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Publication Date

2023

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Design, Fabrication and Analysis of Bending-active Hybrid Gridshells

by

Yasaman Yavaribajestani

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Architecture

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Associate Professor Simon Schleicher, Chair

Professor Khalid M. Mosalam

Professor Claudia P. Ostertag

Fall 2023

Design, Fabrication and Analysis of Bending-active Hybrid Gridshells

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Abstract

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Associate Professor Simon Schleicher, Chair

Concrete is the most widely used material in the world and architecture would be hard to imagine without it. It is valued for its durability, strength, and minimal maintenance and it handles almost every climate. However, the production of concrete and its excessive use for a wide variety of applications in the construction sector is also responsible for 8% of global CO₂ emissions. This number is alarming and calls for an urgent rethink towards a more economical, intelligent, and environmentally conscious use of concrete. However, architectural history, and in particular the development of concrete shells, has shown that it is possible to build efficiently and achieve large spans even with minimal material. While this approach offers tremendous potential for more sustainable construction in the future, shell structures also present some challenges. They typically require excessive scaffolding and labor, resulting in high building costs and significant waste of scaffolding materials, limiting their successful implementation in the construction industry. A renaissance of concrete shells and thus a possible solution for more sustainable construction depends on whether these challenges can be overcome.

Motivated by these pressing questions, this dissertation focuses on the development and analysis of a novel method for the design and fabrication of concrete shells using a stay-in-place bending-active formwork. As part of this dissertation, a feasibility study is conducted to validate the constructability of shells and gridshells using this method. Furthermore, by conducting physical tests, this study shows how the use of CFRP as stay-in-place formwork affects the structural performance of the built shells. Another contribution of this work is a digital study on the influence of the geometrical parameters of such hybrid gridshells produced using bending-active formwork on their structural performance, both from a local and a global point of view. The findings presented in this dissertation provide valuable insights that can be incorporated into the design and early decision-making process of hybrid gridshells. By addressing the challenges associated with the sustainability and design of concrete shells, and by introducing the novel approach of using bending-active CFRP formwork for the design of concrete gridshells, this research contributes to promoting and facilitating the adoption of these material-efficient structures in the construction industry.

To my dear family ...

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Acknowledgments

First and foremost, I would like to thank my Ph.D. advisor, Professor Simon Schleicher, for patiently guiding me throughout my path and empowering me to grow academically and personally. It has been a great honor and learning experience to work under his supervision. Simon has always provided me with the freedom to explore different directions and to follow the one that attracts me the most.

I would like to express my great gratitude to my Ph.D. committee members, Professor Khalid Mosalam, for his constant support in enabling me to cherish and maintain the valuable architectural and structural heritage of my origin and Professor Claudia Ostertag, for her kind support and being a remarkable role model for me as a woman in the engineering field. I would like to thank Professor Gary Black, for believing in me and giving me the opportunity to start my teaching journey and Professor Luisa Caldas for her valuable feedback during my qualifying exam.

I'm greatly thankful to Michael Herrmann, for our fruitful research discussions and his valuable and on-the-point feedback on my research.

I'm also thankful to the entire academic community and my fellow students and colleagues, who through their companionship and exchange of ideas, have enriched my academic journey.

I want to acknowledge my dearest friends at Berkeley, Ferreshteh, Negar, Maryam Sh, Ameneh, Mohaddeseh, Maryam M, Zeinab, Vahid, Payam, Reza, Ali, Mohammad, and Heidar, whose presence made our experience extremely delightful and an unforgettable one. They taught me how true friends support each other during all the ups and downs of a challenging path.

I would like to express my deepest gratitude to my mother, for her unconditional love and support during the most challenging moments of my journey. Her love has given me the strength to overcome obstacles and keep moving me forward. I would like to extend my heartfelt thanks to my father, who from my childhood, instilled in me the idea of pursuing a Ph.D., encouraging me to dream big and reach for ambitious goals. Thank you, Dad, for being my greatest cheerleader.

I'm thankful to my lovely sisters, Mojgan and Maryam, and my brothers-in-law Amir and Mehdi. Thanks to Mojgan, for always making me see the bright side of things and putting a smile on my lips, even during the most challenging times, and Maryam, for consistently pushing me to ignore small distractions and rationally focus on my goals. Thanks to my little ones, Artin and Lydia for always making me the happiest "Khale Yasman", Aunti.

My deepest appreciation to my mother and father-in-law, Zahra Javid and Parviz Keshavarzi, for being a constant source of unconditional support and inspiration. Fatemeh, I am truly grateful for the way you make me feel like an older sister.

Finally, no words can express my appreciation to my beloved and wonderful husband, Mohammad. You made a new definition of love, support, and inspiration in my mind. You have made every minute of my life so colorful and joyful and I'm forever grateful to have you in my life. I wouldn't be here without your endless love and support. This has been an amazing ride and we made it together while constantly holding our hands. As always, let's continue together on this wonderful journey of ours!

Chapter 1

Introduction

1.1 Concrete and Carbon Footprint

Concrete is the most widely used building material in the world [1] and has played an essential role in architecture for centuries, from pre-history to modern times. A remarkable instance of such architectural achievements through the use of concrete is the Pantheon dome (118 CE-128CE) which continues to hold the record for the world's largest unreinforced concrete dome to this day. In the building industry today, concrete is used in three major categories: (i) volumetric foundations; (ii) walls, columns, building cores, etc ; (iii) thin concrete slabs and shells. The high consumption of this man-made material is the result of its low cost, widespread availability, unique structural properties, and great durability. Despite the ubiquitous success of this building material in the past centuries, however, there has been a growing concern recently about high CO₂ emissions and excess water usage in concrete construction. As a matter of fact, almost 40% of the total anthropogenic CO₂ emissions are related to the construction sectors, 8% of which is due to the production and excessive use of concrete alone [2], [3]. This is due to the fact that in the cement production process, which is one of the main ingredients needed for the production of concrete, materials need to be heated at very high temperatures, demanding large amounts of energy input- mostly sourced from fossil fuels. While there have been major advances in developing more sustainable concrete materials such as fly ash concrete, and rice husk ash concrete which use less cement in their mixes, it is essential to take action in lowering related carbon footprint by exploring how concrete structures can be designed and constructed more efficiently. An important way to achieve this is to use as little concrete as possible and to build structures with it that are as strong and load-bearing as they can be. A good example is concrete shells.

1.2 Concrete Shells: Challenges and Opportunities

Concrete shells can be used to build lightweight and material-efficient roof structures that efficiently cover large spans. These types of structures are considered an inspiring field of study as they demonstrate how concrete can be used in the context of structurally efficient and environ-

mentally responsible designs. In recent decades, there have been many advances in the design and construction of such structural systems, which have led to their growing popularity. During the World Wars, when conventional building materials were scarce, concrete became popular as a versatile construction material for various purposes, ranging from concrete ships to plant pots. With low labor costs during that time, the high availability of concrete, and its low cost, concrete shell construction gained popularity as a building solution.

Despite all the economic cost savings and material efficiencies that concrete shells offer, they present a variety of design and construction challenges. The load-bearing capacity of a concrete shell is mainly driven by its geometry and in particular by the possibility to produce concrete shells with high curvatures. However, to pour concrete in curved geometries, excessive falsework, which is a temporary construction to support the permanent structure and formwork until the concrete becomes self-supporting, is required. This becomes particularly crucial because formwork is responsible for retaining plastic concrete in its desired shape until it solidifies. Depending on the complexity of the geometry, it can be the case that the formwork for a concrete shell is not only particularly difficult to manufacture, but also requires scaffolding and consumes many times more material than the concrete shell itself. In particular, in the case of long-span shell structures, the installation of large-scale falsework leads to labor-intensive construction sequences. As shown in Figure 1.1, studies show that more than 80% of the total production cost of curved concrete structures is associated with their falsework [5]. The high cost of fabricating and building falsework, the high cost of labor, and the high waste associated with disposing of the falsework and formwork after the concrete shell is successfully poured can make this construction process extremely inefficient. All these are the reasons why this type of construction, after a peak in the 60s, has declined somewhat in recent times. However, with the increasing demand for material and environmentally friendly construction in times of climate change, it is interesting to revisit concrete shells for their material efficiency and high performance and try to better solve the associated challenges. Therefore, the development of new methods for the fabrication of lightweight concrete shells can provide a useful

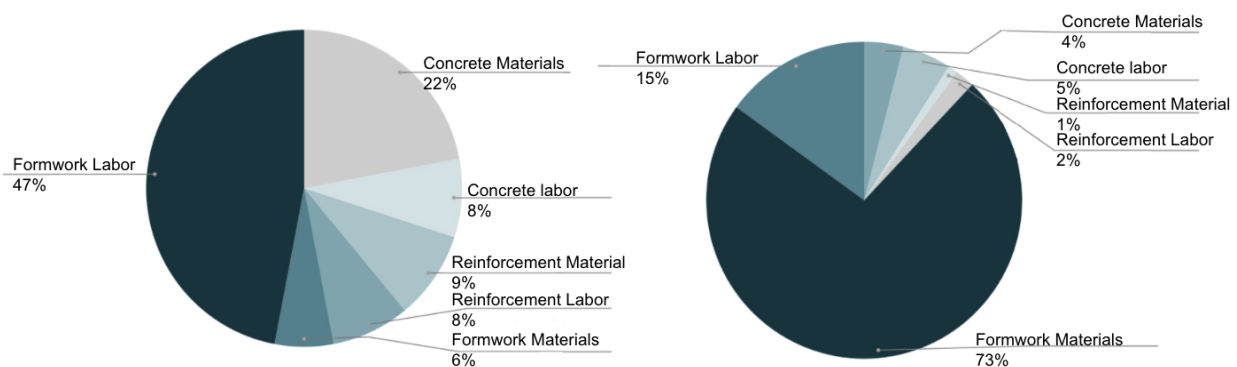


Figure 1.1: Distribution of construction costs of (left) standard and (right) non-standard concrete elements. Adapted from [4].

contribution to the revival of concrete shells and the expansion of our toolbox for environmentally friendly structures.

1.3 Gridshells: Properties and Performance

Gridshells are structures with the shape and strength of a double curvature shell but made of a grid instead of a solid surface. Nowadays, preference is given to using gridshells made from discretized parts instead of dealing with some of the downsides of concrete shells. Gridshell structures can cover large spans while creating openings with an efficient amount of material. Various materials can be used for the construction of gridshells such as steel, aluminum, wood, or even cardboard tubes [6]. Gridshells, in comparison to continuous shells, have both aesthetic and structural advantages. For instance, due to their openings, more natural light can be introduced into the building, creating more comfortable and well-lit spaces [7].

1.4 Elastic Gridshells and Bending-active Structures

Elastic gridshells are a particular subgroup of gridshells, which follow the principles of bending-active structures, where an elastic deformation of initially flat continuous grid results in a three-dimensional gridshell structure [8]. These large deformations are enabled by the comparably low cross-sectional height of the slats, which makes the members easy to bend in a preferred axis. Elastic gridshells can cover long spans with low self-weight and allow for a variety of aesthetical shapes. The shape transformation process facilitates the fabrication and construction of the structure compared to rigid-member gridshells. Since the grid is initially flat rather than three-dimensional, the beams constituting the grid can be constructed straight and the connections between these are both easier to manufacture and assemble [9]. The simplicity of construction of these structures and their ability to generate complex geometrical forms, make them an interesting solution to the problem of conceptualizing formwork for the construction of concrete shells.

1.5 Hybrid Gridshells: Using Bending-active Formwork

In summary, as mentioned above, in conventional practice, rigid timber elements have been used as the falsework for the construction of concrete shell structures. Typically, after the construction of the concrete shell is finished, often the formwork is removed. However, more recently, due to sustainability and cost concerns, a paradigm shift away from using this method in research has been observed. In this context, other promising and more sustainable alternative formwork methods have emerged. In particular, in the realm of flexible formworks, the bending-active structures have shown great potential to be implemented in the construction of free-form concrete shell structures. Certain traditional construction materials such as timber already exhibit an appropriate elastic range to be implemented as bending-active formwork. Furthermore, fiber reinforced polymers (FRP), owing

to their low density and high strength combined with low bending stiffness, are other potential materials for lightweight bending-active structures.

Another key challenge when working with concrete is its relatively low tensile strength compared to its high compressive strength. Conventionally, this has been addressed by installing steel reinforcing bars, which results in thicker cross-sections. In recent decades, FRP composites have gained increasing recognition and application for tensile reinforcement of concrete, both externally and internally, due to their notable mechanical properties and chemical resistance. For instance, carbon fiber reinforced polymers (CFRP) have been used as a suitable external reinforcement for retrofitting concrete bridges due to their high tensile strength.

Motivated by the challenges and opportunities mentioned in this chapter, this dissertation aims to explore the potential of using CFRP bending-active structures as a stay-in-place formwork for concrete gridshells construction. The CFRP strips will both serve as formwork and potentially as external reinforcement. Such an approach can potentially drastically simplify the construction process of concrete gridshells and result in a lightweight hybrid structure with high load-bearing capacity and material efficiency thanks to the combination of the two materials, where the concrete contributes the compressive strength and the lost CFRP formwork contributes the tensile strength.

This study aims to introduce a holistic approach to the design and fabrication of hybrid gridshells based on bending-active formwork by investigating and modeling material properties, cross-sectional compositions, and fabrication strategies. The fabrication challenges of the proposed workflow are studied through feasibility studies and physical prototyping of the concrete gridshell structure to ensure practical viability and real-world applicability. Furthermore, the dissertation will include developing a performance-based generative design system, to facilitate the design and form-finding of high-performance bending-active systems for hybrid gridshell structures. The effect of different geometrical and material parameters on the performance of the gridshells will also be explored in this work.

1.6 Research Focus and Objectives

This dissertation aims to make three main contributions by focusing on three sets of challenges: 1) developing a suitable design and fabrication approach for hybrid gridshells, 2) conducting basic material studies on hybrid cross-sections and incorporating these findings into the design development of hybrid gridshells, and 3) establishing a new workflow and guidelines for the design of hybrid gridshells.

The first research focus is intended to facilitate the construction of shells and gridshells while minimizing waste and material consumption according to the principles of sustainable construction. This method achieves a balance between structural integrity and resource optimization by leveraging the potential of lightweight and high-strength materials.

The Second research focus, the conducting of basic material tests, aims to clarify the mechanical properties and load-bearing capacity of the developed CFRP-concrete hybrid cross-section through comprehensive material testing and analysis. The insights gained from these studies enable

optimized material selection and provide helpful information for the early-phase decision-making process in the design and construction of the gridshell to achieve maximum structural performance.

