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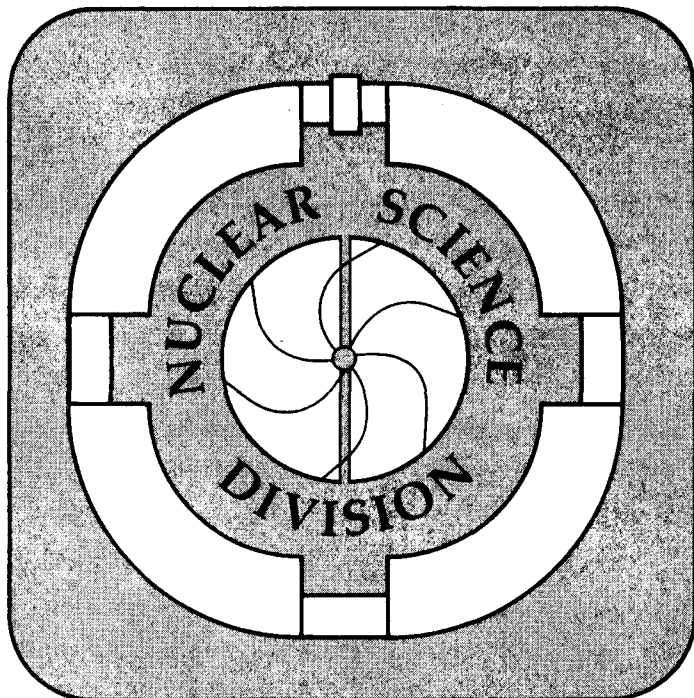
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DROPLET MODEL THEORY OF THE NEUTRON SKIN

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ABSTRACT

The Droplet Model theory of the neutron skin is reviewed and an elementary formula is derived for the associated difference between the RMS radii of the neutron and proton density distributions. The resulting predictions are compared with recent experimental estimates and with Hartree-Fock calculations. There appears to be no serious disagreement with most of the current, tentative data. Improved measurements should eventually make possible an independent estimate of the Droplet Model stiffness coefficient Q , governing the resistance of the nuclear surface against the formation of a neutron skin.

1. INTRODUCTION

In this paper we shall review the Droplet Model theory of the neutron skin and compare the theoretical predictions with recent experimental evidence and with results of Hartree-Fock calculations, as reported in Ref. 1.

Speculations on a possible neutron enrichment of the nuclear surface (or, equivalently, on a larger radius for the neutron distribution than for the proton distribution) go back many years (see, for example, Ref. 2). Various more or less reasonable arguments for such an effect had been advanced, but a reliable estimate for its magnitude was, for a long time, not available. Such an estimate required, first, the isolation of the dominant physical elements governing the formation of a neutron skin and, second, the incorporation of those elements in a self-consistent, quantitative theory.

2. THE DROPLET MODEL FORMULA FOR THE NEUTRON SKIN

According to the Droplet Model of average nuclear properties (see Refs. 3,4,5; "average" implies the disregard of, or an averaging over, nuclear shell effects) the physical elements governing the formation of a neutron skin are very simple. Thus, the well-known preference of bulk nuclear matter for symmetry, i.e., for equality of the neutron and proton densities, will, in the case of a nucleus with a neutron excess ($N > Z$), provide a driving force trying to push the excess neutrons out of the bulk region and into the surface. (The electrostatic energy associated with the protons will reduce this tendency by trying to increase the radius of the proton distribution.) The resulting driving force is

resisted by the nuclear surface energy which, in its turn, is also happiest when conditions in the surface are symmetric, i.e., when there is *no* neutron skin. These opposing tendencies result in a compromise in which, for most nuclei, the ratio of the neutron-to-proton densities in the bulk is somewhat less than the ratio of N to Z, and the effective neutron surface is somewhat outside the effective proton surface, thus leading to a neutron skin with some effective thickness t , say.

(The phrase "neutron skin of thickness t " has sometimes been misunderstood as implying a very unrealistic model with sharp neutron and proton distributions, for which a layer of pure neutrons would appear at the surface. It is important to stress that — certainly in the Droplet Model context — "neutron skin" simply refers to the distance, t , between the locations of the two *diffuse* surface profiles, i.e., to the shift that would be necessary to put one profile on top of the other. This leads to some enrichment of the surface layer in neutrons but not to a pure neutron skin.)

