learning now from the D decays. Furthermore in B decays

there are additional possibilities for confusion if $b \rightarrow u$ is a large mode. In principle, with measurements of the semileptonic to nonleptonic ratios $\Gamma_{SL}(b \rightarrow u)/\Gamma_{NL}(b \rightarrow u)$ and

$\Gamma_{SL}(b \rightarrow c)/\Gamma_{NL}(b \rightarrow c)$, we can measure dynamical enhancements without knowing the K-M angles. But this is easier for theorists to imagine than for experimenters to do. I am doubtful that we will reliably know the strength of enhancements in B decays until we know the K-M angles. Unfortunately, as I will discuss below, it is not easy to measure the K-M angles if there are unknown nonleptonic enhancements.

The cleanest present constraints²⁹ are on θ_1 and θ_3 . They are from beta decay

$$|c_1| = 0.9737 \pm 0.0025 \quad (3.5)$$

and from an analysis of $\Delta S = 1$ decays

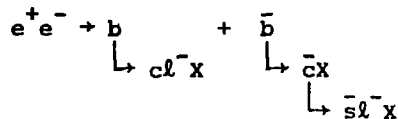
$$|s_3| = 0.28^{+0.28}_{-0.21} \quad (3.6)$$

Equations (3.5) and (3.6) together imply that

$$|v_{ub}| = 0.06 \pm 0.06 \quad (3.7)$$

Thus v_{ub} is very small and is likely to be appreciably smaller than v_{cb} . Notice however in (3.4) that v_{cb} could also be extremely small if, for instance, all $|\theta_1|$ and $|\delta|$ are very small and if $s_2 \cong -s_3$. There are also constraints³⁰ on s_2 , however these constraints require much more theoretical machinery and assumptions. No sacred principles would be violated if they turned out to be wrong.

The key to measuring v_{ub} and v_{cb} is lepton detection. One method is to use the single lepton spectrum³¹ just above $B\bar{B}$ threshold, as at the T¹¹¹. Decays $b \rightarrow u$ have a harder endpoint and almost no soft secondary leptons, while $b \rightarrow c$ has an endpoint 15% softer than for $b \rightarrow u$ and a large number of soft secondary leptons from D decays. Another method³² is to use the like-sign two lepton events generated by the cascade sequence



to determine the ratio $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$.

Both of these methods may be used to extract the K-M angles if there are no large nonleptonic enhancements. But the likelihood of significant enhancements seems greater today than a year or more ago when these analyses were first done - since the betting odds are

coupled to whether there are large enhancements in D decays. And even if there are not large enhancements in D decays they might still occur in B decays; the enhancement mechanism for K decays might be specific to Cabibbo-suppressed decays (e.g. like penguins, which in particular should not be important for B if simple estimates are correct) in which case it could be important in B but not in D decays.

If there are significant but unknown enhancements in $\Gamma_{NL}(b \rightarrow u)$ and/or $\Gamma_{NL}(b \rightarrow c)$ then the method of like-sign dileptons can determine the ratio $\Gamma_{TOT}(b \rightarrow u)/\Gamma_{TOT}(b \rightarrow c)$. (Like-sign dileptons due to $B_0 - \bar{B}_0$ mixing will both be primary or will both be secondary; their momenta will therefore help to separate them from the "usual" like-sign dileptons in which one is primary and the other secondary.) But without knowing the enhancements of $b \rightarrow u$ and $b \rightarrow c$ we cannot extract $|V_{ub}/V_{cb}|$ from this information.

The single lepton spectrum can in principle determine the ratio $|V_{ub}/V_{cb}|$ independent of enhancements. By measuring the yields of leptons near the endpoints for $b \rightarrow u\ell^+X$ and $b \rightarrow c\ell^+X$ one can unambiguously measure $\Gamma_{SL}(b \rightarrow u)/\Gamma_{SL}(b \rightarrow c)$ and extract $|V_{ub}/V_{cb}|$. But it may be impossible to accumulate enough statistics to carry this out in practice, because the signals near the endpoints are very small and because of a background from $e^+e^- \rightarrow \tau^+\tau^-$, cc which is a few times bigger than the signal at the $b \rightarrow (u/c)\ell^+X$ endpoints.

The problem requires more attention than it has been given. I do not know whether there is a practical method to extract $|V_{ub}/V_{cb}|$ from the data which would work given the most general possible pattern of enhancements and K-M angles.