Finally, the third research focus, the development of design workflow and guidelines, aims to formulate comprehensive design guidelines and an efficient workflow for the design and analysis of hybrid gridshells. This type of structural system can present particular challenges to designers and engineers because it involves several design and material uncertainties, and it is a new construction concept that has not yet been implemented in other projects. The guidelines developed therefore seek to address various aspects ranging from material compatibility to connection details and load distribution. The principles developed allow designers to explore different design options while ensuring they comply with material limits.

The three aforementioned research foci of this dissertation lead to the following specific research objectives:

- Clarifying why bending-active structures might be an interesting approach to formwork design for hybrid gridshells.
- Exploring the opportunities and limitations of bending-active structures as formwork by conducting sensitivity analysis and studying material properties, gridshell compositions, and fabrication strategies.
- Reviewing traditional and more recent form-finding techniques for shell and gridshell structures and their suitability for adaptation to bending-active formwork and hybrid gridshell structures.
- Investigating the challenges of fabricating the proposed designs and associated workflow through physical experimentation to further improve the accuracy and constructability of the gridshells generated using the performance-based generative design tool.
- Performing comparative physical tests on hybrid cross-sections to determine limiting material properties and integrating the results into the generative design tool to enable better-informed designs. Exemplifying the improvement through specific examples.
- Developing a performance-based generative design tool for hybrid gridshells based on bending-active formwork. Refinement of this tool to incorporate multiple design factors and performance objectives.
- Informing the design and fabrication process of bending-active shell structures by structural performance to introduce greater possibilities of providing solutions to several shell's design and construction challenges.

1.7 Dissertation Overview

Following this introductory chapter, Chapter 2 will provide an overview of related topics that represent important background knowledge for understanding the research contribution of this

dissertation. Starting with the general topic of long-span structures and the origin of shells and gridshells, the dissertation first will discuss how the need to bridge large spans led to the use of arch and shell geometries and how these structural systems were derived from available materials. Furthermore, Chapter 2 will discuss design and form-finding methods used for shells and gridshells, and the critical role that flexible formwork has played in the past and could play in the future, particularly for the construction of concrete shell structures. This will be followed by an overview of multi-material cross-sections with a focus on concrete and carbon fiber reinforced polymer (CFRP) to enhance the performance of concrete gridshell structures.

Chapter 3 will introduce the topic of bending-active structures in general and their possible use as formwork for the construction of concrete shells and gridshells in particular. To investigate this topic, two feasibility studies were carried out. One focus was on the question of whether and how rigid load-bearing structures can be built from flexible and elastically bent parts. The other investigated the question of whether the bending-active structures produced in this way can support significantly more load and serve as formwork for a concrete gridshell. In both cases, a full-scale prototype is built from millimeter-thin flexible CFRP strips. To achieve these structures, a new workflow is developed that combines physical model-making with new digital form-finding methods that balance the proposed design intent with material and manufacturing constraints. The design principles, material investigations, and manufacturing processes of these two prototypes will be explained and discussed in more detail in this chapter.

Chapter 4 will delve deeper into the idea of creating a stay-in-place bending-active formwork for the construction of hybrid gridshells and therefore examine the material properties of hybrid cross-sections in more detail. In order to achieve this, several specimens with different material and connections are examined and the testing framework and setup will be presented. The chapter will end by discussing the results of the load-displacement graphs of the four-point bending and compression tests performed.

In Chapter 5, a sensitivity analysis is performed to investigate how different geometric parameters affect the global and local structural performance of the hybrid gridshells. The sensitivity analysis is based on a new generative design workflow in which different gridshell geometries with synclastic, monoclastic, and anticlastic curvature are created and defined by their mid-surface height, grid pattern density, and number of edge supports. The system takes the resulting gridshells as input and evaluates their global and local structural performance in the software Karamba3D, including maximum displacement, maximum shear stress, maximum bending stress, and first eigenvalue. This generative workflow is a fully automatic system that can also be controlled manually by the user. This chapter will explain the parameters that affect the geometry and structural design of the gridshells and explain how exactly the structural analysis is performed. Chapter 5 will conclude with a comprehensive discussion of the results of this sensitivity analysis, highlighting some of the key findings that will be useful for future design iterations.

Finally, Chapter 6 will discuss the major findings and research contributions of this dissertation, reflect on the obstacles and limitations discovered during the development of this research, and identify a few possible follow-up investigations that could advance this research direction in the future.

1.8 Related Publication List

The following publications are partially or entirely included in this dissertation.

- **Yavaribajestani, Y.**, Herrmann, M., Schleicher, S. (2022). Parameter Sensitivity Analysis of Hybrid Gridshells with Bending-active Formwork. Paper presented at the *International Association for Shell and Spatial Structures (IASS) Conference*, Beijing, China, 19-22 September 2022.
- **Yavaribajestani, Y.**, Herrmann, M., Mosalam, K. M., Schleicher, S. (2023). Investigation of Cross-sections for Hybrid Gridshells based on Bending-active Formwork. Paper presented at the *International Association for Shell and Spatial Structures (IASS) Conference*, Melbourne, Australia, 10-14 July 2023.
- **Yavaribajestani, Y.**, Schleicher, S. (2019). Bio-inspired Lamellar Structures-Studio One Research Pavilion 2018. In *Proceedings of the 39th Annual Conference of the Association for Computer Aided Design in Architecture (ACADIA)*, Austin, Texas, 24-26 October 2019.

In addition, the list below covers the publications that I co-authored during my Ph.D. studies at UC Berkeley with relevance to the dissertation topic, but are not directly included in this dissertation.

- Schleicher, S., **Yavaribajestani, Y.**, Moustakas, C., Herrmann, M. (2019). Casting Ultra-thin Concrete Gridshells Using Bending-active Formwork. Abstract presented in the *International Association for Shell and Spatial Structures (IASS) Conference*, Barcelona, Spain.
- Darweesh, B., **Yavaribajestani, Y.**, Kurdi, L., Schleicher, S. Conformal 3D Printing with Bending-active Formwork. Manuscript under review.
- Schleicher, S., Kontominas, G., Makker, T., Tatli, I., **Yavaribajestani, Y.** (2019). Studio One: A New Teaching Model for Exploring Bio-Inspired Design and Fabrication. *Biomimetics*, 4(2), 34.

Chapter 2

Background

2.1 Evolution of Shells and Gridshells

Historic Wide-spanning Structures

When building houses or infrastructure projects such as bridges, one of the most important challenges for architects and engineers is the problem of the span - the ability to effectively cover longer distances with very little material and without excessive need for additional support elements. In the past, timber structures were extensively adapted for large-span coverings due to their widespread availability while also maintaining stronger tensile strength when compared to masonry structures [11]. Yet timber structures have limited element dimensions due to the size of the trees, are prone to fungal and insect attacks, and are at risk of fire. In fact, many old timber



Figure 2.1: The wide-span masonry vaults of the Great Hall of the Palace of Taq Kasra in Ctesiphon, the former capital of Persia [10].



Figure 2.2: (a) Pantheon dome in Rome, the world's largest unreinforced concrete dome (b) The Hagia Sophia in Istanbul, the second-largest pendentive dome in the world (c) The Cathedral of Santa Maria del Fiore in Florence, the largest masonry dome in the world. Photographs taken by the author, Yasaman Yavaribajestani.

buildings were destroyed because there were no effective means to extinguish a fire once it started. Therefore, more masonry structures than wood structures have survived to this day. However, when masonry is used in a beam and column system, the undersides of the beams are subjected to tensile stresses, limiting the span of unreinforced masonry beam structures. Alternatively, bridging longer spans with unreinforced masonry can be achieved by a different structural approach, using arches, vaults, and domes [12]. An impressive example of these wide-span masonry vaults is the Great Hall of the Palace of Taq Kasra (Figure 2.1) in Ctesiphon, the former capital of Persia. It was built during the Sassanid dynasty around 550 AD. The Great Hall is a catenary vault made of brick with a span of about 25 meters, which served as a model for many modern thin shell vaults [13]. Each of these buildings is capable of spanning great distances by stacking the masonry in an advantageous (catenary) geometry where the prevailing loads put the building elements under compression. A scenario for which masonry and brick perform really well. Other important examples of historic wide-span masonry domes include the Pantheon in Rome (Figure 2.2a), the Hagia Sophia in Istanbul (Figure 2.2b), and the Cathedral of Santa Maria del Fiore in Florence (Figure 2.2c).

Concrete Shell Structures

Two-dimensional surface structures are characterized as structures with having one dimension, the thickness, significantly smaller than the other two dimensions, which shell structures are considered in this category. Two-dimensional surface structures, similar to linear members, are capable of transferring the loads from one point to the other. However, unlike linear members, which carry the load linearly along the member, surface structures carry them along the surface [14]. This characteristic of surface structures, gained particular significance in the early 20th century with the development of reinforced concrete and its increasing use for various architectural applications began to change the way large shells were built [15]. Architecture and its formal expression were suddenly freed from many geometrical constraints, thanks to the concrete's ability to fill any mold in its liquid state and to become load-bearing after curing. The scarcity of other building materials and the great availability of concrete during the world wars led to higher demand for many construction

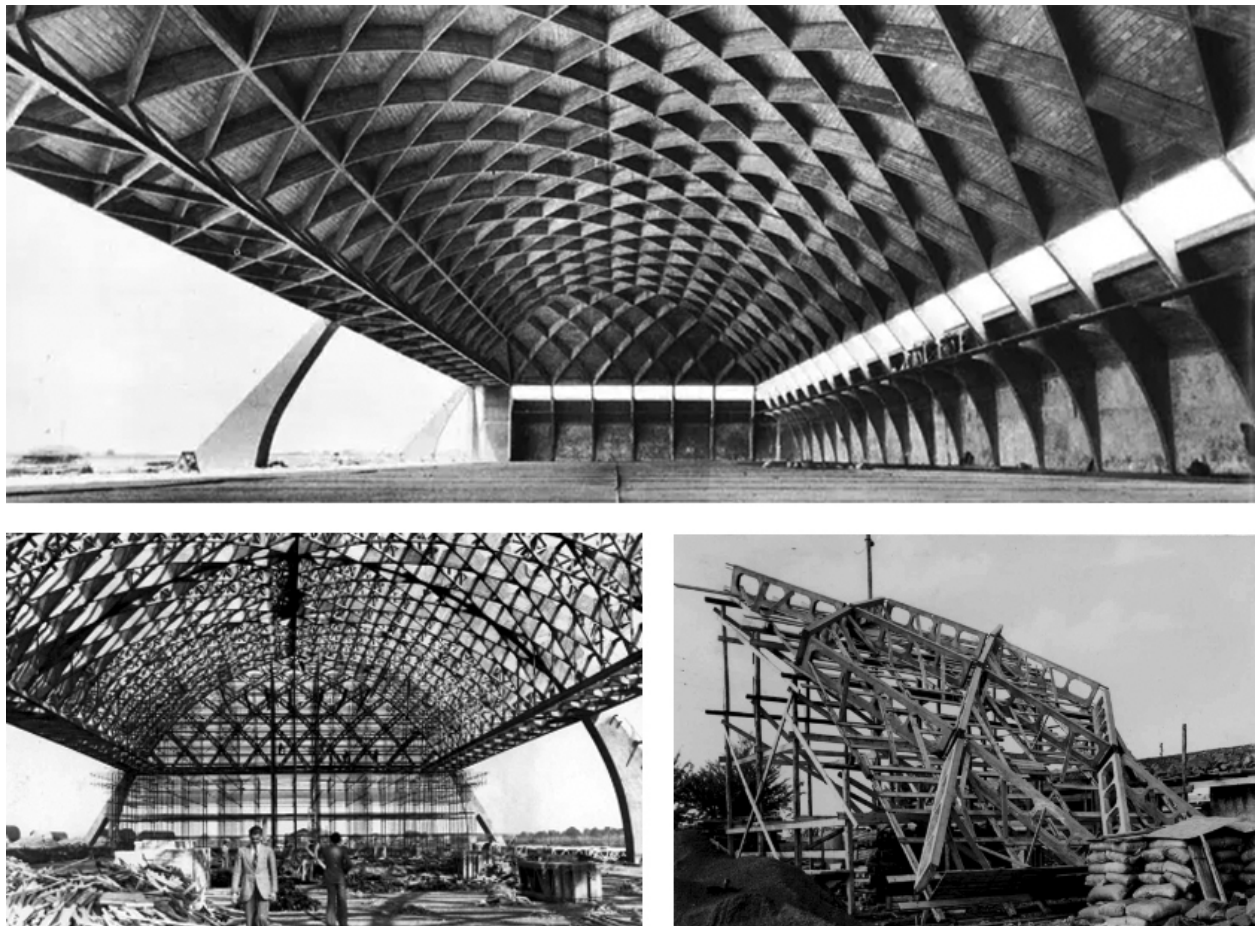


Figure 2.3: Nervi's airplane hanger: (Top) Interior view showing geodesic structure [16] (Bottom) Precasting experiment, Magliana, Rome [17].

purposes. Since concrete was cheaper than metal and steel, which were heavily demanded for war efforts, and stronger than timber, it was an ideal substitution for other building materials. Concrete, however, was widely available and labor costs were low at this time, which further benefited this building material. In addition, the war required construction programs that called for wide-span, column-free structures, and open-air shelters that could be erected quickly and inexpensively. A good example of the innovative use of reinforced concrete in the design of thin-shell structures is the geodesic aircraft hangars of Orvieto Airport (1935-1942) by Italian engineer and architect Pier Luigi Nervi (1891-1979), as shown in Figure 2.3. It efficiently merged the vault configuration with the principles of a space frame, resulting in an efficient, lightweight, and hyperstatic ribbed roof shell [18].

After World War II, reinforced concrete structures gained even more popularity, this time because of the free-form organic shapes that were possible through this material and construction system. Especially in the 50s and 60s, there was an atmosphere of enterprise. Along with technological advances in polymer plastics, concrete became known as a material through which futuristic shapes could be created, not just simple geometries such as squares, rectangles, circles, and cylinders. The period between the 1920s and early 1960s can be called the golden age of concrete shell construction because of the number and variety of shells built during that time. Some of the most iconic concrete shell structures from this era can be seen in the work of Pier Luigi Nervi (1891-1979), Eduardo Torroja (1899 -1961), Felix Candela (1910-1977), and Heinz Isler (1926-2009) in Figures 2.4-2.7 accordingly.

In the years that followed, concrete shells received less and less attention [22], mainly because



Figure 2.4: Palazzetto Dello Sport (1960) by Pier Luigi Nervi, Rome. Photograph taken by the author, Yasaman Yavaribajestani.