The strength of the bulk driving force is characterized by the nuclear symmetry-energy coefficient J (close to 36.8 MeV, Ref. 5, p.5) and so for a nucleus with a relative neutron excess I [equal to $(N-Z)/A$] the driving force will be proportional to $J I$. The resistance of the surface energy against the formation of a neutron skin is characterized, in the Droplet Model, by an effective surface stiffness coefficient Q (with an estimated value of roughly 17 MeV, Ref. 5, p.5). It follows that for a nucleus with a skin thickness t the force resisting skin growth will be proportional to $Q t$. It is then easy to understand that, on balancing these forces, one obtains the following structure for the Droplet Model prediction (Ref. 3, Eq. (2.21)) for the magnitude of the neutron skin thickness of a large (uncharged) spherical nucleus with a relative

neutron excess I:

$$t = \frac{3}{2} r_0 \frac{JI}{Q} \quad (1)$$

Thus the neutron skin thickness, in units of the natural length r_0 (the radius constant of nuclear matter), is predicted to be (apart from the factor 3/2) the ratio of the driving force JI to the restoring force stiffness Q .

The inclusion of the electrostatic energy of the protons (equal to about $c_1 Z^2/A^{1/3}$, with $c_1 = 3e^2/5r_0 \approx 0.7322$ -MeV) reduces the driving force JI by a readily calculable amount and leads to

$$t = \frac{3}{2} r_0 \frac{JI - \frac{1}{12} c_1 Z A^{-1/3}}{Q} \quad (2)$$

(This expression may be verified by applying Eqs. (20) and (59) of Ref. 4 to a spherical nucleus that is large enough to make the second term in the denominator in Eq. (59) small compared to unity.)

According to Eq. (2) the neutron skin will be positive (the nuclear surface will be enriched in neutrons) if the numerator is positive, i.e., if the relative neutron excess I is greater than a certain critical value I_c , given by

$$JI_c = \frac{1}{12} c_1 Z_c A^{-1/3},$$

where $Z_c = \frac{1}{2} A(1 - I_c) \approx \frac{1}{2} A$ for small I_c . It follows that

$$I_c \approx \frac{c_1}{24J} A^{2/3} \quad (3)$$

It is instructive to compare this critical neutron excess I_c with the neutron excess I_β along the valley of beta-stability, obtained by minimizing, at constant A , the sum of the nuclear symmetry energy and the electrostatic energy

$$\frac{\partial}{\partial I} (JI^2A + c_1 Z^2 A^{-1/3}) = 0 \quad ,$$

which leads to

$$2JI_\beta A - 2c_1 Z_\beta A^{-1/3} \cdot \frac{1}{2}A = 0 \quad ,$$

or

$$I_\beta \approx \frac{c_1}{4J} A^{2/3} \quad , \quad (4)$$

as the equation for the valley of β -stability. Comparing Eqs. (3) and (4) we arrive at the following theorem:

"Nuclei whose neutron excess is more than one-sixth of that corresponding to the valley of beta-stability should have a neutron skin. Nuclei with a smaller neutron excess should have a proton skin." (See Fig. 1.)

Apart from this quantitative statement, the most obvious qualitative consequence of the physics embodied in Eq. (2) is that, unless one is willing to postulate that the nuclear surface energy is infinitely stiff against the formation of a neutron skin (i.e., unless $Q = \infty$), there *must*, in general, be a neutron skin. So, from the point of view of theory, the question is not whether there is a neutron skin, but how large it is. (In the first Droplet Model paper [Ref. 3, p.483] the neutron skin for ^{208}Pb was estimated as $t = 0.36$ fm. This implies a difference in the RMS radii of the neutron and proton distributions of about 0.24 fm — see below.)

From the experimental point of view, fairly direct evidence for a neutron skin is only beginning to accumulate as a result of accurate

measurements of the radii of nuclear matter distributions, as deduced from high-energy proton scattering experiments. It is well to remember, however, that indirect evidence for a neutron skin has been available for some time from at least three sources: the sign and magnitude of the surface symmetry energy in the semi-empirical nuclear mass formula (Ref. 3, p.485 and Ref. 6, p.454), the signs and magnitudes of isotope and isotone shifts (Ref. 6) and the evidence for a Goldhaber-Teller component (i.e., a neutron-skin degree of freedom) in nuclear Giant Dipole resonances (Ref. 7). (That these diverse phenomena do, in fact, bear on the question of the neutron skin is brought out by the Droplet Model which, being a rather general theory of average nuclear properties, touches upon many different facets of nuclear structure. On the one hand the Droplet Model is capable of suggesting theoretical relations between these diverse facets and, on the other, it can make use of a multitude of different measurements to firm up estimates of the characteristic nuclear parameters entering into the theory. In the four aspects under discussion, namely, neutron skin, surface symmetry energy, isotope and isotone shifts, and Giant Dipole resonances, a key parameter to which these phenomena are sensitive is the effective surface stiffness coefficient Q .)

