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Authors
Ashofteh, Parisa-Sadat
Bozorg-Haddad, Omid
Loáiciga, Hugo A

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RESEARCH PAPER

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Application of bi-objective genetic programming for optimizing irrigation rules using two reservoir performance criteria

Parisa-Sadat Ashofteh[a], Omid Bozorg-Haddad[b] and Hugo A. Loáiciga[c]

[a]Department of Civil Engineering, University of Qom, Qom, Iran; [b]Department of Irrigation & Reclamation, Faculty of Agricultural Engineering & Technology, College of Agriculture & Natural Resources, University of Tehran, Tehran, Iran; [c]Department of Geography, University of California, Santa Barbara, CA, USA

ABSTRACT
A bi-objective genetic programming (BO-GP) algorithm is developed and applied to optimize the operating rules of the Aidoghmoush reservoir (East Azerbaijan in northeastern Iran). The two-objective optimization problem maximizes reservoir reliability and minimizes the vulnerability index associated with the supply of agricultural water. The developed BO-GP algorithm calculates Pareto possibility frontiers representing loci of optimal operating policies. Any operation policy is calculated by the BO-GP based on the inflow volume to reservoir, the storage volume, and the water demand volume that minimize vulnerability and maximize reliability. The application results show a successful performance of the BO-GP algorithm in solving the bi-objective water supply problem with reservoir operation. This paper's results establish that the system vulnerability and its reliability range between 16–41% and 46–78%, respectively.

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Bi-objective genetic programming; Pareto front; vulnerability; reliability; optimal operating rules

Introduction

Many water allocation problems involve multiple objectives, some of which are in conflict with each other or are valued incommensurably. Therefore, several heterogeneous objective functions (often measured in widely different units) must be taken into account, and a multi-objective optimization problem must be solved. This implies finding a Pareto set (a boundary or surface in two or more dimensions) of optimal solutions defining tradeoffs between the objectives. Within the Pareto set decision makers choose the most desirable solution. Multi-objective optimization methods can be applied to find such Pareto sets. Evolutionary optimization methods are particularly well suited to solve many types of multi-objective optimization problems. Genetic programming (GP) is a leading one among them (Poli *et al.* 2008). One of the advantages of optimization methods with evolutionary algorithms compared to other methods is that these algorithms operate on a population of solutions at each iteration (or generation), which means each iteration generates many potential solutions that are improved upon from generation to the next. A survey of multi-objective GP applications follows to exemplify its wide-ranging capabilities.

Rossi *et al.* (2002) applied multi-objective GP (MO-GP) to design a digital system. The objectives were the suitability of the filter transfer function and the transition activity of digital blocks. Their results demonstrated the performance of MO-GP was appropriate in the automation of electronic design. Hinchliffe (2001) applied GP to steady-state model evolution. Multiple-basis function GP (MBF-GP) was introduced and its performance was compared with the standard GP algorithm. Their results showed the performance of MBF-GP was successful computationally relative to GP in solving multi-objective problems. Dimopoulos (2005) introduced a GP algorithm for the solution of the multi-objective cell-formation problem. They employed MO-GP to identify the Pareto set for a cell-formation problem related to the design of a cellular manufacturing production system. Zhao (2007) proposed a MO-GP approach for developing Pareto optimal decision trees. The Proposed approach allowed the decision-maker to specify partial preferences on the conflicting objectives. A diabetes prediction problem and a credit card application approval problem were used to demonstrate the applicability of the proposed approach. You and Cai (2008a) developed a conceptual two-period model for reservoir operation with hedging. An extended analysis of the model properties was presented with a general utility function, addressing (1) the starting and ending water availability for hedging, (2) the range of hedging that was related to water demand levels, (3) inflow uncertainty, and (4) evaporation loss. Their findings can be applied to improve numerical modelling for reservoir operation. You and Cai (2008b) presented a method that derived a hedging rule from theoretical analysis. Their results show utility improvement with the hedging policy compared to the standard operation policy (SOP). Celeste and Billib (2009) assessed the performance of seven stochastic models used to define for optimal reservoir operating policies operation. The models were based on implicit simulation-optimization (ISO) and explicit stochastic optimization (ESO) as well as on the parameterization–simulation–optimization (PSO) approach. The models were applied to the operation of a single reservoir in northeastern Brazil. The results indicate the ISO and PSO models performed better than SDP and the SOP. Rani and Madalena Moreira (2010) presented a simulation-optimization modelling approaches for reservoir systems. Sreekanth and Datta (2010) conducted multi-objective management of coastal aquifers with GP and modular neural network (MNN) based on surrogate models. The results from GP and MNN were compared with finite element simulations of groundwater flow with the FEMWATER model. Zafra *et al.* (2011) introduced a

CONTACT Omid Bozorg-Haddad obhaddad@ut.ac.ir

multi-objective optimization algorithm based on GP (i.e. MOG3P-MI) for solving a mining problem based on multiple instance learning. The GP-based algorithm performed well when compared with alternative methods. Fallah-Mehdipour *et al.* (2013) developed and extracted a fixed-length gene genetic programming (FLGGP) rule based on GP. The FLGGP rules were employed in an aquifer-dam system with two subsystems. Results demonstrated the FLGGP was more flexible and effective in determining optimal rule curves for a conjunctive aquifer-dam system. Arruda Pereira *et al.* (2014) introduced a multi-objective algorithm based on GP to calculate classification rules in databases composed of hybrid data. A niche technique was employed in their algorithm. Li *et al.* (2014) presented GP to derive the explicit nonlinear formulation of operating rules for multi-reservoir systems in China. The inflow and storage energy terms were selected as input variables for total output of the aggregated reservoir and for decomposition. Hakimzadeh *et al.* (2014) applied GP to simulate outflow hydrograph from earthen dam breach. The results demonstrated the results of the GP method were in good agreement with the observed values. The model was tested for a case study (Teton Dam).

Several studies dealing with reservoir operating rules are reviewed next. Aboutalebi *et al.* (2015) proposed a novel tool that coupled the non-dominated sorting genetic algorithm (NSGAII) with support vector regression (SVR) and nonlinear programming (NLP) to optimize monthly operation rules for hydropower generation. Najl *et al.* (2016) introduced a simulation-optimization model for deriving an operating policy for multi-reservoir systems with a self-adaptive genetic algorithm to maximize the system's hydropower production, subject to the system's physical constraints. Yang *et al.* (2017) used Pareto archived dynamically dimensioned search (PA-DDS) to optimize reservoir operation rules with the objectives of maximizing the power generation and water supply.

The novelty of this paper consists in the development of a bio-objective GP (i.e. BO-GP) to optimize reservoir operation with dual objectives. Previous research has shown the number of iterative solutions with evolutionary algorithms applied to multi-objective problems may grow rapidly without any clear improvement in the fitness function (Poli *et al.* 2008). This anomaly has given impetus to the further development of GP to solve complex multiobjective problems. In the present study GPLAB is applied to solve bi-objective problems. The GPLAB is a GP tool in the MATLAB programming environment (Silva 2007). The developed BO-GP algorithm is first applied to solve a bi-objective problem in the field of mathematics, and its accuracy is demonstrated with this test problem. BO-GP is subsequently applied to calculate reservoir operating rules that optimize the objectives of minimizing the vulnerability of water supply for agriculture and to maximize the reservoir operation's reliability index. The BO-GP is exemplified with the Aidoghmoush one-reservoir system (Iran).