B. What We Learn from $B \rightarrow \psi X$

I want to conclude this sketchy and disjointed discussion of B decays by mentioning what the rate for $B \rightarrow \psi X$ might teach us about the dynamics of nonleptonic decays. In principle we can learn from this inclusive rate about the importance of color rearrangement due to gluonic final state interactions. These were assumed to be negligible in some of the models discussed in Section II, though some authors³³ have suggested that they play an important role in the formation of exclusive final states.

The lowest order effective Hamiltonian is

$$\begin{aligned} \mathcal{H}_{eff} &\propto (\bar{bc})_L (\bar{cs})_L \\ &\propto (\bar{b}_\alpha s_\beta)_L (\bar{c}_\beta c_\alpha)_L \end{aligned} \quad (3.8)$$

where the second line is obtained by Fierz rearrangement and the color indices α, β are summed over. In calculating the rate for

$B \rightarrow \psi X$ the current $\bar{c}_\beta c_\alpha$ is in a color singlet for 1/9 of the final states and in the color octet for 8/9. Therefore there is a factor 9 difference in the predicted rate $B \rightarrow \psi X$ depending on whether we neglect color rearrangement due to gluon f.s.i. (i.e., keep the factor 1/9) or whether we assume that gluons rearrange the $\bar{c}c$ pair into a color singlet with probability one. Making the latter assumption a crude but plausible estimate gives³⁴

$$\begin{aligned} B(B \rightarrow \psi X) &\sim B(b \rightarrow \psi X) \\ &\sim B(b \rightarrow \bar{c}c)B(\bar{c}c \rightarrow \psi) \\ &\sim 3 - 5\% \end{aligned} \quad (3.9)$$

where $|V_{cb}| \gg |V_{ub}|$ is assumed. If color rearrangement is not assumed, the estimate would decrease by 1/9.

A more detailed estimate^{35,36} uses the J/ψ wave functions at the origin as determined from $\Gamma(\psi \rightarrow e^+e^-)$. The result of this calculation is surprisingly large. Assuming no color rearrangement of the final state (i.e., which would have given 1/3 - 1/2% using the method of Eq. (3.9)), the result is

$$B(B \rightarrow \psi X) \sim (.018)(2f_+ - f_-)^2. \quad (3.10)$$

For free quark values $f_+ = f_- = 1$, we get 1.8% from this estimate, which would become 16% if color rearrangement is permitted as in (3.9). At the other extreme if we use the renormalization group values for f_\pm and do not allow color rearrangement, (3.10) yields³⁵ 0.2%. If enhanced values are taken for f_\pm as in Ref. (22), then (3.10) yields 5% (as discussed in Section II, this approach assumes no color rearrangement).

What do we learn from all this? I invite the reader to tell me. It appears that the theoretical estimates are out of control at the level of the factor of 9 that we would like to be able to study.

IV Conclusion

I have tried to emphasize the importance of measuring with greater accuracy the lifetimes and semileptonic branching ratios for D^+ and D^0 . It is clear that $\tau_+/\tau_0 > 1$, but it is not yet clear whether the ratio is $\lesssim 3$ or $\gtrsim 10$ or somewhere in between. If $\tau_+/\tau_0 \lesssim 3$ there need not be any drastic revision of our understanding of nonleptonic decays. Enhancements of this order could be explained by a combination of final state enhancements and QCD corrections. But if $\tau_+/\tau_0 \gtrsim 10$ the conventional picture of nonleptonic decays is called into question. In my

mind this would even raise doubts about the popular semiquantitative picture of the enhancements in $\Delta S = 1$ decays based on the penguin diagrams. If the enhancements in the D systems are not even understood at the level of an order of magnitude, then we are ignorant of important dynamical mechanisms which might also be important in $\Delta S = 1$ and $\Delta B = 1$ decays. I have also tried to emphasize that accurate measurements of exclusive final states can be a powerful probe of the dynamics, though any given prediction must be scrutinized carefully for sensitivity to complicating factors such as final state interactions and SU(3) symmetry breaking.

The possibility of large enhancements in D decays raises the spectre of the same possibility in B decays. One analysis indicates that annihilation diagrams eventually dominate (by a logarithm) as we scale to heavier flavors.³⁷ Even if it turns out that D decays are not drastically enhanced, caution requires that we take account of the possibility of significant enhancements in B decays. In view of their fundamental importance, we want to measure the K-M angles in a way that is independent of possible enhancements and which does not rely on the theoretical prejudice that the enhancements are small. Development of a practical program to accomplish this goal is a problem that merits careful consideration.