In order to proceed with a quantitative application of the Droplet Model theory of the neutron skin it is fairly important to remove the approximation (used in arriving at Eq. (2)) that the second term in the denominator of Eq. (59) in Ref. 4 is small. The physical meaning of this approximation is that the calculated neutron skin is assumed to be small compared to *the maximum value* t_{\max} *that it could have if all the excess*

neutrons, $N-Z$, were pushed into the surface. The value of t_{\max} itself follows from elementary considerations. Thus the ratio of the volume of the effective neutron-skin region (the region enclosed between the effective sharp neutron and proton surfaces) to the total volume, for a nucleus with radius R , is approximately $4\pi R^2 t / \frac{4}{3}\pi R^3$, and since the effective density in the bulk (where there are neutrons and protons) is about twice that in the effective neutron skin (where there would be only neutrons - see Fig. 2) we may write

$$\frac{4\pi R^2 t_{\max}}{2\left(\frac{4}{3}\pi R^3\right)} \approx \frac{N-Z}{A},$$

which leads to

$$t_{\max} = \frac{2}{3} RI \quad . \quad (4)$$

(For ^{208}Pb t_{\max} would thus be about 1.0 fm.)

The generalization of Eq. (2) that is not restricted by the approximation $t/t_{\max} \ll 1$ is obtained by combining Eqs. (20) and (59) of Ref. 4. It reads

$$t = \frac{3}{2} r_0 \frac{JI - \frac{1}{12} c_1 ZA^{-1/3}}{Q + \frac{9}{4} JA^{-1/3}} \quad . \quad (5)$$

This equation correctly predicts that as Q tends to zero and all the excess neutrons are pushed into the surface, t tends to t_{\max} (for an uncharged nucleus). Without the second term in the denominator of Eq. (5) the value of t diverges as Q goes to zero.

Equation (5) is the Droplet Model formula for the thickness of the neutron skin of a spherical nucleus.

For a nucleus of arbitrary shape the equation for t may be obtained by combining Eqs. (20), (59) and (52) of Ref. 4, which leads to

$$t = \bar{t} + \tilde{t} \quad , \quad (5a)$$

where \bar{t} , a number, is given by

$$\bar{t} = \frac{3}{2} r_0 \frac{JI - \frac{1}{12} c_1 ZA^{-1/3} (B_V/B_S)}{Q + \frac{9}{4} JA^{-1/3} B_S} \quad , \quad (5b)$$

and \tilde{t} , a function of position on the surface, is given by

$$\tilde{t} = \frac{3}{8} r_0 \frac{e}{Q} (\tilde{v}_s - \bar{v}) \quad . \quad (5c)$$

In the above, \tilde{v} is the deviation (from its bulk average \bar{v}) of the electric potential v produced by a uniformly distributed charge Ze inside the shape in question; \tilde{v}_s is the value of \tilde{v} on the surface and $\bar{\tilde{v}}$ is the *surface average* of \tilde{v}_s . The quantity B_S is the surface area of the shape in question in units of the area of a sphere of equal volume and B_V is the surface integral of \tilde{v} in units of its value for the sphere. See Ref. 4, especially pp,194, 201 and 206.]

3. THE NEUTRON SKIN AND RMS RADII

The evidence for a neutron skin that is becoming available is usually presented not directly as an estimate of t but as an estimate of the difference between the RMS radii of the neutron and proton density distributions (see Ref. 1). Denoting this difference by Δ_{RMS} one may derive the following expression relating this quantity to the neutron skin thickness t :

$$\Delta_{\text{RMS}} = \sqrt{3/5} \left(t + \frac{5}{2} \frac{b_N^2 - b_Z^2}{R} - \frac{1}{70} \frac{e^2}{J} Z \right) \quad . \quad (6)$$

This expression, which is derived in Appendix A, consists of a (usually dominant) term $\sqrt{3/5} t$ and two (usually smaller) contributions. The factor $\sqrt{3/5}$ simply reflects the circumstance that the RMS radius of a sharp sphere is related to the radius R by $RMS = \sqrt{3/5} R$. The middle term takes account of the diffuseness of the neutron and proton distributions, which increases the RMS radii according to (Ref. 8, Eq. 12)

$$(RMS)_{diffuse} = (RMS)_{sharp} \left(1 + \frac{5}{2} (b/R)^2 \right), \quad (7)$$

where b is the Süssmann width (Ref. 9) of the diffuse surface layer. If the neutron and proton surface widths are assumed equal the diffuseness correction in Eq. (6) drops out. The last term in Eq. (6) is a correction for the redistribution of the proton and neutron densities caused by the electrostatic repulsion. Coulomb redistribution effects are part and parcel of the Droplet Model and the last term in Eq. (6) is the model's prediction for the difference between the neutron and proton RMS radii resulting from the fact that the redistribution of the neutron density is less pronounced than that of the proton density. (This is associated with the finiteness of the symmetry energy coefficient J . If J were infinite the neutrons would follow the protons faithfully and this correction would vanish.)