Figure 2.5: La Zarzuela Race Track (1935) by Eduardo Torroja. Adapted from [19].

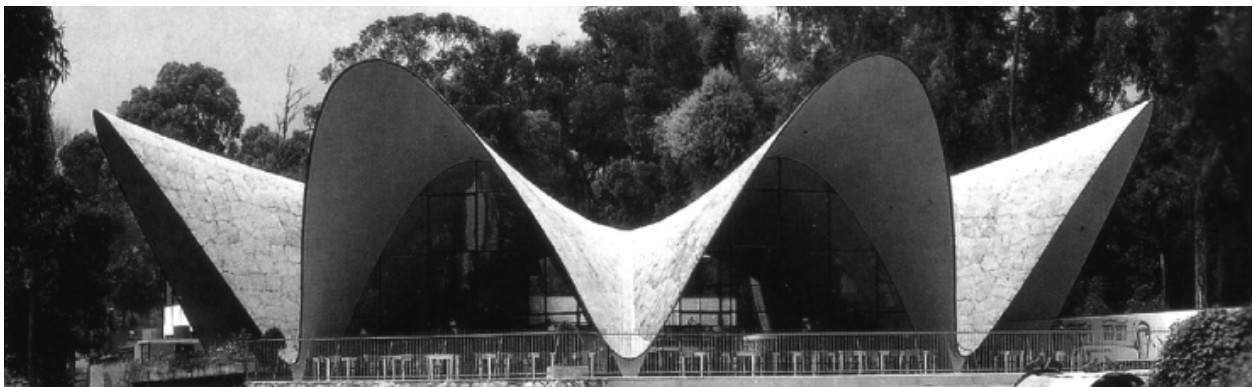


Figure 2.6: Restaurante Los Manantiales (1958) by Félix Candela, Mexico City. Adapted from [20].



Figure 2.7: Deitingen Service Station (1968) by Heinz Isler, Solothurn Switzerland. Adapted from [21].

of the now higher cost of labor, the difficulties in predicting their structural performance by hand calculations, and the advances in steel constructions which enabled the building of other spectacular structures. However, with the advent of the computer age in the mid-20th century and the development of numerical methods for shell design, they again became the focus of interest for many researchers [23]. These methods generate ideal shapes that are the results of stable force equilibrium [24]–[27] and can achieve high structural and material efficiency. However, one of the main problems of shell structures, namely the high labor costs associated with their construction, was still unsolved, leading to a rather rudimentary use of shell structures in architecture in recent decades. Therefore, these types of shells are still not widely used in the construction industry. Apart from these technological advancements, today, in times of climate change and increased awareness of the excessive contribution of the built environment to global CO₂ emissions, we can observe an interesting renaissance of shells and gridshells as representatives of a lightweight, sustainable, and environment-friendly approach to construction. Finally, new possibilities for prefabrication and automated construction are making shells increasingly competitive again, also financially.

Gridshells and Elastic Gridshells

In the context of shell structures, one type of shell, gridshells, is of particular interest. Gridshells are shells in which the structural system consists of a type of grid of linear members rather than a surface, which often consists of straight members connected at nodes. While this design means that loads are transferred through the individual elements of the grid rather than using the surface effect of monolithic shells, gridshells can be advantageous in terms of prefabrication, transportation, and construction. Examples of gridshell structures can be seen as early as the 19th century when Soddingen, a German engineer, built lamellar arch structures for agricultural buildings [28]. As the industrial revolution was showing its impact around the world by providing mass-produced steel, in 1897, Vladimir Shukhov was known to design the first doubly curved gridshell structure for a production hall in the town of Vyksa, Russia [29]. Moreover, the world's first geodesic dome is considered to be the metal lattice shell reinforcement of the first Planetarium in Jena, developed by Walther Bauersfeld, chief engineer of the Carl Zeiss AG, together with civil engineer Franz Dischinger in 1922 [30]. In 1912, Friedrich Zollinger, a German architect, urban planner, and municipal building official, developed and patented the first lightweight barrel vault system, called the "Zollinger Roof," which consists of a large number of uniformly shaped, prefabricated timber lamellas. Compared to conventional roofing systems, this structure offered great benefits, as the total amount of wood required was reduced by 40%. This was especially important in the following years leading up to the World Wars when building materials were in very short supply [31].

Gridshells have been built to some extent since World Wars I and II, but scholars believe that the real advancement of gridshell techniques did not occur until the 60s by German architect Frei Otto [32]. Otto and his research team at the Institut für Leichte Flächentragwerke / Institute for Lightweight Structures (IL) at the University of Stuttgart pioneered a novel construction method for gridshells based on elastically bending a flat lattice of straight, slender wooden laths into a double-curved shell geometry [33]. While Frei Otto constructed some experimental prototypes with a span of less than 10 m from slender steel rebars and wooden slats in the 60s and early 70s [33],

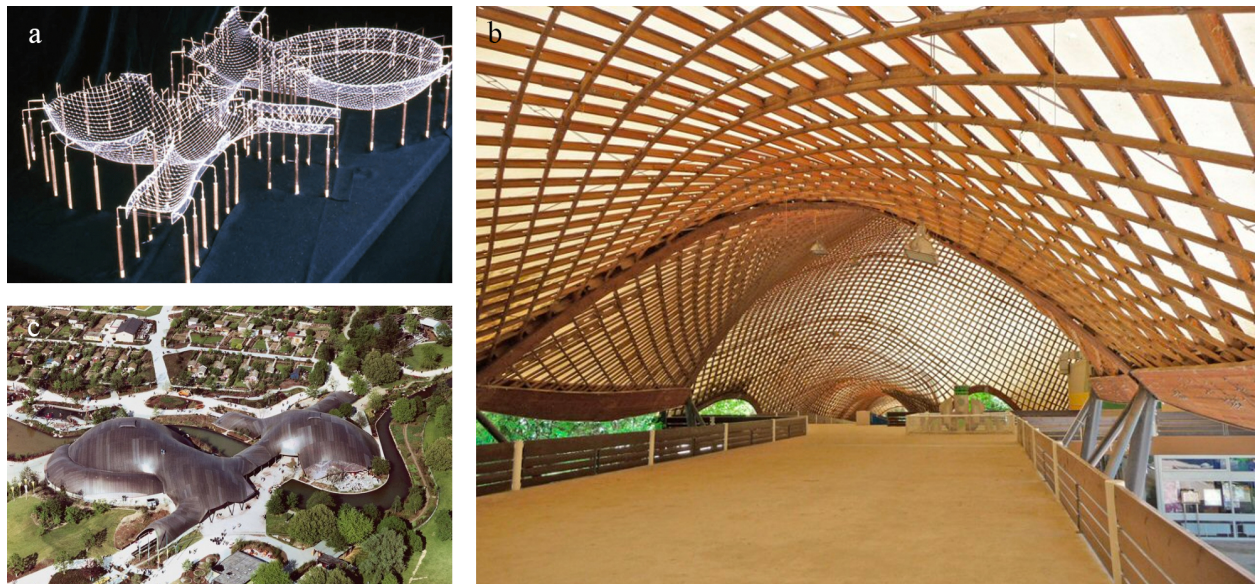


Figure 2.8: Mannheim Multihalle, Germany, designed by Frei Otto in 1975. Otto used a complex model of hanging chains to find the perfect form for its wooden gridshell [34].

the architectural potential of gridshells did not become apparent until 1975, when the multipurpose hall in Mannheim, Figure 2.8, was built with an astonishing span of more than 60 m, yet made of wooden slats with a cross-section of only 5x5 centimeters [28]. The construction method used in the erection of the Mannheim Multihalle began with a flat grid, which was then lifted into the desired shape from below, using hydraulic jacks. This method was challenging in that the hydraulic presses generated localized force concentrations and the gridshell had to be temporarily reinforced at these contact points. Yet, the so-constructed gridshell was unique not only for its extremely light and transparent wooden roof but also for its organic futuristic form, which seemed revolutionary to many architects of the time.

With the evolution of numerical tools in conjunction with the increasing computational power and digital methods for structural design and analysis, especially the development of software based on the dynamic relaxation algorithm, the design and form-finding of gridshells have become much easier for architects and engineers. A good example of a timber gridshell structure that took advantage of advances in computer-aided design methods is the Carpenter Hall of the Waeld and Downland Museum in the UK, shown in Figure 2.9. This gridshell had many notable improvements from its predecessors. In addition to using digital form-finding and structural analysis, the material selection for this project was also very smart. This gridshell was constructed from locally harvested green oak, a wood with a relatively high moisture content that reduces its stiffness and makes it easier to bend. The application of finger and scarf joints and better on-site monitoring of the bending process also significantly reduced the number of broken wood slats. While the construction process began similar to the Mannheim Multihalle with an initially flat lattice structure, the Downland gridshell structure was first raised as a whole to a higher level and then bent downward into the

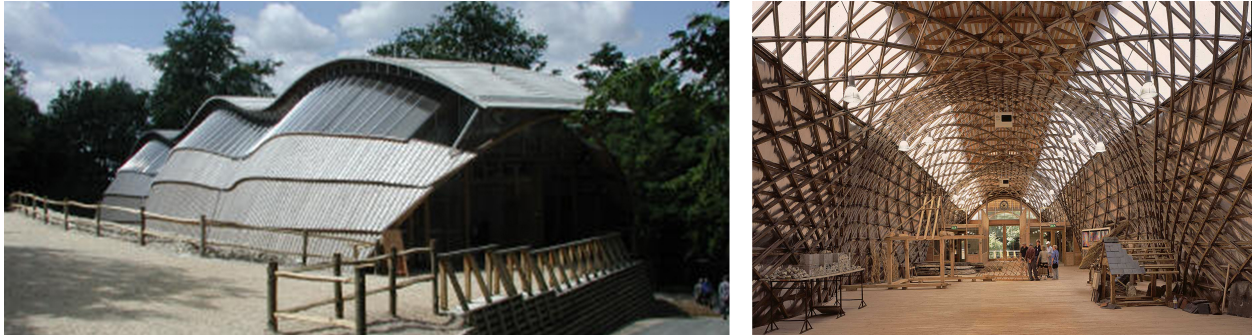


Figure 2.9: Wooden gridshell of the Carpenter Hall of the Weald and Downland Museum [36].

target geometry using gravity [35].

More recently, further advances in materials technology, particularly in composites, have enabled the construction of gridshells from man-made materials with tailored physical properties. Unlike natural materials such as wood, polymer composites are manufactured under controlled conditions and can achieve higher structural performance and more uniform linear-elastic material behavior [6]. Two of the most widely used polymer composites for gridshells are carbon fiber reinforced polymers (CFRP) and glass fiber reinforced polymers (GFRP). An example in which polymer composites have been used for gridshells is the temporary Cathedral of Créteil, Figure 2.10, which was built in 2013 in a Parisian suburb spanning 17x29 meters [9]. The computational tools involved in this project enabled the accurate form-finding of the resulting bent geometry prior to construction, a prediction that was essential for the precise fabrication of the fabric membrane that covered the structure and made it rainwater-tight. As a result, the gridshell and membrane could be prefabricated off-site, resulting in significant time and cost savings [37].



Figure 2.10: The Ephemeral Cathedral of Créteil (Paris, France), a gridshell structure made of composite materials, 2013 [38].

2.2 Form-finding of Shells and Elastic Gridshells

Form-finding (or sometimes called shape-finding) can be defined as the process of seeking an optimal geometry for a form-active structure, which is achieved when the structure is in (or approximates) a state of static equilibrium [23], [39]. Form-finding is a bottom-up process in which control parameters are systematically changed to arrive at an optimal geometry for a structure.

Physical Form-finding

Before the computer age, form-finding was done primarily through scale models made from chains or weighted strings. The weights forced the model into a pure tensile shape, which could be measured and turned upside down to obtain a model geometry that was entirely in compression. A good example of this form-finding technique is the hanging chain model used by Spanish Catalan architect Antoni Gaudi to design the arch-like shapes of the Sagrada Familia [40] (Figure 2.11). Another good example of physical form-finding in the context of shell and gridshell structures are the hanging textile models which Heinz Isler used for the design of his lightweight concrete shells or the hanging chain models that Frei Otto built to find a suitable geometry for the Multihalle Mannheim, as shown in Figure 2.8 [41].

With the emergence of the computer, it was gradually possible to model these physical processes digitally and to calculate form-found shell and gridshell geometries by means of numerical



Figure 2.11: Left: The Sagrada Família in Barcelona [42] Right: The hanging chain model of the Sagrada Família [43].

simulations. In the following subsections, the most commonly used methods for form-finding will be explained in more detail [44].

Force Density Method

Proposed by Linkwitz and Schek [24], the Force Density Method (FDM) is one of the principal form-finding methods for tension structures, aiming to solve the shape equilibrium of tension networks. Within FDM, equations representing the shape equilibrium, are linearized by the introduction of force densities. These force densities are the ratio of the axial force of a branch to its length in a bar or cable segment. These linear equations can be solved efficiently via sparse matrix techniques. For the first-order form-finding, the designer needs to define the initial values for these force densities. An example of FDM form-finding with different density values of the main cables can be seen in Figure 2.12. As seen in the illustration, FDM can generate solutions for a given topology, by varying the force densities and external loads. However, it is not a straightforward process to establish a direct connection between the choice of force densities and the three-dimensional equilibrium solution [45].

Through incorporating loads, FDM also enables the form-finding of synclastic structures which are highly appropriate for designing efficient shell structures. Originally developed for cable nets, this method has become the common practice in the design of tensioned membrane roofs. The ultimate equilibrium values of the force densities are often determined through an iterative optimization process, to meet additional design criteria for having a refined design. The Independence

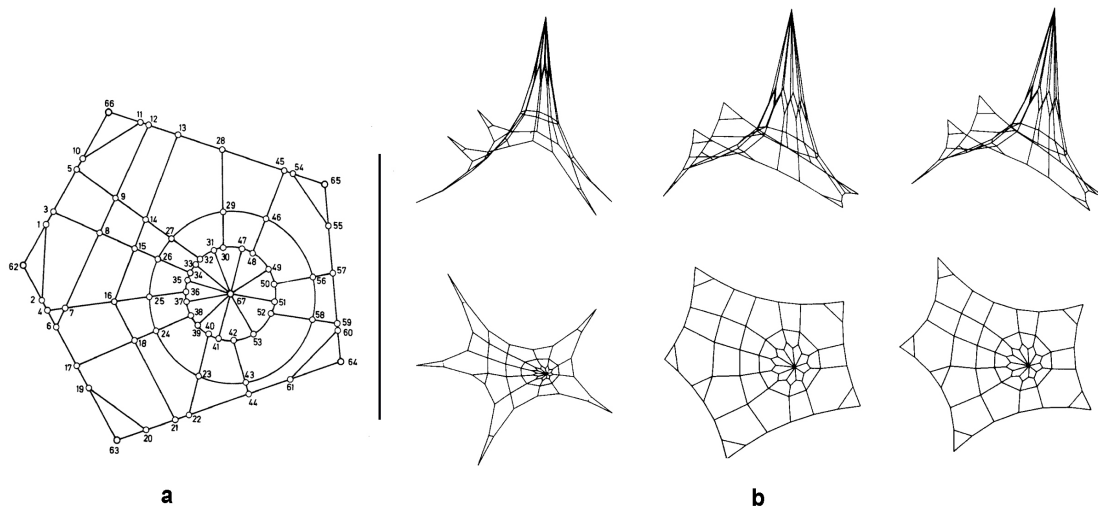


Figure 2.12: An example of FDM form-finding (a) the initial graph (b) global changes in the force densities of the main cables by inputting the initial density values of 10, 5, and 1 (from left to right) for the border branches (main cables). Adapted from [24]

of FDM from material properties, results in two interesting opportunities. First, the ultimate designs can be realized flexibly. This means that by modifying the initial length of the networks in their undeformed shape, the final shape remains unaltered. Second, without any effect on the geometry, the loads can be multiplied to any realistic value and then the internal force distribution can be recalculated [46].