The paper presents the following sections: (1) brief overview of the GP algorithm; (2) description of the BO-GP algorithm for solving of bi-objective problems; (3) proving the efficiency of the BO-GP algorithm in solving a bi-objective mathematical problem; and (4) application of the BO-GP algorithm to calculate reservoir operating rules with the dual objectives of minimizing the vulnerability and maximizing the reservoir reliability. The flowchart of this paper's proposed methodology is portrayed in Figure 1.

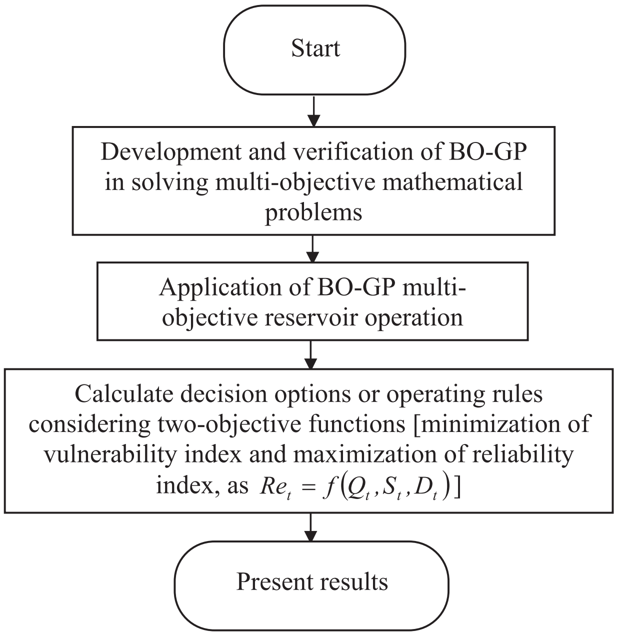

Figure 1. Flowchart of the research approach.

Methodology

Development of the GP tool

The main goal of bi-objective optimization is to find a set of solutions expressed as Pareto possibility frontiers. The presentation of the BO-GP algorithm is preceded by a short summary of the single-objective-GP (SO-GP) algorithm.

(a)#The SO-GP algorithm. The steps of SO-GP are as follows:

(1) Generate the initial random population of solutions. This population consists of mathematical equations (or decision trees), which includes a set of functions (arithmetic and mathematical operators) and terminals (constant parameters and independent variables).
(2) Select parents (with selection operator), perform crossover (with crossover operator); and produce the offspring population of solutions.
(3) Select parents (with selection operator), perform mutation (with the mutation operator); and produce the mutant population of solutions.
(4) Select members of new population from the population of the parents, offspring, and mutants.
(5) If the stopping conditions are not met, go to step (2); otherwise go to step (6).
(6) Stop

A flowchart of the SO-GP algorithm is depicted in Figure 2.

(b)#The BO-GP algorithm. The key difference between the SO-GP and the BO-GP algorithm is in the fourth step of the algorithm described in section '(a) the SO-GP algorithm', which has to do with sorting of the members of a new population. Multi-objective optimization (MOO) performs sorting based on the quality of the solutions and on the order of the solutions.

The main idea in MOO is the theory of Pareto dominance. A first solution dominates a second solution if and only if (1) the first solution is not worse than the second solution with respect to the objectives of a problem, and (2) the first

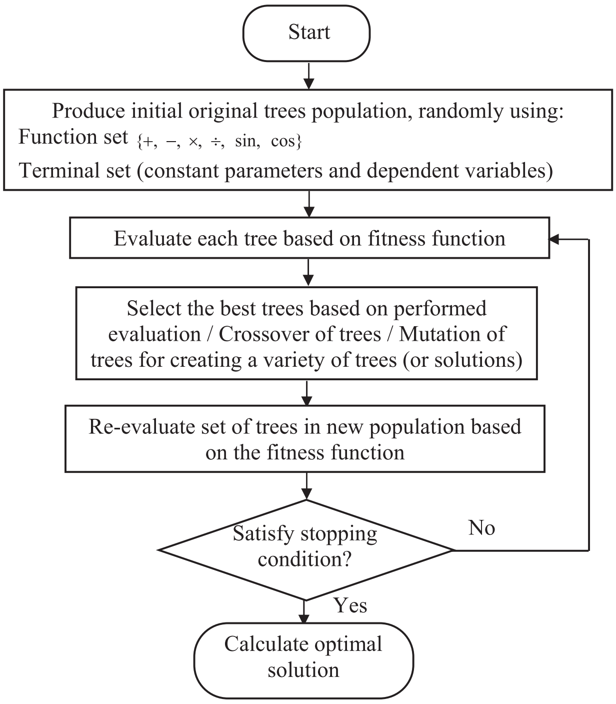

Figure 2. Flowchart of the (single objective) SO-GP algorithm.

solution is strictly better than the second solution with respect to at least one objective. For example, in the Min-Max (minimization of f_1 and maximization of f_2) problem illustrated in Figure 3, solution A dominates solution B based on the maximizing objective function f_2. On the other hand, solution B dominates solution A based on minimizing objective function f_1. However, the solution 'a' (which lies on the Pareto front) is absolutely better than solutions A and B relative to the two objective functions.

Merging, sorting, and truncation scenarios are employed to form a new population of solutions. Figure 4(a,b) illustrates the performance of SO-GP and BO-GP, respectively. It is clear from Figure 4(a,b) the only difference between multi-objective structure relative to single-objective structure is the sorting of solutions (or decision trees), which is a two-state process described below. It is noteworthy in the SO-GP each chromosome or solution represents a tree that is composed of a set of functions and terminals.

Evolutionary algorithms select the best members of one population of solutions to transfer them to the next generation. This selection is not simple given that there are several

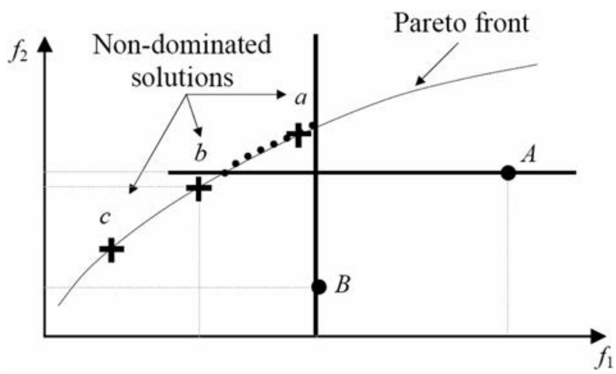

Figure 3. Schematic of the Pareto boundary and non-dominated solutions in a Min-Max problem (minimize f_1 and maximize f_2). Note: the solutions on the dotted line segment dominate A and B.

objective functions in the multi-objective problems. Therefore, the selection of the best solutions is based on the notion of ranking, whereby the solutions are evaluated and ranked based on non-domination. The ranking of the solutions proceeds by comparing the objective function values of each two members of a population. The solutions that are found to be non-dominated receive a rank of 1 (F_1) and are placed on the first front. Subsequently, regardless of the impact on members located in the first front, a series of other non-dominate solutions is determined and receive a rank of 2 (F_2) and placed on the second front. Thus, all solutions are ranked based non-domination. This process is repeated until all the current solutions are ranked and placed on Pareto fronts.