We shall take the following values for the nuclear parameters (see Ref. 5, p.5):

$$\left. \begin{aligned} r_o &= 1.18 \text{ fm (so that, with } e^2 = 1.4400 \text{ MeV fm,} \\ &\quad \text{we have } c_1 = 0.7322 \text{ MeV)} \\ J &= 36.8 \text{ MeV} \\ Q &\approx 17 \text{ MeV} \\ b_N - b_Z &\approx 0 \end{aligned} \right\} (8)$$

The Droplet Model prediction for the difference between the RMS radii of the neutron and proton distributions follows as

$$\Delta_{\text{RMS}} \approx 0.7746 t - (0.000433 \text{ fm}) Z, \quad (9)$$

with

$$t \approx 3.8315 \frac{1 - 0.001658 ZA^{-1/3}}{1 + 4.8706 A^{-1/3}} \text{ fm} . \quad (10)$$

4. COMPARISON WITH EXPERIMENT AND HARTREE-FOCK CALCULATIONS

Table I is based on the recent compilation given in Ref. 1. There are altogether only seven nuclei (all magic) for which experimental estimates of Δ_{RMS} are reported and the uncertainties in the measurements and interpretations are considerable. According to Ref. 1 typical estimates of the error in the experimental determination of neutron radii are of the order of ± 0.06 fm (see Refs. 10,11). Two of the estimates (the 0.08 for ^{208}Pb , Ref. 1, and the 0.10 ± 0.03 for ^{48}Ca , Ref. 11) seem to differ by a factor of two or more from other estimates for the same nuclei. It is clear in view of all this that only a very tentative comparison with theory is possible at this stage.

The Droplet Model predictions with $Q = 17$ MeV shown in Table I give small negative values for Δ_{RMS} in ^{16}O and ^{40}Ca , and positive values for the other nuclei (up to a maximum of 0.27 fm for ^{208}Pb). The order of magnitude is that of the reported experimental estimates, but if the experimental numbers are taken at face value, the theoretical predictions appear to be somewhat high. The same is true of the relation between the Droplet Model predictions (with the parameter set given by Eq. (8)) and the Hartree-Fock calculations: similar magnitudes, but somewhat

higher in several cases. The fact that both calculations give similar orders of magnitude is not surprising: both have in them the relevant physical elements (a bulk symmetry energy, modified by a Coulomb repulsion, working against the surface energy). The fact that both sets of numbers are actually fairly close is interesting. It may be an indication of two things. First, that microscopic (shell) effects, present in the Hartree-Fock calculations but not in the Droplet Model, are not overwhelmingly important for the discussion of RMS radii. That this should be so is not obvious and it will be interesting to follow up this hint. Second, that even though the parameters in the Hartree-Fock calculations were, in general, not adjusted explicitly to reproduce the value of the effective surface stiffness coefficient Q , the values actually used do not, in fact, grossly misrepresent this property of the nuclear surface. (In this connection see Ref. 12, where an attempt is made to investigate this question.) Table I also shows the Droplet Model predictions for ΔR_{MS} and t when Q is increased to 24 MeV. (A value of 21.5 for Q was proposed in Ref. 13, which is based on a careful study of the valley of β -stability.) The last column gives the ratios of these ΔR_{MS} values to typical Hartree-Fock predictions (taken to be numbers in the middle of the range of the reported HF values.) For the heavier nuclei the correspondence is close.

As remarked above the experimental results are still too uncertain and fragmentary to warrant an attempt to extract an independent estimate of Q from the measured values of ΔR_{MS} . Before this will be possible a number of questions will have to be clarified, including the following:

- 1) The firming up of the measured values of ΔR_{MS} .
- 2) An estimate of shell effects and their isolation from the smooth background governed by the value of Q .

- 3) Estimates of the difference between the neutron and proton diffuseness parameters b_N , b_Z . [For example, by taking $b_N = 0.9$ fm and $b_Z = 1.0$ fm (see Ref. 14) the Droplet Model prediction for Δ_{RMS} in ^{208}Pb would go down from 0.2717 to 0.2188. On the other hand, if $b_N \approx 1.07$ and $b_Z \approx 0.89$, as suggested by Ref. 15 (see also Ref. 16), the value of Δ_{RMS} would go up to 0.37, which would begin to suggest a serious disagreement with experiment.]

