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THE ELECTRIC DIPOLE MOMENT OF THE ELECTRON[1]

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ABSTRACT

Recent experimental progress in the search for atomic electric dipole moments (EDMs) d_A of cesium and thallium leads in particular to a substantially increased sensitivity to a possible electron EDM d_e compared with existing upper bounds. Further considerable improvement in the measurement of d_{Tl} is likely. After a brief synopsis of the theory of atomic EDMs we discuss in view of the expected experimental sensitivitiy to d_e the predictions for the electron EDM in various models of CP violation.

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CONTENTS

Introduction

A stable particle, elementary or composite, cannot have an electric dipole moment (EDM) unless both time reversal (T) and parity reflection (P) invariances are broken. It is because the expectation value of the EDM operator $\vec{D} = \int \vec{x}\rho(\vec{x})d^3x$ in a particle state at rest is proportional to the particle's spin (or, more generally, total angular momentum), but spin is odd under T and even under P while $\vec{D}$ is even under T and odd under P [1,2]. This argument applies to atoms and molecules as well.

If the CPT theorem holds, the above statement implies that a nonzero EDM of a particle requires violation of both CP invariance and P invariance. As CPT is known to be a good symmetry for the models of CP violation we consider below, we shall henceforth interchange T and CP violation.

The EDM of a particle is defined by one of its electromagnetic form factors. In particular, for a spin 1/2 particle f the form factor decomposition of the matrix element of the electromagnetic current J_μ is

$$< f(p')|J_\mu(0)|f(p) >= \bar{u}(p')\Gamma_\mu(q)u(p), \tag{1.1}$$

where

$$\begin{aligned}\Gamma_\mu(q) = &F_1(q^2)\gamma_\mu + F_2(q^2)i\sigma_{\mu\nu}q^\nu/2m \\ &+ F_A(q^2)(\gamma_\mu\gamma_5 q^2 - 2m\gamma_5 q_\mu) + F_3(q^2)\sigma_{\mu\nu}\gamma_5 q^\nu/2m\end{aligned} \tag{1.2}$$

with $q = p' - p$ and m denotes the mass of f.

The EDM of f is then given by

$$d_f = -F_3(0)/2m. \tag{1.3}$$

This corresponds to the effective electric dipole interaction,

$$L_I = -\frac{i}{2}d_f\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}, \tag{1.4}$$

which reduces to $L_I = -H_I = d_f\vec{\sigma}\cdot\vec{E}$ in the nonrelativistic limit.

In renormalizable theories of CP violation the interaction (1.4),where f denotes a quark or lepton, must be induced by loop diagrams because it is nonrenormalizable. The EDM interaction (1.4) flips the fermion chirality and is not invariant under the electroweak symmetry group $SU(2)_L$. Hence a nonzero d_f requires besides CP violation also electroweak symmetry breaking, which in a gauge theory must occur spontaneously. The chirality flip which is also necessary to yield a nonzero d_f comes from fermion mass terms. The relevant mass terms can – but need not – arise from spontaneous symmetry breaking of the electroweak symmetry.

In a gauge theory CP invariance may be violated spontaneously (usually parametrized by complex vacuum expectation values of Higgs fields) or it may be broken explicitly for instance if the theory contains CP noninvariant couplings involving scalar fields. This is assumed to be the case in the three–generation standard model (SM) of electroweak interactions. In the SM, CP violation manifests itself by a complex quark mixing matrix, the Kobayashi–Maskawa (KM) matrix [3] which, originates from complex Yukawa couplings. The KM model can accommodate the CP violation found in the neutral kaon system, which is the only place where this phenomenon has been observed so far. According to the KM model, not only EDMs of leptons, but also those of the neutron and other baryons are too small to be observable by experiments in the forseeable future. Therefore, if a nonzero value for the EDM of a particle should be established at the presently discussed levels of sensitivity, it would be evidence for a new CP violating interaction [F1].

Experimentally one can search for a permanent EDM of a particle by placing it into an external field $\vec{E}$ and by looking for a shift ΔE linear in $\vec{E}$ of the interaction energy of the particle with the external field. In the weak field limit,

$$\Delta E = a_i E_i + b_{ij} E_i E_j + \cdots \tag{1.5}$$

where the term linear in $\vec{E}$ is the signature of a permanent EDM. The term quadratic in $\vec{E}$ is an induced EDM contribution which has nothing to do with CP violation.

As to experimental searches, much effort has been and is being made to measure the neutron EDM [4,5]. The Leningrad group obtained [4] $d_n = (-1.4 \pm 0.6) \times 10^{-25}$ $e\ cm$ whereas the Grenoble group recently reported [5] $d_n = (-0.3 \pm 0.5) \times 10^{-25}$ $e\ cm$. This value yields the upper bound

$$|d_n| < 1.2 \times 10^{-25}\ e\ cm. \tag{1.6}$$

The tightest upper limits on the electron EDM d_e were and are being deduced from the null–results so far of the searches for atomic EDMs. However, this assumes that the contribution of d_e to the respective atomic EDM d_A is not accidentally cancelled by other T–violating contributions to d_A (see Sect. 2). Previous searches, for instance for an EDM of Hg [6], resulted in upper bounds of $|d_e|$ of about 2×10^{-24} $e\ cm$. Recently an experiment searching for T violation in thallium fluoride obtained [7] $d_e = (-1.4 \pm 2.4) \times 10^{-25}$ $e\ cm$, and from an experiment which measured the EDM of Cs it was deduced that [8]

$$d_e = (-1.5 \pm 5.5 \pm 1.5) \times 10^{-26}\ e\ cm, \tag{1.7}$$

which corresponds to an upper limit of about 10^{-25} $e\ cm$. An experiment on the EDM of $T\ell$ is in progress and its preliminary result for $d_{T\ell}$ based on a short data taking period gives an upper limit on d_e which is already more restrictive [9] than the one resulting from (1.7). The $T\ell$ experiment is expected to reach an accuracy to d_e of about

$$\delta(d_e) \simeq 10^{-27}\ e\ cm \tag{1.8}$$

within a year or two. Even a null result will provide at this level of accuracy very useful information and will contribute to our understanding of CP–violating forces as we shall review below. In order to appreciate this number, we may compare it with the precision with which the anomalous magnetic moment of the electron, $\frac{1}{2}(g-2) = F_2(0)/e$, is known. (A nonzero contribution to F_2 requires an $SU(2)_L$–breaking and chirality–flipping interaction just as in the case of F_3, but of course no CP violation.) The current precision is [10,11,12]

$$\delta\left(\frac{1}{2}(g-2)\right) = 1 \times 10^{-11}, \tag{1.9}$$

which corresponds to

$$\delta(F_2(0)/2m_e) = 2 \times 10^{-22}\ e\ cm. \tag{1.10}$$

That is, d_e will presumably be known about five orders of magnitude more accurately than the anomalous magnetic moment of the electron within a few years.

In the case of the neutron EDM, uncertainties in low–energy strong interaction physics prevent a precise comparison between an experimental value for d_n and CP–violating parameters at the quark level [13,14]. In contrast, the electron EDM is free from such uncertainties and can be computed unambiguously once a model is fully specified. In this respect, an experimental value for d_e is, in principle, capable of testing models of CP violation more directly. However, in many models, d_e turns out to be smaller than d_n so that a higher experimental accuracy is called for. Moreover, the CP violating parameters of the quark and lepton sectors are a priori unrelated, except in simplified versions of some models. Therefore, the data on the observed CP violation in the K_L decays or the upper limit on d_n cannot be used without further assumptions to constrain the CP–violating couplings which generate d_e and firm predictions about the magnitude of d_e cannot be made. Typically, only upper bounds on d_e are obtained for a given model. Nevertheless, knowledge of d_e with a precision of (1.8) and other existing and upcoming data on CP violation in hadrons will help us in understanding this feeble phenomenon.

Here we attempt to survey models of and ideas on the electron EDM in view of the anticipated experimental sensitivity (1.8). Our review overlaps somewhat with a recent article by Barr and Marciano [15]. This article is organized as follows: In Section 2, we review the relation of the electron EDM to atomic EDMs from which the former is usually deduced. Then we review the predictions for d_e of "nonstandard" models of CP violation. A survey is given in Section 4. In Sections 5,6 and 7 we discuss supersymmetric models, left–right symmetric models and Higgs models of CP violation, respectively. Various interactions, which are CP– and lepton–number nonconserving interactions, are treated in Section 8. Section 9 contains a remark about d_e and CP–violating effective four–electron interactions which may arise if the electron is composed of subconstituents. We end with some conclusions in Section 10.

2. Electric dipole moments of atoms and molecules

A permanent EDM of a stable atomic or molecular state can arise only when P and T invariances are broken. However, it is often said that molecules known as polar molecules have large "permanent" EDMs. We start recalling how this comes about.

2.1 Induced EDMs of polar molecules

Molecules such as ammonia and water have a pair of nearly degenerate states with opposite parities, the lower of which is the ground state. Since their energy splitting is less than the thermal energy kT at room temperature, the two states act practically like a twofold degenerate ground state. When an external electric field $\vec{E}$ is applied, the two states of opposite parities, $|+>$ and $|->$, mix with each other and form new energy eigenstates $|r> \simeq (|+> + |->)/\sqrt{2}$ and $|\ell> \simeq (|+> - |->)/\sqrt{2}$ with energy eigenvalues

$$E_{r,\ell} = \frac{1}{2}(E_+ + E_-) \pm \left[\frac{1}{4}(E_+ - E_-)^2 + (e < \vec{r} > \cdot \vec{E})^2\right]^{\frac{1}{2}}, \tag{2.1}$$

where $< \vec{r} >$ is the transition matrix element between $|+>$ and $|->$ of position $\vec{r}$. Because $E_\pm$ are almost degenerate, $e < \vec{r} > \cdot \vec{E}$ dominates inside the square root in Eq.(2.1) and the energy eigenvalues are given approximately by $E_{r,\ell} = \frac{1}{2}(E_+ + E_-) \pm e < \vec{r} > \cdot \vec{E}$. Since in this approximation the energy shift is linearly dependent on $\vec{E}$, the proportionality constant is called the permanent EDM of this molecule. However, this EDM is not an indication of P and T violation. If measurements were done with an infinitesimally weak $\vec{E}$ at zero temperature, one would find only a quadratic dependence of the energy eigenvalues on $\vec{E}$, *i.e.*, $E_{r,\ell} = E_\pm \pm (e < \vec{r} > \cdot \vec{E})^2/(E_+ - E_-) + \cdots$ by a power series expansion in $\vec{E}$. Thus there is no linear dependence on $\vec{E}$ of the energy shift. If T invariance holds, a molecule acquires only an induced EDM which is enhanced by a small energy difference between opposite parity states.

What we are interested in below is not an EDM of this kind, but a permanent EDM which causes a linear Stark effect even for an infinitesimally weak $\vec{E}$. Such an EDM is a genuine signataure of P and T violation or CP violation.

2.2 Permanent atomic EDMs

A permanent EDM of an atom (or molecule) can be due to EDMs of electrons and/or nucleons, P– and T–violating nucleon–nucleon forces and/or P– and T–violating electron–nucleon and possibly electron–electron forces. In other words, measurements of atomic EDMs provide information about several CP–violating effects. But in general EDM measurements for various atoms and – for a given model of CP violation – reliable atomic and nuclear physics calculations are needed to disentangle the above–mentioned effects. The new improved bounds on the electron EDM d_e referred to in Section 1 rely on the theoretical result that relativistic effects enhance the contribution of d_e to the EDMs of cesium and thallium by two orders of magnitude and more, respectively (see below). For that reason we discuss the contribution of d_e to an atomic EDM d_A in some detail and mention the nuclear contribution to d_A only cursorily.

2.2a Schiff's theorem

To put the relativistic enhancement into perspective it is useful to recall a theorem due to Schiff [16] which, if it applies, would amount to exactly the opposite. Schiff showed that the EDM of a nonrelativistic atom vanishes irrespective of whether the atomic constituents have EDMs or not. The theorem is based on two assumptions:

1. Atoms consist of nonrelativistic particles which interact only electrostatically.
2. The electric dipole moment distribution of each atomic constituent is identical to its charge distribution.

An atomic nucleus is treated here as a single charged particle. The two assumptions are not completely independent of each other.

The theorem can be proven by use of a simple relation between the Hamiltonian H containing the EDMs of the constituents and the Hamiltonian H_0 which does not when an external electric field is present. With the translation operator $Q = -i\sum_j(\vec{d}_j/e_j)\cdot\vec{\nabla}_j$, where e_j and $\vec{d}_j$ are the charge and EDM of the $j-th$ constituent, H can be obtained from H_0 by

$$H = H_0 + H_{EDM} = H_0 + i[Q, H_0]. \tag{2.2}$$

Given the eigenstates ϕ_n of H_0 with eigenvalues E_n, the corresponding eigenstates of H are $e^{iQ}\phi_n$ to the lowest nontrivial order in d_j since

$$\begin{aligned} e^{-iQ}H\, e^{iQ}\phi_n &= \left(H_0 + O\left(d_j^2\right)\right)\phi_n \\ &= E_n\phi_n + O\left(d_j^2\right). \end{aligned} \tag{2.3}$$

That is, the energy eigenvalues of the states ϕ_n and $e^{iQ}\phi_n$ are equal up to $O\left(d_j^2\right)$. There is no energy shift linear in the constituent EDMs even in the presence of an external electric field, which means the constituent EDMs cannot produce a net atomic EDM. Note that the theorem is valid even when a nucleus has an EDM as long as it is treated as a nonrelativistic pointlike particle.

2.2b Relativistic enhancement of the contribution of d_e

The theorem works quite well for the ground state hydrogen atom for instance, but it fails badly for many atoms. In fact, enhancement of the contribution of an individual constituent by more than two orders of magnitude is not uncommon in heavy atoms. Let us consider light atoms first [17,18]. The above assumptions are violated by relativistic effects such as relativistic kinetic energy of electrons and spin–orbit interaction which are formally of $O(\alpha^2)$. The spin–orbit interaction violates in particular the second assumption of the theorem. For instance, the charge distribution of a $p_{1/2}$ state is spherically symmetric while its spin distribution is proportional to cos 2θ.

States with opposite parities mix with each other through P–violating interactions. Such mixing can be caused both by T–conserving and T–violating interactions. However, only the portion of mixing due to P– and T–violating interactions such as those induced by permanent EDMs of electrons and nucleons give rise to an energy shift linear in the external electric field $\vec{E}$ [F2]. The EDM interaction due to $d_e \neq 0$ mixes for instance the hydrogen ground state $1s_{1/2}$ with $2p_{1/2}$ and $2p_{3/2}$. When relativistic effects in the binding force are taken into account, $2p_{1/2}$ and $2p_{3/2}$ are split by the spin–orbit interaction. Then the cancellation which leads to Schiff's theorem is no longer exact. This yields a contribution to the hydrogen EDM d_H of $O((\Delta E_{LS}/R_\infty)d_e)$ where ΔE_{LS} is the spin–orbit energy splitting and $R_\infty = 13.6\ eV$. This means that the contribution of d_e to d_H is suppressed by $\Delta E_{LS}/R_\infty \simeq \alpha^2$. When states of opposite parities are closely spaced such that $\Delta E = O(\Delta E_{LS})$ there is no suppression contrary to a naive expectation from Schiff's theorem [20]. Failure of the theorem is more spectacular for the first excited state of hydrogen, as $2s_{1/2}$ and $2p_{1/2}$ are split only by the Lamb shift. With $\Delta E_{Lamb}/R_\infty = \alpha^3$, we expect that the contribution of the electron EDM to the atomic EDM is actually enhanced by $\Delta E_{LS}/\Delta E_{Lamb} \sim 1/\alpha = 137$, which is confirmed by an explicit calculation [17].

Enhancement occurs most conspicuously in heavy atoms with an unpaired electron. In such an atom a valence electron feels an unshielded strong Coulomb field when it comes close to the nucleus. Since the electron velocity is comparable to the velocity of light in the inner core region of a heavy atom, the nonrelativistic approximation breaks down completely and contribution of d_e to d_A is not suppressed at all. On the contrary, the singular behavior $\propto 1/r^2$ of the electric dipole interaction at short distances makes the mixing between opposite parity states very strong. This results in a strongly enhanced contribution of d_e to d_A. Some of the enhancement factors calculated in the past are tabulated in Table 1. For instance, for thallium where the $6^2p_{1/2}$ state mixes with $6^2s_{1/2}$, an enhancement of 500 to 700 has been predicted. This large enhancement factor and the enhancement factor of about 100 in case of cesiuim were the incentives for undertaking precision measurements of the atomic EDMs of $T\ell$ [7,9] and Cs [8], respectively.

For atoms with electrons paired, electron EDMs sum up to zero in a naive picture. However, a hyperfine interaction prevents the complete cancellation and a small net atomic EDM results from a nonzero d_e [27]. Atomic EDMs of paired electron atoms, *i.e.*, those of Hg and ground state Xe were measured much more accurately than those of unpaired atoms. In fact, before the recent measurement of the EDM of Cs, the best upper bound on the electron EDM had been deduced from the atomic EDM of the $^1S_0$ ground state of Hg. The last column of Table 1 tabulates the values of the electron EDM deduced from the measurements of various atomic EDMs.