Dynamic Relaxation

Introduced by Day in 1965 [27], Dynamic Relaxation (DR) solves a static problem by transforming it into a dynamic problem and iteratively solving the reformulated problem to finally reach a steady-state solution. Initially, the internal forces of a structure do not hold an equilibrium with the externally applied loads. In this method, due to the lack of equilibrium, the structure begins to oscillate until it finally settles into the desired equilibrium position aided by the damping effect. The motion of the structure is determined using time steps, with each time step corresponding to one iteration, as shown in Figure 2.13. In this method, the mass of the structure, the damping, and the length of the time steps dictate the number of time steps required to achieve equilibrium. As these parameters exclusively influence the dynamic behavior of the structure, they can be accounted as tuning parameters, enabling the control of the number of iterations, without influencing the final outcome. Considering the numerical stability of the method is maintained, the parameters' value can be arbitrarily selected to minimize the number of iterations [47].

DR is widely used for the form-finding of tension/compression structures such as shells, gridshells, cable nets, membranes, and tensegrity structures. An example is the software Kangaroo Physics, a plugin for the computer-aided design (CAD) tool, Rhinoceros, which was developed by Daniel Piker and is a particle spring system based on dynamic relaxation [48]. To further refine the Dynamic Relaxation method, various heuristic rules have been introduced to choose the time step and fictitious masses in such a way as to ensure stability for various structural problems [49]. Examples of such heuristics can be found in the work of Barnes et al. [50], they describe numerical procedures for DR with kinetic damping, for the form-finding, analysis, and fabrication patterning of wide-span cable nets and gridshells, uniform or variably pre-stressed fabric membranes, and battened membrane roofs.

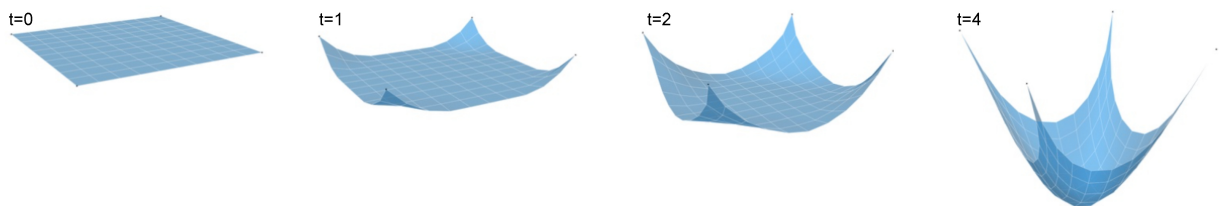


Figure 2.13: An example of DR form-finding. Visualization of DR applied on a plane mesh that is under gravitational load where the corner points are fixed. The structure reaches equilibrium by tracing through time. Initial state at $t=0$ and equilibrium at $t=4$. Adapted from [51]

Finite Element Method

The finite element method (FEM) is an approximation method that divides a continuum into a finite number of elements, each characterized by a finite number of parameters. The solution of the full system as a composition of its constituent elements, can be achieved through the implication of procedures employed for standard discrete problems [53]. In numerous complex structural systems, it would be a challenging task to analyze static displacement, internal stresses, or eigenvalue solutions through classic methods. Consequently, through the subdivision of the system into multiple finite elements, the convergence of the analysis to the exact solution of the total system can be achieved when reducing the sizes of the finite elements. Obtaining the solution of the overall system requires satisfying the equilibrium equations at nodes and the compatibility of displacements between the elements. Since the introduction of FEM, several commercial software packages, such as Ansys, SOFiSTiK, Nastran, and Abaqus have been developed enabling users to address complex engineering problems using this method [54], [55].

Shell form-finding via FEM takes place in three steps: (a) generating the shell shape using a defined geometry representation (b) analysis of the generated shape via FEM (c) using an optimization solver to find the optimum shape. While such a procedure can be generalized for a wide variety of engineering applications, the differentiating aspect for each application can come down to how its geometry is represented. One of the early approaches of exploring shell form-finding with FEM can be seen in the work of Bletzinger and Ramm [52], where they represent the shell using an array of bi-cubic Bezier patches, as shown in Figure 2.14, and utilize their previously developed CARAT tool [56] to find the optimal shape for the shell under its self-weight load. Such an approach was later extended by various researchers in the field, investigating aspects such as isogeometric shape optimization [57], combination with topology optimization [58], [59], and material minimization [60].

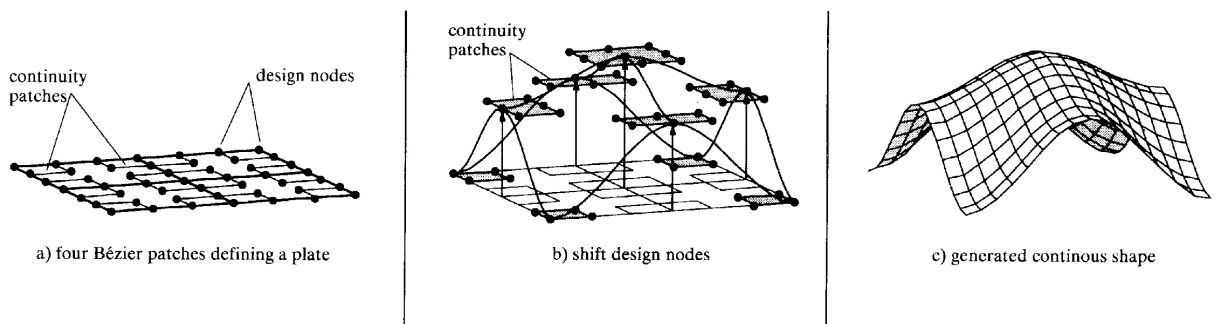


Figure 2.14: Example of a shell representation using continuity patches connecting four Bezier patches for FEM analysis. Adapted from [52]

Thrust Network Analysis

Introduced by Block et al. [26], the Thrust Network Analysis (TNA) is a method to calculate vaulted compression-only networks and surfaces. TNA, similar to graphic statics, employs reciprocal diagrams, to present a visual and intuitive technique. Additionally, TNA can be simply implemented in three-dimensional contexts. The reciprocal diagrams establish a connection between the three-dimensional equilibrium's geometry and their internal forces [45]. The network of compressive forces, a network of thrusts corresponding to the applied loads, acting within the structure can be a representation of the equilibrium of vaulted structures. The network can be defined as a set of nodes and branches. Branches connect each pair of nodes, and they illustrate the direction of the thrust forces. In order to evaluate branch thrusts, heights of internal and external nodes, and the coordinates of boundary nodes, a series of equilibrium conditions (see in detail [26]) are employed. The formulation of these equations primarily focuses on the internal and boundary nodes. Furthermore, support reactions are demonstrated by the external branches, with their endpoints particularly designated as external nodes. Consequently, modeling of the constraints of the vaulted structure takes place through external nodes and branches [61].

Contrary to FEM, TNA is a representational model of the thrust forces that balance the external loadings, rather than a geometrical model of the volume occupied by the vaulted structure. The position of each node in the three-dimensional Cartesian reference frame defines each node. Moreover, the two end nodes of each branch along with the corresponding value of the thrust force, identify each branch in the network. Nodes in each branch, are subject to loading from two sources: external loads whose magnitude is determined by the nodes' area of influence and the thrust forces generated by the branches attached to the node. For each branch, nodes are loaded both by an external force in which its value depends on the region of influence of the node and by the thrust forces pertaining to branches connected to the node. In the case of compression, the thrust forces are directed towards the node. If the branches have a thrust force within the network, they can be categorized as internal branches. On the other hand, if they demonstrate forces acting along a free edge, they are called the edge branches and finally, if they illustrate the support reactions they are named external branches and the same classification exists for the nodes as well [61].

As shown in Figure 2.15, the form-finding through the TNA method involves concurrent manipulation of two diagrams: the form diagram Γ , which represents the horizontal projection of the three-dimensional network G , and the force diagram Γ^* , which comprises the horizontal component of the forces that act on each bar of the compressed network. Form and force diagrams are dual graphs that hold reciprocal relation with every node of Γ corresponding to an equilibrium polygon of Γ^* [62]. In Figure 2.15.b, the number of branches connecting at each node, is identical to the side number of force polygon in equilibrium at that corresponding node, and this relation holds true in reverse as well. It is critical to hold parallel alignment within a determined tolerance angle between each side of all polygons of Γ and the corresponding force of the related equilibrium polygon in Γ^* . The presence of a closed polygon of force vectors for a node in one graph, promises the structural equilibrium of that same node in the other graph and vice versa. The magnitude of the axial force acting in a branch e of the form diagram Γ can be determined by applying a defined scale factor s to the length of each branch e^* of the force diagram Γ^* . This value also corresponds to the

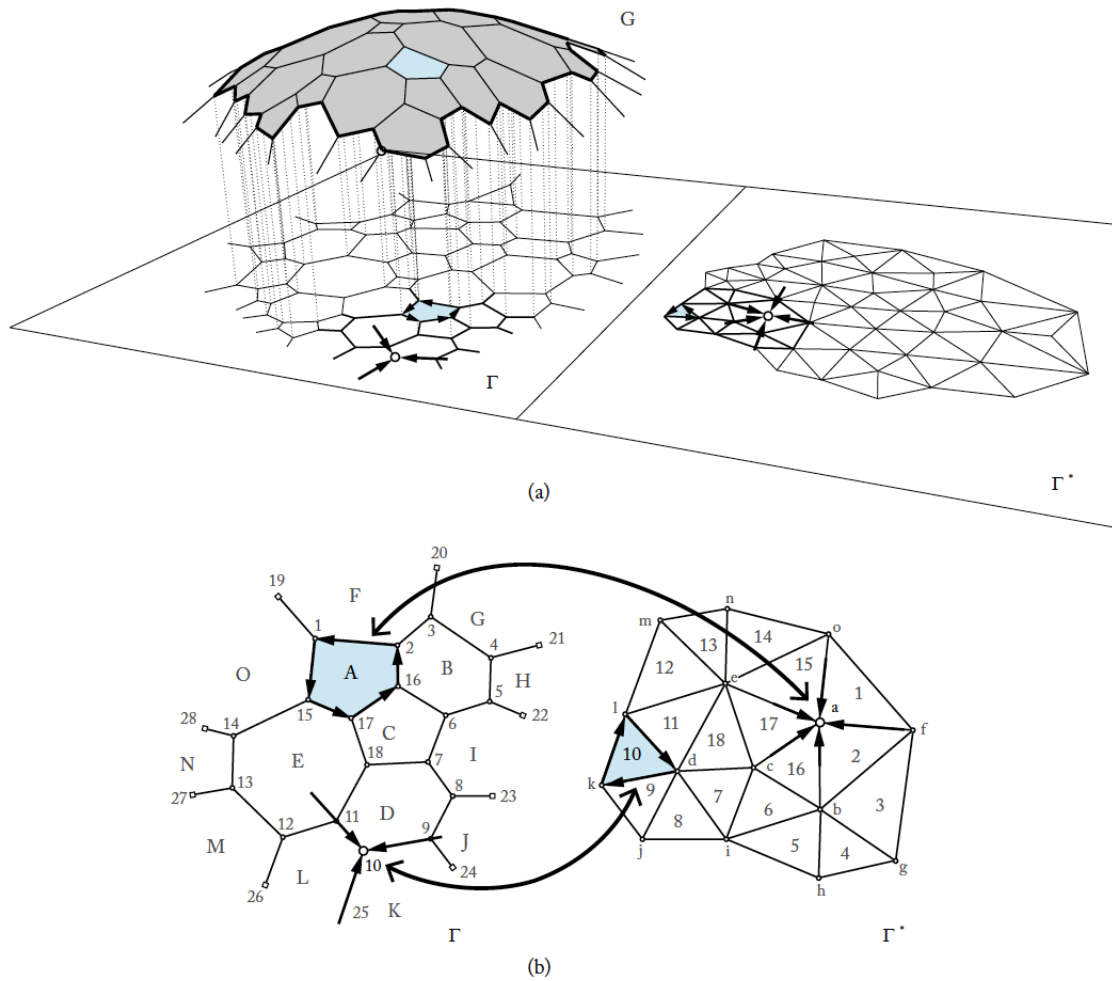


Figure 2.15: An example of TNA form-finding. (a) relationship between the thrust network G , its planar projection, the form diagram Γ and the reciprocal force diagram Γ^* and (b) the reciprocal relation between Γ and Γ^* using Bow's notation to label corresponding elements. Adapted from [63]

horizontal component of the axial forces applied to the respective bar within a three-dimensional thrust network G [24].

Form-finding of Elastic Gridshells

Elastic gridshells are generated by the elastic deformation of initially flat grids into three-dimensional shell-like shapes. Therefore, a sound understanding of bending is required for the design and form-finding of gridshells. The term *elastica* is coined by Leonhard Euler and Jakob

Bernoulli [64] and describes a curve that produces a minimum of potential bending energy in a constrained system. In the context of elastic gridshells form-finding, the equilibrium state is achieved when they closely resemble their elastica curve [65].

