Title

MSW effects in vacuum oscillations

Author

Friedland, Alexander

Publication Date

2000-05-18

MSW Effects in Vacuum Oscillations

Alexander Friedland
Department of Physics, University of California, Berkeley, CA 94720, USA;
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
(July 10, 2004)

We point out that for solar neutrino oscillations with the mass–squared difference of $\Delta m^2 \sim 10^{-10} - 10^{-9}$ eV^2, traditionally known as "vacuum oscillation" range, the solar matter effects are non-negligible, particularly for the low energy *pp* neutrinos. One consequence of this is that the values of the mixing angle θ and $\pi/2 - \theta$ are not equivalent, leading to the need to consider the entire physical range of the mixing angle $0 \leq \theta \leq \pi/2$ when determining the allowed values of the neutrino oscillation parameters.

arXiv:hep-ph/0002063 v2 18 May 2000

1. The field of solar neutrino physics is currently undergoing a remarkable change. For 30 years the goal was simply to confirm the deficit of solar neutrinos. The latest experiments, however, such as Super-Kamiokande, SNO, Borexino, KamLAND, etc, aim to accomplish more than that. By collecting high statistics real–time data sets on different components of the solar neutrino spectrum, they hope to obtain unequivocal proof of neutrino oscillations and measure the oscillation parameters. With the physics of solar neutrinos quickly becoming a precision science, it is more important then ever to ensure that all relevant physical effects are taken into account and the right parameter set is used.

It has been a long–standing tradition in solar neutrino physics to present experimental results in the $\Delta m^2 - \sin^2 2\theta$ space and to treat separately the "vacuum oscillation" ($\Delta m^2 \sim 10^{-11} - 10^{-9}$ eV^2) and the MSW ($\Delta m^2 \sim 10^{-8} - 10^{-3}$ eV^2) regions. In the vacuum oscillation region the neutrino survival probability (*i.e.* the probability to be detected as ν_e) was always computed according to the canonical formula,

$$P = 1 - \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 L}{E} \right), \tag{1}$$

where the neutrino energy E is in GeV, the distance L in km, and the mass–squared splitting Δm^2 in eV^2. Eq. (1) makes $\sin^2 2\theta$ seem like a natural parameter choice. As $\sin^2 2\theta$ runs from 0 to 1, the corresponding range of the mixing angle is $0 \leq \theta \leq \pi/4$. There is no need to treat separately the case of $\Delta m^2 < 0$ (or equivalently $\pi/4 \leq \theta \leq \pi/2$), since Eq. (1) is invariant with respect to $\Delta m^2 \to -\Delta m^2$ ($\theta \to \pi/2 - \theta$).

The situation is different in the MSW region, since neutrino interactions with matter are manifestly flavor-dependent. It is well known that for $|\Delta m^2| \gtrsim 10^{-8}$ eV^2 matter effects in the Sun and Earth can be quite large. In this case, if one still chooses to limit the range of the mixing angle to $0 \leq \theta \leq \pi/4$, one must consider both signs of Δm^2 to describe all physically inequivalent situations. As was argued in [1], to exhibit the continuity of physics around the maximal mixing, it is more natural to keep the same sign of Δm^2 and to vary the mixing angle in the range $0 \leq \theta \leq \pi/2$.

Historically, a possible argument in favor of not considering $\theta > \pi/4$ in the MSW region might have been that this half of the parameter space is "uninteresting", since for $\theta > \pi/4$ there is no level-crossing in the Sun and the neutrino survival probability is always greater than 1/2. However, a detailed analysis reveals that allowed MSW regions can extend to maximal mixing and beyond, as was explored in [2] (see also [3] and [4] for a treatment of 3- and 4- neutrino mixing schemes).

In this letter we point out that for solar neutrinos with low energies, particularly the *pp* neutrinos, the solar matter effects can be relevant even for neutrino oscillations with $\Delta m^2 \sim 10^{-10} - 10^{-9}$ eV^2. These effects break the symmetry between θ and $\pi/2 - \theta$ making it necessary to consider the full physical range of the mixing angle $0 \leq \theta \leq \pi/2$ even in the "vacuum oscillation" case.

