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Abstract

In problem solving situation, cognitive flexibility appears to be a major skill. Fostering cognitive flexibility is therefore a specific stake in mathematics education. This research introduces a learning method to develop mathematical concepts when solving word arithmetic problems. The study was conducted with 8 classes (4^{th}-5^{th} Grades) from high-priority education schools in the Paris area following this protocol: pre-tests, 5 learning sessions for experimental and control groups, post-tests. During learning sessions, students studied arithmetic word problems that can be solved in two different ways: an expansion strategy and a factorization one. The experimental teaching method, based on a recategorization principle, allowed experimental students to improve more than the control students in ability to use the factorization strategy even in contexts where it is the less intuitive and to consider the two successful strategies. Educational entailments of our finding are discussed.

Keywords: cognitive flexibility, evidence-based education, categorization, learning method, word arithmetical problem

Introduction

In mathematics, proposing flexible and adaptive representations and strategies reflects higher problem solving skills (Heinze, Star & Verschaffel 2009). We proposed to study not only strategies in algebra problems but also the related representations derived from word problem. Indeed when solving word problems, novices intuitively induce a superficial structure, triggering a misleading categorization of the situation (Chi, 2008). The present study aimed to improve pupil's cognitive flexibility in problems solving in order to develop an expert categorization on problems that reflects a better mastering of the underlying mathematical notions. Students are encouraged to reelaborate the notion's representation, which leads to recategorize it. Based on recategorization principle, this method is applied on problems involving the distributive property. The distributive property problems admit two solving strategies whose preferential use depends on the representation built by the solver.

An induced representation

Phrasing of mathematical word problems can influence the induced representation of the problem by students (Vergnaud, 1982; Hudson, 1983; De Corte, Verschaffel & De Win, 1985). But in addition to linguistic features, semantic effects also rely on semantic relations or scenario depicted in the problem: when solving a word problem, students build a mathematical representation based on semantic relations inferred from real-word objects (Bassok & Olseth, 1995, Bassok, Chase & Martin, 1998). For instance, a problem involving apples and baskets is likely to evoke the asymmetric "contain" relation. So students align this semantic relation with structurally analogous mathematical relations: apples and baskets support the semantic relation contain (content, container) and thus the mathematical relations of division (dividend, divisor). This spontaneous encoding of problem situations results from the properties and relations of the entities or objects depicted in a problem. Semantic alignment, namely alignment between the semantic and mathematical relations, influences the difficulty of mathematical problems.

A way to study this effect of semantic content on the spontaneous encoding is to use problems solvable by two strategies. Indeed the semantic context can influence the encoding of the problem and thus lead to a preference for one of the two strategies. Several studies showed that the variable involved in the problem impacts the problem's representation built by the solver (Bassok et al., 1995; Vicente, Orrantia &Verschaffel, 2007; Gamo & Sander, 2010). In Gamo et al.'s study, 4th and 5th grade students had to solve isomorphic problems involving one of the three following variables - the number-of- elements, price, and age –. This type of problem *("Antoine took painting courses at the art school for 8 years and stopped when he was 17 years old. Jean began at the same age as Antoine and took the course for 2 years less. At what age did Jean stop?")* can be solved by two strategies: a ''complementation'' strategy (in three steps: *17 – 8 = 9; 8 – 2 = 6; 9 + 6 = 15*) and a ''matching'' strategy (in one step: *17 – 2 = 15*). But the variable involved in the problem fosters one of the two representations of the problem: (a) a part-whole schema that underlies unordered units which triggers the computation of the difference between the part and the whole given in the first half of the problem; and (b) a comparison schema that underlies ordered units. Number-of-elements problems are spontaneously encoded according to the part-whole schema and lead to the complementation strategies whereas the age problems foster the matching strategy but not exclusively.