5. SUMMARY

The main points that we would like to stress are the following:

- 1) According to the Droplet Model the existence of a neutron skin is intimately related to other nuclear properties, in particular to the surface symmetry energy, to isotope and isotone shifts and to the Giant Dipole resonance.
- 2) The physics of the neutron skin consists of the push by the bulk symmetry energy (modified by the electrostatic repulsion) balanced against the neutron skin stiffness characterized by the coefficient Q . The resulting Droplet Model formula for the thickness of the neutron skin is elementary.
- 3) Even without an estimate of Q one predicts that nuclei with a relative neutron excess more than one-sixth of that characterizing the valley of beta-stability should, on the average, have a neutron skin.
- 4) Still without making an estimate of Q one may set an upper limit on the thickness of the neutron skin (divided by the nuclear radius) as two-thirds of the relative neutron excess I .

For nuclei near the end of the periodic table this corresponds to a thickness of about 1 fm.

- 5) The key parameter governing the magnitude of the neutron skin is Q . Using the current Droplet Model estimate of Q the calculated values of the difference between the RMS radii of neutrons and protons for seven magic nuclei are not in serious disagreement with most (but not all) of the tentative experimental estimates or with results of Hartree-Fock calculations, although the Droplet Model tends to give values of ΔR_{MS} that are somewhat higher (when the stiffness coefficient Q is taken to be 17 MeV). If Q is increased to 24 MeV the correspondence with the Hartree-Fock calculations is improved.

TABLE I. Differences between neutron and proton RMS radii and the thickness t of the neutron skin (in fm).^a

| Nucleus | Different experimental estimates of Δ RMS | Hartree-Fock Δ RMS | Droplet Model with $Q = 17$ MeV | | Droplet Model with $Q = 24$ MeV | | |
|-------------------|--|---------------------------|---------------------------------|---------|---------------------------------|---------|---|
| | | | Δ RMS | t | Δ RMS | t | $\frac{\Delta\text{RMS}_{\text{Dr}}}{\Delta\text{RMS}_{\text{HF}}}$ |
| ¹⁶ O | -0.002, ~0 | -(0.02 - 0.03) | -0.0088 | -0.0069 | -0.0081 | -0.0060 | ~0.32 |
| ⁴⁰ Ca | -0.04, -0.04, -0.03, -0.03, -0.07 | -(0.04 - 0.05) | -0.0205 | -0.0153 | -0.0188 | -0.0131 | ~0.42 |
| ⁴⁸ Ca | 0.19, 0.16, 0.17, 0.21, 0.19, 0.10±0.03 ^b | 0.18 - 0.23 | 0.1911 | 0.2579 | 0.1612 | 0.2193 | 0.79 |
| ⁹⁰ Zr | 0.13, 0.09 | 0.07 - 0.12 | 0.1197 | 0.1768 | 0.0971 | 0.1477 | 1.02 |
| ¹¹⁶ Sn | 0.13 | 0.12 | 0.1579 | 0.2318 | 0.1272 | 0.1922 | 1.06 |
| ¹²⁴ Sn | 0.22 | 0.21 | 0.2440 | 0.3429 | 0.1982 | 0.2838 | 0.94 |
| ²⁰⁸ Pb | 0.21, 0.21, 0.08, 0.21, 0.18 | 0.20 - 0.23 | 0.2717 | 0.3966 | 0.2150 | 0.3234 | 1.00 |

^aUnless otherwise noted all data are from Ref. 1.

^bFrom Ref. 11.

APPENDIX A: The RMS Radii According to The Droplet Model

In the Droplet Model the density distributions $\rho_{N,Z}^{\text{actual}}$ (see pp. 188,205 of Ref. 4) are obtained by starting with sharp "generating functions" $\rho_{N,Z}$, whose surfaces are then diffused to the appropriate widths $b_{N,Z}$. The generating functions are slightly perturbed reference distributions (with constant densities $(N,Z)/\frac{4}{3}\pi R_{N,Z}^3$ inside spheres of radii $R_{N,Z}$ and zero outside). The perturbation consists of a slight redistribution of the densities inside these sharp boundaries. For the uniform reference distributions the RMS radii would be

$$\text{RMS}_{N,Z}^{\text{ref}} = \sqrt{3/5} R_{N,Z} \quad , \quad (\text{A.1})$$

where the radii $R_{N,Z}$ are related to the mean effective sharp radius of the matter distribution by

$$R_{N,Z} \approx R \pm \frac{1}{2} t \quad . \quad (\text{A.2})$$

(Compare Eq. (42), Ref. 5 and Eq. (73), Ref. 4. We have put $Z/A \approx N/A \approx 1/2$.)