2. For simplicity, we will only consider here the two-generation mixing. If neutrino masses are nonzero then, in general, the mass eigenstates $|\nu_{1,2}\rangle$ are different from the flavor eigenstates $|\nu_{e,\mu}\rangle$. The relationship between the two bases is given in terms of the mixing angle θ:

$$\begin{aligned} |\nu_1\rangle &= \cos\theta |\nu_e\rangle - \sin\theta |\nu_\mu\rangle, \\ |\nu_2\rangle &= \sin\theta |\nu_e\rangle + \cos\theta |\nu_\mu\rangle. \end{aligned} \tag{2}$$

In our convention $|\nu_2\rangle$ is always the heavier of the two eigenstates, *i.e.* $\Delta m^2 \equiv m_2^2 - m_1^2 \geq 0$. Then, as already mentioned, $0 \leq \theta \leq \pi/2$ encompasses all physically different situations.

Neutrinos are created in the Sun's core and exit the Sun in the superposition of $|\nu_1\rangle$ and $|\nu_2\rangle$. For Δm^2 in the vacuum oscillation region, the neutrino is produced almost completely in the heavy Hamiltonian eigenstate $|\nu_+\rangle$. In this case, if the evolution inside the Sun is *adiabatic*, the exit state is purely $|\nu_2\rangle$. In the case of a *nonadiabatic* transition there is also a nonzero probability P_c to find the neutrino in the $|\nu_1\rangle$ state (a "level crossing" probability). For a given value of P_c, the survival probability for neutrinos arriving at the Earth is determined by simple 2-state quantum mechanics [5,7,8]:

$$\begin{aligned} P = {} & P_c \cos^2\theta + (1 - P_c) \sin^2\theta \\ & + 2\sqrt{P_c(1 - P_c)} \sin\theta \cos\theta \cos\left(2.54 \frac{\Delta m^2 L}{E_\nu} + \delta \right). \end{aligned} \tag{3}$$

Here δ is a phase acquired when neutrinos traverse the Sun. In our analysis it is determined numerically [6]. Units are the same as in Eq. (1).

In the adiabatic limit $P_c = 0$ and Eq. (3) yields $P = \sin^2\theta$. Neutrinos exit the Sun in the heavy mass eigenstate and do not oscillate in vacuum. In the opposite limit of small Δm^2, when the neutrino evolution in the Sun is "extremely nonadiabatic", $P_c \to \cos^2\theta$. It is trivial to verify that Eq. (3) in this limit reduces to Eq. (1). It has been assumed that in the vacuum oscillation region this limit is reached. Remarkably, however, this is not always the case for the low energy solar neutrinos, especially the *pp* neutrinos ($E_\nu \leq 0.42$ MeV).

The most reliable way to compute P_c is by numerically solving the Schrödinger equation in the Sun for different values of Δm^2 and θ. We do this using the latest available BP2000 solar profile [9]. Fig. 1 shows contours of constant P_c for the energy of ^{7}Be neutrino (solid lines). Note that the variable on the horizontal axis is $\tan^2\theta$. With this choice, points θ and $\pi/2-\theta$ are located symmetrically on the logarithmic scale about $\tan^2\theta = 1$ (see [10]) [11]. The figure demonstrates that the contours are not symmetric with respect to the $\tan^2\theta = 1$ line, except in the region of $\Delta m^2/E_\nu \lesssim 10^{-10}$ eV^2/MeV, where the extreme nonadiabatic limit is reached. This simple observation is the crucial point of this letter.