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REFERENCES

1. For a review see M. K. Gaillard, Proceedings of the 1978 SLAC Summer Institute.
2. S. Glashow, J. Iliopolous, and L. Maiani, Phys. Rev. D2, 1285, 1970.
3. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652, 1973.
4. M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108, 1974; G. Altarelli and L. Maiani, Phys. Lett. 52B, 351, 1974.
5. M. Shifman, A. Vainshtein, and V. Zakharov, J.E.T.P. Lett. 22, 553, 1975; Nuc. Phys. B120, 316, 1977.
6. J. Donoghue, E. Golowich, W. Ponce, and B. Holstein, Phys. Rev. D21, 186, 1980.
7. L. Abbott, P. Sikivie, and M. Wise, Phys. Rev. D21, 1768 (1980).
8. J. Ellis, M. K. Gaillard, and D. Nanopolous, Nucl. Phys. B100, 313, 1975.
9. G. Altarelli, N. Cabibbo, and L. Maiani, Nucl. Phys. B88, 285, 1975.

10. N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418, 1978.
11. M. Suzuki, Nucl. Phys. B145, 420, 1978; N. Cabibbo and L. Maiani, Phys. Lett. 79B, 109, 1978.
12. J. Kirkby, Proceedings of the IX International Symposium on Lepton and Photon Interactions at High Energies, Batavia, IL.
13. V. Luth, ibid.
14. N.W. Reay Colloquium presented at the Lawrence Berkeley Laboratory, April, 1980.
15. M. Gaillard, B. Lee, and J. Rosner, Rev. Mod. Phys. 47, 277, 1975.
16. T. Hayashi, M. Nakagawa, M. Nitto, and S. Ogawa, Prog. Theor. Phys. 49, 351, 1973; 52, 636, 1974.
17. See also E. Ma, S. Pakvasa, and W. Simmons, University of Hawaii preprint UH-511-369-79, 1979; M. Matsuda, M. Nakagawa, and S. Ogawa, preprint, 1979; I. Bigi and L. Stodolsky, SLAC-PUB-2410, 1979.
18. M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7, 1980, and erratum, ibid, 44, 962 (1980).
19. H. Fritzsche and P. Minkowski, Phys. Lett. 90B, 455, 1980 and Bern University preprint, unpublished, 1980.
20. I. Bigi, CERN preprint TH 2798-CERN, 1979.
21. S. P. Rosen, Phys. Lett. 89B, 246, 1980.
22. B. Guberina, S. Nussinov, R. Peccei, and R. Rückl, Phys. Lett. 89B, 111, 1979.
23. M. Katuya and Y. Koide, Phys. Rev. D19, 2631, 1979; Y. Koide, Phys. Rev. D20, 1739, 1979.
24. SU(3) predictions for decays into two pseudoscalars in the 6 quark model are given by M. Suzuki, Phys. Lett. 85B, 91, 1979; L.-L. Wang and F. Wilczek, Phys. Rev. Lett. 43, 816 1979; C. Quigg, Z. Phys. C4, 55, 1980.
25. H. Lipkin, Phys. Rev. Lett. 44, 710, 1980.
26. J. Donoghue and B. Holstein, Phys. Rev. D21, 1334, 1980.
27. M. Einhorn and C. Quigg, Phys. Rev. D12, 2015, 1975.
28. I am grateful to F. Gilman for calling this relationship to my attention.
29. R. Shrock and L.-L. Wang, Phys. Rev. Lett. 41, 1692, 1978.
30. J. Ellis et. al., Nucl. Phys. B131, 285, 1977; V. Barger, W. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585, 1979; R. Shrock S. Treiman, and L.-L. Wang, ibid. 42, 1589, 1979.
31. V. Barger, T. Gottschalk, and R. Phillips, Phys. Lett. 82B, 445, 1979; A. Ali, Z. Physik C1, 25, 1979; A. Ali and Z. Aydin, Nuc. Phys. B14B, 165, 1979.
32. C. Quigg and J. Rosner, Phys. Rev. D19, 1532, 1979.
33. N. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. 90B, 431, 1980.
34. H. Fritzsche, Phys. Lett. 86B, 164, 1979; ibid, 343, 1979.
35. M. Wise, Phys. Lett. 89B, 229, 1980.
36. T. De Grand and D. Toussaint, Phys. Lett. 89B, 256, 1980.
37. M. Suzuki, LBL preprint Univ. of California, Berkeley pre-print, UCB-PTH-80/4, (submitted to Nucl. Phys. B), 1980.