When comparing elastica and catenary curves, studies [65] have shown that the catenary proves to be an ideal geometry for a rigid arc subjected to uniformly distributed vertical loading. When looking into various rise-to-span ratios and their resultant maximum displacement of catenary and elastica curves under vertical loading, as shown in Figure 2.16, in the catenary curve, even with a very large rise-to-span ratio, the resulting deflections are relatively small. The main reason for the small deformations is per definition, the axial load transfer of the catenary, putting the arc purely in compression with little to no bending moments. Up to a rise-to-span ratio of about 0.3, the deformations of an elastica curve and bending-active arc are highly close to the deflections of the catenary. This means that structures made by elastic bending with this rise-to-span ratio can, first, use a geometry that causes mainly compression forces, and, second, can be determined using catenary scale models. Thanks to this characteristic, Frei Otto and his team, for example, were able to determine the curved geometry of the Multihalle Mannheim by means of a hanging chain model. However, with a larger rise-to-span ratio, the resulting deformations of elastica curves increase exponentially and significantly exceed the deformations of the catenary. Above a rise-to-span ratio of 0.3, loads are increasingly carried by bending moments, which rapidly reduce the stiffness of the arch. This highlights the critical role of selecting the appropriate rise-to-span ratio in the structural design process of arcs.

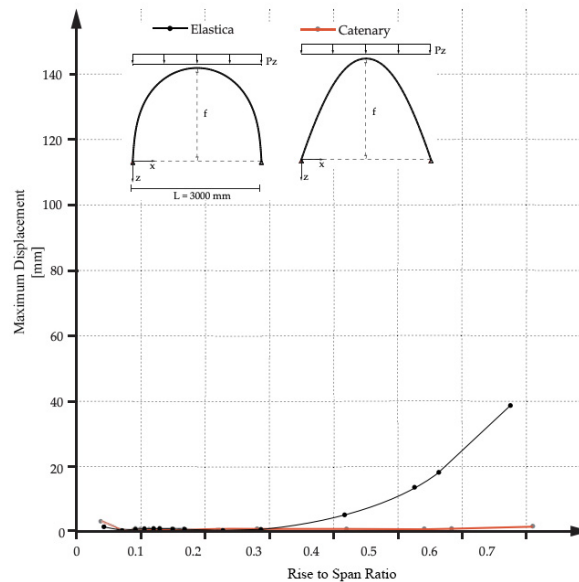


Figure 2.16: Load-deflection comparison of elastica, catenary, and circular arc for various rise-to-span ratios. Adapted from [65]

2.3 Construction and Formwork

The conventional approach for constructing concrete shells is by pouring or spraying the material mix in the liquid state onto one big mold that acts as formwork or by assembling several smaller, prefabricated modular parts into a segmented concrete shell. In this process, the formwork provides structural stability to the liquid concrete and holds it in shape until the concrete is fully cured and can support itself and the formwork can be removed. However, due to the heavy weight of the material, the formwork itself must often be supported by a substructure system known as scaffolding or falsework. Both the formwork and the supporting scaffolding may consist of either (a) rigid elements such as straight timber boards and planks, or (b) flexible materials such as textiles, inflatable foils, or curved boards or planks. Rigid techniques which mainly involve excessive scaffolding can become often a challenging and costly process both in terms of labor effort and assembly complexity depending on the general geometry or the size of the designed shell. The following sections will explain in more detail the various design strategies for rigid and flexible formwork from the early days to the current state of the art.

Rigid Formwork Systems

Timber Formwork

The use of rigid timber planks as formwork is considered the traditional method in concrete construction. Timber formwork can be arranged in various combinations to create planar and complex curved shapes. They are suitable and effective to support the weight of the construction workers during assembly and the applied concrete during the curing process [67]. The first concrete shells designed by German civil engineers Franz Dischinger and Walther Bauersfeld in Jena in 1922 were backed with timber to prevent the concrete from falling behind the shell, a process known as Torkret, a forerunner of today's shotcrete process [68]. While the thin-shell Planetarium in Jena was based on a geodesic hemisphere and thus most of the elements of the formwork had almost identical lengths, more complex geometries were explored in the shell structures designed in the following years. Notably, the Spanish and Mexican architect Félix Candela was one of the first to use the advantages of ruled geometries for timber formwork in the construction of thin concrete shells. Ruled geometries are curved surfaces that are formed from straight lines. This geometrical feature has the great advantage that a formwork for these complex surfaces can be built simply from a series of straight timber planks. Typical examples of ruled surfaces are shapes such as cylinders, cones, conoids, hyperboloids, and hyperbolic parabolas also often referred to as hypars. Especially the latter formwork geometries have the additional property of having a high double curvature, which gives the concrete shells built with them a particularly high stiffness. Furthermore, Candela was aware that using ruled geometries, which have at least one straight line through every point in their surface which also lies in the surface, also meant that the load could be calculated and analyzed. Finally, by using a dry concrete mix with reinforcement bars laid out close together, he was able to cast concrete on steep surfaces while avoiding wet concrete from sliding down. While the shells by Dischinger, Bauersfeld, and Felix Candela demonstrate unique aesthetic success, the



Figure 2.17: Highly complex and intensive labor casting process of Felix Candela's shells in Mexico. Adapted from [66]

technical innovations in the design and construction of thin concrete shells are always guided by the technology available at the time [69].

The construction of concrete shells required erecting a scaffolding system, also referred to as falsework, a temporary structure of wood or metal that supports the form boards and wet concrete. Timber boards are placed on top of the scaffolding to allow the concrete to take its proper form. As seen in Figure 2.17, Candela's shell construction often required a complex scaffolding-intensive method of building where dense forests of timber scaffolding were used which involved an expensive and labor-intensive process.

With the surge of computational power and the advancement of computer-aided design (CAD) and computer-aided manufacturing (CAM) techniques, building custom timber panels using com-



Figure 2.18: 'Meiso no Mori' Municipal Funeral Hall, Kakamigahara Japan. Concrete roof.
Adapted from [70]

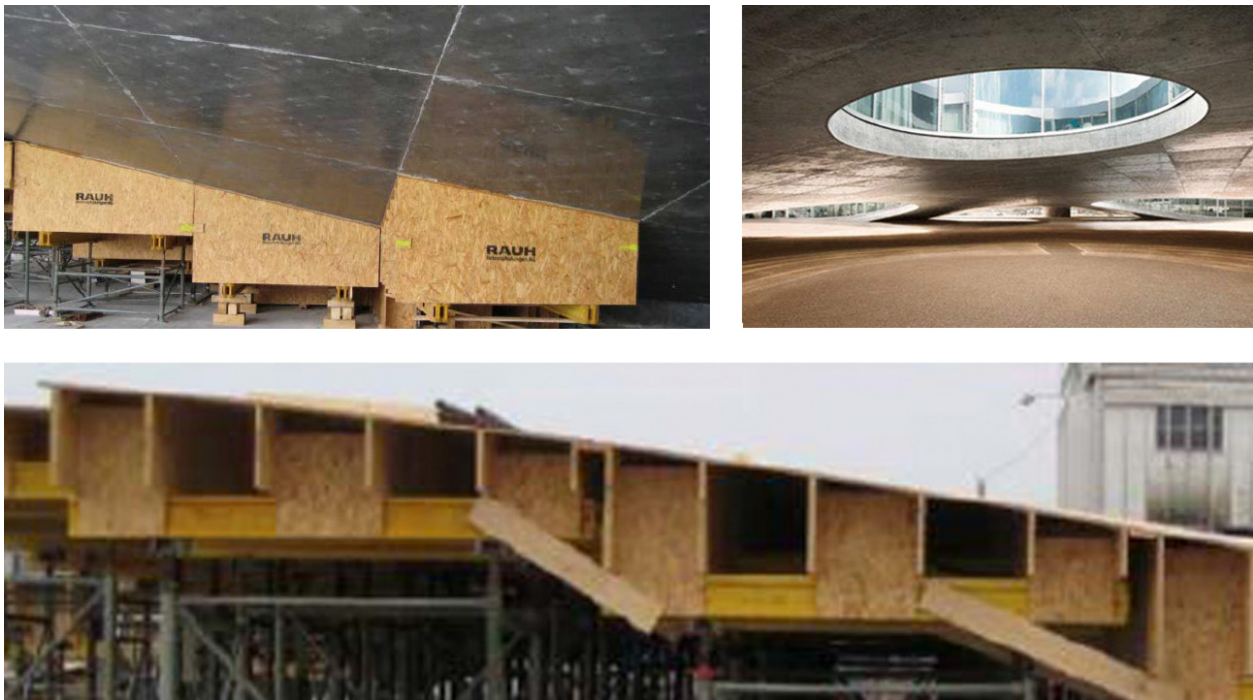


Figure 2.19: Special adjustable casting tables had to be designed and constructed in the construction of The Rolex Centre Shell. Adapted from [71]

puter numerical control (CNC) and rapid construction workflows provided the ability to manufacture a large number of elements in a precise manner to serve as the formwork system of concrete shells. Such a process can also enhance economic efficiency as material wastage could be highly optimized. Another example of utilizing CAM techniques for formwork construction can be seen in the concrete roof shell of the Kakamigahara Crematorium which is shown in Figure 2.18. Custom-cut plywood was used in areas with increased curvature to result in a finer and smoother finish [72].

A similar approach is also seen in the Rolex Learning Centre by SANAA at EPFL where a field of 1400 bespoke casting tables was used, each having a unique and precise curved geometry that formed a smooth 800 mm deep concrete slab. The architects used this continuous surface to create an undulating indoor topography blurring the boundary between room and circulation, with complex level changes [72]. Also in this structure, the architects sought a polished finish as seen in Figure 2.19, somehow hinting at the grid of formwork trestles that once supported the concrete.

Pre-cast Elements

A precast concrete structure is an assemblage of precast elements which, when suitably connected together, form a three-dimensional framework capable of resisting loads [75]. Precast elements are usually used as lost formwork, meaning they become part of the shell and are not removed later. The other advantage of pre-cast elements is that they can be prefabricated off-site under better construction conditions, which often results in much better surface finishes such as the surface finish of Nevi's structures. The innovative use of precast panels stemmed from the need to construct rapidly and efficiently. As one of the early pioneers for using pre-cast shell elements, Pier Luigi Nervi's construction innovation responded directly to the given economic situation requirements of post-war construction economics. Nervi's method involved concrete panels known as *travelloni*, pre-cast with ferrocemento on ground level first, and then raised into position (Figure 2.20). This reduced the use of vertical supports and with the reinforcement bars tying them together, concrete was then poured in grooves between. Nervi's shells were highly economical as they used available materials and were fast to build [76].

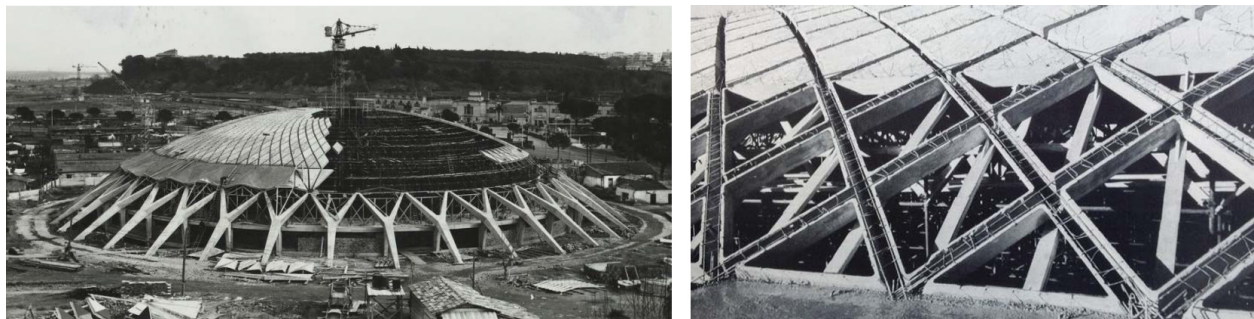


Figure 2.20: Palazetto dello Sport in Rome (left) precast panels assembled and ready for stitching concrete by pouring concrete in the valleys between the *travelloni* (right). Adapted from [73]



Figure 2.21: Nine hundred precast concrete panels are hoisted into position before concrete is poured onto the top surface to stitch the panels together. Adapted from [71]

The construction of the Duxford Aircraft Museum by Foster and Partners is a more recent example of employing pre-cast techniques for shell construction. In contrast to Nervi's approach, in which the precast concrete modules create a lost formwork onto which the concrete is poured, here the actual shell structure is broken down into several pre-cast elements and only their structural connection at the contact seam is filled with concrete. The process of constructing the shell included 900 precast panels laid out in a double-layer system and stitched together by pouring concrete over the top to connect the concrete shells together [67] as seen in Figure 2.21.

Another more recent, precasting technique can be seen in the work of Enrique et al. [74], called CASTonCAST shown in Figure 2.22, using precast stackable components for building free-form shells. In this method, they developed a manufacturing technique of curved building components which relies on casting a series of components one on top of another in stacks by using the previous component as the bottom mold of the next component. Although this manufacturing process inevitably requires some time, since the components must be cured before they can serve as formwork for the next component that is stacked on top, it has the great advantage that no additional formwork has to be manufactured.

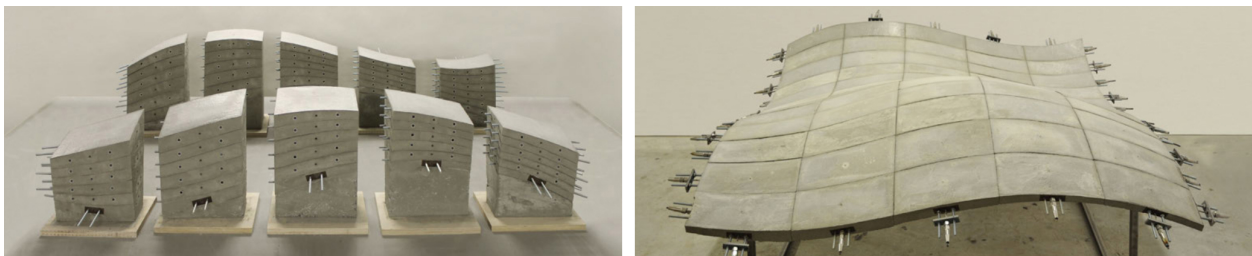


Figure 2.22: 1:10 Prototype of a post-tensioned shell structure built from precast stackable components: stacks of components (right) and post-tensioned shell (left). Adapted from [74]

Earth Mound as Formwork

Earth mound formwork refers to the use of moist granular natural materials, such as earth, sand, and clay, as formwork materials for concrete structures [77]. After the hardening of the concrete, the earthwork could be excavated. The obvious advantage of this technique, using earth as a formwork material, is that it is relatively cheap and widely available, and may even be harvested directly from the construction site. The disadvantage here is the challenge of getting the soil into the right shape, ensuring its stability and surface quality during the casting process, and the labor-intensive removal of the formwork since a lot of material was required to fill the entire volume of the formwork mold. This method was formalized by Heinz Isler, who experimentally used earthworks as formwork for a free-form shell in 1961 [78]. Other notable examples of this technique can be seen in the construction of the Phillips Pavilion for the Expo 1958 in Brussels, built with the use of sandhill [79] and the Albuquerque City Hall constructed in 1955 [80]. The Teshima Art Museum (Figure 2.23) constructed in 2010 also used earth mound as its formwork [72]. Designed by SANAA, the open-air building sits like a water droplet in the landscape on the island of Teshima. Another example is Ulrich Mütter's precast shell components on sand mounds for the lifeguard rescue tower at Binz, East Germany, Figure 2.24. The later version of the shell had thinner walls as he used a special ferrocemento with a hexagonal mesh [81].