2.2c Nuclear contributions

Schiff's theorem also fails for realistic nuclei. A nucleus is not a pointlike particle. Once the structure of a nucleus is taken into account, the first assumption of the theorem is violated because nuclear forces have nothing to do with electrostatic forces. Furthermore, if the proton and neutron have EDMs, the EDM distribution of a nucleus is quite different from its charge distribution because nuclear forces are strongly spin–dependent. Nuclear contributions to an atomic (or molecular) EDM d_A are usually discussed by considering P– and T–odd nuclear multipoles which interact with the atomic electrons. These P– and T–odd interactions can induce mixing between opposite parity states and can thus lead to a nonzero d_A. Two T–odd nuclear moments are usually taken into account in this context: a nuclear magnetic quadrupole moment (MQM) [33] and a so–called nuclear "Schiff moment" [34–38] which arises if the charge and EDM distributions of a nucleus are different. (A total EDM d of a nucleus is not relevant here: In a stationary atomic or molecular state the average electric field $\vec{E}$ at the nucleus vanishes; *i.e.*, the interaction $\vec{d} \cdot \vec{E}$ is absent. However, nucleon EDMs distributed over a finite size in a nucleus can contribute to the Schiff moment.) The MQM of a nucleus contributes to an atomic EDM only if the electron cloud has nonzero angular momentum. Furthermore it should be recalled that atoms with spin 1/2 nuclear ground states, *e.g.*, $^{129}Xe, ^{199}Hg, ^{203}T\ell$, and $^{205}T\ell$ have zero MQMs.

At the nuclear level these moments can be generated by P– and T–violating effects such as proton and neutron EDMs and P– and T–violating nucleon–nucleon interactions. For instance, calculations of the MQMs and Schiff moments of various nuclei in terms of the parameters of a general P– and T–odd nucleon–nucleon interaction were made in Ref.[39]. A systematic attempt to identify the contribution to the Schiff moments at the level of quarks and gluons and to estimate the strength of these P– and T–odd hadronic interactions in some models of CP violation was made in [40].

Besides P– and T–violating hadronic interactions also P– and T–violating electron–nucleon (or quark) interactions can produce a nonzero d_A. One can define tensor– pseudotensor and scalar–pseudoscalar electron–nucleon interactions $a_T(i\bar{N}\sigma_{\mu\nu}\gamma_5 N)(\bar{e}\sigma^{\mu\nu}e)$ and $a_S(i\bar{N}\gamma_5 N)(\bar{e}e)$, respectively [28,41,42] and d_A can be calculated in terms of the coefficients a_S and a_T.

In view of the above discussion the EDM of an atom (or molecule) can be written schematically:

$$d_A = R\, d_e + c_N \tag{2.4}$$

where the enhancement/suppression factor R depends on the given atom, whereas the contribution c_N involving nucleons depends on the given atom and on the mechanism of CP nonconservation. Obviously, if a nonzero d_A for some atom should be found, elaborate theoretical input would be necessary but possibly not sufficient to pin down its origin. So far only d_A's consistent with zero have been measured. It is customary to deduce from these measurements upper bounds on the electron EDM (see Table 1) and on the parameters appearing in c_N (see *e.g.*, the compilation in [15]), barring accidental cancellations between the different contributions in (2.4). We may feel less uncomfortable with this approximation for unpaired electron atoms such as Cs and $T\ell$ where the electron EDM contribution is enormously enhanced. However, from a measurement of, say, $d_{T\ell}$ with a sensitivity of order 10^{-25} $e\ cm$ one can infer a sensitivity to d_e of a few times 10^{-28} $e\ cm$ only if $c_N \lesssim 10^{-25}$ $e\ cm$

can be established for $T\ell$. Further theoretical studies are thus desired on this point. The danger of an accidental cancellation can be reduced by analyzing the implications for d_e and c_N from d_A's of several different atoms.

2.2d Future possibilities

Hadronic P–and T–nonconserving interactions can be considerably enhanced in certain rare and actinide nuclei where nearly degenerate opposite–parity ground-state doublets exist which are mixed by these CP–violating forces. Ref.[43] finds nuclear EDMs and MQMs which are $10 - 10^3$ and $10^3 - 10^4$ times larger, respectively, than the respective moments generated by the unpaired valence nucleon. Whether the EDMs of these atoms can be measured with high precision remains to be seen.

Spectacular enhancements of the contribution of d_e to d_A can occur in certain diatomic molecules with very closely spaced rotational levels of opposite parities [44,45]. For instance for BiS it was estimated [44] that enhancement factor $R = 10^7 - 10^{11}$. If experiments are feasible this opens the possibility of a substantial increase of the sensitivity to d_e even compared with (1.8).

3. The electron EDM in the Standard Model

In the remainder of this article we review the predictions of various models of CP nonconservation for the electric dipole moment of the electron. We begin with the Standard Model of particle physics.

In the three–family $SU(3)_C \times SU(2)_L \times U(1)_Y$ model of electroweak interactions, CP violation arises - apart form the "θ term" in quantum chromodynamics, which is of no concern to us here - from the complex couplings of the charged weak quark currents, *i.e.*, the Kobayashi–Maskawa matrix V. All CP–violating phenomena observed so far in the neutral kaon system can be accounted for by the KM mechanism. This mechanism generates however only tiny electric dipole moments of baryons. For instance, for the neutron one expects $(d_n)_{KM} < 10^{-30}\ e\ cm$ (cf. [13,14,46–55]). If neutrinos are massless, no CP–violating couplings occur among leptons. Nevertheless CP violation in the hadron sector can induce nonzero EDMs of leptons; in particular, of the electron. This effect was recently calculated within the SM in [56]. In the SM with massless neutrinos CP violation in the lepton sector originates from quark loops. The Feynman diagrams which generate a nonzero d_e must be at least of three–loop order (see Fig. 1). (If only two W bosons couple to the quark loop the diagram is independent of the CP–violating KM phase as its dependence on the KM matrix is of the form $|V_{ij}|^2$.) In the limit that two charge 2/3 quark masses or two charge –1/3 quark masses are equal, CP violation vanishes in the quark sector and d_e must vanish, too. Ref.[56] summarizes its numerical investigation in the form:

$$d_e = -1.7 \times 10^{-38} (m_t/100\, GeV)^2 \left(J/10^{-4}\right)\ e\ cm, \qquad (3.1)$$

where m_t is the mass of the t quark and $J = c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta (c_i = \cos\theta_i, s_i = \sin\theta_i, s_\delta = \sin\delta)$ is the invariant combination of the KM angles to which all observable CP–nonconserving effects are proportional in the SM [57]. Using $m_t < 200$ GeV [58] and $J < 2 \times 10^{-4}$, we obtain

$$|d_e| < 1.4 \times 10^{-37}\ e\ cm. \tag{3.2}$$

At this point we may note that it is possible to set a quite model–independent upper limit on the electron EDM arising from hadronic CP violation through an induced EDM of the W boson. CP nonconservation in the hadron sector can induce CP–odd terms in the $\gamma W^+ W^-$ vertex. In particular, it can generate an EDM of the W boson which corresponds to an interaction term of the form $i(e/2)\lambda_W\ \epsilon^{\mu\nu\kappa\rho}\ W_\mu^\dagger W_\nu F_{\kappa\rho}$, where $F_{\kappa\rho}$ is the electromagnetic field tensor. This interaction will in turn lead to EDMs of fermions, in particular of the neutron and the electron. From the upper bound on the neutron EDM Ref.[59] estimated that $\lambda_W < 10^{-3}$. This limit implies that the electron EDM generated by this interaction is smaller than $10^{-27}\ e\ cm$ [59].

After this digression let us now discuss the possible CP–violating leptonic couplings in the SM. If at least two of the three neutrinos are massive and their masses are different, then CP violation can occur in the lepton sector - in analogy to the quark sector - through complex couplings of the weak leptonic currents due to a lepton mixing matrix V_ℓ. The charged current interaction is

$$L_I = -\left(g/\sqrt{2}\right) \bar{N}_L \gamma^\mu V_\ell E_L W_\mu^+ + h.c., \tag{3.3}$$

where $E = (e, \mu, \tau)$ and $N = (\nu_1, \nu_2, \nu_3)$ are mass eigenstates. If neutrinos are Dirac particles, then in complete analogy to the KM matrix of the quark sector, V_ℓ has four observable parameters; three Euler angles and one CP–violating phase. If the neutrinos are Majorana particles, V_ℓ contains two more CP–violating phases [60]. However, the resulting lepton EDMs are too tiny to be interesting: To one–loop order the lepton–photon vertex cannot produce an EDM because it is proportional to $(V_\ell)_{1i}(V_\ell^*)_{1i}$ and possible CP– violating phases cancel (see Fig.2). In two–loop order with respect to the weak couplings each single diagram can contribute to an EDM, but the sum of all diagrams yields a zero EDM. This was shown for the electron [61] and for quarks [46]. As no symmetry argument is known which extends to higher orders, one expects the EDM of a lepton (or a quark) to be nonvanishing in three–loop order. The estimate of the leptonic three–loop contribution to d_e can be expressed in the form

$$d_e \simeq e\left(\alpha^2/\pi^4\right) G_F m_e f_e = 6 \times 10^{-29} f_e\ e\ cm, \tag{3.4}$$

where f_e denotes a product of small mass ratios and lepton mixing angles which must also be small. For comparison the corresponding factors f for quarks are of the order of 10^{-9} or smaller. From data on the $e - \mu - \tau$ universality and from the experimental upper bounds on $m_{\nu e}, m_{\nu\mu}$ and $m_{\nu\tau}$, one concludes that $|f_e| \ll |f_q|$ [F3]. This conclusion remains valid even if extra generations with heavy neutrinos exist. Therefore, if future experiments should find a nonzero EDM of the electron of $O(10^{-27}\ e\ cm)$ or larger, it would signal a new CP–violating interaction. Of course, failure to observe d_e at this level cannot necessarily be regarded as a positive proof of the KM model of CP violation.

4. Nonstandard models of CP–violation and d_e: Overview

Many "nonstandard" CP–violating interactions involving leptons are conceivable once we depart from the SM with a single complex Higgs doublet. Various models of CP non-conservation have been proposed and analyzed in the literature. A posteriori CP–violating interactions are weaker than CP–conserving weak interactions. In view of the experimental sensitivitiy we are therefore mainly interested in models which generate an electron EDM to one–loop order. However, higher loop effects on d_e may also be important. In fact, it was recently pointed out [164] that in Higgs models of CP violation some two–loop contributions to d_e are by far more important than the one–loop effect (cf. Section 7.2). In renormalizable gauge models the generic one–loop diagrams which can give rise to a nonzero EDM d_e are depicted in Fig.3. The boson B must couple both to e_L and e_R with complex couplings g_L and g_R, respectively, such that $\mathrm{Im}(g_L g_R^*) \neq 0$. Moreover, the necessary chirality flip must come from the mass term of the intermediate fermion F which can be much larger than m_e. The formulae for d_e corresponding to the diagrams of Fig.3 are given in the Appendix.

It is convenient [15] to distinguish between flavor–conserving and flavor–changing models of CP–violation. Models whose most significant one–loop effect on the EDM of the electron (and/or of the neutron)is represented by the amplitudes of Fig.3 where F is not necessarily a fermion from the second or higher generation are assigned to the first category. Among them are some popular models: (1) Supersymmetric models, where F can be the scalar electron (scalar electron–neutrino) and B can be a neutralino (chargino); (2) Left–right symmetric models, where F can be the electron–neutrino (more precisely, the light ν_{eL} slightly mixed with a heavy N_{eR}) and B can be a charged weak vector boson; (3) Higgs models, where F is the electron and B is a Higgs particle with indefinite parity. These models will be discussed in the following sections. On the other hand there are many models which can generate a large electron EDM, *i.e.*, $|d_e| \gtrsim 10^{-27}\ e\ cm$ by the exchange of an intermediate heavy fermion F from a higher generation in the diagrams of Fig.3. These models are put into the second category and some of them will be discussed in the section on lepton–flavor–changing models.

5. Supersymmetric models

One of the main theoretical motivations for considering supersymmetry (SUSY) in particle physics is the aim to understand the large hierarchy between the electroweak mass scale and the Planck scale. In the SUSY approach to this so–called gauge hierarchy problem the electroweak scale is generated by the dynamics of the supersymmetric theory which at the Planck scale is usually assumed to be N=1 supergravity. One usually considers models which are a minimal supersymmetric extension of the SM where SUSY is broken by soft terms induced by N=1 supergravity [62–67]. (The word "soft" refers to terms which break SUSY without reintroducing quadratic divergences into the unrenormalized theory.) What are the sources of CP violation in these models? As in the SM there is the KM phased δ in the quark mixing matrix, possibly an analogous phase (or phases in the case of massive Majorana neutrinos) arising from a lepton mixing matrix and the QCD θ parameter. In addition SUSY models can have a few more interesting CP–violating phases which arise from complex parameters in the superpotential and in the soft SUSY–breaking terms (see below). While the KM mixing is of importance for CP nonconservation in quark–flavor changing processes, its effect on EDMs is bound to be very small [68,69]. However, nonzero "SUSY phases" generate fermion EDMs already to one–loop order - irrespective of generation mixing [70–76]. Since the purpose of this Section is to focus on predictions on the electron EDM which are characteristic of SUSY models, neutrino masses are not of primary interest in what follows. We therefore set them to zero and comment on the effects which result from nonzero neutrino masses at the end of this section. Then our survey is based on a popular SUSY model, often referred to as the supersymmetric standard model, which is specified below (for reviews, see [77–79].)

The model involves gauge supermultiplets of the gauge group $G_s = SU(3)_c \times SU(2)_L \times U(1)_Y$ and three generations of left chiral matter supermultiplets for quarks, leptons and their SUSY partners and two Higgs supermultiplets. The quantum numbers of the matter supermultiplets with respect to G_s are:

$$\begin{aligned} \hat{Q}_i \left(3,2,\frac{1}{6}\right), \hat{U}_i^c \left(3^*,1,-\frac{2}{3}\right), \hat{D}_i^c \left(3^*,1,\frac{1}{3}\right), \\ \hat{L}_i \left(1,2-\frac{1}{2}\right), \quad \hat{E}_i^c(1,1,1), \\ \hat{H}_1 \left(1,2,\frac{1}{2}\right), \quad \hat{H}_2 \left(1,2,-\frac{1}{2}\right), \end{aligned} \tag{5.1}$$

where $\hat{Q}_i = \left(\hat{U}_i, \hat{D}_i\right)$, $\hat{L}_i = \left(\hat{N}_i, \hat{E}_i\right)$ with the index i referring to generations and each supermultiplet consists of a particle and its SUSY partner such as $\hat{E}_1 = (e_L, \tilde{e}_L)$ and $\hat{E}_1^c = (e_R^c, \tilde{e}_R^*)$ with e and $\tilde{e}$ denoting the electron and its spinless SUSY partner, respectively. The Lagrangian of the model is

$$L + L_0 + L_W + L_{soft}, \tag{5.2}$$

where L_0 denotes the kinetic terms and gauge interactions and L_W is obtained from the superpotential W of the Higgs multiplets

$$-W = \hat{U}^c h_U \hat{Q} \hat{H}_1 + \hat{D}^c h_D \hat{Q} \hat{H}_2 + \hat{E}^c h_E \hat{L} \hat{H}_2 + \mu \hat{H}_1 \hat{H}_2 + h.c. \tag{5.3}$$

The SUSY breaking terms are

$$
\begin{aligned}
-L_{soft} &= \widetilde{U}_R^* \xi_U \widetilde{Q}_L H_1 + \widetilde{D}_R^* \xi_D \widetilde{Q}_L H_2 + \widetilde{E}_R^* \xi_E \widetilde{L}_L H_2 \\
&+ \mu B H_1 H_2 + \frac{1}{2} \sum_i \mu_i^2 z_i^* z_i + \frac{1}{2} \sum_a \widetilde{m}_a \lambda_a \lambda_a + h.c.
\end{aligned}
\tag{5.4}
$$

In Eqs. (5.3) and (5.4), h and ξ are 3×3 matrices in generation space, H_1 and H_2 denote the scalar Higgs doublets, z_i is the scalar partner of any matter field, and the last sum in (5.4) is over the Majorana mass terms of the gauginos.

Following Ref.[80], let us now identify the CP–violating phases: The complex Yukawa coupling matrices h_U and h_D lead after the diagonalization of the quark mass matrices to the KM phase δ. Here h_E will be taken to be real and diagonal. Furthermore, in Eqs.(5.3) and (5.4) the matrices $\xi_{U,D,E}$, the mass parameter μ, B and the Majorana masses $\widetilde{m}_a$ are complex in general. Moreover, there may be off–diagonal complex scalar mass terms μ_{ij}^2 for z_i in (5.4). By redefining the phase of, say H_1, the term μB in (5.4) can be made real and therefore the mass μ has a fixed phase $\mu = |\mu| \exp(-i\varphi_B)$. The Majorana masses $\widetilde{m}_a$ can also be made real by absorbing their phases into λ_a. These phases are then shifted into interaction terms (see below). Often one considers models in which at tree level all $\widetilde{m}_a$ have a common phase and

$$
\xi_X = A\, h_X \;(X = U, D, E), \tag{5.5}
$$

where A is some complex mass parameter. Then apart from the KM phase δ and the QCD parameter θ, there are two more CP–violating phases [80,81], namely those of A and B, which can be expressed in terms of $\phi_A = \arg(A\widetilde{m}_a^*)$ and $\phi_B = \arg(B\widetilde{m}_a^*)$ without a specific phase convention [80]. However, in general the phases of ξ_U, ξ_D and ξ_E are not related to each other, nor are those of $\widetilde{m}_a$.