We are now required to write down the two corrections to Eq. (A.1) caused by the diffuseness of the surface and by the non-uniformity of the generating densities. Since the corrections will be treated as small and only lowest-order formulae will be derived, the two effects may be calculated one at a time and the results added. Thus in treating the diffuseness correction we are allowed to disregard the non-uniformity and use the standard result, Eq. (7), for uniform spheres. Similarly, in treating the non-uniformity we may disregard the diffuseness. Furthermore, by a similar argument relying on the smallness of the difference between R_N and R_Z , we may calculate the redistribution correction due to the

non-uniformity as if the generating densities $\rho_{N,Z}$ were bounded by the single mean surface R (rather than by R_N and R_Z).

The small non-uniformities of the neutron and proton generating densities are given by

$$\tilde{\rho}_{N,Z} \equiv \rho_{N,Z} - \bar{\rho}_{N,Z} = \frac{1}{2} \rho_0 e \left(-\frac{9}{2K} \pm \frac{1}{4J} \right) \tilde{v}$$

where $\bar{\rho}_{N,Z}$ are the averages of $\rho_{N,Z}$, ρ_0 is the density of standard nuclear matter, K is the compressibility coefficient and \tilde{v} is the deviation (from its average value \bar{v}) of the electric potential v . (Equation (A.3) is obtained by combining Eqs. (5), (6), (7), (42), (47) in Ref. 4. The upper and lower signs refer to neutrons and protons, respectively.)

To lowest order in small quantities the correction due to the non-uniformity is given by

$$\begin{aligned} (\text{RMS})_{\text{non-uniform}}^2 - (\text{RMS})_{\text{uniform}}^2 &= \frac{\int_0^R 4\pi r^4 \tilde{\rho}_{N,Z} dr}{\int_0^R 4\pi r^2 \rho_{N,Z} dr} \\ &= \frac{1}{N,Z} \frac{1}{2} \rho_0 e \left(-\frac{9}{2K} \pm \frac{1}{4J} \right) \int_0^R 4\pi r^4 \tilde{v} dr . \end{aligned} \quad (\text{A.4})$$

Inside a sphere with radius R and charge Ze the deviation of the electric potential from its average value is

$$\tilde{v} = \frac{Ze}{R} \left(\frac{3}{10} - \frac{1}{2} \left(\frac{r}{R} \right)^2 \right) , \quad (\text{A.5})$$

where r is the distance from the center. It follows that Eq. (A.4) gives

$$(\text{RMS})_{\text{non-uniform}}^2 - (\text{RMS})_{\text{uniform}}^2 \approx \frac{6}{175} \left(\frac{9}{2K} \mp \frac{1}{4J} \right) Ze^2 R .$$

[In discussing this correction we have allowed ourselves, as usual, the

consistent approximation $N \approx Z \approx \frac{1}{2} \rho_0 \left(\frac{4}{3} \pi R^3 \right)$.] Since $(\text{RMS})_{\text{uniform}}^2 \approx \frac{3}{5} R^2 \approx \frac{3}{5} R_{N,Z}^2$ the correction for redistribution may be written as

$$(\text{RMS})_{\text{non-uniform}} - (\text{RMS})_{\text{uniform}} \approx \frac{1}{35} \sqrt{3/5} \left(\frac{9}{2K} \mp \frac{1}{4J} \right) Ze^2 . \quad (\text{A.6})$$

The final Droplet Model formula for the RMS radii of the neutron and proton distributions reads

$$\text{RMS}_{N,Z} = \sqrt{3/5} \left[R \pm \frac{1}{2} t + \frac{5}{2} \frac{b_{N,Z}^2}{R} + \frac{1}{35} \left(\frac{9}{2K} \mp \frac{1}{4J} \right) Ze^2 \right] , \quad (\text{A.7})$$

with t given by Eq. (5). The difference $\text{RMS}_N - \text{RMS}_Z$ then leads to Eq. (6). If the separate RMS radii are required one needs R , for which the Droplet Model expression is given in Ref. 5, p.4, and is reproduced here for convenience

$$R = r_0 A^{1/3} (1 + \bar{\epsilon}) ,$$

with

$$\bar{\epsilon} = (-2a_2 A^{-1/3} + L\bar{\delta}^2 + c_1 Z^2 A^{-4/3}) / K , \quad (\text{A.8})$$

and

$$\bar{\delta} = \left[I + \frac{3}{16} (c_1/Q) ZA^{-2/3} \right] / \left[1 + \frac{9}{4} (J/Q) A^{-1/3} \right] .$$

In the above, a_2 is the surface energy coefficient (20.69 MeV) and K and L are the compressibility and density-symmetry coefficients (nominally 240 MeV and 100 MeV, respectively - see Ref. 5, p.5).