Figure 2.23: Teshima Art Museum: Earth mound as formwork. Adapted from [82]

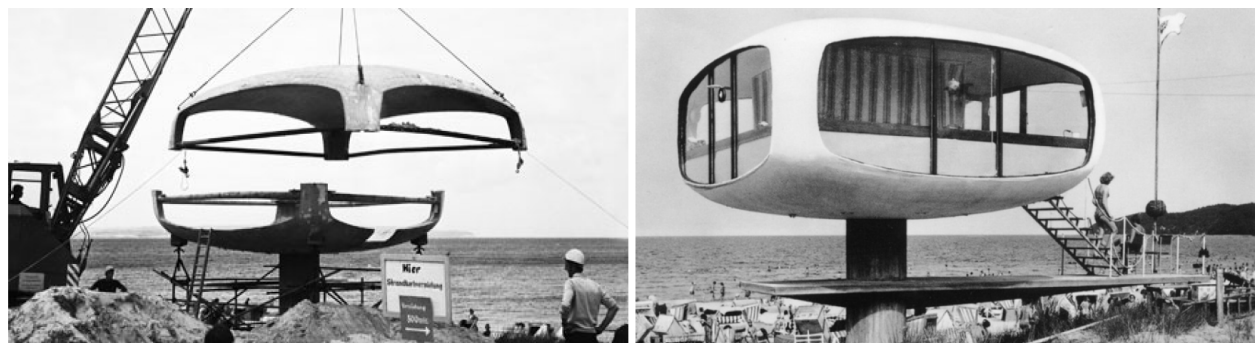


Figure 2.24: The water rescue station, built using earth mound formwork. Adapted from [81]

Flexible Formwork Systems

Replacing conventional rigid molds with high-strength flexible material can utilize the fluidity of the concrete to create structurally optimized concrete structures [83]. Complex, curved, or funicular geometries can create the potential to design more material-efficient and aesthetically pleasant structural forms. Flexible formwork has been used successfully, and profitably in a wide range of structures since the late 1800s. However, new techniques in producing synthetic fiber textiles and recent digital modeling techniques have created a vast array of new possibilities and fueled recent interest and innovation in flexible formwork.

Flexible formwork can be categorized into two groups based on the nature of the loading of the formwork: (a) filled formwork which is used for floors, beams, trusses, columns, walls, marine applications, etc.; and (b) surface formwork which is used in roofs, canopies, and shell structures [84]. Surface molds are used predominantly to form shell structures. Usually, only a single forming surface is required, onto which the concrete is applied. If the surface is inclined, the concrete must be self-supporting in order to prevent flow. Geometry is again dictated by the relationship between applied forces and internal stresses in the formwork. Different types of flexible surface molds in shell construction which have been either explored by researchers or have been utilized in the industry can be classified as follows: Fabric [76], [85]–[87], pneumatic [88], cable-net [87], adaptive molds [89] and bending-active formworks [71]. The following subsections elaborate more on each flexible formwork system by mentioning some of their early examples and more recent state-of-the-art approaches.

Fabric Formwork

Fabric formwork is constructed using textile sheets made of synthetic fibers such as nylon, polyester, or polypropylene that are erected to contain concrete during its placement and curing. [85]. Using this method, the concrete shell's form corresponds to the resulting fabric shape which comes under the load of the wet concrete's pressure in combination with the elasticity of the plastic foil or fabric. Manipulation of the fabric formwork is possible to control the shape before, during, and after casting [90]. Unlike traditional formwork, fabric formwork is highly flexible and can deflect under the pressure of fresh concrete. The resulting forms exhibit curvature as well as excellent surface finishes that are generally not associated with concrete structures [84]. In the mid-20th century, hundreds of fabric-formed shells were constructed by James Waller [91] using fabric hanging from rigid arches to create ribbed single-span domes. This was followed by the work by Kersavage [92] and Knott and Nez [93] led to dozens of fabric-formed roofs such as TSC Global. In TSC Global, the flexible reinforcing mesh was stretched around a timber frame and coated with concrete. The prestress in the flexible mesh created a double-curvature anticlastic shell form, which, combined with a low self-weight, improved the structure's earthquake resistance [94]. In the past decade, prototypes of anticlastic, flexibly formed shells have been constructed by West [95], Pronk et al. [96], Tysmans [97], Pedreschi and Lee [98], Seracino et al. [99] and Veenendaal and Block [87].

Pneumatic Formwork

Pneumatic formwork refers to the application of concrete to the inside or outside of an inflated membrane. It is therefore an example of a surface formwork system as presented in the previous section on flexible formwork. In pneumatic formwork systems, the concrete is poured inside or outside a membrane that is firmly connected to the ground and supported by air pressure [101]. This technique is known to have been pioneered by Wallace Neff, a Californian architect who used inflatable domes as formwork for concrete bubble houses [102]. The work of Dante Bini et al. is another example of the construction of hemispherical thin-shell structures with inflated formwork. What is interesting about his approach is that the placement of the reinforcing steel and the pouring of the concrete was done in the flat state, onto a pneumatic formwork anchored to a circular ring beam, before the formwork was inflated, and then the whole structure was pneumatically lifted from the ground by inflating the formwork before the concrete was fully cured and thereby resulting in a hemispherical dome [103].

This was later followed by the work of David and Barry South, where polyurethane foam was sprayed on the inside of an inflated fabric formwork. The foam provided stiffness and support from the inside. Shotcrete was then applied to the interior of the form and eventually formwork was either removed, reused, or left in place [67].

Another method of using pneumatic formwork is to lay on the concrete while the formwork membrane is deflated. Before the concrete cures, it is inflated with special reinforcement patterns used to control the displacement and sliding of the bars while formwork is inflated [67]. The work of Zimmermann et al. at the University of Kassel is another example of using pneumatically formed concrete gridshells where a double-layered membrane was welded together to create a network of interconnected tubes [67]. Once sufficient negative pressure was achieved, the interconnected tubes were inflated and incrementally filled with ultra-high strength fiber cement (UHSFC) eradicating the need for steel reinforcement.

In 2014, researchers at TU Vienna developed a system to create a concrete shell from flat plates



Figure 2.25: Pneumatic wedge system. Adapted from [100]

[104]. This method called the pneumatic wedge method, consists of a concrete slab resting on a pneumatic cushion. The flat formwork tray slab was wedged with spaces in between the segments so that when the middle section is lifted, the segments will fit together perfectly as seen in Figure 2.25. A shell of a height of 2.9m was achieved within a lifting period of 2 hours.

Finally, instead of inflating the membrane formwork, 3D formwork can be built from vacuumatically pre-stressed structures. Vacuumatic is a theoretical counterpart of pneumatic structures as they rely on the negative difference in air pressure for their structural integrity. Vacuumatics rely on the structure principle of pre-stressing incoherent elements inside an enclosed skin by introducing a vacuum inside this enclosure [105]. An example of utilizing this technique can be seen in Huijben et al. work, where they created three-dimensional temporary formwork structures for concrete [106].

Adaptive Formwork

Adaptive formwork aims to overcome one of the limitations of conventional formwork by introducing the concept of reusability and reconfigurability of the mold into the manufacturing process. In this method, a set of adjustable actuators called a pin-bed, is placed on a regular grid with the tips close together to form the contours of a double-curved shape. The shape of this formwork can be freely adjusted by changing the exact positions of the pins. To allow for a smooth surface finish, the pins are usually connected and covered with plastic foil or a flexible rubber mat, which also prevents contact between the concrete and the pin-bed mechanism. Adaptive molds can be reshaped between uses, taking advantage of a flexible mold's ability to conform to multiple geometries depending on support conditions [107]. Several prototypes for a flexible mold system have been designed, and in some cases built, by researchers and manufacturers over the years [89], [108], [109]. An example of such prototypes can be seen in the Kine-Mould project as shown in Figure 2.26. A number of commercial applications have also been developed for flexible molds [110], [111]. Schipper [90] presents a comprehensive overview of historical patents for adaptive flexible molds.

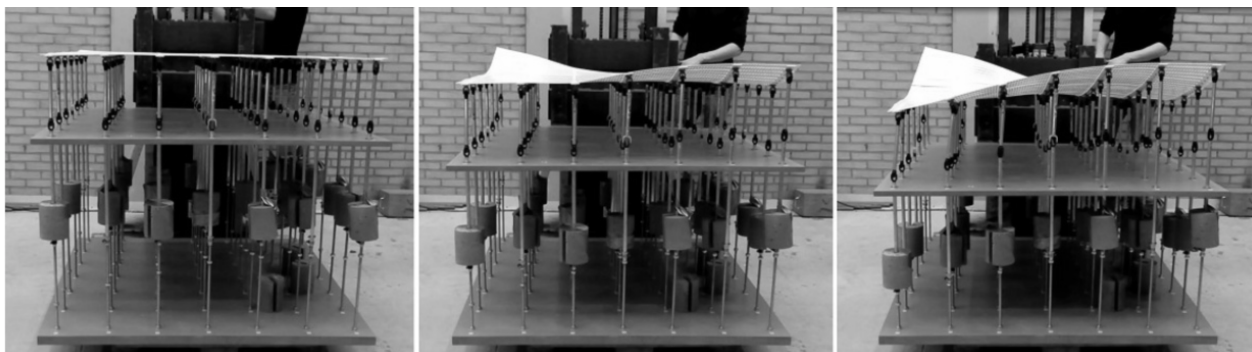


Figure 2.26: An adaptive formwork prototype built in the project Kine-Mould funded by STW (Dutch Technology Foundation). In this prototype, the tips of the actuators can sway in one or two directions to allow for the necessary horizontal displacements. Adapted from [109]

Cable-net Formwork

Cable net constructions are pre-stressed structures that consist of cable-nets and supporting frames, in which the interactions between the elements result in a state of equilibrium for the entire structure [113]. The use of cable-net constructions as formwork for concrete shells is achieved by tensioning a cable-net between a timber or steel frame and placing a layer of finer mesh or textile shuttering on top to prevent the concrete from slipping through the openings of the cable-net. The concrete can either be poured directly or, as is more common, sprayed on as shotcrete. Cable nets are considered an efficient and reusable approach to formwork construction capable of supporting the weight of concrete with minimal use of materials. However, the ability of the cable-net to achieve sufficient stiffness to support the concrete with acceptable tensile forces depends on the geometry and curvature of the target shape, as well as the amount of pre-stressing in the cable-net. High pre-stressing forces usually result in the need for very stiff frames, which has a negative impact on the material efficiency of the overall structure. However, the real strength of the cable-net lies in its ability to span large distances with a very lightweight structure. Geometries with high double curvature are the most influential factor in giving the cable-net its stiffness. For the design this means that the shape of the cable-net and the resulting concrete shell are directly dependent on each other [112]. Echenagucia et al. describe the construction of a thin-shell concrete roof with a cable-net and fabric formwork system shown in Figure 2.27 [112]. Furthermore, Veeneendal et al. presented a prototype where a combination of a cable-net system with fabric formwork was used

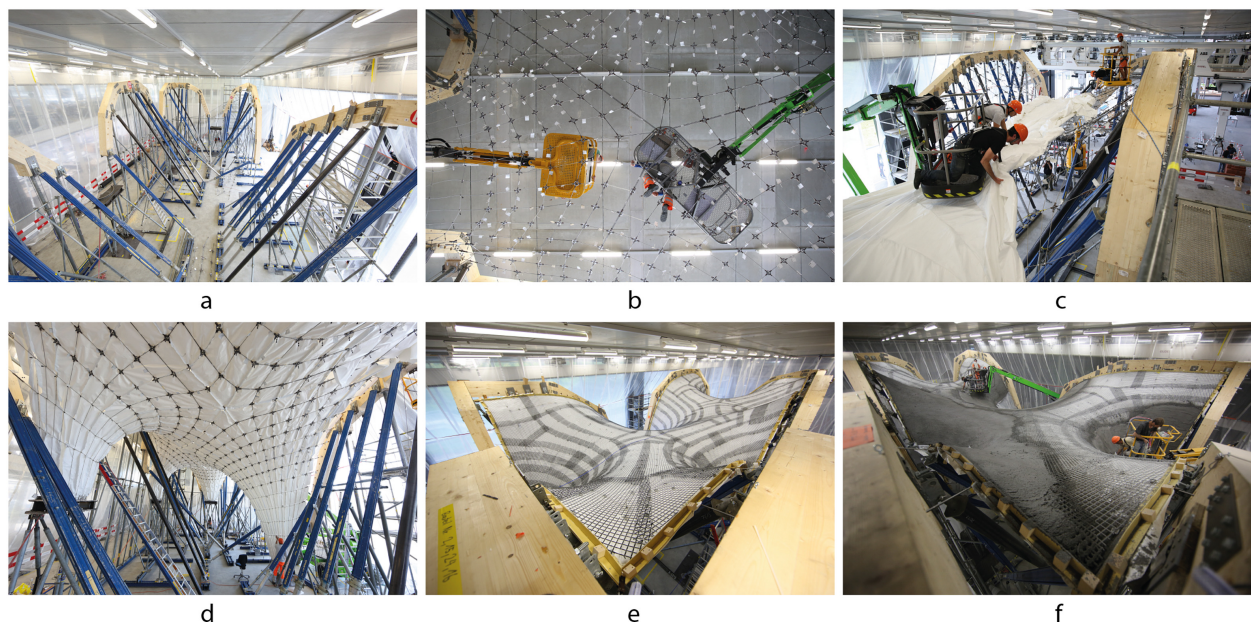


Figure 2.27: Prototype construction sequence. a) Scaffolding and boundary beam, b) cable-net, c) fabric shuttering, d) tensioned system, e) reinforcement, and f) concrete spraying. Adapted from [112]

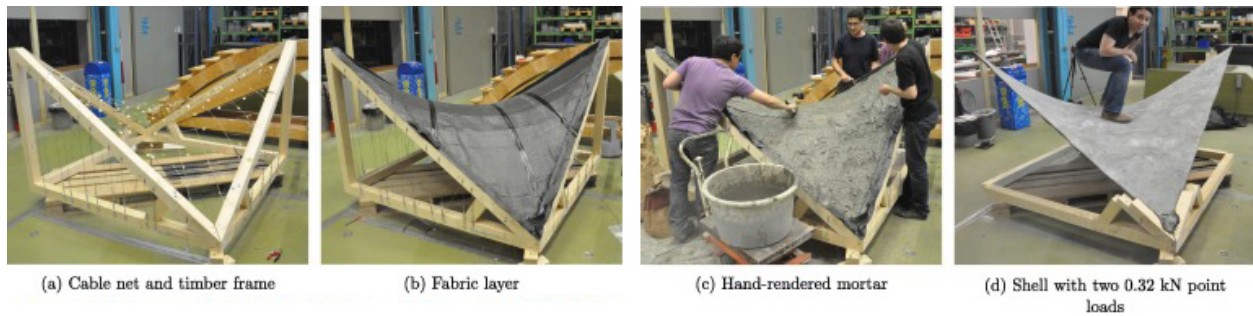


Figure 2.28: Construction of the formwork and shell using cable-net method. Adapted from [87]

for the construction of shell structures [87], [114] as illustrated in Figure 2.28.