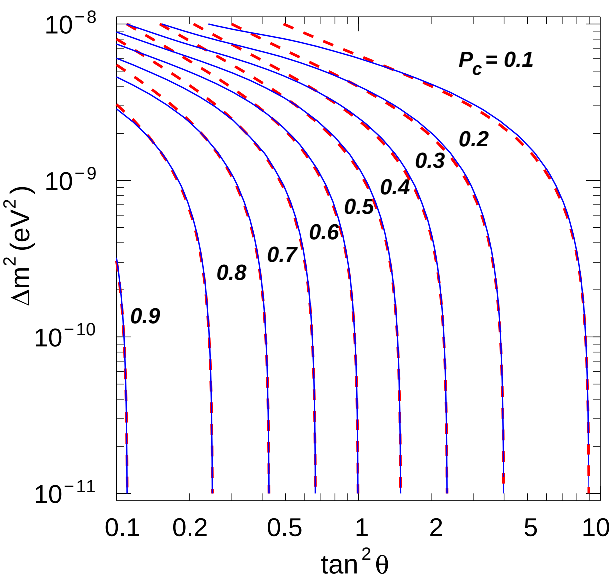

FIG. 1. Contours of constant level crossing probability P_c for neutrino energy of 0.863 MeV (^{7}Be line). The solid lines are the results of numerical calculations using the BP2000 solar profile. The dashed lines correspond to using the exponential profile formula with $r_0 = R_\odot/18.4 = 3.77 \times 10^4$ km.

In the MSW region the value of P_c is often computed using the analytical result [12]

$$P_c = \frac{e^{\gamma \cos^2\theta} - 1}{e^\gamma - 1}, \tag{4}$$

where

$$\gamma = 2\pi r_0 \frac{\Delta m^2}{2E_\nu}, \tag{5}$$

valid for the exponential solar profile $n_e \propto \exp(-r/r_0)$, with $r_0 = R_\odot/10.54 = 6.60 \times 10^4$ km [13]. Although originally derived for $\theta \leq \pi/4$, it also applies when $\theta > \pi/4$, as was demonstrated in [1]. In the region relevant for vacuum oscillation, however, $0.9R_\odot \lesssim R \lesssim R_\odot$, the profile falls off faster than the exponential with $r_0 = R_\odot/10.54$. Nevertheless, Eq. (4) can still be used with the appropriately chosen value of r_0. The dashed lines in Fig. 1 show the contours of P_c computed using Eq. (4) with $r_0 = R_\odot/18.4 = 3.77 \times 10^4$ km. As can be seen from the figure, the agreement between the two sets of contours for $\Delta m^2 \lesssim 4 \times 10^{-9}$ eV^2 is very good. Note that a similar result was arrived at in [14] for $\theta \leq \pi/4$, where the value of $r_0 = R_\odot \times 0.065 = 6.5 \times 10^4$ km was obtained.

Fig. 1 can also be used to read off the values of P_c for different neutrino energies, since P_c depends on E_ν through the combination $\Delta m^2/E_\nu$. It is obvious that for neutrinos of lower energies P_c starts deviating from its "extreme nonadiabatic" value at even smaller values of Δm^2, and vice versa. Consequently, as will be seen later, the solar matter effects on vacuum oscillations are most important at the gallium experiments, which are sensitive to the *pp* neutrinos, while the Super-Kamiokande experiment is practically unaffected.

Using Eqs. (4,3), it is possible to derive a corrected form of Eq. (1), by retaining in the expansion terms linear in γ:

$$P = 1 - \left(1 + \frac{\gamma}{4}\cos 2\theta\right)\sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right) + \\ + O(\gamma^2) \tag{6}$$

Notice that the first order correction contains $\cos 2\theta$ and hence is manifestly not invariant under the transformation $\theta \to \pi/2-\theta$. Using Eq. (5) with $r_0 = 3.8\times10^4$ km, we see that for the *pp* neutrinos ($E_\nu \leq 0.42$ MeV) this correction is indeed non-negligible already for $\Delta m^2 \sim 10^{-10} - 10^{-9}$ eV^2.

With matter effects being relevant already at $\Delta m^2 \gtrsim 10^{-10}$ eV^2 one might wonder if the separation between vacuum oscillation solutions and MSW solutions is somewhat artificial. To fix the terminology, we will adopt a definition of vacuum oscillations as the situation when the value of neutrino survival probability depends on the distance L from the Sun, regardless of whether matter effects are negligible or not. The transition between the vacuum and the MSW regions will be discussed shortly.

