## Title

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# Borda Scores and Aggregation of Preference: <br> A Geometric-Combinatoric and A Topological Approach 

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Borda scores are defined over ranking probabilities ("voter profiles"), that is, the probability distribution over all rankings of a set of candidates. It will first be shown that the space of Borda scores form a convex polytope called "permutahedron". Investigating the various ways a permutahedron as a geometric-combinatoric object may arise allows one to appropriately define Borda scores for binary choice probabilities, for subset choice probabilities, and for rank-position probabilities. A topological approach is then adopted to study aggregation of preference ("social choice") when each voter's preference is given as a vector whose components are defined on an interval scale (a generalization of ordinal ranking). It turns out that any well-behaved (continuous, anonymous, and respecting unanimity) aggregation map has to allow for null outcome in the social choice while not allowing for null preference by an individual voter. This result complements the impossibility theorems of Arrow (1963) and Chichilnisky (1980).

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