Cognitive flexibility in problem solving

Even after instruction, non-relevant representations remain: experts do not systematically use the most efficient strategy when solving arithmetic problems (Star & Newton, 2009) even though they master it. Therefore, arithmetic problem solving raises the question of the influence of problem representations on the possibility to choose flexibly the most efficient strategy. Cognitive flexibility seems to be critical while solving problems (Clement, 2006). Indeed, it refers to the ability to select adaptively among multiple representations of an object, perspectives or strategies in order to adjust to the demands of a situation (Cragg & Chevalier, 2012; Diamond, 2013). Through the problem of 'water-jug measuring problems' (Luchins, 1942), Clement (2006) proposed the concept of representational flexibility in problem solving: following an impasse situation, individuals recode the situational properties and adopt a new representation that leads to transfer the right strategy. Hence, cognitive flexibility is related both to abstraction and transfer. Cognitive flexibility can therefore be measured through the mastering of multiple strategies and of their appropriate use (Rittle-Johnson & Star, 2007; Star & Seifert, 2006). Students with high flexibility in problem solving are more likely to adapt existing strategies when faced with unfamiliar transfer problems and to better understand domain concepts (Hiebert & Wearne, 1996; Rittle-Johnson & Star, 2007).

Recategorization in problem solving

Studies on cognitive flexibility in mathematics focused either on interpretation of the situation or on strategies (Heinze et al., 2009). Being able to adopt a multiplicity of categorization makes it possible to change point of view according to the needs of the situation. For example, a physicist who sees a glass falling down does not need to categorize a glass as a body under the law of gravitation and on which forces are exerted. Categorizing a glass only as "an object made of a fragile material" is sufficient to act in the appropriate way, namely to catch up the glass. Thus, the more an individual diversifies his repertoire of categorization, the more he is able to adopt different perspectives. By articulating different points of view on the same situation, the individual can embrace its complexity (Hofstadter & Sander, 2013).

In the present study, we proposed to focus on recategorization as a mechanism to recode a representation and transfer the adapted strategy to a new context. Evidences from social psychology showed that if an individual seems to be inconsistent with his/her category membership, perceivers would integrate other information and recategorize the individual in the newly applied category (Gawronski & Creighton, 2013). When it comes to solving problems, a same situation or entity can be categorized at different levels of abstraction in multiple ways. The categorization adopted has been identified as an indicator of expertise. For example, in physics, novices categorize problems according to the objects used (problems of pulley or inclined planes), while experts categorize problems according to the physical principle (e.g. Newton's third law) (Chi, Feltovich, and Glaser, 1981, Chi et al., 1989). Unlike the experts, the novices therefore construct their categories mainly on the basis of superficial information, such as specific objects (Schoenfeld & Herrmann, 1982). Experts rely on a greater number of categories and levels of categorization than novices to represent a situation. Since novices use some superficial cues, they can make negative transfers, using an irrelevant strategy by analogy with a problem that share the same superficial traits (vocabulary, object, theme) (Chen, 2002). Teaching to recategorize in a relevant manner seems to be a lever to develop students' ability to transfer strategies.

Thus, training cognitive flexibility is a challenge for developing learning method. Teaching experiments in mathematics mainly studied number calculations: multi-digit addition and subtraction (Carpenter et al., 1997), decomposition (Klein, Beishuizen, & Treffers, 1998) and linear equations (Rittle-Johnson & Star, 2007). Evidences have therefore been obtained for the algorithmic aspects but are sparser when it comes to word problems. One method consists in comparing two strategies for the same problem (Brissiaud, 1994). Gamo et al. (2010) proposed a training method in order to develop mathematical concepts through the semantic recoding of the word problem. The principle is to recode the semantically induced structure into a more apt mathematical structure. By recoding the problem, the students adopts a new point of view, which leads them to develop a representation of the problem that corresponds to the mathematical structure and succeed to use more expert solving strategies. In Gamo et al.'s study, students in Grade 4 and 5 had to solve problems sharing the same deep structure but being spontaneously categorized as problems solvable by complementation strategy and not by the most efficient one (the matching strategy). During the training session, students compared the problems to stress the common structure. At post-test, students improved their use of matching strategy. Whereas interpretation initially realized at a level of abstraction based on the semantic structure, students acquired an additional degree of abstraction based on the mathematical structure, after semantic recoding. Comparison fosters a more abstract representation of the problems.

The present study: a training method based on recategorization

Because of the lack of understanding of abstract ideas, when they are not operationalized (Willingham, 2009), and the difficulties of transferring solving strategies (Ross, 1984), the learning method to develop students' cognitive flexibility is applied in a specific school context: word arithmetic problems on the distributive property. This type of problems -listed in the French curricula in 4^{th} and 5^{th} grade- has the methodological interest to be solvable by two strategies. Moreover, the nature of the variable involved in the problems favor one of the strategies (expansion or factorization) (Sander, 2008; Moreau & Coquin-Viennot, 2003).