3. To illustrate the role of matter effects in vacuum oscillations, we present fits to the total rates of the Homestake [15], GALLEX [16] and SAGE [17], and Super-Kamiokande [18] experiments. We combine experimental rates and uncertainties for the two gallium experiments and use the latest available 825-day Super-Kamiokande

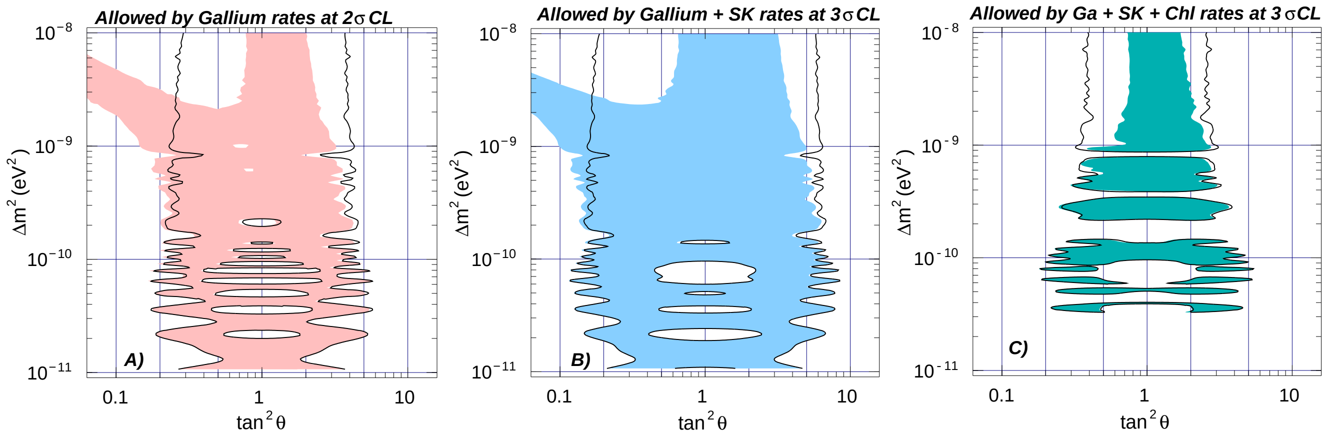

FIG. 2. Regions allowed by total rates of GALLEX and SAGE only (A), GALLEX, SAGE, and Super-Kamiokande (B), and GALLEX, SAGE, Homestake, and Super-Kamiokande (C). Black outlines correspond to neglecting the solar matter effects.

data set. The experimental results are conveniently collected and tabulated in [3].

We fit the data to the theoretical predictions of the BP98 standard solar model [19]. Predicted fluxes and uncertainties for various solar reactions were kindly made available by J. N. Bahcall at [20]. To compute the rate suppression caused by neutrino oscillations, we numerically integrate the neutrino survival probability, Eq. (3), over the energy spectra of the *pp*, ^{7}Be, ^{8}B, *pep*, ^{13}N, and ^{15}O neutrinos. In addition, to account for the fact that the Earth–Sun distance L varies throughout the year as a consequence of the eccentricity of the Earth's orbit

$$L = L_0(1 - \epsilon\cos(2\pi t/\mathrm{year})) \qquad (7)$$

we also integrate over time to find an average event rate. In Eq. (7) t is time measured in years from the perihelion, $L_0 = 1.5\times10^8$ km is one astronomical unit, and $\epsilon = 1.7\%$.

In Fig. 2 (A) we show the vacuum oscillation regions allowed by the total rates of GALLEX and SAGE. For comparison, we also show the regions one would obtain by neglecting the neutrino interactions with the solar matter (dark outlines), *i.e.* by setting $P_c = \cos^2\theta$ (black contours). The allowed regions were defined as the sets of points where the theoretically predicted and experimentally observed rates are consistent with each other at the 2σ C.L. for 1 d.o.f. ($\chi^2 = 4.0$) [21]. The plot demonstrates that the matter effects at the gallium experiments are quite important, with their contribution being significant for $\Delta m^2 \gtrsim 2\times10^{-10}$ eV^2.

The remaining two plots in Fig. 2 show the vacuum regions allowed at 3σ C.L. by the rates of GALLEX, SAGE, and Super-Kamiokande (B) (2 d.o.f., $\chi^2 = 11.83$, in the same convention as before), and all four experiments combined (C) (3 d.o.f., $\chi^2 = 14.15$). In order to properly account for the correlation between the theoretical errors of the different experiments, we followed the technique developed in [22] and [3]. The matter effects are noticeable for $\Delta m^2 > 6\times10^{-10}$ eV^2.