Let us now come to the electron EDM. In the model specified above, it is generated by the one–loop neutralino and chargino exchanges depicted in Figs. 4 and 5. (More precisely, one should consider neutralino and chargino mass eigenstates, respectively, rather than treating gaugino mixing to first order as indicated in Figs.4 and 5.) As too many unknown mass and mixing parameters are involved, the general expression for d_e resulting from these diagrams is not very illuminating. In order to assess the typical order of magnitude of a SUSY contribution to d_e, we restrict ourselves to the photino exchange contributions in Figs. 4a and 4b. (This may be justified by assuming that the photino is the lightest supersymmetric particle.) As $h_E = h_i \delta_{ij}$ and $\xi_E = \xi_i \delta_{ij}$, the leptonic terms in Eqs.(5.2)-(5.4) are flavor diagonal. In particular, L_0 in (5.2) contains the photino-electron-selectron coupling (in the convention of Ref.78)

$$
L_{\widetilde{\gamma}} = \sqrt{2}\, e\, \bar{\widetilde{\gamma}}\, (e_L \widetilde{e}_L^* - e_R \widetilde{e}_R^*) + h.c., \tag{5.6}
$$

where $e > 0$ is the positron charge, $\widetilde{\gamma}$ is the four-component Majorana spinor field $\widetilde{\gamma} = (-i\lambda_\gamma, i\bar{\lambda}_\gamma)$ of the photino and $e_{L,R} = \frac{1}{2}(1 \mp \gamma_5)e$. In the ground state of the model, where the Higgs fields H_1 and H_2 acquire VEVs v_1 and v_2, respectively, the terms in (5.3) and (5.4) yield the following selectron mass matrix;

$$L_{M\tilde{e}} = -(\tilde{e}_L^*, \tilde{e}_R^*) \begin{pmatrix} \mu_L^2 + m_e^2 & A_e^* m_e \\ A_e m_e & \mu_R^2 + m_e^2 \end{pmatrix} \begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix}, \tag{5.7}$$

where we have defined

$$\begin{aligned} A_e m_e &= \xi_e v_2 + \mu^* h_e v_1 \\ &= m_e \left(\xi_e / h_e + \mu^* v_1 / v_2\right) \end{aligned} \tag{5.8}$$

by use of $m_e = h_e v_2$. In the following we shall put

$$A_e = |A_e| \, exp(i\varphi_A). \tag{5.9}$$

The mass parameter $|A_e|, \mu_L, \mu_R$ – and others appearing in $L_{\mathbf{soft}}$ – are expected to be of the order of the W–mass [F4]. We may transform $\tilde{e}_L$ and $\tilde{e}_R$ to mass eigenstates $\tilde{e}_1$ and $\tilde{e}_2$:

$$\begin{aligned} \tilde{e}_L &= exp(-\frac{1}{2}i\varphi_A)(c_\theta \tilde{e}_1 + s_\theta \tilde{e}_2) \\ \tilde{e}_R &= exp(\frac{1}{2}i\varphi_A)(c_\theta \tilde{e}_2 - s_\theta \tilde{e}_1). \end{aligned} \tag{5.10}$$

Then the mass matrix in Eq.(5.7) has the eigenvalues

$$M_{1,2}^2 = 1/2 \left\{ \mu_L^2 + \mu_R^2 + 2m_e^2 \mp \left[\left(\mu_L^2 - \mu_R^2 \right)^2 + 4m_e^2 |A_e|^2 \right]^{1/2} \right\} \tag{5.11}$$

and the mixing angle θ is given by

$$\tan 2\theta = 2|A_e| m_e / \left(\mu_L^2 - \mu_R^2 \right). \tag{5.12}$$

The photino mass term, resulting from (5.2), is in two–component notation:

$$L_M = -\frac{1}{2} \tilde{m}_\gamma \lambda_{\tilde{\gamma}} \lambda_{\tilde{\gamma}} + \; h.c. \tag{5.13}$$

The Majorana mass $m_{\tilde{\gamma}}$ is in general complex:

$$\tilde{m}_\gamma = M_{\tilde{\gamma}} \, exp\left(i\varphi_{\tilde{\gamma}} \right) \tag{5.14}$$

where $M_{\tilde{\gamma}} > 0$. In the basis of mass eigenstates with real mass eigenvalues, the photino interaction reads:

$$L = \sqrt{2} e \sum_{a=1,2} \bar{\tilde{\gamma}} (e_L \Gamma_{La} + e_R \Gamma_{Ra}) \tilde{e}_a^* \; + h.c., \tag{5.15}$$

where

$$\begin{aligned} \Gamma_{La} &= \exp\left(\frac{1}{2} i \left(\varphi_A - \varphi_{\tilde{\gamma}} \right) \right) (c_\theta, s_\theta), \\ \Gamma_{Ra} &= \exp\left(-\frac{1}{2} i \left(\varphi_A - \varphi_{\tilde{\gamma}} \right) \right) (s_\theta, -c_\theta). \end{aligned} \tag{5.16}$$

The EDM d_e generated by the interaction (5.15) arises from the diagram Fig. 13a of the Appendix. Using (5.15) and (A.5) we obtain

$$\begin{aligned} d_e &= -e(\alpha/2\pi)M_{\tilde{\gamma}} \sum_{a=1,2} \left[\mathrm{Im}\,(\Gamma_{La}\Gamma^*_{Ra})/M_a^2\right] I_3(r_a,0) \\ &= -e(\alpha/2\pi)M_{\tilde{\gamma}} c_\theta s_\theta \sin\left(\varphi_A - \varphi_{\tilde{\gamma}}\right) \left[I_3(r_1,0)/M_1^2 - I_3(r_2,0)/M_2^2\right], \end{aligned} \tag{5.17}$$

where $r_a = (M_{\tilde{\gamma}}/M_a)^2$. As mentioned above, we expect $\mu_L \approx \mu_R \approx |A_e| = O(M_W)$. Then $m_e|A_e|/M_{1,2}^2 \ll 1$, so we can expand Eq.(5.17) to first order in this quantity. For simplicity, we set $\mu_L = \mu_R = \mu$ and therefore $M_{1,2}^2 = \mu^2 \mp 2m_e|A_e|$ and $c_\theta = s_\theta = 1/\sqrt{2}$. In this case, we obtain d_e to first order in $m_e|A_e|/M_{1,2}^2$:

$$d_e = -e(\alpha/24\pi)(m_e|A_e|/M_{\tilde{\gamma}}^3)\sin(\varphi_A - \varphi_{\tilde{\gamma}}) f\left(M_1^2/M_{\tilde{\gamma}}^2\right), \tag{5.18}$$

where

$$f(x) = \frac{12}{(x-1)^2}\left(1/2 + \frac{3}{x-1} - \frac{2x+1}{(x-1)^2}\ln\, x\right). \tag{5.19}$$

The function $f(x)$ is smooth across $x = 1$ where $f(x) = 1$. Formula (5.18) corresponds to Figs.4a and 4b.

Estimating d_e numerically is not straightforward because no completely model-independent experimental information is available on $M_{\tilde{e}}$ and $M_{\tilde{\gamma}}$. Experimental analyses usually assume that $\tilde{\gamma}$ is the lightest stable SUSY particle. With this proviso the tighest limits on $M_{\tilde{e}}$ and $M_{\tilde{\gamma}}$ to date were recently obtained by experiments at LEP [82]. For instance, for the mass-degenerate case $M_{\tilde{e}1} = M_{\tilde{e}2}$ the ALEPH and OPAL experiments exclude $M_{\tilde{e}} <$ 43 GeV for photino masses up to 35 GeV and 30 GeV, respectively, with 95% CL. On the other hand, it is appealing to postulate $|A_e| \approx M_{\tilde{e}} \approx M_{\tilde{\gamma}} = O(M_{W,Z})$ from the viewpoint of "naturalness". With this postulate, Eq.(5.18) becomes

$$d_e \simeq 1.0 \times 10^{-25} \times (M_{\tilde{\gamma}}/100\text{ GeV})^{-3}(|A_e|/100\text{ GeV})\sin(\varphi_A - \varphi_{\tilde{\gamma}})\; e\; cm. \tag{5.20}$$

For comparison we estimate the SUSY contribution to the EDM of the neutron. First we consider the valence quark contribution to d_n. Among the various contributions to the EDM d_q of a quark, gluino contributions are expected to be the most important ones as gluinos couple with the strong interaction coupling constant. (See Figs. 4a and 4b with $\tilde{\gamma} \to \tilde{g}$, $e \to u, d$, and $\tilde{e} \to \tilde{u}, \tilde{d}$.) Neglecting generation mixing and denoting the parameters of left-right squark mixing $\tilde{u}_L \leftrightarrow \tilde{u}_R$ and $\tilde{d}_L \leftrightarrow \tilde{d}_R$ by $A_u m_u$ and $A_d m_d$, respectively in analogy to Eq.(5.7), we can compute d_u and d_d in analogy to d_e. In the nonrelativistic valence approximation $d_n = 4d_d/3 - d_u/3$, we obtain

$$\begin{aligned} d_n = &- e(2\alpha_s/81\pi)(m_d|A_d|/M_{\tilde{g}}^3)\sin(\varphi_{Ad} - \varphi_{\tilde{g}}) f(M_{\tilde{d}}^2/M_{\tilde{g}}^2) \\ &- e(\alpha_s/81\pi)(m_u|A_u|/M_{\tilde{g}}^3)\sin(\varphi_{Au} - \varphi_{\tilde{g}}) f(M_{\tilde{u}}^2/M_{\tilde{g}}^2). \end{aligned} \tag{5.21}$$

Although the recently published experimental lower bounds on $M_{\tilde{q}}$ and $M_{\tilde{g}}$ are still model dependent [83], the region $M_{\tilde{q}}, M_{\tilde{g}} < 75$ GeV seems to be excluded on fairly mild assumptions. For an estimate we substitute $m_u = 5$ MeV, $m_d = 10$ MeV, $\alpha_s = 0.1, |A_u| = |A_d|, \varphi_{Au} = \varphi_{Ad}$, and $M_{\tilde{u}} = M_{\tilde{d}} = M_{\tilde{g}}$. Then

$$d_n = -2 \times 10^{-23}(M_{\tilde{g}}/100 \text{ GeV})^{-3}(A_d/100 \text{ GeV})\sin(\varphi_{Ad} - \varphi_{\tilde{g}})\ e\ cm. \qquad (5.22)$$

Long-distance strong interaction effects tend to enhance this valence-quark estimate [14]. Comparison with $|d_n|_{\exp} < 1.2 \times 10^{-25}$ $e\ cm$ suggests that, barring accidental cancellations between two terms in Eq.(5.21), the SUSY phases φ_{Aq} and $\varphi_{\tilde{g}}$, more precisely the SUSY phase difference $\varphi_{Aq} - \varphi_{\tilde{g}}$, must be very small, or the masses of SUSY particles must be much larger than 100 GeV, or the squark mixing parameters $|A_q| \ll 100$ GeV. However, choosing $M_{\tilde{g}}$ to be much larger than 1 TeV or choosing $|A_q| \ll 100$ GeV runs against the naturalness of the SUSY-SM. If $M_{\tilde{q}} \simeq |A_q| \simeq 100$ GeV and $\sin(\varphi_{Aq} - \varphi_{\tilde{\gamma},\tilde{Z}})$=O(1), then the photino and zino contributions to d_n already contradict with its experimental upper limit. If the phase difference $\varphi_A - \varphi_{\tilde{\gamma}}$ in the lepton sector is comparable to those in the quark-gluon sector and if all SUSY particles have roughtly the same masses, Eqs.(5.20) and (5.21) imply

$$d_e \simeq 10^{-2} d_n. \qquad (5.23)$$

Substituting the experimental upper bound on $|d_n|$, we find from Eq.(5.23) $|d_e| < 10^{-27}$ $e\ cm$. Note however that Eq.(5.23) involves many assumptions. For instance, if it happens that the gluinos are substantially heavier than the photino, the ratio $|d_e/d_n|$ would be much closer to unity.

The present experimental upper bound on $|d_n|$ – and to a lesser degree that on $|d_e|$ – indicates that the SUSY phases times the sfermion mixing parameters A may be quite small. Although no compelling reason exists why this should be the case in general, it appears that some mechanism ought to operate to suppress these SUSY phases in viable SUSY models of electroweak interactions.

Finally a remark about the effect of generation mixing on EDMs: Suppose that all intrinsic SUSY phases, in particular those of the left–right sfermion mixing terms A, were zero but the fermion and sfermion mass matrices are complex. Because the quark and squark mass matrices are diagonalized in general by different sets of unitary rotation matrices, complex flavor–nondiagonal quark–squark–gluino (photino or zino) couplings arise in the mass eigenbasis. These couplings lead to quark EDMs at two–loop order [69]. With very generous assumptions about the strength of the flavor–changing gluino couplings, Ref.[69] estimates the resulting contribution to the neutron EDM to be less than 8×10^{-29} $e\ cm$.

If the neutrinos are massive Dirac particles then there can also be CP–violating flavor–nondiagonal lepton–slepton–photino (zino) couplings which generate a contribution to d_e in two–loop order. However, we expect it to be of little relevance because we have for the corresponding EDM contributions $d_e/d_n \propto (\alpha/\alpha_s)^2$.

As to the charged currents which couple to the W bosons and its SUSY partners: Ref.[68] showed that this contribution to quark and lepton EDMs vanishes to two-loop order – as in the SM. Hence KM-type contributions are expected to be as small as those estimated in Section 3.

6. Left–right symmetric models

Left–right symmetric models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ [84-90]. They are invariant under parity reflection before spontaneous symmetry breaking. In the minimal version [89] the large vacuum expectation values (VEVs) of two Higgs multiplets break the gauge symmetry. A triplet Higgs χ_R transforming like (1,3) under $SU(2)_L \times SU(2)_R$ is assumed to develop a large VEV to break parity symmetry at the scale of 1 TeV or above, generating masses of the right weak bosons which are much larger than the electroweak scale. The VEVs of a complex multiplet ϕ transforming like (2,2) contribute to the masses of both left and right weak bosons and cause mixing between them. The VEVs of ϕ are also responsible for the masses of quarks and leptons. Because of parity symmetry, models contain right–handed neutral leptons, i.e., right–handed neutrinos as parity partners of left–handed neutrinos. The right–handed neutral leptons acquire large Majorana masses from the VEV of χ_R and mix with left-handed neutrinos through Dirac masses which are generated by the VEVs of ϕ. The VEV of a left–handed triplet $\chi_L \sim$ (3,1) must be very small, if nonzero, in order to keep the left–handed neutrinos light. We will ignore the VEV of χ_L in the following.

In left–right symmetric models, CP violation may exist in the Higgs couplings even before spontaneous symmetry breaking or may arise spontaneously, *i.e.*, from the phases of the complex VEVs of χ_R and ϕ upon symmetry breaking. CP violation manifests itself in particular through phases of the complex $W_L - W_R$ transition mass term and of the complex Dirac masses of neutral leptons. Not all of these phases are physical, however (see below). The electron EDM arises to one–loop order from mixing between the left and right weak bosons and from complex neutral lepton masses (see Fig.6) [91–93]. For contributions to d_e from Higgs exchange, see *e.g.* [92]. In order to generate an EDM of the magnitude which is of interest for experiments in the near future, one needs a sizable Dirac mass connecting the left–handed electron neutrino ν_{eL} and a right–handed heavy neutrino N_R. Such a large Dirac mass term can be accommodated only if N_R has a large Majorana mass and the mass eigenvalue of the light neutrino is suppressed by the seesaw mechanism [89,94,95].