The main goal of this method is to allow students to overcome the spontaneous encoding of problem situations. In order to develop learning methods favoring abstraction, while taking into account the difficulties of transfer, the training of recategorization was conducted through a semantic analysis and was supported by an explicit method built with students. This method consisted in allowing students to switch between two conceivable points of view on the same situation. Then they were prompted to choose their own one. This choice of point of view by students is related to a reflexive level and is consistent with previous work (Siegler, 1999; Blöte, 2001) that stressed the flexible use of strategies and encouraged students to think about the value of different procedures for solving a given problem. The different steps of the experimental training are detailed in *Method, Training sessions*.

Hypothesis

We therefore hypothesized that the experimental training method should favor students' cognitive flexibility on a mathematical concept- the distributive property- involved in arithmetic word problems.

Students should be able to adopt the two points of view on the problem and use the two strategies (expansion and factorization) without depending on the semantic context. For the training problems, no significant difference in factorization and dual strategies use between the two groups should be observed, since they are both trained to solve this kind of problems (Hypothesis 1). Yet, the experimental method based on recategorization should favor far-transfer compared to the traditional method. Thus the experimental group should propose significantly more factorization and dual strategies than the control group at the post-test and higher progression should be observed for the experimental group for the non-trained problems (Hypothesis 2).

Method

Participants

The experiment was conducted with eight classes from four elementary schools located in high-priority education network in Paris region during regular classwork school hours. 142 students took part in the study: 74 were fifth graders and 68 were fourth graders (mean age=10 years and 3 months, SD=6 months, 78 boys, 64 girls).

The experimental group included 66 students (37 5th graders from 2 classes and 29 4^{th} graders from 2 classes. The control group included 76 students (37 5^{th} graders from 2 classes and 39 were 4th graders from 2 classes).

Design

The experiment included three phases: pretest, training sessions, post-tests. The pre- and posttest were strictly identical. Both the experimental group (EG) and the control group (CG) followed training sessions taught by the experimenter. Within each group, training sessions were identical in their duration, organization and problems statements.

Material

Pre and post-tests

The material was composed by 8 isomorphic distributive word problems (Table 1) and 5 filler problems. Indeed, filler problems were proposed between distributive problems, in order to make the structural similarities between the distributive problems less salient for the students.

Each distributive problem describes a situation involving one factor and three summands. The final question whose structure is “How much/many … in all?” was placed at the end of the text. Two main solving strategies make it possible to reach the solution: Expansion strategy (sum of each part multiplied by the factor: 4x6 + 4x7 + 4x8) and Factorization strategy (sum of the parts, then multiplied by the factor: (6+7+8) x4). We selected four different variables (Numbers, Duration, Price, Weight). For each variable, a statement whose summands are said specific categories and a problem whose summands are called general categories were proposed (Table 1). The summands of the specific category problem are grouped at a base level, while those of the general category problem at a more abstract level than the base level (in the context of a treat cone, 3 balloons, 8 cookies, 4 figurines or 7 lollipops, 8 8 candies, 3 chocolates). So summing them as a whole is easier for specific than general category, and could influence the strategy.

The distributive problems were constructed by controlling the familiarity of the vocabulary and the numerical values at stake. The numerical values had different features in common in order to limit numbers' effects. Factor value was between 4 and 8 (5 was excluded since the associated multiplication table is easier). The three summands were between 2 and 8 (5 is excluded). Their sum lied between 11 and 21. And the result was inferior to 100 (between 72 and 98) in order to control the level of calculation difficulty.