Bending-active Formwork

Bending-active structures refer to the use of large elastic deformation of initially straight or planar building elements in order to construct complex curved geometries [8]. The main motivation for using the bending-active approach is the simplicity of generating curved elements. In the past, the bending-active method was widespread and accepted building practice due to the lack of alternative construction procedures. The main driving forces of using bending-active systems are economic factors, ease of transportation and assembling procedure, performance, and adaptability of the structure [65]. Various implementations of this approach have been explored by researchers in building lightweight architectural installations, temporary pavilions, kinetic structures, and compliant mechanisms [115], [116]. Lienhard et al. provide a comprehensive review of structures that actively use bending as a self-forming process [117]. In another recent review by the same authors [118], they highlight the potential efficiency of bending-active gridshells for large-span structures. Examples of such an approach can be seen in the gridshell at La Closca [119] and the Cathédrale éphémère de Créteil [120].

In the past, bending-active structures made of timber have also been used as formwork, for example as structural support and guide in the construction of masonry vaults, domes, and shells. An early example of such an approach can be seen in the roof construction of an office and house in Hirituka City, Kanagawa Japan. In this project which is designed by Shinji Yoshino a latticed bending-active timber gridshell supported the formwork that formed five shell roofs [121]. Another example of such a method can be seen in the work of Japanese architect Shoen Yoh, where concrete structures were constructed by applying concrete onto steel meshes laid onto a bamboo gridded sub-frame. Bamboo was first sliced into thin lath members and woven by hand in the spirit of Japanese craft tradition by the local community.

More recent examples of such an approach can be seen in the work of Tang et al. where they use a deployable gridshell for the fabrication of concrete shells. Their work can be seen in Figure 2.29 [122]. Furthermore, Cuvilliers et al. presented a shell fabrication method that combined a textile and bending-active formwork [123]. The gridshell was made from flexible GFRP rods providing

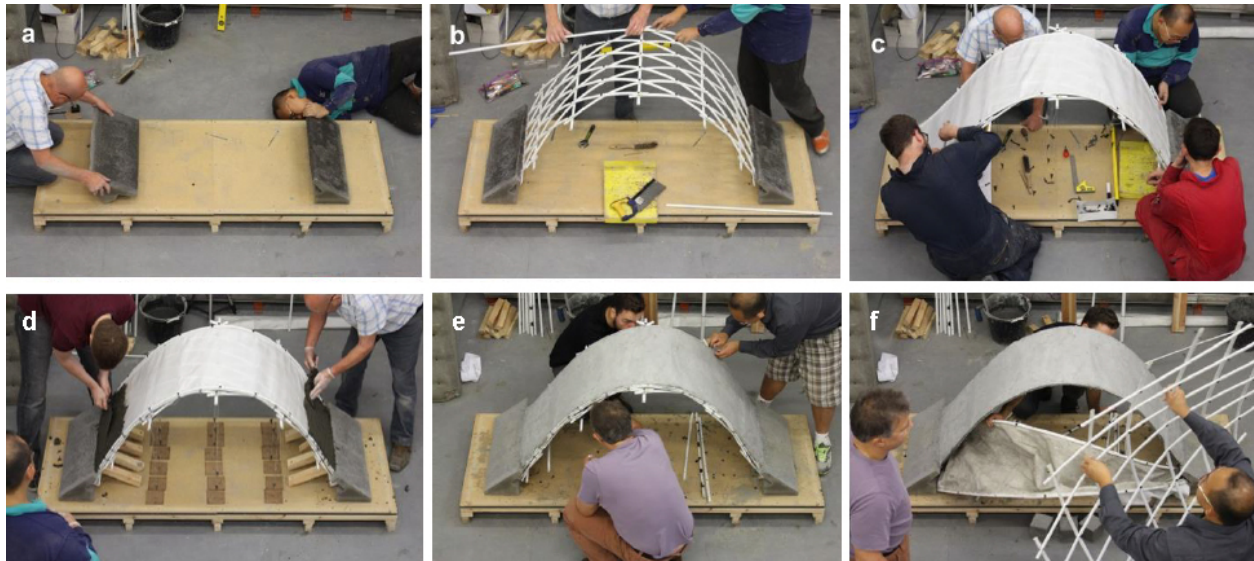


Figure 2.29: Stages of construction with bending active formwork: a) Abutments are affixed onto the baseboard. b) grid mat is subtended between the grid mats. c) Fabric hemmed at 2 edges is secured onto gridmat tightly. d) Once the fabric is secured with an edging piece on both sides, mdf measuring boards are arranged to match plumb lines. e) concrete applied to both sides and top f) formwork gridshell removed easily to be reused. Adapted from [71]

the structure its funicular shape while the concrete helped to rigidity the structure, distribute the stresses, and reduce the danger of local buckling. Finally, the work of Schleicher and Herrmann explores the novel idea of using bending-active structures, made from millimeter-thin CFRP strips, as lost formwork and structural reinforcement for the production of a hybrid gridshell prototype [124]. The latter project will be discussed in more detail in the following Chapter 3, as it is part and result of the research conducted in this dissertation.

2.4 Concrete and Carbon Fiber Reinforced Polymers

After this detailed insight into the relationship between construction and formwork, the final section of this background chapter will discuss the interesting combination of concrete with carbon fiber reinforced polymers as a construction method. Structural systems with multi-material cross-sections of two or more materials are usually referred to as hybrid structures and can achieve higher performance than the individual materials by themselves. In hybrid systems, the goal is to take advantage of the strengths and characteristics of different materials to improve their functional role and overall material efficiency [125]. Reinforced concrete is a good example to illustrate the advantages of a hybrid system. While the concrete component in this material mix is good under compression, it is very weak under tension. Therefore it requires the placement of reinforcement to resist tension forces. Typically, steel rebar is used to reinforce concrete, as steel has a much

higher tensile strength than concrete. Although this approach has become widely accepted and countless buildings around the world have been constructed using this method, it also has some significant drawbacks. The challenges lie particularly in the susceptibility of steel to corrosion, the different coefficients of thermal expansion and contraction of steel and concrete, the relatively high weight of steel, and the material and labor costs of using steel reinforcement. It is therefore not surprising that many alternative reinforcement methods have been developed and tested in the past. One of the most interesting is the use of different fibers as strong, lightweight, cheap, and resilient reinforcing materials. For example, one of these reinforcement materials is fiber-reinforced polymer (FRP). It is made of a thin polymer layer that can be composed of various fiber materials such as glass FRP (GFRP), carbon FRP (CFRP), aramid FRP (AFRP), or basalt FRP (BFRP) [126]. FRP composites come in various forms such as laminates, rods, dry fibers, or sheets, which can either be placed in the concrete during casting or retrofitted to the concrete from the outside after curing using adhesives and fasteners [127].

When the concrete is subjected to compression and the FRP composites to tension, the mechanical advantages of the two materials can be fully exploited. Compared with conventional profiles and cross-sections, such as steel-reinforced concrete, the advantages of FRP-concrete composites can be listed as follows: [128], [129] (i) enhanced flexural performance in positive moment regions (those areas in which the material would need to stretch when bending occurs) [130]–[133]; (ii) high whole-life cost-effectiveness due to their corrosion resistance characteristics; (iii) ease of installation and transportation due to their light-weight nature [133]; and (iv) extended durability in aggressive environments [134].

Applications of concrete CFRP composite material in construction can be divided into two broad categories [135]: (i) applications for new construction and (ii) for repair and retrofit applications. Custom-made FRP elements act as a barrier for protecting the concrete, and the concrete on the other side improves the abrasion, fire, and impact resistance of custom-made FRP elements.

Using internal FRP reinforcement as a replacement for traditional steel reinforcement offers some advantages. Each of the two most common types of fiber-reinforced polymers, CFRP and GFRP, has its own unique properties. In general, carbon fiber reinforced polymers (CFRP) are more efficient compared to glass fiber-reinforced polymers (GFRP) due to their significantly higher tensile strength. However, CFRP comes at a significantly higher cost. GFRP, while less efficient than CFRP and even steel, is more economical because it is cheaper than the other two and much lighter. A key advantage of CFRP and GFRP over steel is that their higher corrosion resistance reduces the need for concrete cover. Unlike steel bars, which require significant concrete protection, the use of FRP materials allows for lower section heights and therefore results in thinner and lighter structures. Moreover, retrofitting concrete members such as columns, beams, slabs, and walls with externally bonded FRP has proven to be an effective method. For instance, numerous techniques are used to strengthen reinforced concrete beams with FRP materials. These techniques involve the use of externally bonded laminates, near-surface mounted bars/strips, mechanical anchorage systems, or grooving methods with or without adhesives [136], [137]. FRP strengthening can be performed with the desired number of strengthening layers on a beam in any configuration, such as side bonding, U-wrapping, or full wrapping [127] which Zou et al. provide a comprehensive review of such utilization of FRP materials with concrete [138]. FRP composites are also widely



Figure 2.30: Reinforcement and repair of concrete structure with carbon fiber materials. Adapted from [139]

used for retrofitting bridge structures in various formats, such as long unidirectional FRP lamellas for strengthening and reinforcement. In addition, wide FRP sheets made of carbon, glass, or aramid fibers bonded with a polymer matrix are used as a substitute for steel for structural rehabilitation of bridges when a larger area of the concrete structure needs to be reinforced [129]. Examples of such an application can be seen in Figures 2.30-2.32.

In particular, for the construction of concrete shell structures, the use of FRP opens up various interesting possibilities, which can be summarized as follows: 1) FRP is lighter than steel and requires less concrete protection when placed internally. This significantly reduces the cross-sectional thickness of the concrete, which in turn makes the shell much thinner and lighter. 2) By attaching the FRP reinforcement to the concrete structure from the outside, it is visible to the naked eye, which facilitates maintenance and safety evaluation. 3) The external FRP reinforcement itself could serve as a lost formwork during the construction process of a shell or gridshell. A provocative and groundbreaking new idea, which will be discussed in more detail in the following chapter.

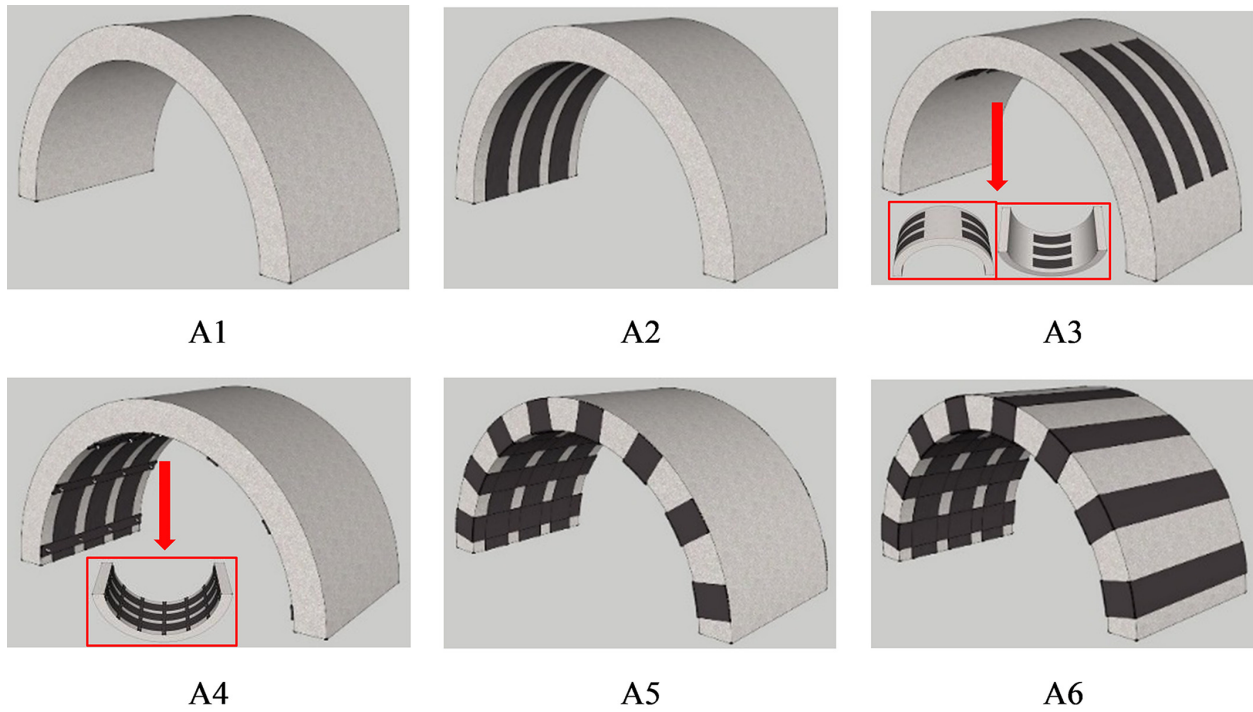


Figure 2.31: Different schemes of bonding carbon fiber reinforced polymer (CFRP) with additional wraps or anchors to strengthen reinforced concrete (RC) arches [140].