Let us parametrize the relevant interactions in the minimal $SU(2)_L \times SU(2)_R \times U(1)$ model. The charged weak current interaction of the leptons is given by

$$L_I = -(g/\sqrt{2}) \sum_i \left(\bar{\ell}_{iL} \gamma^\mu \nu_{iL} W^-_{L\mu} + \bar{\ell}_{iR} \gamma^\mu N_{iR} W^-_{R\mu} \right) + h.c., \tag{6.1}$$

where the summation is over the lepton families and the charged lepton ℓ_i have been chosen to be mass eigenstates. Since W_L and W_R mix with each other through the mass matrix

$$\left(W_L^+,\ W_R^+\right)\begin{pmatrix} M_L^2 & \Delta \\ \Delta^* & M_R^2 \end{pmatrix}\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix}, \tag{6.2}$$

the two mass eigenstates W_1 and W_2 are related to W_L and W_R by a unitary matrix U,

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = U \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}, \tag{6.3}$$

where we require for the eigenvalues of (6.2); $M_1 \simeq M_W < M_2$. The off-diagonal element Δ of the mass matrix is complex in general. However, Δ can always be chosen to be real by a suitable redefinition of the relative phases of the W_L and W_R fields. We will adopt this phase convention for the W fields in the following. Then the unitary matrix U is actually an orthogonal matrix,

$$U = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ sin\zeta & \cos\zeta \end{pmatrix} \tag{6.4}$$

The neutrino mass matrix is represented by [89]

$$(\bar{\nu}^c\ ,\bar{N})_R \begin{pmatrix} \mu_\nu & \mu_D \\ \mu_D^T & \mu_N \end{pmatrix}\begin{pmatrix} \nu \\ N^c \end{pmatrix}_L + h.c., \tag{6.5}$$

where both ν and N carry a generation index $i = 1 \cdots n$. This $2n \times 2n$ mass matrix is a complex symmetric matrix. The phases of its elements generate CP violation. Note that a genuine CP violating phase exist even in the case of a single generation because the Dirac mass μ_D can be complex. (More precisely, there are two independent phases: That of μ_D and, conventionally, that of μ_ν.) By diagonalizing the neutral lepton mass matrix by a $2n \times 2n$ unitary matrix V

$$\begin{pmatrix} \nu \\ N^c \end{pmatrix}_L = V\psi_L,$$
$$(\bar{\nu}^c,\ \bar{N})_R = \bar{\psi}_R V^T, \tag{6.6}$$

with $V = \begin{pmatrix} V_L \\ V_R^* \end{pmatrix}$, or explicitly

$$\begin{aligned} \nu_{iL} &= \sum_{j=1}^{2n} V_{Lij}\psi_{jL} \\ N_{iL}^c &= \sum_{j=1}^{2n} V_{Rij}^*\psi_{jL}, \end{aligned} \qquad (i = 1 \cdots n) \tag{6.7}$$

one obtains the charged current interaction in terms of the mass eigenstates

$$L = -(g/\sqrt{2})\sum_{i=1}^{n}\sum_{j=1}^{2n}\sum_{a=1,2} W_a^\mu(U_{La}V_{Lij}\bar{\ell}_{iL}\gamma_\mu\psi_{jL} + U_{Ra}V_{Rij}\bar{\ell}_{iR}\gamma_\mu\psi_{jR}) + h.c. \tag{6.8}$$

Comparison of Eq.(6.8) with the standard form in Eq.(A.1) of the Appendix gives us

$$G^a_{Lij} = -(g/\sqrt{2})U_{La}V_{Lij},$$
$$G^a_{Rij} = -(g/\sqrt{2})U_{Ra}V_{Rij}. \tag{6.9}$$

With $Q_i = -e$ and $Q_j = 0$, we obtain from formula (A.4):

$$d_e = \frac{eM_W^2 G_F}{4\sqrt{2}\pi^2} \sum_{a=1,2} (U_{La}U_{Ra}/M_a^2) \sum_j m_j \,\mathrm{Im}(V_{L1j}V^*_{R1j}) I_1(m_j^2/M_a^2, m_e^2/M_a^2). \tag{6.10}$$

The parameters appearing in Eq.(6.10) are constrained by data from low-energy weak interaction experiments (see Table 2) [97-99]. The constraints imposed by nonleptonic processes are based on the assumption that the quark mixing matrices are identical for the left and right sectors. On the other hand, semileptonic and leptonic decays can constrain the parameters without such assumptions. If the right-handed neutral leptons are too heavy to be produced in known weak decays, one can set a stringent limit on $|\zeta|$ because these leptons would nevertheless cause a departure from universality. This limit is

$$|\zeta| \lesssim 0.004. \tag{6.11}$$

Lower limits on the mass of W_2 have been derived under various assumptions [90]. Unless one requires a high numerical precision, it is safe to assume $(M_1/M_2)^2 \ll 1$ and to ignore the W_2 exchange processes compared with the $W_1(\simeq W_L)$ exchange processes.

The simplest case of a single lepton generation – or of many generations with negligible generation mixing – deserves a detailed study since it illuminates quantitative implications of the formula (6.10). In this case, we may keep in the sum over j only the heavy neutral lepton of the first generation. In order to keep the electron neutrino light, we must exploit the seesaw mechanism. With $\mu_\nu = 0$ and $|\mu_D| \ll |\mu_N|$ in Eq.(6.5), the electron neutrino mass is given by

$$m_{\nu_e} \simeq |\mu_D^2/\mu_N| \simeq |\mu_D^2|/m_{Ne}, \tag{6.12}$$

where m_{Ne} is the mass of the heavy right-handed neutrino. The lepton mixing is then

$$V_{Lij}V^*_{Rij} \simeq (\mu_D/m_{Ne})\delta_{ij}. \tag{6.13}$$

By substituting Eq.(6.12) and $U_{L1}U_{R1} = \frac{1}{2}\sin 2\zeta$ in Eq.(6.10), one obtains

$$d_e = \frac{e\,G_F}{8\sqrt{2}\pi^2} I_1(m_{Ne}^2/M_W^2, 0) \sin 2\zeta \,\mathrm{Im}\,\mu_D. \tag{6.14}$$

Numerically

$$d_e = 2.1 \times 10^{-24} I_1(m_{Ne}^2/M_W^2, 0) \sin 2\zeta \; (\mathrm{Im}\,\mu_D/1 \text{ MeV}) \; e\ cm. \tag{6.15}$$

The integral $I_1(x,0)$ takes values from 2 to 1/2 as x varies from 0 to ∞. With the current experimental upper limit $|\sin 2\zeta| < 0.008$ from Eq.(6.11), Eq.(6.14) gives

$$|d_e| < \begin{cases} 8.2\times 10^{-27}(\mathrm{Im}\,\mu_D/1MeV) \quad e \quad cm \quad \text{for} \quad (m_{Ne}/M_W)^2 \gg 1, \\ 3.3\times 10^{-26}(\mathrm{Im}\,\mu_D/1MeV) \quad e \quad cm \quad \text{for} \quad (m_{Ne}/M_W)^2 \ll 1. \end{cases} \tag{6.16}$$

It is often speculated the μ_D should be comparable to a charged lepton mass, namely the electron mass in our case. From tritium beta decay, we have the upper limit $m_{\nu_e} < 18eV$[$F5$]. For m_{Ne}, a theoretical argument, namely vacuum stability against the N_R loop correction to the Higgs potential requires that μ_N should be less than or at most of the order of 1 TeV [102,103]. Combining these bounds, Eq.(6.12) implies that $|\mu_D| \lesssim 4$ MeV, which is consistent with the speculation. Therefore μ_D= O(1 MEV) seems to be reasonable.

In some simple versions of left–right models, we can relate $|d_e|$ to the ϵ' parameter of the $K \to 2\pi$ decay and to d_n. Let us consider for example a model with no explicit CP violation in which CP violation is spontaneously broken by the VEVs of the Higgs fields. Such a model is often referred to as a pseudo–manifest left–right symmetric model [90]. For simplicity, we assume that mixing of the first and the second quark generation to the third generation can be ignored. Furthermore we do not take into account possible generation mixing in the lepton sector. In this model the ϵ' parameter arises entirely from $W_L - W_R$ mixing. Therefore a nonzero value of ϵ' would imply a lower bound on the mixing parameter ζ [14] which in turn would yield, through Eq.(6.14), a lower bound on $|d_e|$. Unfortunately, present data are inconclusive on whether ϵ' is nonzero or not. Whereas the NA31 experiment at CERN obtained [104]

$$\epsilon'/\epsilon = (3.3 \pm 1.1) \times 10^{-3}, \tag{6.17}$$

the E731 experiment at Fermilab recently announced [105]

$$\epsilon'/\epsilon = -(0.4 \pm 1.4 \pm 0.6) \times 10^{-3}. \tag{6.18}$$

In the model specified above one may also relate d_e and d_n. If d_n is computed in the valence quark approximation [91,96] one gets

$$d_e/d_n = \frac{9 \text{ Im } \mu_D}{40 \sin\theta_L \sin\theta_R m_c \sin(\gamma+\delta_1)} I_1(m_{Ne}^2/M_W^2, 0), \tag{6.19}$$

where γ and δ_1 are CP–violating phases from the quark mixing matrices [12,90], $\theta_{L,R}$ are the Cabibbo angles for the left and right–handed quarks, respectively, and only the c quark intermediate state has been retained in obtaining Eq.(6.19). Although the sources of CP violation are common in the quark and lepton sectors, the relation between $\mathrm{Im}\,\mu_D$ and the angles (γ, δ_1) is nontrivial because of the difference in the lepton and quark mass matrices. Therefore, the CP–violating phases do not cancel out in the ratio in Eq.(6.19). With $|\sin(\gamma+\delta_1)| < 1$ and $\theta_L = \theta_R = \theta_C$, Eq.(6.19) implies

$$|d_e/d_n| > \begin{cases} 1.5\times 10^{-3}|\mathrm{Im}\,\mu_D/1MeV| \quad \text{for} \quad m_{Ne}^2 \gg M_W^2, \\ 6\times 10^{-3}|\mathrm{Im}\,\mu_D/1MeV| \quad \text{for} \quad m_{Ne}^2 \ll M_W^2. \end{cases} \tag{6.20}$$

In more general models of left–right symmetry, there is no simple relation between d_e and d_n nor a reliable bound on $|\zeta|$ imposed by ϵ' that leads to a lower bound on $|d_e|$.

When mixing between different lepton generations is included, the numerical analysis is complicated. However, if the squares of all intermediate lepton masses m_j^2 are either much larger or much smaller than M_W^2, the formula for d_e simplifies thanks to the relation [93]

$$\sum_j m_j V_{L1j} V^*_{R1j} = (\mu_D)_{11} \tag{6.21}$$

and Eqs.(6.14)–(6.16) remain valid. On the other hand, if only a single term other than the first generation dominates in the summation j over generations in Eq.(6.10), it is likely that the $\mu \to e\gamma$ decay is induced by a flavor–changing counterpart of the diagrams generating d_e. According to the argument in Section 8.1, $|d_e|$ is naturally bounded by 2.8×10^{-26} $e\ cm$ in this case. If one adopts the hypothesis that $|V_{ij}| \simeq (m_i/m_j)^{1/2}$ for $m_i \ll m_j$, this upper bound is lowered to 2.5×10^{-27} $e\ cm$.

One can extend left–right symmetric models by incorporating more exotic fermions. Then a large electron EDM can be generated by processes other than W exchange. One model which was recently proposed [106] contains charged leptons $E_{L,R}$ which are singlets of $SU(2)_L \times SU(2)_R$. They couple to the light leptons through Higgs doublets ϕ_L and ϕ_R which transform as (2,1) and (1,2) respectively. Upon symmetry breaking, ϕ_L and ϕ_R mix with each other and the electron EDM is generated by the diagram shown in Fig. 7. CP violation arises from the Yukawa couplings and the mass matrices. The electron EDM d_e is given by

$$d_e = \sum_{a=1,2} \frac{e}{16\pi^2 M_a^2} \sum_j m_j \,\mathrm{Im}\left(\Gamma^a_{Lej}\Gamma^{a*}_{Rej}\right) I_4\left(m_j^2/M_a^2, 0\right) \tag{6.22}$$

in the notation of the Appendix, where the summation j is over the exotic singlet charged leptons. When the $\phi_L - \phi_R$ mixing is small, the two terms in Eq.(6.22) tend to cancel each other. It was suggested [106] that if the W_R mass is about 1 TeV, then $m_j \simeq 10$ TeV and $|\Gamma^a_{Lej}\Gamma^{a*}_{Rej}| \simeq 4 \times 10^{-5}$. With these parameter values Eq.(6.22) gives $d_e = O(10^{-27}\ e\ cm)$. We mention this model as an illustration that within the basic idea of left–right symmetry nonminimal models can be built which produce an electron EDM larger than the prediction of Eq.(6.14).

To summarize, in left–right symmetric models of CP violation the electron EDM can be naturally in the range of the order 10^{-27} to 10^{-28} $e\ cm$. As is the case in most models of CP violation, d_e tends to be smaller than d_n because the mass scale responsible for the electron chirality flip is generally smaller than the mass term which causes the quark chirality flip.

7. Higgs models

Higgs models of CP violation are motivated by the idea [107] of linking the origin of CP nonconservation to the mechanism which is also responsible for the absence of $SU(2)_L \times U(1)$

gauge symmetry in the spectrum of states, *i.e.*, spontaneous symmetry breaking (SSB). One considers gauge theory models with several Higgs multiplets whose Lagrangians are CP invariant before SSB. (P and C invariances are, however, explicitly broken.) The ground state of such a model is assumed to break CP invariance. This is parametrized by vacuum expectation values (VEVs) of Higgs fields which are complex relative to each other. The complex VEVs lead, after the diagonalization of the fermion mass matrices, to CP–violating Yukawa couplings of Higgs particles to fermions. W boson exchange may be an additional source of CP violation, depending on the models under consideration. The simplest models of spontaneous CP violation (SCPV) are extensions of the $SU(2)_L \times U(1)$ standard model by two or more Higgs doublets of $SU(2)_L$. A major concern in the construction of these models is flavor–changing neutral currents induced by neutral Higgs boson exchanges. Natural flavor conservation (NFC) is usually enforced in these models by imposing a set of discrete symmetries on their Lagrangians. Then at least three Higgs doublets are needed in order to have SCPV [108].

7.1 Lee model

The two–Higgs doublet model of Lee [107], which is the simplest model of SCPV, has flavor–changing neutral Higgs exchanges. Two possibilities were discussed in the literature in order to bring this model in accord with experimental constraints on NFC: *(i)* to assume large Higgs masses of order 10 TeV [109–111], or *(ii)* to assume that the Yukawa couplings which lead to the violation of NFC are very small [112]. Neither aproach is unproblematic theoretically. A source of leptonic CP violation in the Lee model can arise from flavor–conserving neutral Higgs couplings. Such couplings will be discussed in the context of the Weinberg model in Section 7.2. Moreover, there are also CP–violating and lepton–flavor–changing neutral Higgs couplings. The effect of such couplings to the electron EDM will be studied in a general framework in Section 8.

7.2 Weinberg model [108]

This model contains three Higgs doublets Φ_i which allow for NFC and spontaneous CP violation simultaneously. Several coupling schemes of the doublets Φ_i to the right–handed fermion fields are possible. Let us mention of only two possibilities here:

$$\Phi_1 \leftrightarrow U_R, \;\; \Phi_2 \leftrightarrow D_R, \;\; \Phi_3 \leftrightarrow N_R, E_R \tag{7.1a}$$

$$\Phi_1 \leftrightarrow U_R, \;\; \Phi_2 \leftrightarrow D_R, E_R, \;\; \Phi_3 \leftrightarrow N_R, \tag{7.1b}$$

where $U = (u, c, t)$, $D = (d, s, b)$, $E = (e, \mu, \tau)$ and $N = (\nu_e, \nu_\mu, \nu_\tau)$. We assume for definiteness that the neutrinos are massive Dirac particles. The coupling schemes (7.1a) and (7.1b) are enforced by imposing an appropriate set of discrete symmetries. Before SSB the Lagrangian of the model is CP–invariant. For the number of generations $n_G = 3$, it turns out that NFC leads to a a real quark mixing matrix [113]. CP violation arises then from neutral and charged Higgs boson exchange only. (However, this need not be true for $n_G \geq 4$ [114].) The model contains four charged and five neutral physical Higgs bosons, $H^{\pm}_{1,2}$ and ϕ_i, respectively. In the basis where all fermion and scalar boson mass matrices are diagonal, the Yukawa interactions of the physical Higgs bosons are given by [115–117]

$$L_H = -\left(2\sqrt{2}G_F\right)^{1/2} \sum_{i=1,2} \Big\{ \bar{U}[\alpha_i V M_D P_R + \beta_i M_U V P_L] D\, H_i^+ + \bar{N}[\gamma_i V_\ell M_E P_R + \delta_i M_N V_\ell P_L] E\, H_i^+ \Big\} + h.c. \tag{7.2}$$

and

$$\begin{aligned} L_\phi = -\left(\sqrt{2}G_F\right)^{1/2} \sum_{j=1}^{5} \Big(& \xi_{Uj} \bar{U} M_U U + i\tilde{\xi}_{Uj} \bar{U}\gamma_5 M_U U \\ & + \xi_{Dj} \bar{D} M_D D + i\tilde{\xi}_{Dj} \bar{D}\gamma_5 M_D D \\ & + \xi_{Ej} \bar{E} M_E E + i\tilde{\xi}_{Ej} \bar{E}\gamma_5 M_E E \\ & + \xi_{Nj} \bar{N} M_N N + i\tilde{\xi}_{Nj} \bar{N}\gamma_5 M_N N \Big) \phi_j . \end{aligned} \tag{7.3}$$

In Eqs. (7.2) and (7.3), M_U, M_D, M_E and M_N are diagonal quark and lepton mass matrices, respectively, and V denotes the real orthogonal KM mixing matrix and V_ℓ is its leptonic analogue. The parameters $\alpha, \beta, \gamma, \delta$ and the real parameters $\xi, \tilde{\xi}$ depend on the magnitudes and phases of the three VEVs $< 0|\Phi_i|0 >$, on the parameters of the Higgs potential and on the coupling scheme. For instance, in the case of (7.1a) one obtains $\gamma_i = \delta_i$, whereas in the case of (7.1b) one gets $\gamma_i = \beta_i$.