The BO-GP logic (see Figure 4(b)) indicates first creating in iteration or step t the offspring tree population $Q(t)$ (with a number of trees nc) and then the mutants tree population $R(t)$ (with a number of trees nm) from the original tree population $P(t)$ (with a number of trees $npop$). The three tree populations are merged and the tree population (of solutions) $P'(t)$ is created (with a number of trees $npop + nc + nm$). The trees of population $P'(t)$ are ranked based on non-domination of the solutions after evaluating the objective functions. Several classes of solutions are created based on the priority of classes (as shown in Figure 4(b)). Next, the population of the next trees is filled one-by-one with these classes of solutions. Recall the number of original trees in population $P(t)$ equals $npop$. Therefore, not all the trees of the population $P'(t)$ could be placed in the population of new trees $P(t+1)$, and those trees that do not have space are removed. The trees belonging to the front F_2 in Figure 4(b) are selected for transfer to new population $P(t+1)$ based on the proper dispersion of trees (F'_2 in Figure 4(b), see Deb *et al.* 2002).

The BO-GP selects parents based on the binary tournament method, whereby the first two members of the population of solutions are selected randomly. If the ranks of the two selected members are not equal, the higher-ranking member wins the tournament. Otherwise, the member that creates appropriate spread of solutions is selected (Deb *et al.* 2002). These binary comparisons continue until all the parents are selected.

Verification of the BO-GP algorithm with a bi-objective mathematical problem

The BO-GP algorithm is tested by solving of a bi-objective mathematical problem whose objective functions are minimizing the root mean square error (*RMSE*) and minimizing the inverse *RMSE* (1/*RMSE*) given by Equations (1) and (2), respectively:

$$\text{Minimize} \quad \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(y_o - y_c)^2}{n}} \tag{1}$$

$$\text{Minimize} \quad 1/\text{RMSE} = \sqrt{\frac{n}{\sum_{i=1}^{n}(y_o - y_c)^2}} \tag{2}$$

in which the following relational formula holds:

$$y_o = f(x_o) = x_o^3 - 2x_o^2 + x_o - 4 \qquad -6 \le x_o \le 8 \tag{3}$$

and where $RMSE$ and $1/RMSE$ = objective functions of the problem; y_o = observed data; y_c = calculated data by the BO-GP algorithm; and n = the number of observed data in the desired range (for this problem the domain of the independent variable x_0 is $-6 \le x_o < 8$).

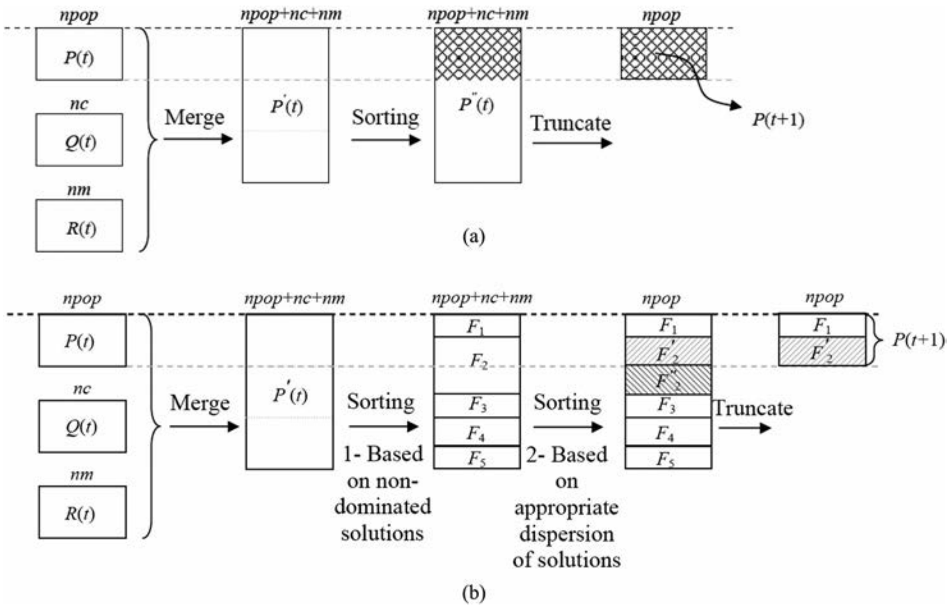

Figure 4. Schematic of the algorithms for (a) SO-GP and (b) MO-GP.

Reservoir operation

Conservation of water is expressed by the continuity equation according to Equation (4):

$$S_{t+1} = S_t + Q_t - \mathrm{Re}_t - Sp_t - LE_t \qquad t = 1, 2, \ldots, T \quad (4)$$

in which S_t and S_{t+1} = storage volume of reservoir at the beginning and ending of period t (10^6 m^3), respectively; Q_t = inflow volume to reservoir during period t (10^6 m^3); Re_t = release volume of reservoir during period t (10^6 m^3); Sp_t = spill volume of reservoir during period t (10^6 m^3); T = total number of operating periods; and LE_t = volume of evaporation from the reservoir surface during period t (10^6 m^3).

The LE_t is determined with Equation (5):

$$LE_t = E_t \times \left(\frac{A_t + A_{t+1}}{2}\right) \qquad t = 1, 2, \ldots, T \quad (5)$$

in which E_t = evaporation depth during period t; A_t and A_{t+1} = area of reservoir lake water surface at the start and end of period t, respectively; A_t and A_{t+1} are determined according to Equations (6) and (7):

$$A_t = a_0 + a_1 S_t \qquad t = 1, 2, \ldots, T \quad (6)$$

in which a_0 and a_1 = constants in surface-volume curve of reservoir.

The Sp_t is subject to the following constraint:

$$Sp_t = \begin{cases} S_{t+1} - S_{\max} & S_{t+1} \geq S_{\max} \\ 0 & S_{t+1} < S_{\max} \end{cases} \qquad t = 1, 2, \ldots, T \quad (7)$$

in which S_{max} = maximum volume (capacity) of the reservoir (10^6 m^3).

Other additional constraints related to the reservoir operation, are given by Equations (8) and (9):

$$S_t \geq S_{\min} \qquad t = 1, 2, \ldots, T \quad (8)$$

$$\mathrm{Re}_t \geq 0 \qquad t = 1, 2, \ldots, T \quad (9)$$

in which S_{min} = minimum (dead) volume of reservoir (10^6 m^3).