A pedagogical advisor of the French National Education was involved in the conception of all sessions, in order to assure ecological material and ecological pedagogical acts. Therefore, 8 booklets were constructed by controlling the numerical values, the order of presentation of the problems variable and problems versions. On each page of the booklet, the problem was presented in written form with two sections in order to propose two strategies. The instruction for solving the problem with two strategies was both orally given by the experimenter and written on each page:

Variables	Duration	Numbers	Weight	Price
Context	X has made a list of purchase for y years. Each year, X's purchases are:	X wants to prepare a treat cone per child. There are y children. Making a treat cone requires the following items:	X wants to fill his/Her pencil case with items that weight y grams each. In the pencil case, there are:	X is at the checkout of a supermarket. He bought some items. For each item, he took y in his basket:
Specific Categories	Printers Computers Scanners	Lollipops Candies Chocolates	Pens Gums Markers	Buns Cakes Pies
General Categories	Microscopes Desks Hamsters	Balloons Cookies Figurines	Shells, Key-rings Candies	Ice Plants Plates
	In all, how many purchases has X bought?	In all, how many objects does X need?	In all, how much does these items weight?	In all, how much did X spend?

Table 1: The 8 problems at pre and post-tests

Write all your calculations and the result in the following section:

Do you see any other method to come up with the same result? If yes, write it down while writing all calculations you performed to find the result.

Tests lasted 45 minutes. Students were given 3 minutes to solve each problem. Students were instructed that they could ask the experimenter or the teacher to read aloud the problem in order to bypass reading difficulties, and that they had to write down all calculations. When the time was over, students had to turn the page and begin the new problem when the experimenter gave the instruction. They could not modify their answer once they turned the page.

Training sessions

Training sessions took place in 5 sessions over 5 weeks (45 minute session each week) for each class (Table 2).

Usual French textbooks inspired the pedagogical method used by the control group for Grades 4 and 5 (*Vive Les Maths* and *Companion Math*). The experimental method was built for this study. The two approaches did only differ in their treatment of the problem. In the control group, students learnt to select relevant information in statements and choose operations. And the experimental group looked for the semantic relations (the sum of the parts forms a whole) and chose the point of view it wished to adopt. For instance, one of the training problem was the following: *« A team of 5 athletes participated in a relay: each athlete ran on a loop of 8 km, then on a straight line of 2 km and finally on a loop of 6km. How many kilometers has the team traveled? »*

To find the number of kilometers, two strategies are possible. Experimental group learnt to choose between:

- adopting the point of view of each part of the relay (loop and line): each loop/line is a separate part and we realize an expansion strategy: 5x8 + 5x2 + 5x6 = 80km
- adopting the more abstract point of view of the relay: the different parts (loops and line) form the relay, we carry out a factorization strategy: (8 + 2 + 6) x 5 = 80 km

Thus whereas categorizing each addend as a part leads to an expansion strategy, categorizing them as a whole leads to a factorization strategy.

Regarding all other aspects, session organization was similar between the two groups: students began by exercising on the slate, in order to engage them in the task. Then the students had to answer on an exercise sheet, whose support was also projected on the blackboard. Finally, students ended the session by answering the question "What did I learn today? " and then a general conclusion was proposed by the experimenter and was written by the students. The distributive problems studied in the sessions 3, 4 and 5 were identical between the two groups and involved only two types of variables: Number-of-Elements and Distance.

Coding and scoring

For pre and post-test, problems were analyzed under two criteria:

- the use of factorization strategy (correct reasoning and calculation) to solve the problem as a first or second strategy
- the use of double strategy (reasoning and correct calculation)

Then a global improvement score was calculated. At pre and post-test, each problem was coded as 1 when it was solved by an expansion, by 2 when it was solved by a factorization, by 4 when it was solved by dual-strategies and 0 if otherwise. Then the difference between post and pre-test was computed. Each student got therefore an improvement score, reflecting his/her progress between the pre and post-test.

Results

Hypothesis 1 stated that the frequency of factorization and dual-strategies by students was not expected to be different between the two groups for the training problem (Number-of-elements) due to the effect of training in both groups. The improvement score was 0.34 for the control group and 0.37 for the experimental group (p>0.5) for factorization strategy and 0.30 for dual-strategy for each group (p=1).