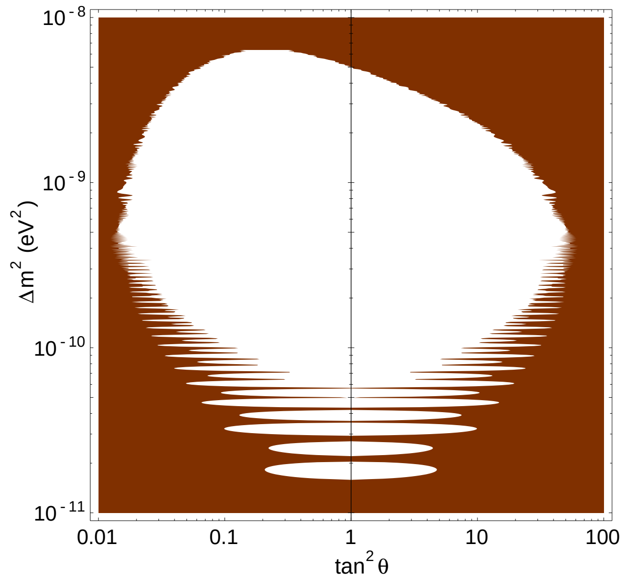

FIG. 3. The sensitivity region of the Borexino experiment to anomalous seasonal variations for the full range of the mixing angle (95% C.L.). Notice the asymmetry for large Δm^2.

4. An important question is how well future experiments will be able to cover vacuum oscillation solutions with $\theta > \pi/4$. In Fig. 3 we show the sensitivity of the Borexino experiment to anomalous seasonal variations for the entire physical range of the mixing angle $0 \le \theta \le \pi/2$. This is an extension of the analysis performed in [7], where the details of the procedure are described. The sensitivity region shows a clear asymmetry as a result of the solar matter effects.

Fig. 3 gives us an opportunity to discuss the extent of the vacuum oscillation region. There are two primary physical reasons why the neutrino event rate becomes independent of L (and anomalous seasonal variations disappear) for sufficiently large Δm^2:

- *Adiabatic evolution in the Sun.* As $P_c \to 0$ the last term in Eq. (3) vanishes.

- *Integration over neutrino energy spectrum.* To compute the event rate one has to integrate Eq. (3) over neutrino energies. For sufficiently large Δm^2 the last term averages out to zero, leading effectively to the loss of coherence between the two mass eigenstates.

As Δm^2 increases, coherence is first lost for reactions with broad energy spectra, such as *pp* and ^{8}B, and persist the longest for neutrinos produced in two-body final states. The most important such reaction is ^{7}Be+e^- $\rightarrow ^7$Li $+\nu_e$, which produces the ^{7}Be neutrinos. The ^{7}Be neutrinos have an energy spread of only a few keV, arising from the Doppler shift due to the motion of the ^{7}Be nucleus and the thermal kinetic energy of the electron. A detailed discussion of this phenomenon can be found in [7,5].

In order to properly take these effects into account, in our codes we numerically integrate over the exact ^{7}Be line profile, computed in [23]. As Fig. 3 shows, the neutrino survival probability becomes independent of L for $\Delta m^2 \gtrsim 6\times 10^{-9}$ eV^2. For this reason, we present our fits for Δm^2 ranging from 10^{-11} eV^2 to 10^{-8} eV^2. Unfortunately, in the literature vacuum oscillations are usually studied in the range from 10^{-11} eV^2 to 10^{-9} eV^2 [24,25,4], although the allowed regions in all these papers seem to extend above 10^{-9} eV^2.

5. In summary, the preceding examples clearly illustrate the importance of including the solar matter effects when studying vacuum oscillation of solar neutrinos with $\Delta m^2 \gtrsim 10^{-10}$ eV^2. Because to describe such effects one has to use the full range of the mixing angle $0 \leq \theta \leq \pi/2$, future fits to the data should be extended to $\theta > \pi/4$. This seems especially important in light of the latest analyses [25], [4], which in addition to the total rates also use the information on the neutrino spectrum and time variations at Super-Kamiokande. In this case the allowed vacuum oscillation regions are mostly located in the $\Delta m^2 \gtrsim 4\times 10^{-10}$ eV^2 region [4], precisely where the matter effects are relevant. (The best fit to the Super-Kamiokande electron recoil spectrum is achieved for $\Delta m^2 = 6.3\times 10^{-10}$ eV^2, $\sin^2 2\theta = 1$ [25].) It would be very desirable to repeat these analyses with the solar matter effects included.