Penalty functions are introduced to guarantee satisfaction of constraints (8) and (9) by adding them to the objective functions:

$$\mathrm{Penalty1}_t = A'.\left(\frac{S_{\min} - S_t}{S_{\max} - S_{\min}}\right)^2 + B' \qquad t = 1, 2, \ldots, T \quad (10)$$

$$\mathrm{Penalty2}_t = C' \cdot \left(\frac{\mathrm{Re}_t}{D_{\max}}\right) + D' \qquad t = 1, 2, \ldots, T \quad (11)$$

in which $\mathrm{Penalty1}_t$ = penalty function imposed on violations of constraint (8); $\mathrm{Penalty2}_t$ = penalty function imposed on violations of constraint (9); D_{max} = maximum downstream demand (by the irrigation network) in the operating interval; and A', B', C', D' = positive constants of the penalty functions.

The first objective function is the minimization of the vulnerability index (Hashimoto *et al.* 1982, Ajami *et al.* 2008) of water allocation to the agricultural sector. Thus, the pertinent objective function is given by Equation (12):

$$(\mathrm{O.F.})_1\text{:Minimize } \frac{\sum_{t=1}^{T} [\max(D_t - \mathrm{Re}_t, 0)]}{[Num(D_t - \mathrm{Re}_t | \mathrm{Re}_t < D_t)] \cdot D_{\max}} \qquad t = 1, 2, \ldots, T \quad (12)$$

in which $(O.F.)_1$ = first objective function; D_t = demand volume in during period t; and $Num(D_t - \mathrm{Re}_t|\mathrm{Re}_t < D_t)$ = the number of deficiency months during period t. The vulnerability index measures the magnitude of the water deficiency of supplying water for irrigation downstream of the reservoir system.

The second objective function is the maximization of the reliability index (Hashimoto *et al.* 1982, Ajami *et al.* 2008) for supplying water demand. The pertinent objective function is expressed by Equation (13):

$$(\mathrm{O.F.})_2\text{:Maximize } \frac{Num(D_t - \mathrm{Re}_t | \mathrm{Re}_t \geq D_t)}{T} \qquad t = 1, 2, \ldots, T \quad (13)$$

in which $(O.F.)_2$ = second objective function; and $Num(D_t - \mathrm{Re}_t|\mathrm{Re}_t \geq D_t)$ = the number of supply months in during period t.

The penalty functions (Equations (10) and (11)) are added to the first objective function according to Equation (14), and subtracted from the second objective function according to Equation (15):

$$(\text{O.F.})_1 = (\text{O.F.})_1 + \text{Penalty}1_t + \text{Penalty}2_t \quad t = 1, 2, \ldots, T \tag{14}$$

$$(\text{O.F.})_2 = (\text{O.F.})_2 - \text{Penalty}1_t - \text{Penalty}2_t \quad t = 1, 2, \ldots, T \tag{15}$$

The reliability index is a measure of the number of months where the downstream irrigation water use is fully met scaled by the total number of operating months.

The volume of water released from the reservoir is a function of the inflow volume to the reservoir, the storage volume, and the volume of water demand, according to Equation (16). Any non-dominated solution (or point) on the Pareto set corresponding to the stated optimization objectives represents a viable strategy that can be obtained with the BO-GP algorithm.

$$[\vec{R}_{(\text{O.F.})_1(\text{O.F.})_2}]t = \text{Re}_t = f(Q_t, S_t, D_t) \quad t = 1, 2, \ldots, T \tag{16}$$

in which $[\vec{R}_{(\text{O.F.})_1(\text{O.F.})_2}]_t$ = a set of decision options or the optimal solutions (a set of points on the Pareto set); and $f(\,)$ = rule curve calculated by the BO-GP.

Performance criteria

This study applies as performance criteria the correlation coefficient (R) [or coefficient of determination (R^2)], the *RMSE,* and the Nash-Sutcliffe coefficient of efficiency (*NSE*) that compare the BO-GP algorithm' results for the mathematical problem with the corresponding known values (Ashofteh *et al.* 2013a). However, there are other performance metrics for hydrosystems (see, for example, Gupta *et al.* 2009).

Case study

The Aidoghmoush one-reservoir system and its downstream irrigation network of 13,500 ha (located in East Azerbaijan, northeastern Iran) (Figure 5) were used to evaluate the performance of the proposed BO-GP algorithm in determining optimal reservoir operating policies to supply agricultural water. It is noted that previous studies have shown the GP model has performed successfully (e.g. Fallah-Mehdipour *et al.* 2015, Akbari-Alashti *et al.* 2014).

The normal level of the Aidoghmoush reservoir is 1,341.5 m above sea level. The total capacity of the reservoir and its dead volume are 145.7 and 8.7 (10^6 m^3), respectively. The a_0 and a_1 constants of the reservoir surface-volume curve are equal to 0.03 and 0.8, respectively. The inflow, demand, and evaporation data correspond to the 14-year interval 1987–2000 (Ashofteh *et al.* 2013a, 2013b).

The Aidoghmoush reservoir's downstream irrigation network is one of the largest pressurized irrigation networks in Iran. Several studies of this reservoir system have been reported (see, e.g. Ashofteh *et al.* 2017). The standard operation policy (SOP) is the current operating rule for the Aighmoush reservoir to supply water for irrigation.

Data pertinent to the parameters and termination criteria of the BO-GP algorithm

GPLAB is the toolbox for the developed GP algorithm (BO-GP) available in the MATLAB 11.0 software (Silva 2007). The values of the parameters used in the BO-GP algorithm for solving the mathematical problem and the reservoir problem are listed in Table 1.

The evolutionary search for optimal solutions is carried out until new algorithmic iterations do not cause any improvement in the objectives' values. A computer with Intel (R) Core (TM) I7, CPU 2.20 GHz and RAM 6.00 GB, was employed in this study as the computational engine.

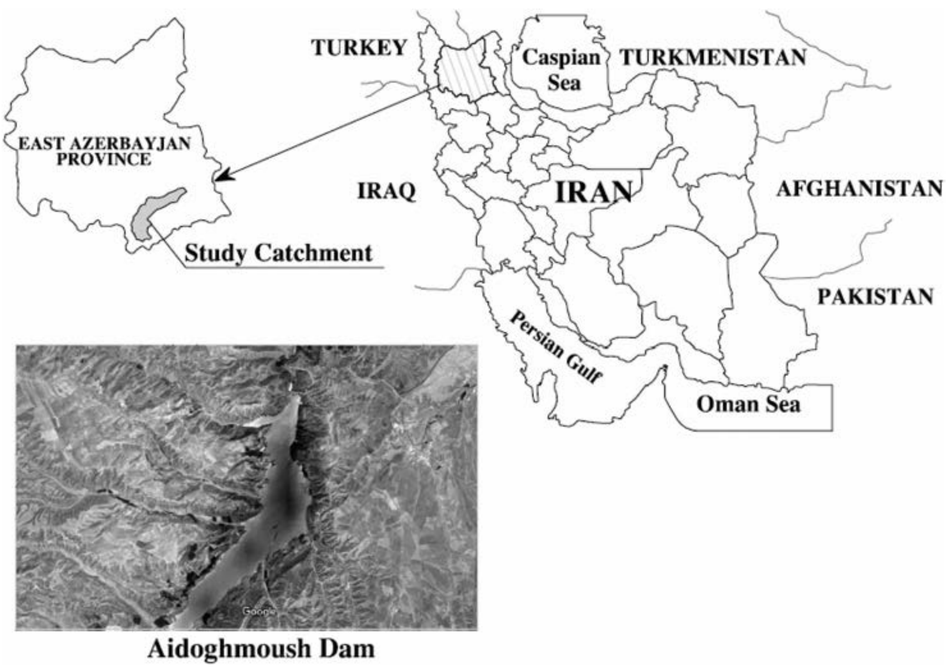

Figure 5. Map of the study area.