	Experimental Group	Control Group
Session 1	A problem: way of seing a situation, parts and whole	A problem: a question, useful data, operations
Session 2	Multiplication and commutativity by semantic recoding	Multiplication and commutativity by repeated addition
Session 3	Distributivity: semantic relations and choice in point of view	Distributivity: useful data and choice in operations
Session 4	Dual strategies by change in point of view	Dual strategies by equivalency of procedures
Session 5	Distributivity problems: choice in points of view	Distributivity problems: useful data, operations

	PreTest		PostTest	
	Factorization	Dual Strategy	Factorization	Dual Strategy
CG	0.14 (0.24)	0.05 (0.14)	0.47 (0.40)	0.36 (0.40)
EG	0.18 (0.27)	0.07 (0.18)	0.63 (0.38)	0.47 (0.43)

Table 2: The training sessions

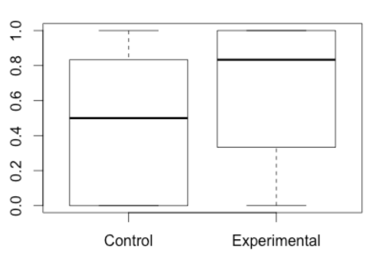
Table 3: Means (and SD) of factorization and dual strategy frequency by students at pre and post-test in use of in the experimental group and control group

This absence of difference between the two groups for the trained problems reflects the similarity in terms of learning method between the two groups: they both learned to use dual-strategies (expansion and factorization) for the distributive problems. The control group focused on relevant information in statements, digits and operations whereas the experimental group focused on semantic relations, words and point of view. At the pretest, repeated measure ANOVAs with group as the between factor and problem variable as the within-subjects factor showed that there was no difference regarding the use of factorization ($F<1$, ns) or dual strategies, ($F < 1$, ns). Both groups showed the same pattern of choice of strategies for each type of problem.

Hypothesis 2 stated that a better transfer should appear in the experimental group for non-training problems. At the posttest, reapeated measure ANOVAs with group as the between subjects factor and problem variable as the within-subjects factor were performed: the experimental group was significantly superior to the group control for the use of factorization ($M=0.63$ vs $M=0.47$ $F(1,140) = 6.15$, $p=0.01$) and we observed a marginal trend for the use of dual strategies ($M=0.47$ vs $M=0.36$, $F(1,140) = 2.72$, $p=0.1$) (Figure 1 and Table 3).

Then we analyzed the improvement score. The global improvement score raised 1.46 for the experimental group and the control group's one raised 1.10. A repeated measure ANOVA was performed. The difference in improvement between the groups got a statistically significant trend ($F(1,140)=3.5$, $p=0.06$). For problems from general category, we observed an improvement score by 1.59 for the experimental group compared to 1.10 for the control group. A repeated measure ANOVA was performed (Table 4). Therefore, the improvement score for the more abstract problems (general category) was significantly higher for the experimental group ($F(1,140)= 6.12$, $p=0.014$) .

Figure 1: Mean in factorization by students at Post-Test



Discussion

The learning method based on both the resolution strategy comparison and the explicit analysis of semantic relations during classroom activities showed its success in promoting transfer. The experimental group was more successful in transferring the factorization strategy and dual strategies to non-trained problems in the post-test. The Price, Duration and Weight variables in post-test problems had not been trained during the learning sessions. The progression in terms of factorizing strategy and dual strategy suggests that the experimental group became less dependent on the choice of the variable than the control group. That means that the experimental group shows a greater ease in independence to context. They were successful switching from the spontaneous representation influenced by the variables of the problem to flexible representation based on the mathematical structure. Since progress for trained problems are similar between groups, the added value of the recategorization method lies in the success of far transfer.

The use of isomorphic problems made it possible to identify more precisely the robustness of the transfer effects from the learning method. Indeed as we studied the non-trained problems, the greater progression for the experimental group shows that the training was not superficial. This transfer reflects a semantic change by students that could adopt a double point of view on the problem.

In addition, our findings support the work of Vicente et al. (2007) who pointed out that the difficulty for students lies in developing the conceptual relations between the entities of the problem. Thus, in their study, the success rate of problems whose rewording shed light on "part-whole" relationships was higher than problems with additional information about the problem's situation. The properties and relations of the entities or objects depicted in a problem are therefore key in the choice of strategies. In our study, we did not use a conceptual rewording that underlines the underlying semantic relations but the experimental method consisted in orienting students to establish these relations because their categorizations of the elements of the problems were based on them.