APPENDIX B: A Guide to Misprints and Obscurities in the
Droplet Model Papers

For the interested reader who makes the effort to look up the original Droplet Model papers the following hints will be helpful.

A misprint that could be especially confusing because it occurs twice should be corrected in Eq. (3.7) and on p.426 in Ref. 3: the factor $(1 - \frac{2}{3} \frac{P}{J})$ should read $(1 - \frac{2}{3} \frac{P}{J})^{-1}$. A minor misprint occurs on p.425 of Ref. 3, where, one-third of the way down the page, $\partial/\partial\sigma$ was written instead of $\partial/\partial\tau$. In Ref. 5, the factor $(1+2\theta^2)$ in Eq. (26) should be replaced by $(1-2\theta^2)$.

The reader may be puzzled by the difference in the expressions for $\bar{\delta}$ in Ref. 3 on the one hand, and Refs. 4 and 5 on the other. In the former case the numerator has a term $\frac{3}{8} (c_1/Q) Z^2 A^{-5/3}$, in the latter $\frac{3}{16} (c_1/Q) Z A^{-2/3}$. The ratio is $2Z/A$, which differs from unity by a formally small quantity, of order I , and corresponds to a term of higher order than those retained in the Droplet Model scheme. In this sense either formula is acceptable, but we have come to prefer the latter version.

A similar comment applies to the variety of equations given for the radii R_N, R_Z (e.g., Eqs. (2.3) in Ref. 3, Eqs. (9) and (42) in Ref. 5, and, implicitly, Eqs. (73) in Ref. 4 and Eq. (A.2) in the present paper.) Again, it will be found that the differences involve quantities that are formally of higher order in the small expansion parameters of the theory. Unless there is a special reason not to do so, the simplest looking alternative is probably the one to use, for example, Eq. (A.2) in this paper.

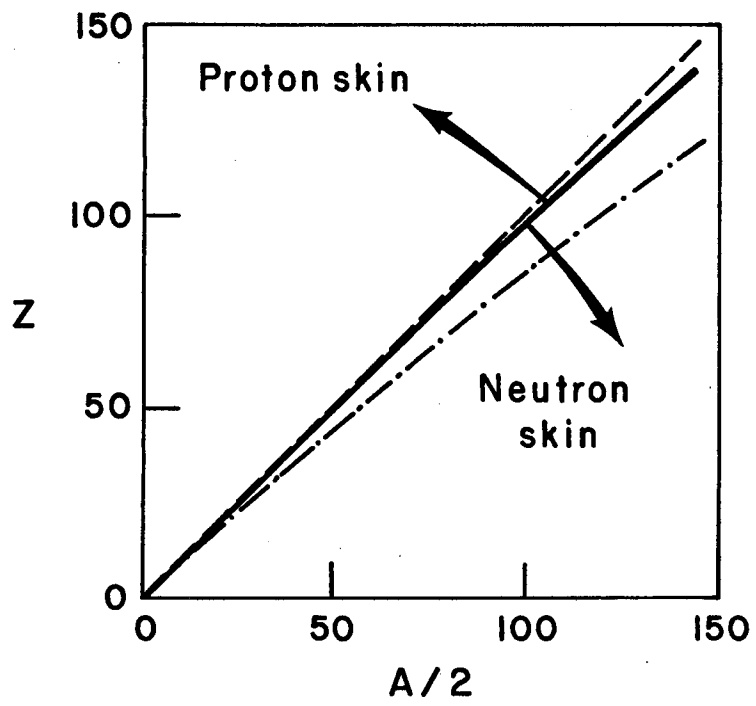
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FIGURE CAPTIONS