CP violation generated by charged Higgs exchanges to one–loop is characterized by the parameters $\mathrm{Im}(\alpha_i\beta_i^*)$ in the quark sector and by $\mathrm{Im}(\gamma_i\delta_i^*)$ in the lepton sector. The relations $\mathrm{Im}(\alpha_1\beta_1^*) = -\mathrm{Im}(\alpha_2\beta_2^*)$ and $\mathrm{Im}(\gamma_1\delta_1^*) = -\mathrm{Im}(\gamma_2\delta_2^*)$ hold [111,116,118]. (Note that $\mathrm{Im}(\gamma_i\delta_i^*)$ = 0 in the coupling scheme (7.1a).) The neutral Higgs particles ϕ_i can generate P– and CP–violating interactions as they couple both to CP–even scalar and CP–odd pseudoscalar densities. The mass eigenstates ϕ_i are realized by the mixing of CP–even and CP–odd states [115].

In the Weinberg model, strangeness changing $|\Delta S| = 1$ and $|\Delta S| = 2$ charged Higgs exchange amplitudes at one–loop must account for the observed CP nonconservation in the K_L decays; *i.e.*, for the parameter ϵ. Moreover, these amplitudes must account for the fact that $|\epsilon'/\epsilon| \ll 1$ if nonzero at all (cf.(6.17) and (6.18)). Several investigations [111,118–120] indicate that this is possible in a semiquantitative way, although predictions are that $|\epsilon'/\epsilon| >$ a few $\times 10^{-3}$ which is barely compatible with the data (6.17) and (6.18). Fitting ϵ to its experimental value requires a relatively light charged Higgs particle, say H_1, with sizable coupling $\mathrm{Im}(\alpha_1\beta_1^*)$. However, recent searches at LEP [165] for charged scalars exclude charged Higgs particles with mass below 43 GeV. (The precise limits depend on the decay modes of the Higgs particles being investigated.) Then if $m_{H1} > 45$ GeV one obtains from the analysis of [118]

$$\mathrm{Im}(\alpha_1 \beta_1^*) > 9, \tag{7.4}$$

which is uncomfortably large.

As for EDMs, let us examine that of the neutron first. The charged Higgs interaction (7.2) generates EDMs of quarks to one–loop order. The resulting EDM estimated in the nonrelativistic valence quark approximation; $d_n = O(10^{-25}\ e\ cm)$ [118,121], is dangerously close to the present experimental upper limit $1.2 \times 10^{-25}\ e\ cm$. The estimate in Ref. [14], which takes into account long–distance strong interaction effects on d_n, is even larger: $d = O(10^{-24}\ e\ cm)$. Equally important are contributions from the neutral bosons ϕ_i. Naively one expects their contributions to d_n to be nonhazardous because they generate EDMs of light quarks q which vanish like $d_q \sim \xi_{qi}\tilde{\xi}_{qi} G_F m_q^3/m_{\phi i}^2$ as $m_q \to 0$ (see below). However, it was pointed out [122] that the correct estimate of the low–energy ϕ_i–nucleon couplings yields couplings proportional to the nucleon mass. Hence these couplings are considerably enhanced with respect to the quark couplings and do not vanish in the chiral limit m_u and $m_d \to 0$. Assuming that one of the neutral Higgs particle, say ϕ_1, dominates the contribution to d_n and choosing $\xi_{q1}\tilde{\xi}_{q1} = O(1)$ one obtains, following [118,122,123], approximately

$$|d_n| \simeq 10^{-25} \times (100\ GeV/m_{\phi 1})^2\ e\ cm, \tag{7.5}$$

which requires ϕ_1 to be heavier than 100 GeV. (Recent results from LEP [126] imply the lower bound $m_{\phi 1} > 24$ GeV.)

Moreover it has been pointed out [124] that the dimension–six, P– and T–violating effective gluon interaction $O = c\ f_{abc}\ G^{a\mu\rho}\, G^{b\nu}{}_{\rho} \tilde{G}^{c}{}_{\mu\nu}$ (where $G_{\mu\nu}$ is the gluon field strength tensor and $\tilde{G}_{\mu\nu}$ its dual), which is generated in a large class of models of CP violation, can have a sizable effect on the neutron EDM. In Higgs models of CP violation the coefficient c is generated by two–loop diagrams with a top quark in the loop and a neutral Higgs particle with indefinite parity being exchanged (Fig.8a). Specifically in the Weinberg model of CP violation, c can also be generated to two–loop order by charged Higgs exchange [166]: a t quark being converted into a b quark in the loop by emitting a charged Higgs H^+ and being transformed back into a t quark by reabsorbing H^+ (Fig.8b). With the correct anomalous dimension for the operator O [167] which is necessary to scale c to low energies, and with $\xi_{ti}\tilde{\xi}_{ti} \leq O(1)$ the effect of O on d_n for neutral Higgs exchange is expected not to exceed $10^{-25}\ e\ cm$. However, when c is generated by CP–violating $H^\pm$ exchange in the Weinberg model, it is proportional to $\mathrm{Im}(\alpha_i \beta_i^*)$ which is bounded from below by Eq.(7.4). In this case the value for d_n generated by O can potentially be larger than the present experimental upper limit. Precise statements are hampered by the fact that the matrix element of O between neutron states cannot be evaluated reliably.

In view of all these difficulties, especially with CP violation with $H^\pm$ exchange the

Weinberg model hardly seems to be viable any longer. The next round of experiments on d_n and ϵ'/ϵ should decide conclusively over the fate of this model.

Let us now discuss the contributions of charged and neutral Higgs exchanges arising from Eqs.(7.2) and (7.3), respectively, to the electron EDM. The one–loop effects are bound to be very small. Using Eqs.(A.2) and (A.5), we obtain for the contribution of the charged Higgs boson H_1.

$$d_e(H_1) \simeq \frac{e\sqrt{2}G_F\, m_e}{16\pi^2 m_{H_1}^2} \sum_i m_i^2\, \mathrm{Im}(\gamma_1 \delta_1^*) |(V_\ell)_{1i}|^2. \tag{7.6}$$

Since the product $m_i(V_\ell)_{1i}$ is severely bounded by the experimental data on the $\pi \to \mu\nu$ branching ratio [125]

$$|m_i(V_\ell)_{1i}|^2 < 3 \times 10^{-4}\ MeV^2, \tag{7.7}$$

the value of $d_e(H_1)$ computed from Eq.(7.6) cannot be larger than $10^{-36}\ e\ cm$.

The one–loop contribution of a neutral Higgs particle is shown in Fig.14b. Using Eqs.(7.3), (A.2), and (A.5), we obtain for a neutral Higgs boson ϕ_1

$$\begin{aligned} d_e(\phi_1) &= -\frac{e\sqrt{2}G_F\, m_e^3}{8\pi^2 m_{\phi 1}^2}\, \xi_{e1}\tilde{\xi}_{e1} I_4\left(m_e^2/m_{\phi 1}^2, m_e^2/m_{\phi 1}^2\right) \\ &\simeq -\frac{e\sqrt{2}G_F\, m_e^3}{4\pi^2 m_{\phi 1}^2}\xi_{e1}\tilde{\xi}_{e1} \ln(m_{\phi 1}/m_e), \end{aligned} \tag{7.8}$$

where I_4 is defined in Eq.(A.10). With $m_{\phi 1} = 100$ GeV (cf. Eq.(7.5)) and $\xi_{e1}\tilde{\xi}_{e1} \simeq 1$, Eq. (7.8) yields

$$d_e = -4.4 \times 10^{-34}\ e\ cm. \tag{7.9}$$

However, recently it has been observed [164] that the suppression by m_e^3/m_ϕ^2 of the one–loop neutral Higgs contribution is overcome at two–loops. A representative diagram is depicted in Fig.9. The chirality flip necessary for generating d_e is provided by the $\phi_1 ee$ vertex and yields $d_e \propto m_e$. Ref.[164] finds that the amplitude of Fig.9 contributes

$$(d_e)_{t-\text{loop}} = \frac{16\sqrt{2}\ e\alpha}{3(4\pi)^3} G_F m_e\ F\left(m_t^2/m_\phi^2,\ \xi_{ei}\tilde{\xi}_{ti}, \xi_{ti}\tilde{\xi}_{ei}\right), \tag{7.10}$$

where the function F is of order one or larger if $m_t/m_\phi \gtrsim 1$ and $\xi_{e1}\tilde{\xi}_{e1} \approx \xi_{t1}\tilde{\xi}_{t1} = O(1)$. The factor in front of F in Eq.(7.10) is about $3 \times 10^{-27}\ e\ cm$. Besides the t quark W and charged Higgs bosons in the loop are also significant. Ref.[164] finds that the W contribution is about five times larger than (7.10). Moreover Higgs models can induce CP violation in the $W^+W^-\gamma$ vertex to one–loop which in turn induces a two–loop EDM d_f of a fermion f where $d_f \propto m_f$ (cf. Sect.3). This indicates that the Weinberg or other Higgs models of CP nonconservation can yield a substantial electron EDM at the level of the present experimental sensitivity - contrary to naive expectations.

7.3 Hybrid models

The Weinberg model assumes SCPV as the sole origin of CP violation. In view of the difficulties of this model discussed in the previous subsection it is reasonable to consider a more general class of models; *i.e.*, Higgs models having also hard CP violation through the couplings of the scalar fields. (After all, CP violation may not have an aesthetically satisfactory "unique" explanation.) That is, one may consider Higgs models with NFC where CP nonconservation results from W exchange as well as from the charged and neutral Higgs exchanges. For three generations, the KM phase δ provides an extra CP–violating parameter. W exchange, which involves δ, alone, can explain the observed CP violation in K_L decays. Because of the experimental constraints arising from d_n and ϵ'/ϵ, CP violation in charged Higgs particle mixing parametrized by $\mathrm{Im}(\alpha_i\beta_i^*)$ is likely to be small as discussed above. It may be avoided altogether by considering models with just two Higgs doublets Φ_1 and Φ_2 (and any number of singlets) [F7]. CP violating neutral Higgs particles mixing is however not constrained to be small. That is, in these models one still expects sizable EDMs of the neutron and the electron due to neutral Higgs exchange. Effects might be as large as 10^{-25} $e\ cm$ for d_n and 10^{-26} $e\ cm$ for d_e, respectively. This means that measuring d_e at the level of a few $\times 10^{-27}$ $e\ cm$ is a very important and clean test of these models of CP violation. Recall that the calculation of the neutron EDM involves hadronic effects which are at present not under control. As Higgs–fermion couplings grow with the mass of the fermion the EDMs of heavy quarks and leptons may become substantially larger than d_e. However, because the one–loop contributions to d_f of a fermion f are proportional to m_f^3/m_ϕ^2 (cf. Eq.(7.8)) whereas the two–loop contributions discussed above (cf. Eq.(7.10)) are proportional to αm_f there is no simple scaling relation. For the EDMs of the muon and the tau lepton, this manifests itself as follows: For illustration let us assume that the lightest Higgs particle with indefinite parity, say ϕ_1, has a mass of 50 GeV and that $\xi_{f1}\tilde{\xi}_{f1} = O(1)$. Then $d_e \simeq 1 \times 10^{-26}$ $e\ cm$ is generated by the two–loop contribution. For the muon we obtain: $(d_\mu)_{1-\mathrm{loop}} \simeq 2 \times 10^{-26}$ $e\ cm$ and $(d_\mu)_{2-\mathrm{loop}} \simeq 2 \times 10^{-24}$ $e\ cm$. Eventually, for the tau lepton the one–loop contribution is somewhat larger than the two–loop effect, namely; $(d_\tau)_{1-\mathrm{loop}} \simeq 1 \times 10^{-22}$ $e\ cm$, whereas $(d_\tau)_{2-\mathrm{loop}} \simeq 3 \times 10^{-23}$ $e\ cm$.

What about the experimental sensitivity to d_μ and d_τ? A forthcoming experiment [127] aims at improving the measurement of the anomalous magnetic moment of the muon by about a factor of 20. As a byproduct, sensitivity to d_μ will increase by a similar factor [128]. At present one has the 95% CL upper bound $|d_\mu| < 7.3 \times 10^{-19}$ $e\ cm$ [128].

As to the τ lepton: Information on d_τ can be obtained by measuring CP–odd correlations (involving τ momenta and polarizations) in $e^+e^- \to \tau^+\tau^-$ [129]. Because we expect that a large number of $\tau^+\tau^-$ pairs are produced at the Z resonance by the LEP collider, it is sensible to examine another CP–violating form factor of the τ; namely its electric dipole form factor $d_\tau^{(Z)}(q^2)\sigma_{\mu\nu}\gamma_5 q^\nu$, which can be present in the $Z\tau^+\tau^-$ vertex. If 10^7 Z bosons are produced a sensitivity to d_τ $(q^2 = M_Z^2)$ of a few times 10^{-18} $e\ cm$ might be attainable by measuring appropriate CP–odd correlations in $Z \to \tau^+\tau^-$. Although there is no model–independent relation between d_τ and $d_\tau^{(Z)}$, most models of CP–violation predict d_τ and $d_\tau^{(Z)}$ to be of the same order of magnitude. Specifically, the interaction (7.3) generates a form factor $d_\tau^{(Z)}$ whose magnitude is of the order of d_τ given above. Therefore it is unlikely that this interaction can generate a nonzero d_μ, d_τ and/or $d_\tau^{(Z)}$ at the sensitivity level of present experiments or of experiments in the near future.

8. Lepton flavor changing models

Let us now come to interactions which may generate a sizable electron EDM to one–loop order by a generation–changing transition from the electron to some heavy fermion F from a higher generation in the amplitude depicted in Fig. 3. Before surveying specific models it is appropriate to discuss the constraint on $|d_e|$ which, as noted in Ref.[130], arises for such interactions under fairly general assumptions from the experimental upper limit on the branching ratio of the rare decay $\mu \to e\gamma$.

8.1 d_e and $\mu \to e\gamma$. [130]

If the interaction vertex eFB exists, it is likely that the transition μFB also occurs. This means that the decay $\mu \to e\gamma$ is induced by one–loop magnetic and electric transition dipole moments which arise from diagrams analogous to Fig.3 where the incoming electron is replaced by a muon. Note that a nonzero transition EDM does not signal CP violation. If we define in analogy to (1) the amplitude $< e|J_\alpha^{em}|\mu >= \bar{u}_e\Gamma_\alpha u_\mu$ with $\Gamma_\alpha = F_2^{\mu e} i\sigma_{\alpha\beta}q^\beta/(m_\mu + m_e) + F_3^{\mu e}\sigma_{\alpha\beta}\gamma_5 q^\beta/(m_\mu + m_e) + \cdots$, then the branching ratio for $\mu \to e\gamma$ is given by

$$B(\mu \to e\gamma) = \left[24\pi^2/G_F^2 m_\mu^2(m_\mu + m_e)^2\right]\left[|F_2^{\mu e}(0)|^2 + |F_3^{\mu e}(0)|^2\right]^2. \tag{8.1}$$

The experimental bound [131]

$$B(\mu \to e\gamma) < 5 \times 10^{-11} \tag{8.2}$$

implies

$$\left[|F_2^{\mu e}(0)|^2 + F_3^{\mu e}(0)|^2\right]^{1/2}/(m_\mu + m_e) < 3.7 \times 10^{-26}\, e\, cm \tag{8.3}$$

As our experience with quark generation mixing suggests that the $\mu \to F$ transition should be favored over the $e \to F$ transition, we expect the electron EDM $d_e = -F_3(0)/2m_e$ to be smaller in magnitude than $F_{2,3}^{\mu e}(0)/(m_\mu + m_e)$ even if the CP–violating phase involved in d_e is of order one. If so, we obtain from (8.3) that

$$|d_e| < 2.6 \times 10^{-26}\, e\, cm \tag{8.4}$$

On the other hand we now know directly from the experiment on Cs that $|d_e|$ cannot be larger than 10^{-25} $e\, cm$ [8] and the preliminary value of the upper limit set by the ongoing $T\ell$ experiment [9] is $(0.1 \pm 3.2) \times 10^{-26}$ $e\, cm$. Therefore, unless we introduce a stronger assumption on the ratio of the μF and eF transitions, the $\mu \to e\gamma$ decay does not impose a much stronger constraint on d_e in flavor–changing models of CP violation. Several examples of such interactions will be discussed in the following subsections.

8.2 Flavor–changing neutral Higgs couplings

In the Lee model [107] of spontaneous CP violation flavor–changing neutral Higgs (FCNH) couplings arise because both Higgs doublets couple to right–handed quark and lepton fields. Many variations of models with FCNH exchanges can be constructed. Taking for simplicity a single FCNH particle H° of definite mass, we parametrize its couplings to charged leptons $E = (e, \mu, \tau)$ and quarks $U = (u, c, t)$, $D = (d, s, b)$ as follows:

$$L_{H^\circ} = -\left(\sqrt{2}G_F\right)^{1/2} \sum_{f=E,U,D} \bar{f}_R M_f \alpha_f f_L H^\circ + h.c. \tag{8.5}$$

where M_f are the diagonal fermion mass matrices.