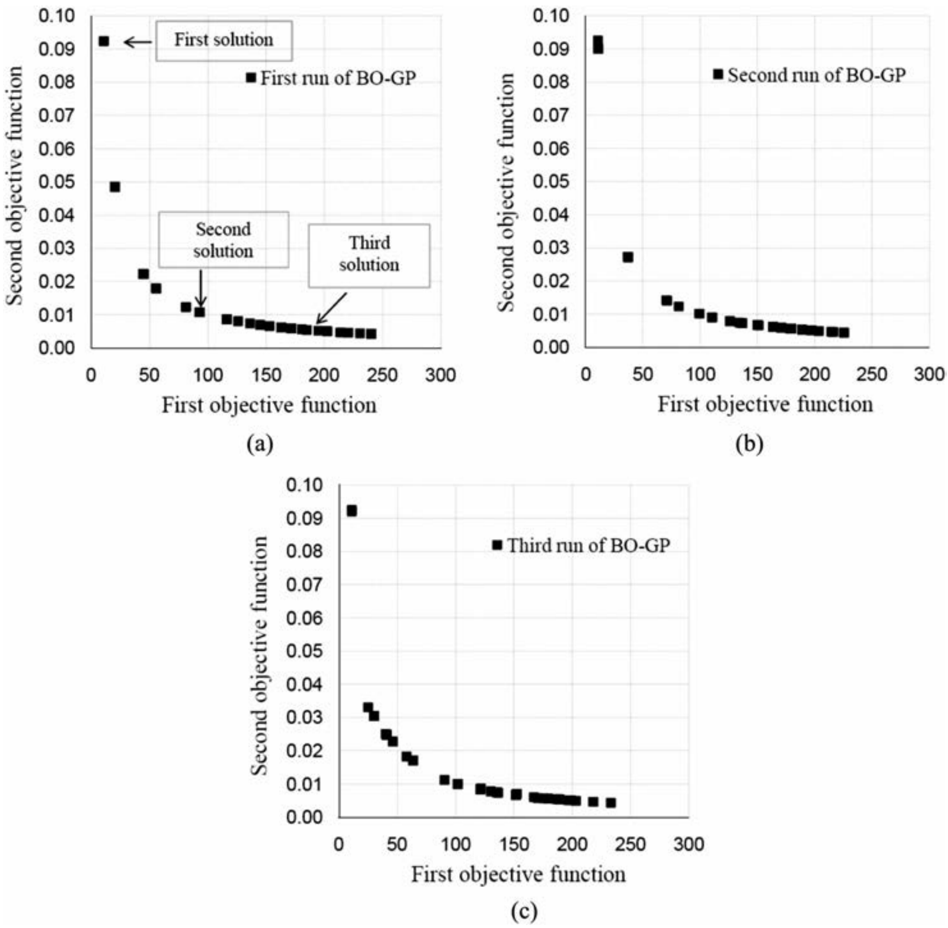

Figure 6. Comparison of the Pareto fronts obtained from the application of the BO-GP algorithm to the mathematical Problems (1)–(3); (a) the first run; (b) the second run; and (c) the third run.

Results

The mathematical problem

The results of the application of the BO-GP algorithm to the mathematical problem stated by Equations (1)–(3) are depicted in Figures 6 and 7. The values of the objective function are shown in Figure 6(a–c) calculated in three different runs of the BO-GP algorithm.

Figure 6 shows there is minimal difference between the graphs of the objective function obtained from the three runs. However, due to appropriate spread of solutions obtained with the first run, this run was chosen for further analysis. Three optimal solutions were randomly chosen from Pareto sets obtained from the first run. The calculated results corresponding to the three optimal solutions (Figure 6(a)) are graphed in Figure 7(a–f) for the first run (the best run of the three runs).

It is shown in Figure 7(a) the first solution is desirable from the perspective of the first objective function because it minimizes the *RMSE* and maximizes the *NSE*. Figure 7(e) shows the third solution is desirable form the viewpoint of the second objective function because it minimizes the inverse *RMSE* and the *NSE*.

The equations identified by the BO-GP algorithm for the selected first, second, and third solutions are given by Equations (17)–(19), respectively:

$$\begin{aligned} y_0 = &-x_0 - \cos(x_0 - \cos(x_0)) \cdot (x_0 + \cos(x_0)) \\ &+ \sin(\sin(\cos(\cos(x_0) + \cos(x_0 + \cos(x_0))) \\ &+ \cos(\sin(\cos(x_0))) + x_0^2 - \sin(\sin(x_0)) \\ &+ \cos(\cos(x_0 + \cos(x_0))) - \cos(\sin(x_0) + \cos(x_0)) \\ &+ x_0^2 - (x_0 - \sin(x_0)) \cdot x_0 - (x_0 - \cos(x_0 \cdot (x_0 + \\ &\sin(\sin((x_0 + \cos(x_0) - \sin(\sin(x_0))) \cdot x_0)) \\ &- x_0/\cos(\cos(x_0) \cdot \sin(x_0))))) \cdot x_0 + \cos(x_0) \\ &- x_0 \cdot (x_0 + \cos(x_0)) - \cos(x_0/\cos(\cos(\sin(\sin(x_0))))) \\ &\cdot \cos(x_0) + \cos(x_0) + \sin(x_0)/(x_0 - \sin(x_0)))/x_0 \cdot \cos(x_0) \end{aligned} \tag{17}$$

$$\begin{aligned} y_0 = &-\cos(\sin(x_0)) - 1 + \sin(\sin(x_0)) - \cos(x_0) - 4x_0 \\ &+ \cos(\sin(x_0) + \cos(x_0)) - x_0^2 - x_0 \cdot (x_0 + \cos(x_0)) \\ &- (x_0 - x_0/(1 - x_0) \cdot \cos(x_0) - (x_0 - \sin(x_0)) \cdot x_0) \cdot x_0 \\ &+ (2x_0 - x_0/(1 - x_0) \cdot \cos(x_0) - (x_0 - \sin(x_0)) \cdot x_0)/\cos(x_0)/ \\ &x_0 - x_0^2 \cdot (x_0 - \sin((x_0^2 - \sin(x_0) + \cos(-\sin(x_0) \\ &+ x_0^2/\sin(x_0) + x_0))/(x_0 \cdot (\sin(\sin(\sin(x_0) + x_0)) \\ &+ x_0 - \sin(x_0))/\cos(x_0)/\sin(x_0) - \sin(x_0) + \cos(x_0))) \\ &- \cos(\cos(-2x_0 + x_0^2 + 2x_0/\sin(x_0))) \cdot x_0) \end{aligned} \tag{18}$$

