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ABSTRACT

An alternative definition of the gradient terms in the lattice space quantization method is used to calculate the energy of a nonlinear meson. The expansion in powers of momentum agrees somewhat better with the relativistic expansion than the result previously reported by Schiff.

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KINETIC ENERGY IN NONLINEAR MESON THEORY

D. H. Holland

In his paper¹ dealing with the quantization of a nonlinear meson theory, Schiff has derived an approximate relationship between the momentum and energy of the mesons which appear as quanta of the nonlinear field. Quantization is carried out by the lattice space method, which entails the definition of a lattice analog of the gradient operator. More recently, the writer has applied the lattice space method to a different problem,² and found an alternative definition of the gradient operator to be both more convenient and more plausible than the one employed by Schiff. It is shown below that with this definition, the calculation of the kinetic energy is considerably simplified, and the agreement with the relativistic momentum-energy relationship somewhat improved.

We define the i -component of the vector $\underline{p(k)}$

$$p_i = \frac{\sin k_i \ell}{\ell}, \quad (1)$$

where ℓ is the lattice spacing, and $\underline{k}$ is a reciprocal lattice vector.

The quantity A_{st} used by Schiff is redefined as

$$A_{st} = N^{-1} \sum_{\underline{k}} p^2(\underline{k}) \exp i \underline{k} \cdot (\underline{r}_s - \underline{r}_t), \quad (2)$$

where N is the number of points in the lattice, and the sum is extended over a cell of the reciprocal lattice.

The calculation of the kinetic energy proceeds exactly as before, with the new definition of A_{st} used throughout. In Schiff's notation we have

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$$P \equiv A_{ss} = N^{-1} \sum_{\underline{k}} p^2(\underline{k}) = \frac{3}{2\ell^2}, \quad (3)$$

where Eq. (3) is obtained by taking the linear dimension L of the lattice to be large and replacing the sum by an integral. Similarly

$$Q = N^{-1} \sum_{\underline{k}} p^4(\underline{k}) = \frac{7}{6} P^2, \quad (4)$$

$$R = N^{-1} \sum_{\underline{k}} p^6(\underline{k}) = \frac{3}{2} P^3. \quad (5)$$

Also appearing is the quantity

$$\begin{aligned} Z(\underline{K}) = N^{-4} \sum_{\underline{s}} \sum_{\underline{t}} \sum_{\underline{k}} \sum_{\underline{k}'} \sum_{\underline{k}''} p^2(\underline{k}) p^2(\underline{k}') p^2(\underline{k}'') \exp \left[i(\underline{k} + \underline{k}' + \underline{k}'' + \underline{K}) \cdot (\underline{r}_{\underline{s}} - \underline{r}_{\underline{t}}) \right] \end{aligned} \quad (6)$$

where $\underline{K}$ is the approximate momentum of the particle. The $\underline{k}$ sums are taken over a cell of the reciprocal lattice, and the $\underline{s}, \underline{t}$ sums over a cell of the space lattice, the prime denoting that the points $\underline{r}_{\underline{s}} = \underline{r}_{\underline{t}}$ are to be excluded. Evaluation of the $\underline{s}, \underline{t}$ sums yields

$$Z(\underline{K}) = P^3 + N^{-2} \sum_{\underline{k}} \sum_{\underline{k}'} \sum_{\underline{k}''} p^2(\underline{k}) p^2(\underline{k}') p^2(\underline{k}'') \delta(\underline{k} + \underline{k}' + \underline{k}'' + \underline{K} - \underline{K}_1), \quad (7)$$

where $\underline{K}_1$ is a vector each of whose components is 0 or $\pm 2\pi/L$.

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The appearance of $\underline{k}_1$ is due to the fact that the sum of more than two $\underline{k}$ vectors can reach to the origin of an adjacent cell of the reciprocal lattice. It is the presence of $\underline{k}_1$ which complicates the analysis if the $\underline{k}$'s are used in place of the $\underline{p}$'s, since then one must evaluate integrals with a discontinuous integrand. When the $\underline{p}$'s are used, however,

$$p^2(\underline{k}) \delta(\underline{k} + \underline{k}' + \underline{k}'' + \underline{K} - \underline{K}_1) = p^2(\underline{k}) \delta(\underline{k} + \underline{k}' + \underline{k}'' + \underline{K}), \quad (8)$$

since p is periodic in $\underline{k}$ space.

Substituting Eq. (8) in Eq. (7) and performing the $\underline{k}''$ sum, we have

$$Z(\underline{K}) = P^3 + N^{-2} \sum_{\underline{k}} \sum_{\underline{k}'} p^2(\underline{k}) p^2(\underline{k}') p^2(\underline{k} + \underline{k}' + \underline{K}), \quad (9)$$

which is readily evaluated and yields

$$Z(\underline{K}) = \frac{P^2}{36} \left[\frac{p^2(\underline{K})}{\ell^2} - P \right] \simeq \frac{P^2}{36} (K^2 - P). \quad (|K| < \frac{1}{\ell}) \quad (10)$$

The final expression for the kinetic energy, correct to third order in the gradient term of the Hamiltonian can be written

$$E = a + bP + cK^2 + dP^2 + eK^2P + fK^4 + gP^3 + hK^2P^2 + iK^4P + jK^6. \quad (11)$$

A similar expansion can be found assuming the energy is of the form

$$E = (m^2 + K^2)^{\frac{1}{2}}. \quad (12)$$

With m chosen in analogy with the linear case, as discussed in (A),

the coefficients a , b , etc. derived from Eqs. (11) and (12) may be compared.³ Such a comparison is shown in Table I for the case $g = \infty$, where g is the coefficient of the nonlinear term in the meson Hamiltonian.

As discussed in (A), the errors are zero for $g = 0$, hence this calculation probably overestimates the errors for finite g . The agreement could also be improved by changing the definition of m in Eq. (12).

The writer wishes to thank Professor L. I. Schiff for helpful comments on this work.

TABLE I

Coefficient	Percent error this calculation	Percent error from (A)
a	0	0
b	0	0
c	0	0
d	12.2	19.4
e	0	0
f	0	0
g	15.8	19.3
h	5.3	24.5
i	0	1.5
j	0	0.9

Table I. Percent error in the coefficients in Eq. (11).

REFERENCES

1. L. I. Schiff, Phys. Rev. 92, 766 (1953). Referred to herein as (A).
2. D. H. Holland, Phys. Rev. 98, 788 (1955).
3. The expansions (11) and (12) are given in Eqs. (A26) and (A28).