The interaction (8.5) can generate one–loop quark and lepton EDMs both through the flavor–diagonal and flavor–changing couplings contained in Eq.(8.5). The flavor–diagonal contributions have already been discussed in Section 7 in the context of flavor–conserving Higgs models. Here we consider only the off–diagonal couplings in Eq.(8.5). The effective couplings α_F^2/M_H^2 of the flavor–changing four–fermion interaction generated by (8.5) are constrained by various data. In the quark sector the most stringent constraint comes from the $K^\circ - \bar{K}^\circ$ transition. By requiring that the transition amplitude through H° must not be larger than the value determined by the $K_1 - K_2$ mass difference, we obtain

$$|(\alpha_f)^2_{sd}/M_H^2| < 3 \times 10^{-7}\, GeV^{-2}. \tag{8.6}$$

The imaginary part of $(\alpha_f)^2_{sd}$ contributes to the CP–violating amplitudes of K_L decays. Comparison of the CP–violating $|\Delta S| = 2$ amplitude mediated by H° with the experimental value of the ϵ parameter gives the estimate

$$|\,\mathrm{Im}(\alpha_f)^2_{sd}/M_H^2| < 10^{-9}\, GeV^{-2}. \tag{8.7}$$

Detailed analyses of FCNH couplings with two neutral Higgs bosons can be found in Refs.[110–112,132]. If the FCNH couplings to leptons are of the same order as those of quarks, their contribution to d_e is too small to be observable. By substituting the inequality $|\,\mathrm{Im}(\alpha_f)^2_{e\tau}/M_H| < 10^{-8}\, GeV^{-2}$ in the formula for d_e, we obtain $|d_e| < O(10^{-32}\, e\, cm)$ from the τ intermediate state. Since d_e is proportional to m_ℓ^2 of the intermediate lepton ℓ, a lepton much heavier than τ, if it exists, can enhance d_e.

If we abandon the assumption that the FCNH couplings of the quark and lepton sectors are comparable, the upper bound on $|d_e|$ is relaxed significantly. The direct experimental constraints on the FCNH couplings to leptons are available from the data on rare decays of leptons. The constraint from the $\mu \to e\gamma$ decay [131] has been given in Eq.(8.4). The experimental upper limit on the $\mu \to e\bar{e}e$ branching ratio [133] does not give a stringent bound on the coupling α_f because the coupling of H° to the electron is severly suppressed by the electron mass. The required upper bound on the effective four–fermion coupling is

$$|(\alpha_f)_{ee}(\alpha_f)_{e\mu}/M_H^2| < 3 \times 10^{-2}\ GeV^{-2}, \tag{8.8}$$

which leads to $|d_e| < O(10^{-24}\ e\ cm)$. Therefore, flavor–changing neutral Higgs models of CP violation still have a chance to generate an electron EDM large enough to be observed in the near future if the FCNH couplings to leptons are much larger than those to quarks.

8.3 Dilepton models

A more exotic possibility of lepton–flavor changing interactions is encountered in so–called dilepton models. Dileptons are bosons which carry two units of lepton numbers and up to two units of electric charge. To be specific, we examine the model of Zee [134]. This model introduces two sets of dileptons, a singlet and a triplet of $SU(2)_L$, which mix with each other when the $SU(2)_L \times U(1)$ gauge symmetry is broken. Denoting these scalar dileptons by κ and t, their $SU(2)_L \times U(1)$ symmetric interaction with leptons reads:

$$L_I = \sum_{ij} g_{Rij} \kappa \bar{e}'^c_{Li} e'_{Rj} + \sum_{ijA} g_{Lij} t^A \bar{\ell}'^c_{Ri} \tau^A \ell'_{Lj} + h.c., \tag{8.9}$$

where ℓ'_{Lj} are lepton doublets and e'_{Rj} are lepton singlets (the primes denote weak eigenstates), i and j are generation indices and τ^A are the Pauli matrices. Upon symmetry breaking, κ^{++} and t^{++} mix with each other to form the mass eigenstates δ_1 and δ_2. In terms of the mass eigenstates for dileptons and leptons, the relevant part of the interaction is

$$L_I = \sum_{aij} \delta_a \left(\bar{\ell}^c_{Li} \Gamma^a_{Rij} \ell_{Rj} + \bar{\ell}^c_{Ri} \Gamma_{Lij} \ell_{Lk} \right) + h.c., \tag{8.10}$$

where the couplings Γ^a_{Rij} and Γ^a_{Lij} are complex in general as a result of mass diagonalization.

The dileptons $\delta_{1,2}$ generate many flavor–changing neutral interaction processes. The experimental upper limits on rare leptonic decays set upper bounds on the effective four–fermion couplings mediated by $\delta_{1,2}$. Barring an accidental cancellation as usual, we find from the experimental upper bounds on rare μ decays, *e.g.*, $B(\mu \to e\bar{e}e) < 1 \times 10^{-13}$ [133];

$$|\Gamma_{11}\Gamma^*_{12}|/M_\delta^2 < 4 \times 10^{-11}\ GeV^{-2}, \tag{8.11}$$

and from rare τ decays, *e.g.*, $B(\tau \to e\bar{e}e) < 4 \times 10^{-5}$ [135];

$$|\Gamma_{11}\Gamma^*_{13}|/M_\delta^2 < 5 \times 10^{-7}\ GeV^{-2}. \tag{8.12}$$

In Eqs. (8.11) and (8.12) Γ_{ij} stand for the Yukawa couplings defined in Eq.(8.10) with chirality L or R and dilepton index a =1 or 2.

The dilepton interaction of Eq.(8.9), or equivalently of Eq.(8.10) generates an electron EDM through the diagrams shown in Fig.10. We obtain from Eq.(A.5)

$$d_e = -\sum_a \sum_j \frac{e}{16\pi^2 M^2_{\delta a}} m_j Im\left(\Gamma^a_{L1j}\Gamma^{a*}_{R1j}\right) \left[2 I_3\left(m_j^2/M^2_{\delta a}, 0\right) + I_4\left(m_j^2/M^2_{\delta a}, 0\right)\right]. \tag{8.13}$$

The two contributions from δ_1 and δ_2 tend to cancel each other, in particular when the mass difference between δ_1 and δ_2 is smaller than the masses of δ_1 and δ_2 themselves. If we consider for simplicity a special case where the diagonal elements of the $\kappa - t$ mass matrix are equal, we obtain by expanding Eq.(8.13) to first order in the $\delta_1 - \delta_2$ mass difference:

$$d_e \simeq \sum_j \frac{e}{16\pi^2} m_j \frac{\mathrm{Im}\left(\Gamma^1_{L1j}\Gamma^{1*}_{R1j}\right)}{M^2_{\delta 1}} \frac{(M^2_{\delta 1} - M^2_{\delta 2})}{M^2_{\delta 1}}, \tag{8.14}$$

which corresponds to the diagrams of Fig.10.

By use of Eq.(8.11) the contribution of the $\bar{\mu}$ intermediate state is bounded by

$$|(d_e)_\mu| < 1 \times 10^{-27} |M^2_{\delta 1} - M^2_{\delta 1}| / \left(M^2_{\delta 1} + M^2_{\delta 2}\right) \; e\; cm. \tag{8.15}$$

If one uses the bound (8.12) deduced from rare τ decays, one may conclude that the $\bar{\tau}$ intermediate–state contribution could be gigantic. However, the experimental sensitivity to lepton–flavor violation is so far much lower in τ decays than in μ decays. If we assume that the bound (8.11) applies to the $\bar{\tau}$ intermediate state as well, the $\bar{\tau}$ contribution is bounded by

$$|(d_e)_\tau| < 1 \times 10^{-26} |M^2_{\delta 1} - M^2_{\delta 1}| \left(M^2_{\delta 1} + M^2_{\delta 2}\right) \; e\; cm. \tag{8.16}$$

Since the dilepton coupling are not constrained by μe conversion or $|\Delta S| = 2$ processes, this dilepton model is capable of generating a large electron EDM without introducing a heavy fourth generation.

8.4 Leptoquark models

Let us now turn to leptoquarks. Leptoquarks are spin–zero or spin–one bosons which turn leptons into quarks (or antiquarks) and vice versa. Leptoquarks with CP–violating couplings arise in a variety of models [136–138]. Recently, the "superstring inspired" E(6) gauge model has attracted much attention among model builders [139,140] and some CP–violating effects arising from scalar leptoquarks were pointed out [141,142]. This model has a set of color–triplet $SU(2)_L$–singlet and charge (-1/3) scalar leptoquarks ϕ_i. Their couplings to charged leptons are:

$$L_I = \sum_{ijk} \phi_i \left(\bar{\ell}_{Lj}\Gamma_{Lijk}U^c_{Rk} + \bar{\ell}_{Rj}\Gamma_{Rijk}U^c_{Lk}\right) + h.c. \tag{8.17}$$

in the mass eigenstate basis after symmetry breaking. In Eq.(8.17), U_j stands for the up quark of the j-th generation. As to the bounds on the effective flavor–changing four–fermion couplings generated by the interaction (8.17): This specific model does not induce the decay $K_L \to \mu\bar{e}(\bar{\mu}e)$. However, μe conversion can occur and its experimental upper bound sets the stringent bound [143]

$$|\Gamma_{Li11}\Gamma^*_{Ri21}|/M^2_{\phi i} < 2 \times 10^{-11}\; GeV^{-2}. \tag{8.18}$$

With this bound, we obtain an upper bound on d_e by taking into account only the $\bar{c}$ quark intermediate state in Fig.11,

$$|d_e| < 1 \times 10^{-27} \sin\varphi \ln(M_\phi/m_c)\ e\ cm, \tag{8.19}$$

where $\varphi = \arg(\Gamma_{Li12}\Gamma^*_{Ri12})$. Because of its large mass the contribution of the $\bar{t}$ intermediate state could be much larger than that of the $\bar{c}$ intermediate state.

One interesting feature of the leptoquark models of EDMs is that chirality flip is caused by an antiquark for a lepton EDM and by an antilepton for a quark EDM. Therefore, contrary to most other models, in the leptoquark models it is likely that d_e is larger in magnitude than d_n. In the E(6) model the d, s and b quarks cannot have large EDMs through leptoquark exchange because the accompanying intermediate states are antineutrinos, while the u, c and t quarks acquire EDMs through e^+, μ^+ and τ^+ intermediate states. Because the signs of the electric charges of $\bar{u}, \bar{c}, \bar{t}$ and e^+, μ^+, τ^+ are opposite, the sign of d_e is also opposite to that of d_n in the valence approximation [142].

8.5 Mirror fermion models

For the known fermions the left-handed states are assigned to $SU(2)$ doublets and the right-handed states are assigned to singlets, but there is no a priori reason why this rule should apply to new fermions yet to be discovered. The electroweak $SU(2)$ assignment is determined by the weak interaction properties of new particles. Some models actually postulate heavy fermions with $SU(2)$ assignment opposite from that of the known quarks and leptons; namely left-handed fermions being singlets and right-handed fermions being doublets. Such fermions are called "mirror fermions" [144–148].

In mirror fermion models weak-isospin-conserving $\Delta I = 0$ terms are allowed for off-diagonal elements in the mass matrices of the ordinary and the mirror fermions. In order to render the ordinary fermions light enough and the mirror fermions heavy enough to be compatible with experiment, some tuning of the mass matrix elements is required. In the charged lepton sector, e, μ and τ can stay light if one assumes the $\Delta I = 0$ off-diagonal mass matrix elements to be negligible or else one may invoke the seasaw mechanism [94,95] with $\Delta I = 0$ off-diagonal masses much smaller than the $\Delta I = 1/2$ diagonal terms of the mirror fermions. In the neutral lepton sector, three generations of left-handed neutrinos can remain light if, for instance, one adds a right-handed (left-handed) $SU(2)$ singlet neutrino to each (mirror) family [147].

After diagonalizing mass matrices, a generalized KM matrix appears in the charged weak currents. Since the charged weak currents of the mirror fermions are right-handed, mixing between the ordinary and the mirror fermions generates some amount of $W^\pm$ couplings to the right-handed states of the light fermions. The neutral weak current is not flavor-diagonal after mixing because the $SU(2) \times U(1)$ quantum numbers of the ordinary and the mirror fermions are different. Furthermore, since the mass matrices contain terms other than those originating from the VEV of the Higgs doublet, the Yukawa couplings of neutral Higgs particle(s) H° are flavor–nondiagonal in general. Therefore, H°, W, and Z couple to both the left– and right–handed states of fermions and their couplings are complex in general. Consequently the one–loop diagrams of W, Z, and H° exchanges depicted in Figs. 13 and 14 can generate EDMs of fermions. We are interested in the contribution of the mirror fermion intermediate states to the one–loop amplitudes.

The contribution of W exchange to d_e can be obtained from Eq.(A.4),

$$d_e(W) = -\frac{eg^2}{32\pi^2 M_W^2}\sum_j m_j \,\mathrm{Im}\left(V_{L1j}V_{R1j}^*\right) I_1\left(m_j^2/M_W^2, 0\right), \tag{8.20}$$

where V_L and V_R are analogues of the KM matrix in the left and right sectors, respectively and in the sum over j only mirror fermion contributions are of interest. Mixing between the ordinary and mirror fermions is subject to various experimental constraints [144–149]. The upper bound $\sum_j |V_{L,R1j}|^2 \lesssim 0.02$ has been obtained from a detailed analysis of experimental data [149]. However, theoretical considerations indicate much tighter bounds on such mixing: When two fermions mix with each other slightly and produce two mass eigenstates with vastly different mass eigenvalues m and $M(\gg m)$, the sine of the mixing angle is of the order of $(m/M)^{1/2}$ or less. Therefore we expect

$$|V_{L,R1j}| \lesssim O\left((m_e/m_j)^{1/2}\right) \tag{8.21}$$

With this bound and taking into account only one mirror generation with $m_j \approx M_W$, we estimate

$$|d_e(W)| \lesssim \frac{3eg^2 m_e}{64\pi^2 M_W^2}\sin\varphi = 3\times 10^{-24}\sin\varphi \; e\; cm, \tag{8.22}$$

where $\varphi = \arg\left(V_{L1j}V_{R1j}^*\right)$.

Z exchange can generate d_e through the neutral flavor–changing interactions between the electron and charged mirror fermions. The μ and τ intermediate states are unimportant because not only are their masses much smaller than those of mirror fermions but also their flavor–changing couplings to e are severely constrained by rare decay data. The contribution of Z exchange to d_e takes a form similar to Eq.(8.20) where $g^2 V_{L1j}V_{R1j}^*$ is replaced by the corresponding expression for the neutral current. In general $d_e(Z)$ can be of the same order of magnitude as $d_e(W)$ estimated in (8.22).

The contribution of a Higgs boson H° is obtained from Eq.(A.5) and reads

$$d_e(H) = \frac{eg^2 m_e}{64\pi^2 M_H^2}\sum_j \left(\frac{m_j}{M_W}\right)^2 \mathrm{Im}\left(\alpha_{L1j}\alpha_{R1j}^*\right) I_4\left(m_j^2/M_W^2, 0\right), \tag{8.23}$$

where $\alpha_{L,R1j}$ are the Higgs couplings defined in Eq.(8.5). By the argument leading to (8.21), we expect for mirror fermion j that

$$|\alpha_{L1j}\alpha_{R1j}^*| \lesssim O\left((m_e/m_j)^{1/2}\right). \tag{8.24}$$

Since the mirror fermion mass terms are not $SU(2)$ invariant, the mirror fermion masses cannot be much larger than M_W. Therefore, $d_e(H)$ is expected to be smaller than $d_e(W)$ and $d_e(Z)$. It is not easy to improve the estimate of d_e unless parameters of models are specified in detail.

If one modifies mirror fermion models such that both chiral states are either $SU(2)$ singlets or doublets [61,150,151] quite different conclusions emerge in some cases on the relative importance of $d_e(W), d_e(Z)$, and $d_e(H)$. If for instance only one heavy generation is added with both chiral states being doublets [151], $d_e(W)$ and $d_e(Z)$ become negligible and only the Higgs contribution $d_e(H)$ remains relevant. Then this case reduces to that of the flavor–changing neutral Higgs couplings in Section 8.2.

8.6 Horizontal gauge interaction models

Before spontaneous symmetry breaking, the standard model with n generations is symmetric under global $SU(n)$ rotations among generations. When this global symmetry, often referred to as horizontal symmetry, is gauged, an interaction arises which is mediated by spin–one neutral gauge bosons [152–157]. One motivation for introducing a horizontal gauge symmetry was to explore the possibility of explaining quark and lepton mass spectra by the self–energies they receive from horizontal gauge boson exchange. For this purpose, it is necessary for the horizontal gauge bosons to couple to both chiral states of quarks and leptons. These couplings are

$$L_I = -g_H \sum_a \sum_{ij} X_\mu^a \left(\bar{\psi}'_{Li} L^a_{ij} \gamma^\mu \psi'_{Lj} + \bar{\psi}'_{Ri} R^a_{ij} \gamma^\mu \psi'_{Rj} \right), \tag{8.25}$$

where g_H is the coupling of the horizontal gauge group G_H; L^a and R^a are the $n \times n$ representations of the hermitian generators of G_H associated with left– and right–handed quarks and leptons ψ'_{Li} and ψ'_{Ri} in the weak eigenstate basis, respectively, and X^a_μ are the neutral horizontal gauge bosons. Because of the flavor–changing couplings contained in Eq.(8.25) the horizontal gauge symmetry must be broken at a rather high mass scale, *i.e.*, the horizontal gauge boson mass must be very heavy. When quarks and leptons are rotated from weak eigenstates ψ'_i to mass eigenstates ψ_i the interaction Eq.(8.25) turns into

$$L_I = -g_H \sum_a \sum_{ij} X_\mu^a \left(\bar{\psi}_{Li} G^a_{Lij} \gamma^\mu \psi_{Lj} + \bar{\psi}_{Ri} G^a_{Rij} \gamma^\mu \psi_{Rj} \right), \tag{8.26}$$

where $G_L = V_L L V_L^\dagger, G_R = V_R R V_R^\dagger$, and $V_{L,R}$ are the unitary matrices which diagonalize the fermion mass matrices. As the mass matrices need not be hermitian, $V_L \neq V_R$ in general. The flavor–changing gauge couplings in Eq.(8.26) can be complex and therefore CP–violating. The interaction (8.26) can generate fermion EDMs to one–loop order. The diagram relevant to the electron is depicted in Fig.12.