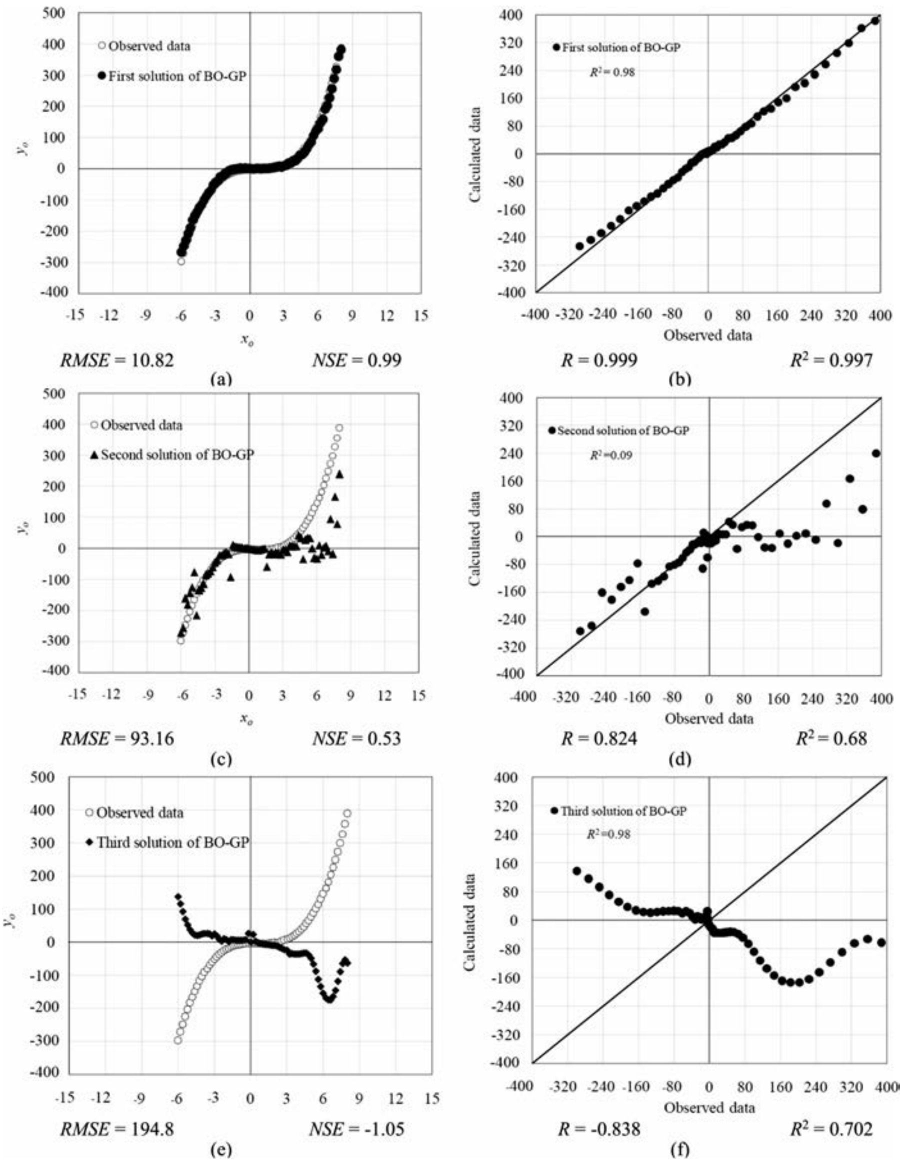

Figure 7. Comparison of the results: (a) y_0 vs x_0 for the first solution, (b) scattergram of calculated vs observed data for the first srun, (c) y_0 vs x_0 for the second solution, (d) scattergram of calculated vs observed data for the second run, (e) y_0 vs x_0 for the third solution, (f) scattergram of calculated vs observed data for the third run.

$$y_0 = -\cos(\sin(x_0)) - 5x_0 + \sin(\sin(x_0)) - \cos(\cos(x_0 + \cos(x_0))) + \sin(x_0) + \cos(1/2 \cdot (\cos(x_0) - \sin(\sin(x_0) + x_0))/\cos(\sin(2x_0)))) - x_0^2 - (1+\cos(x_0)) \cdot x_0 - \cos(x_0) \cdot x_0 + 1/\sin(x_0)/x_0 - x_0^2 \cdot (x_0 - \sin((x_0^2 - \sin(x_0) + x_0 \cdot (\sin(\sin(\sin(x_0) + x_0)) + x_0 - \sin(x_0))/\cos(x_0)/\sin(x_0) + \sin(-2x_0/x_0^2 - \cos(x_0)) + 1 + \cos(x_0) + x_0^2) + x_0)/(x_0 \cdot (\sin(\sin(2x_0)) + x_0 - \sin(x_0))/\cos(x_0)/\sin(x_0) - \sin(x_0) + x_0)) - \cos(\cos(x_0)) \cdot x_0 \quad (19)$$

Results for the Aidoghmoush reservoir

The results obtained by the application of the BO-GP algorithm to calculate the operating rule of the Aidoghmoush one-reservoir system are summarized in Figure 8, which depicts the calculated Pareto front for the minimization of vulnerability index and maximization of reliability index. Each of the solutions that form the Pareto set represents

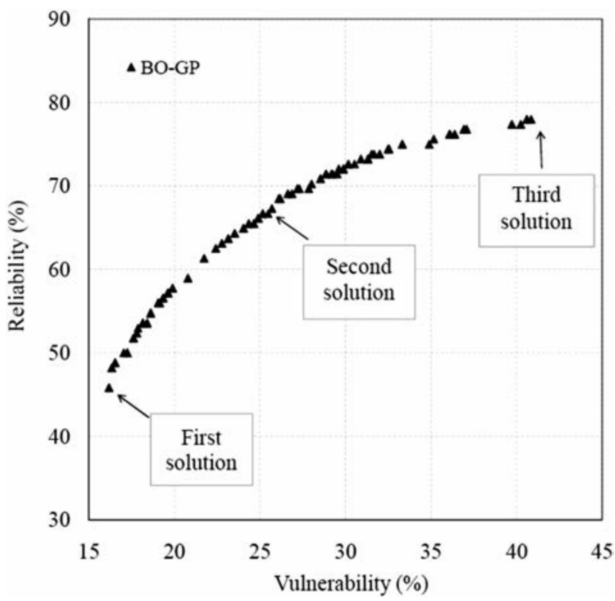

Figure 8. Pareto front calculated with the BO-GP algorithm for the operating rule of the reservoir with two-objective (minimization of vulnerability and maximization of reliability).