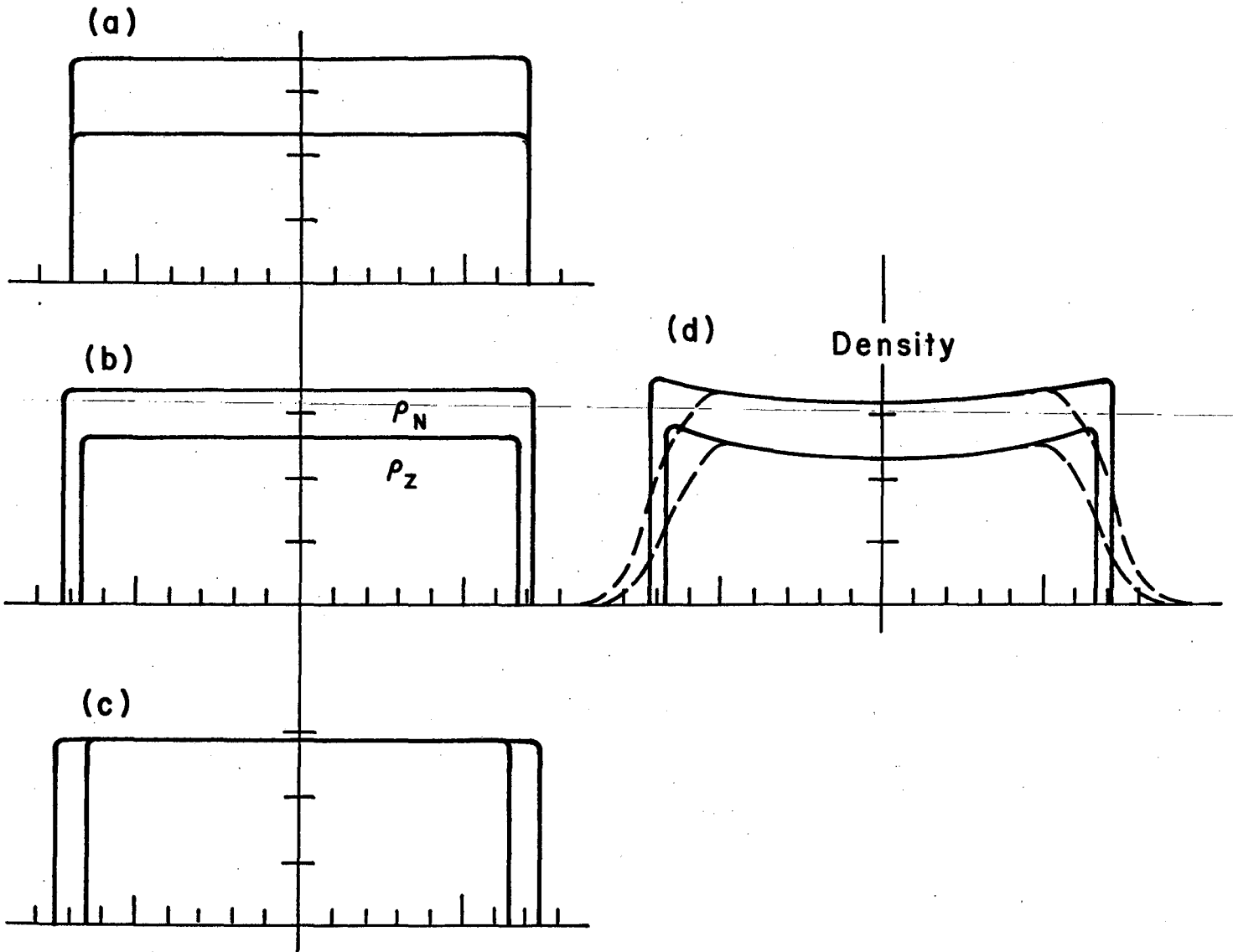
Fig. 1. A plot of Z vs. $A/2$ for three different cases. The dashed line corresponds to $N = Z$. The solid line corresponds to the locus of nuclei for which the neutron skin thickness t is predicted (by Eq. 3) to be zero. As neutrons are added beyond this point the skin becomes thicker and thicker. For nuclei with fewer neutrons a proton skin is predicted. The dot-dashed line corresponds to the location of the valley of β -stability predicted by Eq. (4).

Fig. 2. Schematic plots of the nuclear neutron and proton density distributions for ^{208}Pb vs. the radial distance. The surfaces are drawn sharp (zero diffuseness) to help illustrate the points being made. Part (a) represents the strict "liquid drop model" in which $R_N = R_Z = R = r_0 A^{1/3}$. Part (c) represents the opposite extreme where the central neutron and proton densities are the same and all the excess neutrons are placed in the surface. Part (b) represents the actual (intermediate) situation predicted by the Droplet Model, where the neutrons are partially pushed into the surface by the volume symmetry energy. To the right of part (b), in part (d), the Droplet Model prediction for the central depression in the neutron and proton density distributions (caused by the Coulomb repulsion) is also shown. The diffuseness of the actual surfaces is given approximately by the dashed lines in part (d).



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Fig. 1



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Fig. 2

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