The magnitude of the effective four–fermion couplings mediated by X^a exchange is severely constrained by experimental bounds on flavor–changing neutral current processes

[153,158]. Let us assume for simplicity that all the X^a masses are approximately equal. The experimental upper limit [159] on the μe conversion $\sigma(\mu Ti \to eTi)/\sigma(\mu Ti \to all) < 4.6 \times 10^{-12}$ requires

$$g_H^2/M_X^2 < 5 \times 10^{-12}\, GeV^{-2}. \tag{8.27}$$

Strangeness–changing $|\Delta S| = 2$ processes which can occur at tree level when $V_L \neq V_R$ impose tighter bounds on the couplings if the off–diagonal elements of $V_L V_R^*$ are nonnegligible. The $K_1 - K_2$ mass splitting demands

$$g_H^2 \left| \sum_a G_{Lsd}^a G_{Rsd}^a \right| / M_X^2 < 1 \times 10^{-13}\, GeV. \tag{8.28}$$

The ϵ parameter of K_L decays sets a stringent bound on the CP–violating part of these couplings. It was estimated that [160]

$$g_H^2 \left| \sum_a \mathrm{Im}\left(G_{Lsd}^a G_{Rsd}^a \right) \right| / M_X^2 \lesssim 10^{-15}\, GeV^{-2}. \tag{8.29}$$

The sum $\sum_a G_{Lsd}^a G_{Rsd}^a$ is nonzero only to the extent that V_L differs from V_R. If the non-diagonal elements of $V_L V_R^*$ are of the same order of magnitude as those of the KM matrix, $\sum_a G_{Lsd}^a G_{Rsd}^a = O(\sin^2\theta_C)$ where θ_C is the Cabibbo angle. Then the constraint (8.28) turns into

$$g_H^2/M_X^2 < 2 \times 10^{-12}\, GeV^{-2} \tag{8.30}$$

and the constraint (8.29) reads

$$g_H^2 |\sin\varphi| / M_X^2 < 10^{-14}, \tag{8.31}$$

where φ is the phase angle of $G_{Lsd}^a G_{Rsd}^a$.

Let us now examine the electron EDM. We obtain from Eq.(A.4)

$$d_e = -\sum_a \sum_j \frac{e g_H^2}{16\pi^2 M_X^2} m_j \,\mathrm{Im}\left(G_{Lej}^a G_{Rej}^{a*} \right) I_2\left(m_j^2/M_X^2, 0 \right), \tag{8.32}$$

where the function $I_2 \to 2$ when $M_X \to \infty$. From the constraint (8.27) we obtain

$$|d_e| < 2 \times 10^{-27}\, e\, cm. \tag{8.33}$$

With the bound (8.30) which involves a plausible, but experimentally untested assumption, the upper bound on d_e gets a little more stringent:

$$|d_e| < 1 \times 10^{-27}\, e\, cm. \tag{8.34}$$

If we assume further that the phase of $\sum_a G_{Lej}^a G_{Rej}^{a*}$ is comparable to that of $\sum_a G_{Lsd}^a G_{Rsd}^a$, (8.31) leads to the upper bound on d_e

$$|d_e| < 5 \times 10^{-30}\, e\, cm. \tag{8.35}$$

Horizontal interaction models often postulate very heavy generations beyond the third one in order to generate masses for quarks and leptons of the first three generations. If (8.31) applies it needs however a charged lepton with mass of about 1 TeV in order to push (8.35) to the level of 10^{-27} $e\ cm$.

Horizontal interactions may also be mediated by spinless bosons. However, such couplings are indistinguishable from those of flavor–changing Higgs models (cf. Sect. 8.1).

9. Composite electron

So far, we have treated models of CP nonconservation in which the electron is considered to be an elementary particle. There are speculations that the electron - among other particles - is composed of subconstituents. This substructure would first of all affect its anomalous magnetic moment at some level. The dynamics of composite models, characterized by the energy scale Λ_c, is usually assumed to conserve electron chirality and lepton flavor. Then the electron's substructure leads to a correction to the magnetic form factor $F_2/2m_e$ of the order m_e/Λ_c^2 [161] which yields a contribution of the order $(m_e/\Lambda_c)^2$ to $(g-2)_e$. When this argument is applied to the muon, comparison of the current experimetal value of $(g-2)_\mu$ with its Standard Model prediction leads to $\Lambda_c \gtrsim 1$ TeV. For the electron, the most stringent bound on Λ_c has been deduced from wide–angle Bhabha scattering by comparing the experimental cross section with the Standard Model prediction. If, for instance, $L_I = \lambda(2\pi/\Lambda_c^2)(\bar{e}_L\gamma_\mu e_L)(\bar{e}_L\gamma^\mu e_L)$ is chosen to represent the effective four–electron interaction induced by the compositeness dynamics [162], one obtains from the data [163] $\Lambda_c > 1.4$ TeV for $\lambda = +1$ and $\Lambda_c > 3.3$ TeV for $\lambda = -1$.

The relation between the electron EDM and Λ_c is more model–dependent since the dynamics of subconstituents need not violate CP invariance. If it does, it should induce CP–violating effective four-fermion interactions among composite leptons. As an example, let us consider the following effective four–electron interaction

$$L_I' = \left(2\pi/\Lambda_c^2\right)\left[\frac{1}{2}\eta(\bar{e}_L e_R) + \frac{1}{2}\eta^*(\bar{e}_R e_L)\right]^2 \tag{9.1}$$

where η is a complex parameter normalized to unity. This interaction contains the P– and CP–violating term $(\bar{e}e)(\bar{e}i\gamma_5 e)$. In fact, this is the only independent CP–violating operator of dimension six that involves four electrons. (However $L_I' \sim \Lambda_c^{-2}$ is an Ansatz. The dynamics of subconstituents might actually lead to $L_I' \sim m_e^2/\Lambda_c^4$ because L_I' does not conserve chirality.) Although the interaction (9.1) does not conserve chirality, it can be accommodated in composite models since its contributions to the electron self–energy and $(g-2)_e$ are proportional to m_e and $(m_e/\Lambda_c)^2 \ln(\Lambda_c/m_e)$, respectively when the divergent integrals in the loop diagrams are cut off at Λ_c. With this ultraviolet cutoff, the interaction (9.1) yields the one–loop EDM,

$$d_e = e\left(m_e/8\pi\Lambda_c^2\right)\sin 2\delta\ \ln(\Lambda_c/m_e), \tag{9.2}$$

where $\delta = \arg\ \eta$. Given an experimental value of d_e or an upper bound on $|d_e|$, Eq.(9.2) implies

$$\Lambda_c \gtrsim 400 \times |\sin 2\delta|^{1/2} \left(|d_e|/10^{-28}\, e\ cm\right)^{-1/2}\ TeV. \tag{9.3}$$

When η is real or purely imaginary, the interaction (9.1) reduces to a CP-conserving scalar or pseudoscalar interaction, respectively so that no lower bound on Λ_c results from (9.3).

10. Concluding remarks

In a spontaneously broken gauge theory with scalar fields, CP nonconservation occurs quite naturally either through complex vacuum expectation values of scalar fields or through explicit CP noninvariance in nongauge couplings - apart from the P- and T-violating QCD "θ-term" of nonperturbative origin. In the SM which contains only a single Higgs doublet, CP nonconservation in the Yukawa coupling of quarks is transformed into the KM phase and is related to the hierarchy of the quark mass spectrum. However, if the nongauge sector (*i.e.*, Yukawa interactions and scalar self-interactions) is richer than that of the SM, the CP violation is not necessarily connected with the nondegeneracy of the quark mass spectrum. Then CP violating effects are potentially much larger than in the SM. In such nonstandard models near- degeneracy of the neutrino mass spectrum and lack of experimental evidence for lepton generation mixing do not imply that CP-violating phenomena among leptons are doomed to be unmeasurably small. This has been reviewed in detail above.

The ongoing measurement of the EDM of thallium and future measurements of atomic EDMs and possibly of molecular EDMs are important in that they provide a unique means for studying the question whether CP-violating forces among leptons actually exist or not. It should be emphasized that these measurements also yield information of P- and T-violating hadronic and semileptonic interactions. If in the future the experimental upper bound on the electron EDM is lowered to the level of 10^{-27} $e\ cm$ (recall however the caveats involved in the extraction of d_e from atomic EDMs), such a bound would impose useful constraints on parameters of supersymmetric models, left-right symmetric models, Higgs models, and lepton-flavor changing models of CP violation.

In many models the electron EDM is expected to be at least two orders of magnitude smaller than the neutron EDM. The reasons are smaller chirality flip and weaker gauge couplings for leptons. However, these estimates usually rely on a naturalness argument or an educated guess about magnitudes of relevant parameters, in particular CP-violating phases, for leptons in comparison with corresponding quantities for quarks. There are no conclusive arguments leading to $|d_e/d_n| \ll 1$ that result from solid experimental information. If a model contains nonstandard interactions and/or exotic particles, this assumption often fails. For instance, d_e can be made as large as d_n in left-right symmetric models without sacrificing much of naturalness. In leptoquark models neither chirality flip nor coupling strength suppresses d_e relative to d_n. Therefore the present experimental limit on the neutron EDM, $|d_n| < 1.2 \times 10^{-25}$ $e\ cm$, does not imply that d_e must be below the sensitivity level of the ongoing measurement of the EDM of thallium. If a nonzero EDM of an atom, for instance, of thallium would be found, it would be a clear evidence for a new CP–violating interaction other than the one due to the KM phase. Yet even if high–sensitivity measurements of other atomic EDMs and the neutron EDM would eventually conclude that the thallium EDM is due to a nonzero d_e, it would be impossible to trace back the origin of this symmetry violation. But in conjunction with ongoing and future searches for CP–violating interactions in other places such as K and B decays and with searches for lepton–flavor changing decays, it would make an important contribution to a deeper understanding of this feeble phenomenon.

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Appendix

One–loop contributions to the EDM of an elementary fermion

The EDM d_f of a Dirac fermion f is defined by means of the form factor decomposition of the electromagnetic current, Eqs.(1.1)–(1.3). In a general gauge theory, d_f can be generated to one–loop order by exchange of spin–one gauge bosons or spin–zero bosons. Their contributions to d_f have been calculated in Ref. [91], whose formulae are presented here for convenience. Let ψ_i, W_a^μ, and H_a denote the fields of the fermion (*e.g.*, leptons, quarks, photino, zino, Higgsinos, etc.), spin– one boson (*e.g.*, $W^\pm, Z$, spin–one leptoquarks, etc.), and spin–zero boson (*e.g.*, Higgs bosons, sfermions, scalar leptoquarks, etc), respectively. These fields are assumed to be those of mass eigenstates at the tree–level. The relevant interaction Lagrangian for the boson–fermion couplings are written

$$L_I = -\sum_{ija}\left\{\bar\psi_i\gamma_\mu\left(G^a_{Lij}P_L + G^a_{Rij}P_R\right)\psi_j W^\mu_a + \bar\psi_i\left(\Gamma^a_{Lij}P_L + \Gamma^a_{Rij}P_R\right)\psi_j H_a\right\} + \text{h.c.} \quad (A.1)$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. If the fields W_a^μ and H^a are chosen to be hermitian, then the hermitian conjugate terms are absent in Eq.(A.1) and $G^a_{L,Rij}$ and $\Gamma^a_{L,Rij}$, considered as matrices in the space of all fermions $f_i (i = 1\cdots n)$, have the property

$$G^{a\dagger}_L = G^a_L,\ \ G^{a\dagger}_R = G^a_R,\ \ \Gamma^{a\dagger}_L = \Gamma^a_R. \quad (A.2)$$

The electromagnetic couplings of W_a^μ are taken to be the gauge couplings of $SU(2)_{L,R}\times U(1)$ and the electromagnetic couplings of H_a are the minimal couplings of scalar fields.

The one–loop contributions of W_a^μ and H_a to the electromagnetic form factors of a fermion f_i arise from the amplitudes depicted by the Feynman diagrams of Figs. 13 and 14. Specifically, the EDM d_i of the fermion f_i is found to be

$$d_i = \sum_a \left[d_i(W_a) + d_i(H_a)\right], \quad (A.3)$$

where the W–loop contributions are

$$d_i(W_a) = \frac{1}{16\pi^2 M_W^2}\sum_j m_j \,\text{Im}\left(G^a_{Lij}G^{a*}_{Rij}\right)\left[(Q_j - Q_i)I_1(r_j,s_i) + Q_j I_2(r_j,s_i)\right], \quad (A.4)$$

with $r_j = m_j^2/M_W^2$ and $s_i = m_i^2/M_W^2$ and the H–loop contributions are

$$d_i(H_a) = -\frac{1}{16\pi^2 M_H^2}\sum_j m_j \ \text{Im}\left(\Gamma^a_{Lij}\Gamma^{a*}_{Rij}\right)\left[(Q_j - Q_i)I_3(r_j,s_i) + Q_j I_4(r_j,s_i)\right], \quad (A.5)$$

with $r_j = m_j^2/M_H^2$ and $s_i = m_i^2/M_H^2$.

In the formulae (A.4) and (A.5), the electric charge of f_i is denoted by Q_j. The function $I_k(r,s)$ are defined by

$$\begin{aligned} I_1(r,s) &= 1/2 + 3F_0(r,s) - 6F_1(r,s) + (3-s)F_2(r,s) + sF_3(r,s) \\ &\simeq \frac{2}{(1-r)^2}\left[1 - \frac{11}{4}r + \frac{1}{4}r^2 - \frac{3r^2\ln r}{2(1-r)}\right] \end{aligned} \quad (A.6)$$

$$I_2(r,s) = (4+r-s)F_1(r,s) - 4F_2(r,s)$$
$$\simeq \frac{2}{(1-r)^2}\left[1+\frac{1}{4}r+\frac{1}{4}r^2+\frac{3r\ \ln\ r}{2(1-r)}\right] \tag{A.7}$$

$$I_3(r,s) = F_1(r,s) - F_2(r,s) \simeq \frac{1}{2(1-r)^2}\left(1+r+\frac{2r\ \ln\ r}{1-r}\right) \tag{A.8}$$

$$I_4(r,s) = F_2(r,s) \simeq -\frac{1}{2(1-r)^2}\left(3-r+\frac{2\ \ln\ r}{1-r}\right) \tag{A.9}$$

where

$$F_a(r,s) = \int_0^1 dx\ x^a/[1-x+rx-sx(1-x)].$$

The approximate expressions in Eqs.(A.6)–(A.9) hold for $s \simeq 0$, that is, if the external fermion is very light compared with the boson in the loop. Finally, the integral $I_4(s,s)$, needed in Section 7.2, is

$$I_4(s,s) = 1+\frac{1}{2s}\ln\ s+\frac{1-2s}{2s}K(s) \tag{A.10}$$

with

$$K(s) = \begin{cases} \frac{1}{(1-4s)^{1/2}}\ln\frac{1+(1-4s)^{1/2}}{1-(1-4s)^{1/2}} & \text{for } s < 1/4, \\ 2(4s-1)^{1/2}\ \arctan(4s-1)^{1/2} & \text{for } s > 1/4. \end{cases}$$

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Footnotes

[F1] If a nonzero EDM of the neutron or some other baryon should be observed, it might be due to the P– and T–violating gluonic interaction $(\theta/32\pi^2)G^{\mu\nu}\widetilde{G}_{\mu\nu}$ which can be present in the SM. If so, we might call this also a new CP–violating interaction.

[F2] The P–violating and CP–conserving standard neutral current, that is Z boson exchange, can produce a nonzero EDM $< \vec{D}_A >$ of an unstable state [2,19]. However, this does not lead to a linear Stark effect.

[F3] The authors of [15] give an estimate $|d_e| \ll 10^{-50}$ $e\ cm$, which translates to $|f_e| \ll 10^{-22}$

[F4] This is demanded by "naturalness", *i.e.*, the requirement that the electroweak symmetry breaking scale is stable against radiative corrections up the Planck scale.

[F5] Neutrinoless double beta decay does not provide us with a direct constraint on $m_{\nu e}$. Only if we are willing to assume that generation mixing is negligible, the upper limit $m_{\nu e} <$ $(1 \sim 2)$ eV is obtained [100,101].

[F6] Again barring an accidental cancellation among contributions from different ϕ_i.

[F7] With the proviso not to enforce NFC by a discrete symmetry in order to allow for CP violation in neutral Higgs particle mixing.

Figure Captions

Fig. 1: An example of a quark loop contribution to the electron EDM in the Standard Model. The cross denotes a mass insertion.

Fig. 2: One–loop SM contribution to the lepton–photon vertex.

Fig. 3: Generic one–loop diagrams which can generate a nonzero EDM of the electron. F denotes a fermion and B denotes a boson of spin zero or one.