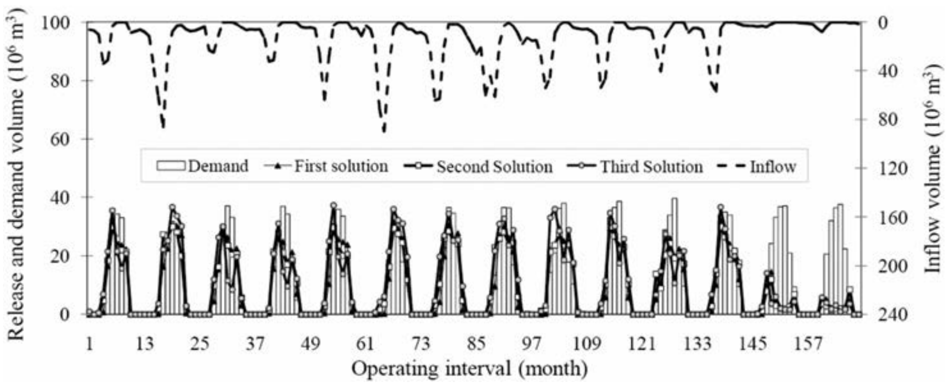

Figure 9. Reservoir releases and water demand based on rule calculated with BO-GP for the three runs.

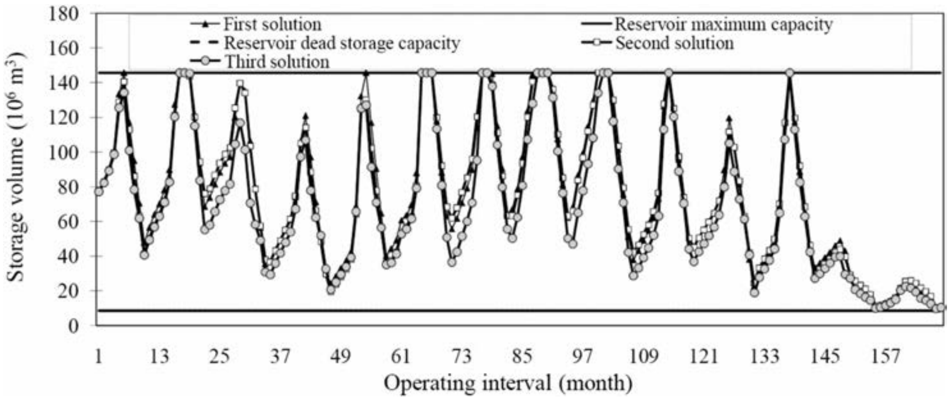

Figure 10. Reservoir storage based on rule calculated with BO-GP for the three runs.

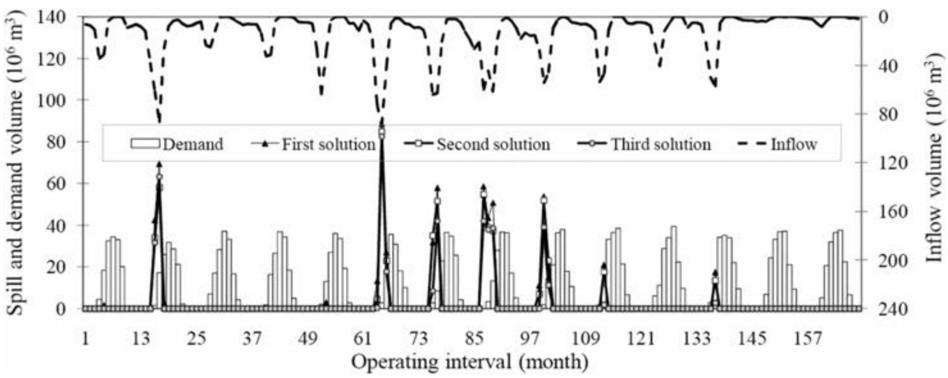

Figure 11. Reservoir spill and water demand based on rule calculated with BO-GP for the three runs.

an optimal solution or optimal operating policy that specifies the release volume from the reservoir as a function of the inflow volume to the reservoir, the storage volume, and the water demand volume. It is seen in Figure 8 that changes of the system vulnerability range between 16% and 41%, and changes of the system reliability range

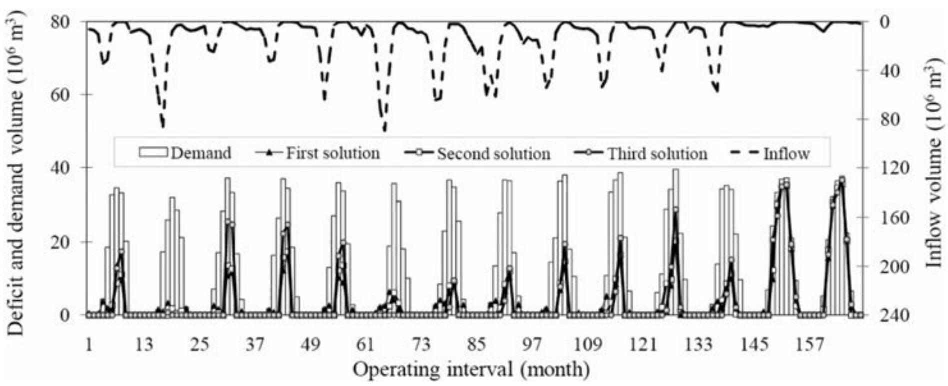

Figure 12. Deficit and demand volume based on rule calculated with BO-GP for the three runs.