Conclusion

The students from the training group became less dependent from semantic context. Their choice of strategy was less constrained by the nature of the variable. The substantial transfer of the non-preferred strategy (factorization) illustrates the ability to adopt a new point of view on the situation. Thus students were able to change their encoding based on spontaneous representations to an encoding based on conceptual relations. To adopt this flexible and multiple points of view on a problem, the training method based on recategorization seems to be promising. In addition to improve semantic analysis, students were encouraged to adopt a reflexive attitude thanks to the notion of point of view. Thus students developed their cognitive flexibility: developing flexible strategies with the ability to transfer them to new problems. Yet studying the extent of this transfer could be the goal of further research. The teaching method appears to be a useful framework to identify if cognitive flexibility is domain-general or domain-specific. Indeed, fostering cognitive flexibility takes part of a broader goal, namely promoting conceptual development.

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References

Bassok, M., & Olseth, K. L. (1995). Object-based representations: Transfer between cases of continuous and discrete models of change. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *21*(6)

Bassok, M., Chase, V. M., & Martin S. A. (1998). Adding apples and oranges: Alignment of semantic and formal knowledge. *Cognitive psychology*, vol. 35, no2, 99-134.

Blöte, A., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems. *Journal of Educational Psychology*, *93*(3), 627.

Brissiaud, R. (1994). Teaching and development: solving 'missing addend' problems using substraction. *European Journal of Psychology of Education*, 9, 343-365

Chen, Z. (2002). Analogical Problem Solving: A Hierarchical Analysis of Procedural Similarity. Journal of Experimental Psychology, 28, 81-98

Chi, M. T., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science*, 13, 145–182.

Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive science*, *5*(2), 121-152.

Chi, M. T. (2008). Three types of conceptual change: *International handbook of research on conceptual change*, 61-82.

Clément, E (2006). Approche de la flexibilité cognitive en résolution de problème. *L'Année Psychologique, 106*

Cragg, L., & Chevalier, N. (2012). The processes underlying flexibility in childhood. *The Quarterly Journal of Experimental Psychology*, *65*(2), 209-232.

DeCorte, E. Verschaffel, L., & De Win, L. (1985) Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology*, 77. 460-470

Diamond, A. (2013). Executive functions. *Annual review of psychology*, *64*, 135-168.

Gamo, S., Sander, E., & Richard, J-F. (2010). Transfer of strategies by semantic recoding in arithmetic problem solving. *Learning and Instruction*, *20*, 400-410.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology,* 12, 306-355.

Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and instruction*, *14*(3), 251-283.

Hofstadter, D., & Sander, E.(2013). *Surfaces and Essences*New York, Basic Books.

Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. Child Development, 54(1), 84–90.

Klein, A. S., Beishuizen, M., & Treffers, A. (1998). The empty number line in Dutch second grades: Realistic versus gradual program design. *Journal for Research in Mathematics Education*, 443-464.

Lautrey, J., Rémi-Giraud, S., Sander, E., & Tiberghien, A. (2008). *Les connaissances naïves.* Armand Colin.

Lusk, Cynthia M., and Charles M. Judd. "Political expertise and the structural mediators of candidate evaluations." *Journal of Experimental Social Psychology* 24.2 (1988)

Richland, L. E., & Simms, N. (2015). Analogy, higher order thinking, and education. *Wiley Interdisciplinary Reviews: Cognitive Science*, *6*(2), 177-192.

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*(3), 561.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. Journal of Experimental Psychology. Learning, Memory and Cognition, 8, 484.

Siegler, R. S. (1999). Strategic development. *Trends in Cognitive Sciences*, *3*(11), 430-435.

Star, J. R., & Newton, K. J. (2009). The nature and development of experts' strategy flexibility for solving equations. *ZDM*, *41*(5), 557-567.

Star, J. R., & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology*, *31*(3), 280-300.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, *18*(6), 565-579.

Vergnaud, G. (1982). A classification of cognitive tasks and operations of 1195 thought involved in addition and subtraction problems. In T. P. Carpenter, 1196 J. M. Moser, & T. A. Romberg (Eds.), Addition and subtraction: A 1197 cognitive perspective (pp. 39e59). Hillsdale, NJ: Erlbaum.

Vicente, S., Orrantia, J., and Verschaffel. L., Influence of situational and conceptual rewording on word problem solving. *British journal of educational psychology* 77.4 (2007): 829-848.

Willingham, D. T. (2007). Critical thinking: Why is it so hard to teach? *American Educator*, 8–19.