Fig. 4: One–loop neutralino exchanges which contribute to d_e. Diagrams where $\widetilde{H}_2^0$ couples to electron–selectron at both ends are much smaller because they are of a higher order in m_e.

Fig. 5: One–loop chargino contribution to d_e.

Fig. 6: Diagram for the electron EDM in the minimal left–right symmetric model drawn in weak eigenstates.

Fig. 7: Diagram for the electron EDM generated by Higgs mixing in weak eigenstates.

Fig. 8: Examples of two–loop contributions of neutral and charged Higgs particles to the effective gluon interaction $f_{abc}G^{a\mu\rho}G^{b\nu}{}_{\rho}\widetilde{G}^{c}{}_{\mu\nu}$. Here ϕ_1 and $H_1^{\pm}$ are neutral and charged Higgs particles of indefinite parity, respectively.

Fig. 9: A two–loop diagaram contributing to the electron EDM. Here ϕ_1 denotes a neutral Higgs particle of indefinite parity.

Fig. 10: The diagrams which generate d_e through dilepton exchange. A chirality flip is understood along the anti lepton intermediate line.

Fig. 11: One diagram which generates an electron EDM through leptoquark exchange. A chirality flip is understood along the intermediate antiquark line. The diagram where the photon couples to ϕ_i is not shown.

Fig. 12: The diagram to generate an electron EDM by a horizontal gauge boson X. A chirality flip is understood along the internal lepton line.

Fig. 13: Gauge boson diagrams for the EDM of fermion i. The W bosons include the ghost Higgs fields associated with them in general gauges.

Fig. 14: Scalar boson diagrams for the EDM of fermion i.

Tables

Table 1: The enhancement/suppression factor R is defined by $d_A = Rd_e +$ (nuclear contribution), where d_A is the atomic EDM and d_e is the electron EDM. The calculated values of R are subject to uncertainties due to methods of calculation. Some references quote more than one value of R for a given atom. For the uncertainties involved in the calculation of R, the references should be consulted. The last column lists the values for d_e deduced from the experimental results on atomic EDMs.

Atom	Enhancement/suppression factor R	d_e (e cm)
Li	4.5×10^{-3} [20], 4.19×10^{-3} [22]	—
Na	0.33 [20,22]	—
K	2.65 [20], 3.04 [22]	—
Rb	27.5 [20], 27.2 [22], $16 \sim 24$ [26]	—
Cs	133 [20], 159 [22], 131 [23], $80 \sim 106$ [26]	$< 3 \times 10^{-24}$ [30] $(-1.5 \pm 5.5 \pm 1.5) \times 10^{-26}$ [8]
Fr	1150 [20]	—
Tl	-700 ± 100 [21], -500 [24], $(-502) \sim (-607)$ [26]	$(1.9 \pm 3.4) \times 10^{-24}$ [21,31] $(-1.4 \pm 2.4) \times 10^{-25}$ [7] $(0.1 \pm 3.2) \times 10^{-26}$ [9]
Xe($^3P_2$)	130 [25]	$(0.7 \pm 2.2) \times 10^{-24}$ [25]
Xe($^1S_0$)	-0.8×10^{-3} [28,29]	$(4 \pm 14) \times 10^{-24}$ [28,32]
Hg	-1.4×10^{-2} [28,29]	$(-0.5 \pm 1.1) \times 10^{-24}$ [6]

Table 2: The upper limit on the $W_L - W_R$ mixing parameter S.

$$|S| \quad < \quad \begin{cases} 0.041 & \text{for } m_R \to \infty \quad (\xi \text{ parameter of } \mu \to e\nu\nu)\,[97] \\ 0.0055 & \qquad\qquad\quad \text{(nonleptonic decays)}\,[98] \\ 0.05 & \qquad\qquad\quad \text{(validity of Adler-Weissberger relation)}\,[99] \\ 0.004 & \qquad\qquad\quad \text{(KM matrix elements for } b \text{ quark)}\,[99] \end{cases}$$

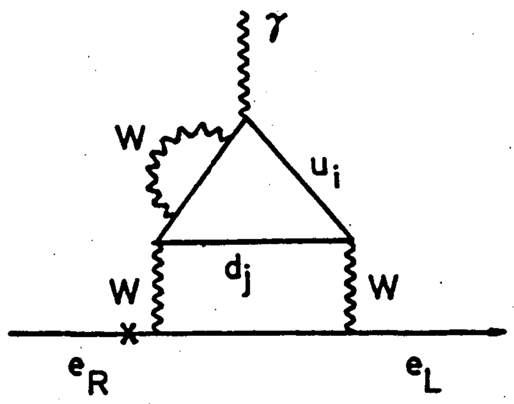

FIG.1

Fig. 1: An example of a quark loop contribution to the electron EDM in the Standard Model. The cross denotes a mass insertion.

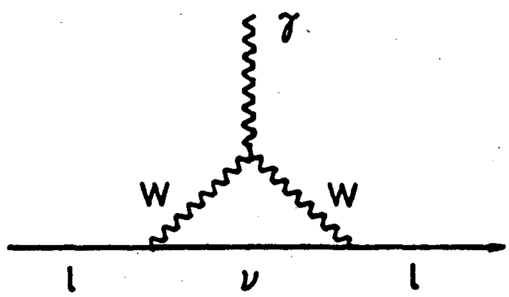

FIG.2

Fig. 2: One–loop SM contribution to the lepton–photon vertex.

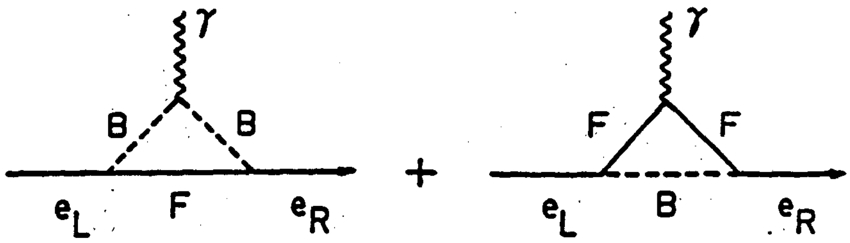

FIG.3

Fig. 3: Generic one–loop diagrams which can generate a nonzero EDM of the electron. F denotes a fermion and B denotes a boson of spin zero or one.

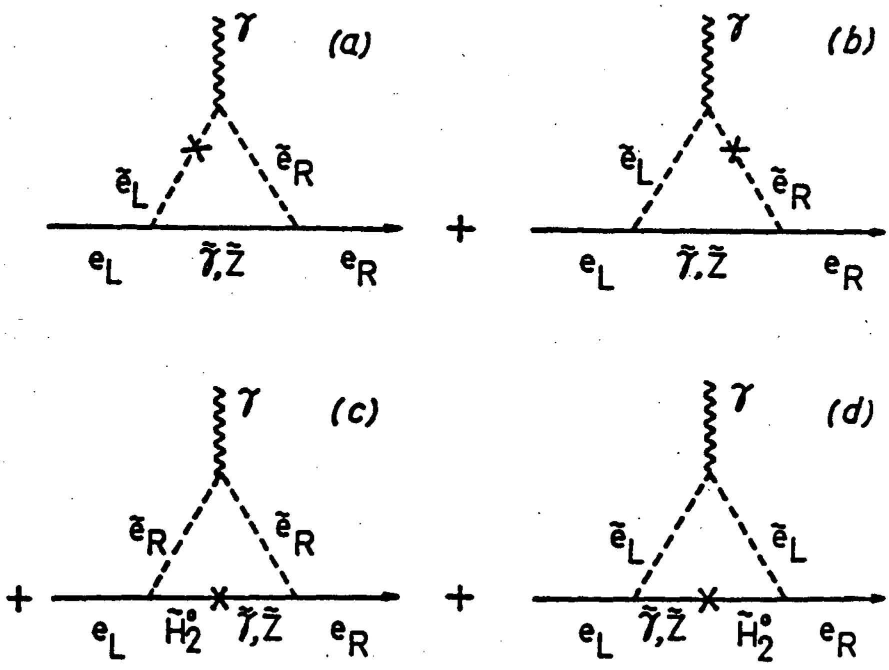

FIG.4

Fig. 4: One–loop neutralino exchanges which contribute to d_e. Diagrams where $\tilde{H}_2^0$ couples to electron–selectron at both ends are much smaller because they are of a higher order in m_e.

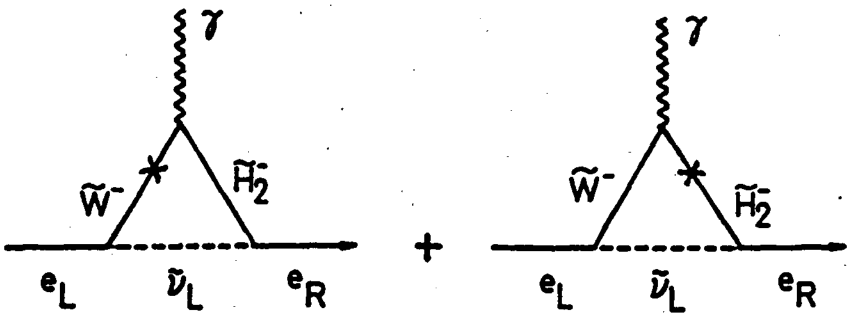

FIG.5

Fig. 5: One–loop chargino contribution to d_e.

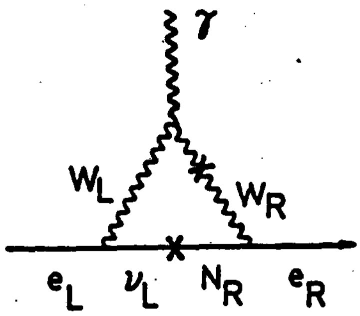

FIG.6

Fig. 6: Diagram for the electron EDM in the minimal left–right symmetric model drawn in weak eigenstates.

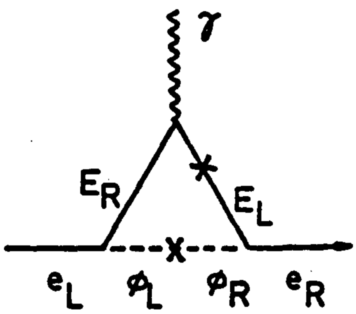

FIG.7

Fig. 7: Diagram for the electron EDM generated by Higgs mixing in weak eigenstates.

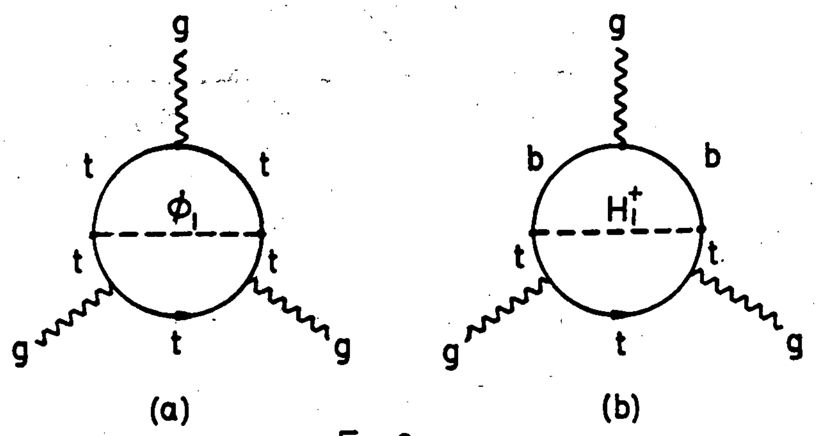

FIG. 8

Fig. 8: Examples of two–loop contributions of neutral and charged Higgs particles to the effective gluon interaction $f_{abc}G^{a\mu\rho}G^{b\nu}{}_{\rho}\tilde{G}^{c}{}_{\mu\nu}$. Here ϕ_1 and $H_1^{\pm}$ are neutral and charged Higgs particles of indefinite parity, respectively.

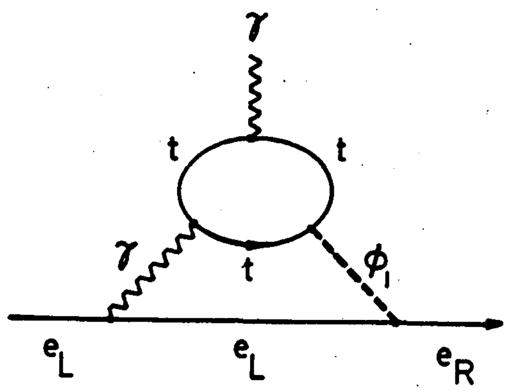

FIG. 9

Fig. 9: A two–loop diagaram contributing to the electron EDM. Here ϕ_1 denotes a neutral Higgs particle of indefinite parity.

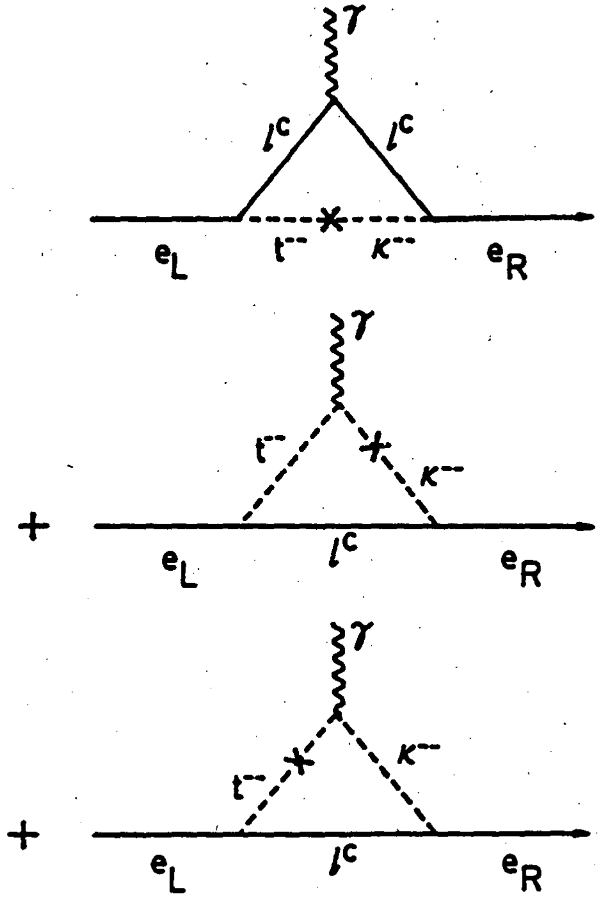

FIG. 10

Fig. 10: The diagrams which generate d_e through dilepton exchange. A chirality flip is understood along the anti lepton intermediate line.

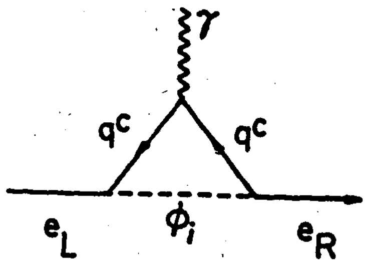

FIG. 11

Fig. 11: One diagram which generates an electron EDM through leptoquark exchange. A chirality flip is understood along the intermediate antiquark line. The diagram where the photon couples to ϕ_i is not shown.

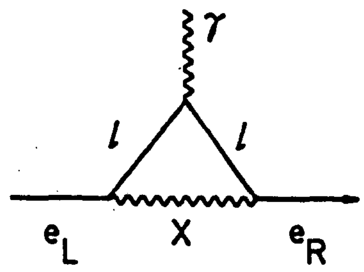

FIG. 12

Fig. 12: The diagram to generate an electron EDM by a horizontal gauge boson X. A chirality flip is understood along the internal lepton line.

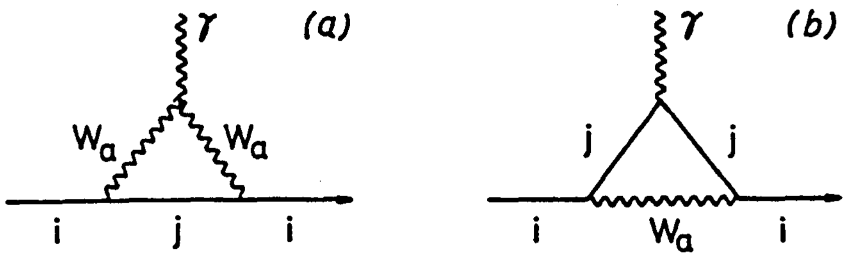

FIG. 13

Fig. 13: Gauge boson diagrams for the EDM of fermion i. The W bosons include the ghost Higgs fields associated with them in general gauges.

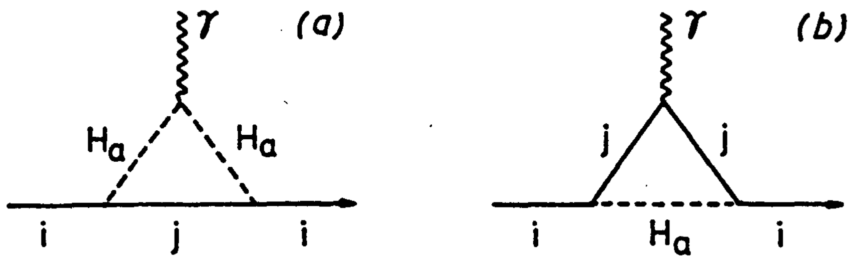

FIG. 14

Fig. 14: Scalar boson diagrams for the EDM of fermion i.