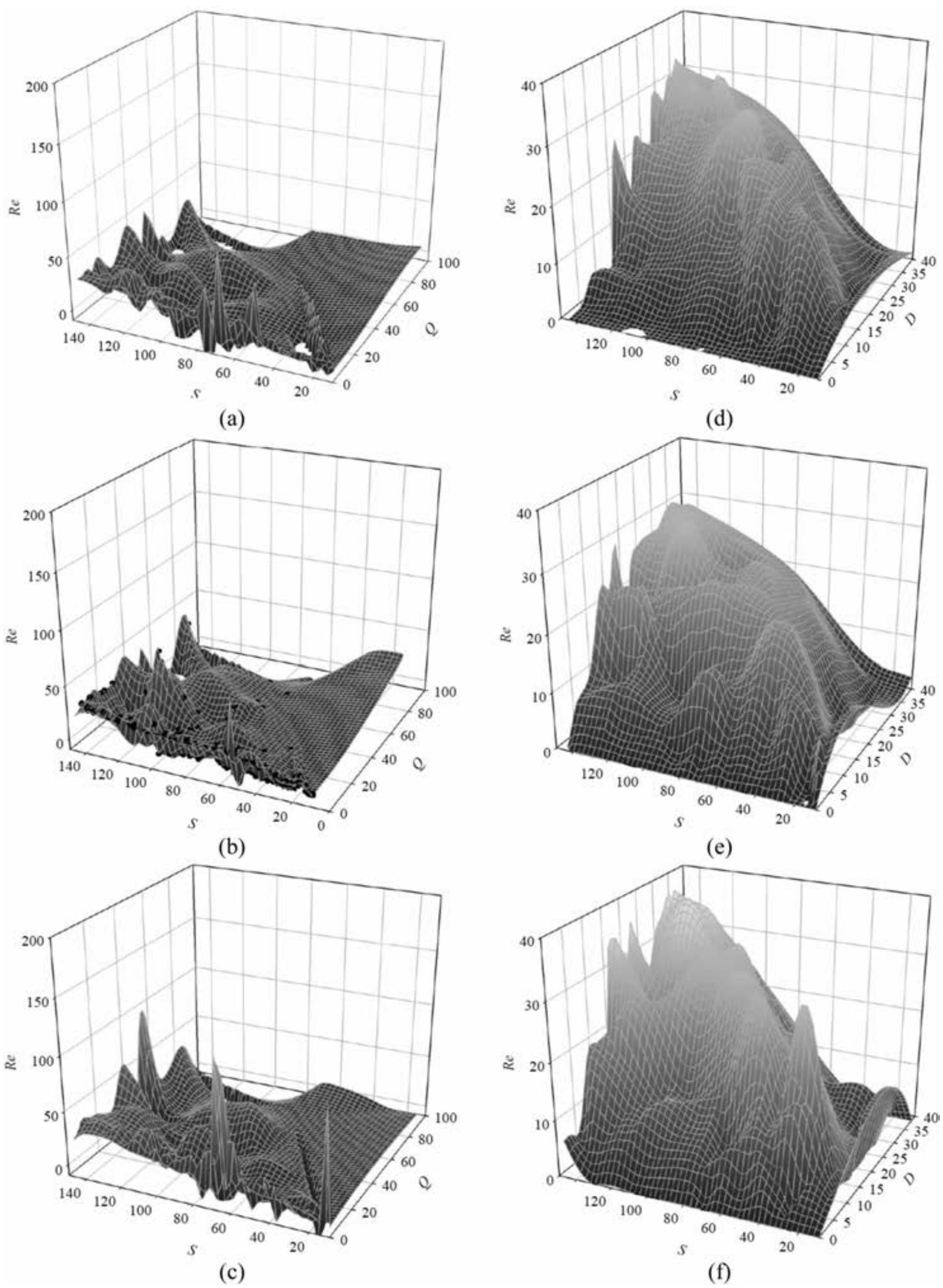

Figure 13. Surfaces of rules resulting from mathematical equations calculated with BO-GP for input variables S_t and Q_t in (a) first run; (b) second run; (c) third run; and for input variables S_t and D_t in (d) first run; (e) second run; (f) third run.

between 46% and 78%. Three solutions, named Solutions 1, 2, and 3 were chosen for further analysis and are shown in Figure 8. Changes of the release volume, storage volume, spill volume, and deficit volume corresponding to Solutions 1, 2, and 3 are depicted in Figures 9–11.

Comparison of Figures 9–11 establish the first, second, and third solutions of operating policy feature reliabilities of water supply for irrigation are equal to about 46%, 67%, and 78%, respectively, and their vulnerabilities to water shortages equal 16%, 26%, and 41%, respectively. It is noteworthy none of these solutions have advantages relative to each other. Instead, they are options for decision making that might become more or less attractive with changing conditions over time. For example, the first solution induces less shortage severity than the second and third solutions. Also, the third solution is desirable in terms of the second objective function (i.e. maximization of the reliability). This means the third solution prescribes releases that are larger and spills that are smaller than those of the first and second solutions (Figure 12).

The operating developed rules by the BO-GP algorithm for the selected first, second, and third solutions of reservoir operation are listed in Equations (20)–(22), respectively. Also, the rules developed by the BO-GP have been plotted in Figure 13.

$$(\mathrm{Re}_t)_1 = 1/\cos(\cos(Q_t/D_t))/\cos(\cos(\cos((S_t+D_t/S_t)))/\cos(\cos(\cos((S_t+Q_t+D_t)/S_t)))/\cos(\cos(\cos(D_t/S_t)/\sin(D_t)/D_t/\sin(\cos(1))/\cos(D_t)))/(S_t\cdot(S_t+D_t+(2S_t+Q_t)/D_t)/D_t/(D_t/\cos(\cos(S_t\cdot(S_t+D_t+Sin(D_t)-\cos(-S_t+D_t))/D_t/(D_t/\cos(\cos(Q_t))+D_t)))+D_t)+D_t)\cdot S_t \quad t=1,2,\cdots,T \tag{20}$$

$$(\mathrm{Re}_t)_2 = 1/\cos(\cos((\cos((S_t+2D_t+2\sin(D_t))/Q_t)-Q_t^2-Q_t)/D_t^2/\cos(2D_t)))/\cos(\cos(\cos((S_t+D_t+\sin(D_t))/S_t)))/\cos(\cos(\cos((S_t+D_t+\sin(\sin(S_t))^2)/S_t)))/\cos(\cos((\cos(D_t)-Q_t^2-Q_t)/D_t/(D_t+Q_t)/\cos(S_t\cdot\cos(D_t)+D_t)/Q_t/\cos(\cos(\cos((S_t+D_t+\sin(D_t))/S_t)))/\cos(D_t)))/(3S_t/D_t+D_t)\cdot S_t \quad t=1,2,\cdots,T \tag{21}$$

$$(\mathrm{Re}_t)_3 = 1/\cos(\cos(\cos(\cos(Q_t)/D_t)-D_t\cdot Q_t-Q_t)/D_t^2/\cos((S_t+2D_t+\sin(\cos(Q_t/S_t))\cdot D_t)/D_t/(2D_t+\sin(\cos(Q_t/S_t))\cdot Q_t))))/\cos(\cos(\cos((S_t+2D_t+2\sin(D_t))/S_t)))/\cos(\cos(\cos((S_t+2D_t)/S_t)))/\cos(\cos(\cos(\sin((\sin(Q_t)+\cos(S_t))\cdot S_t))/\cos(\cos(\cos(D_t)))/\sin(D_t)/D_t/\sin(\cos(S_t/((Q_t+D_t)\cdot D_t-Q_t)))/\cos(D_t)))/(2S_t/D_t+D_t)\cdot S_t \quad t=1,2,\cdots,T \tag{22}$$

Concluding remarks

This study developed a BO-GP algorithm for solving a mathematical problem and for operating the Aidoghmoush one-reservoir system (located in East Azerbaijan, northeast of Iran). Results demonstrated the BO-GP algorithm was successful in solving efficiently a bi-objective mathematical problem, and that it performed satisfactorily in calculating the optimal operating policy of the Aidoghmoush reservoir. This work considered reservoir release rules as a function of decision parameters such as the inflow volume to the reservoir, the storage volume, and the water-demand volume.

This paper's results indicate the reservoir system vulnerability and reliability vary between 16–41% and 46–78%, respectively. Three solutions were selected from set of Pareto solutions and were employed to assess their corresponding changes in reservoir releases, storage volume, volume of spill, and volume of water deficits.

Vulnerability is a key index for measuring the failure severity of water resources systems in supplying water demand (the first objective). It is also useful to know the percentage of water demand supplied during an operating period in terms of the timing of supply (the second objective). The joint consideration of these criteria is necessary for correctly solving water resources systems problems. Water systems' objectives and policies related to reservoir operation are best solved with multi-objective algorithms of the type presented in this work.

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