Additionally, since the ^{7}Be neutrinos remain (partially) coherent for $\Delta m^2 > 10^{-9}$ eV^2, it is desirable to present the results of the fits in the range 10^{-11} eV^2 $<$ $\Delta m^2 < 10^{-8}$ eV^2, as was done in [2].

ACKNOWLEDGMENTS

I am very grateful to Hitoshi Murayama and John Bahcall for their support. I would like to thank John Bahcall for including in the BP2000 solar model the data for the outer regions of the Sun. I would also like to thank James Pantaleone, Plamen Krastev, M.C. Gonzalez-Garcia, and Yosef Nir for their valuable input. This work was in part supported by the U.S. Department of Energy under Contract DE-AC03-76SF00098.

[1] A. de Gouvea, A. Friedland, and H. Murayama, (1999), hep-ph/9910286.
[2] A. de Gouvea, A. Friedland, and H. Murayama, (2000), hep-ph/0002064.
[3] G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, (1999), hep-ph/9912231.
[4] C. Giunti, M. C. Gonzalez-Garcia, and C. Pena-Garay, (2000), hep-ph/0001101.
[5] S. Pakvasa and J. Pantaleone, Phys. Rev. Lett. **65**, 2479 (1990); J. Pantaleone, Phys. Rev. **D43**, 2436 (1991).
[6] For a good analysis of this phase see J. Pantaleone, Phys. Lett. **B251**, 618 (1990).
[7] A. de Gouvea, A. Friedland, and H. Murayama, Phys. Rev. **D60**, 093011 (1999), hep-ph/9904399.
[8] Since for vacuum oscillations neutrinos are produced in $|\nu_+\rangle$, the simple quantum mechanical argument is enough to give Eq. (3). For $\Delta m^2 \gtrsim 10^{-5}$ eV^2 the analysis is a bit more complicated. See, for example, [1], S. T. Petcov, Phys. Lett. **B406**, 355 (1997) [hep-ph/9910335].
[9] J. N. Bahcall, M. Pinsonneault, and S. Basu, to be published, http://www.sns.ias.edu/∼jnb.
[10] G. L. Fogli, E. Lisi, and D. Montanino, Phys. Rev. **D54**, 2048 (1996), hep-ph/9605273.
[11] Other possible choices include θ or $\sin^2\theta$ on a linear scale, as was done in [1]. The log scale here was chosen to unify the treatment with [2].
[12] S. T. Petcov, Phys. Lett. **B200**, 373 (1988).
[13] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).
[14] P. I. Krastev and S. T. Petcov, Phys. Lett. **B207**, 64 (1988), Erratum-ibid. **B214**, 661 (1988).
[15] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998).
[16] W. Hampel *et al.*, Phys. Lett. **B447**, 127 (1999).
[17] J. N. Abdurashitov *et al.*, Phys. Rev. **C60**, 055801 (1999), astro-ph/9907113.
[18] Y. Suzuki, talk presented at the XIX International Symposium on Lepton and Photon Interactions at High Energies, Stanford University, August 9-14, 1999.
[19] J. N. Bahcall, S. Basu, and M. H. Pinsonneault, Phys. Lett. **B433**, 1 (1998), astro-ph/9805135.
[20] http://www.sns.ias.edu/∼jnb/.
[21] Notice that this is different from the conventional approach (see, for example, [3,2]).
[22] G. L. Fogli and E. Lisi, Astropart. Phys. **3**, 185 (1995).
[23] J. N. Bahcall, Phys. Rev. **D49**, 3923 (1994).
[24] J. N. Bahcall, P. I. Krastev, and A. Y. Smirnov, Phys. Rev. **D58**, 096016 (1998), hep-ph/9807216.
[25] J. N. Bahcall, P. I. Krastev, and A. Y. Smirnov, Phys. Lett. **B477**, 401 (2000), hep-ph/9911248.