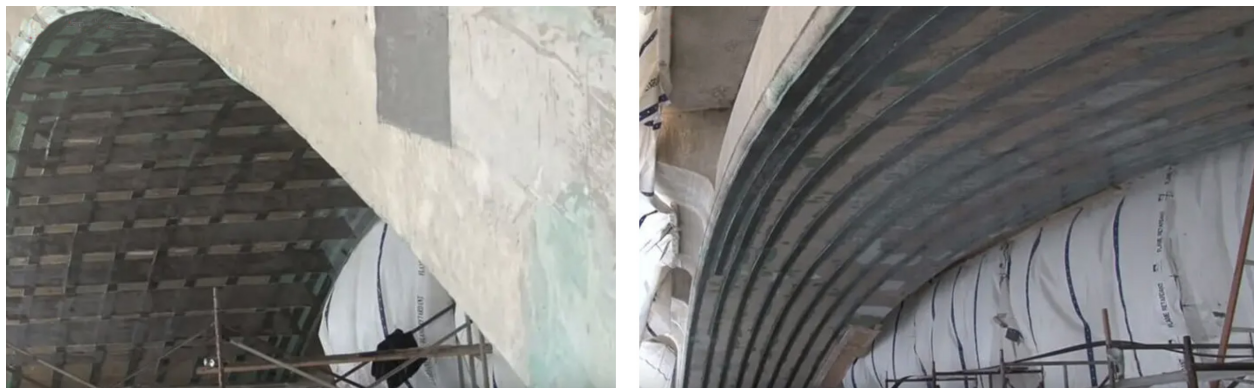


Figure 2.32: Retrofitting of reinforced concrete bridge with carbon fiber reinforced polymer.
Adapted from [141]

Chapter 3

Feasibility Study

This chapter builds on the previous sections by continuing the discussion on lightweight concrete shells and gridshells and investigating the thought-provoking idea of using CFRP lamellas as a lost formwork in the construction process. To investigate the feasibility of this approach, two prototypes, shown in Figure 3.1, were designed, built, and studied in more detail. While the first prototype focuses on the general question of how to gain structural stability from flexible elements, the second prototype more specifically addresses the challenge of using bending-active structures as formwork for the construction of concrete gridshells.



Figure 3.1: Left: Feasibility study on gaining stability from flexible material Right: CFRP bending-active formwork for the construction of concrete shells.

3.1 Bio-inspired Lamellar Structure: Gaining Rigidity from Flexible Materials

The first project was developed as part of UC Berkeley's post-professional degree program, called Studio One (MAAD), and was completed as a team effort by nine students and the instructor, Prof. Simon Schleicher. Over the course of a year, the students studied biological models and developed bio-inspired fabrication and construction methods that can be used in architecture. At the end of the program, a collaborative research pavilion was built that illustrated the technology transfer developed and demonstrated the feasibility of the ideas in a prototypical way. The resulting 2018 Studio One Research Pavilion is a bio-inspired lamellar structure that delves into the idea of constructing a rigid yet lightweight, bending-active structure from flexible parts. This project began from two starting points. A "bottom-up" study of light yet strong biological models specifically in plant kingdom and a "top-down" material investigation of flexible CFRP strips provided by our industry partner Kryslar and Associates. These CFRP strips have interesting material properties as they combine high tensile strength with low bending stiffness, mainly due to their low thickness of only 1.2 mm. These characteristics make the material an ideal candidate for research on bending-active structures. As explained in the previous chapter, the goal of using bending-active structures is to take advantage of large elastic deformations of originally straight or planar components to construct complex curved geometries that gain stability when bent and connected together, or to allow for controlled elastic deformations and flexible movements. Recent work by research teams around the world has demonstrated several successful applications of bending-active structures for lightweight architectural installations, temporary pavilions, kinetic structures, and compliant mechanisms [115]–[117], [142], [143]. However, this construction process is also associated with some challenges. One of these is related to the fundamental contradiction of trying to build a rigid and stable structure from soft and pliable parts. The second challenge is associated with the question of how to predict the complex equilibrium shape that results when several elements are bent and joined together. The third challenge in the design of bending-active structures is how best to take advantage of the high flexibility and resilience of the materials and what types of shapes and building structures can be achieved if this design process is followed systematically. To address these research questions, this study took a somewhat unconventional approach and tapped into new sources of inspiration by exploring soft and elastic structures in nature and investigating the underlying principles by which flexible plants gain their stability and functionality.

Biological Inspiration

Nature, and in particular the plant kingdom, is full of fascinating lightweight structures that can carry substantial loads despite their often delicate and fragile composition. A good example is the fibrous anatomy of cactus skeletons like *Cylindropuntia acanthocarpa* (commonly known as Buckhorn Cholla) and *Opuntia basilaris* (known as Prickly Pear), as shown in Figure 3.2. These wooden structures are fascinating because they are astonishingly robust while using only a minimal amount of material. Unlike smaller, round, or barrel-shaped cacti that only consist of water-storing

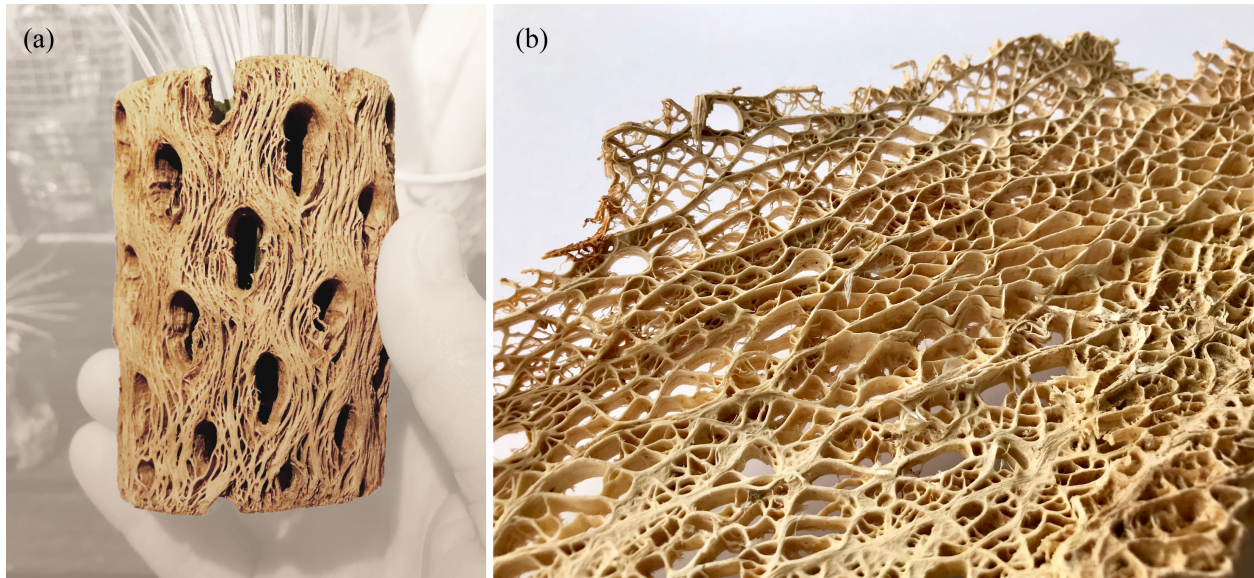


Figure 3.2: Study of the dried cactus skeletons of the (a) Buckhorn Cholla and (b) Prickly Pear.

tissue, these cacti can grow much taller and effectively combine water storage with structural support by means of an inner lignified skeleton. Inside the cylindrical stems of the Cholla or the flattened pads of the Prickly Pear, for instance, are fine reticulate vascular bundles that form lightweight yet very strong lattice structures [144], [145]. Over the past years, botanists and biomechanics have studied these wooden skeletons with great interest and investigated how their shape and material composition are likely the result of load-adaptive growth processes [146], [147]. Inspired by these structures, the study focused on the fiber arrangement and material distribution in various dried cactus skeletons, leading to the derivation of the following bio-inspired design principles. First, the tall skeletons feature long continuous fibers that give the structure a clear alignment and help to effectively transmit forces along the structure's main axis. Second, smaller cross-connecting fibers create an interlaced network that braces the structure laterally and significantly increases the overall stiffness. Thirdly, some skeletons seem to feature sandwich-like material compositions with stronger fibers covering a lighter core. Finally, the observations revealed that the lignified material in the skeletons is not distributed uniformly, but instead visibly thicker in regions of higher anticipated stresses. This can be seen, for example, when comparing hole sizes and material density. The skeleton features notably smaller openings and more solid material at the lower part of the stem while being significantly lighter and thinner towards the top.

Material Exploration

Carbon fiber reinforced polymers (CFRP) are composite materials that can be produced by means of a pultrusion process. This is a highly automated, low-labor manufacturing process that converts fiber reinforcements and resin matrices into finished composite parts by pulling the

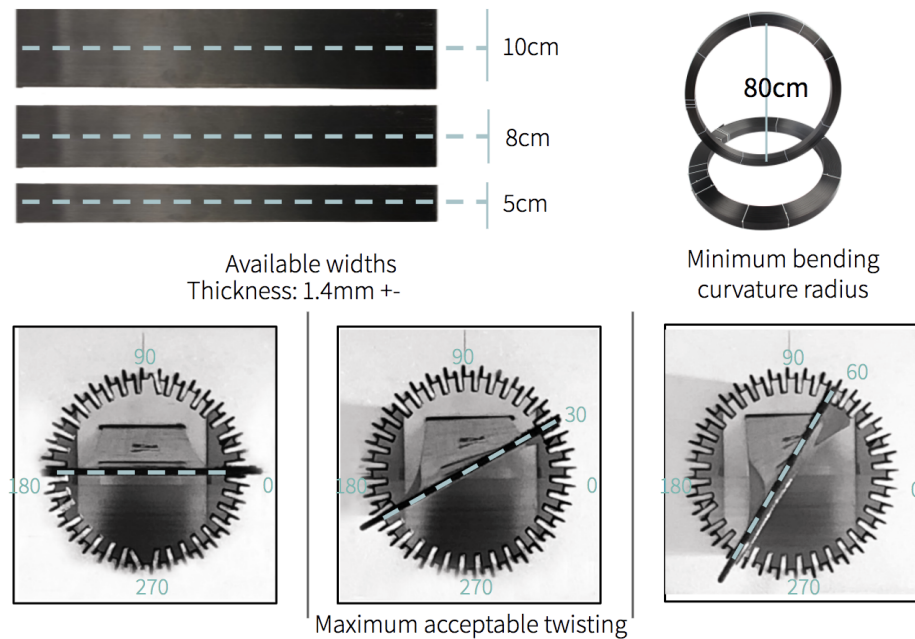


Figure 3.3: Initial material testings to understand the constraints of handling the CFRP strips.

materials through a heated die to produce continuous composite profiles such as strips and beams. As mentioned in Section 2.4, these CFRP profiles are often used for architectural retrofitting to either repair or reinforce concrete structures. The pultruded carbon fiber strips used for this study are available in rolls of 10 and 5 cm width and 1.2 mm thickness. Due to the small thickness of the material, the CFRP strips are very flexible. A side effect of the pultrusion process used to produce the CFRP strips is that they consist of only a single layer of lateral fibers and no cross-bonding fibers, making them particularly susceptible to fracture. Given this property, it was very important to further investigate the possibilities and limits of such a material. In addition to the material data sheets provided by the manufacturer, physical experiments were performed to measure the twisting and bending properties of the CFRP material. As shown in the Figure 3.3, the physical material tests revealed that CFRP strips have a minimum bending radius of 40 cm and a maximum twist angle of 30° for a 5 cm wide and 100 cm long specimen.

Design and Construction Methods

Impressed by the intriguing shapes and structural performance of the cactus skeletons, the derived design principles were transferred into bio-inspired formal and structural explorations. In this phase, the investigations primarily consisted of using physical and digital model-making. Physical models were made from thin strips of plywood veneer and were particularly helpful in gaining a quick tactile understanding of how multiple strips can be bent and coupled together to create a series of possible pavilion designs, as shown in Figure 3.4. In comparison, the goal of the

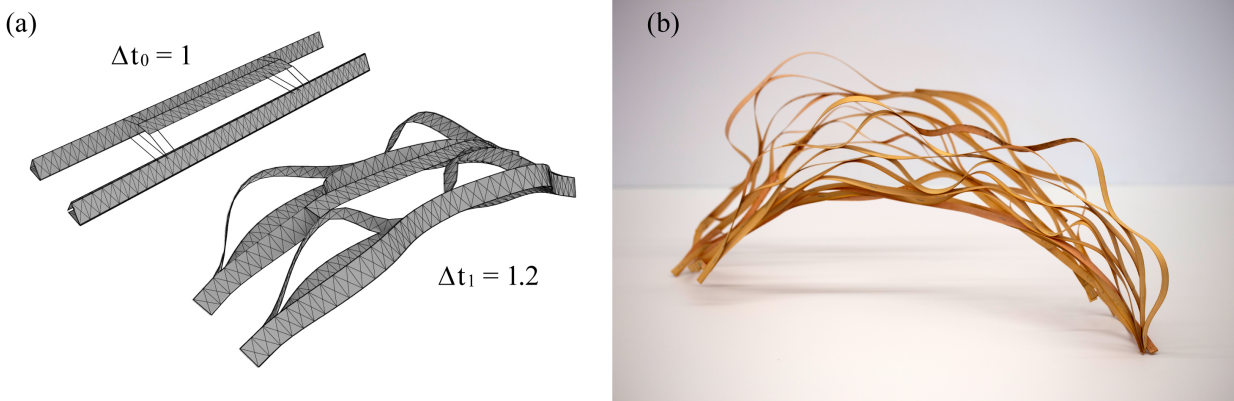


Figure 3.4: Using digital growth simulations in (a) Kangaroo Physics to design and (b) build models from thin strips of plywood veneer.

digital models was to find new ways of generating lattice-like structures that feature characteristics similar to cactus skeletons. The starting point for these explorations was the work by Lienhard et al. [148] and Nabaei et al. [149], who developed digital form-finding strategies for the design of interlaced strip systems. Based on their methods, the design approach involved working in two different modeling environments. The first approach took advantage of the opportunities provided by the Rhinoceros plugin Kangaroo Physics [48] for Grasshopper. This interactive tool allowed for real-time simulations of particle-spring systems and made it very easy to produce different design iterations during the early phase of the creative process. Figure 3.4a, for example, shows a model in which different growth rates were assigned to a bundle of densely packed strips, causing a delamination effect that forced the entire structure to bend. The individual strip lengths were then measured and transferred to a physical model, as shown in Figure 3.4b. While this approach created very appealing results and reflected the actual growth processes in the cactus well, it was also very difficult to control the final shape. Thus, the second approach aimed to control the form-finding process by using the software SOFiSTiK [150], which is based on finite element methods (FEM). While this engineering software was originally conceived for post-design structural analysis and can be computationally heavy, in this study it was used as an early-stage design tool to accurately simulate the geometry of elastically deformed strips and to ascertain their internal stresses. To produce a bent geometry, as shown in Figure 3.5, the method of contracting cables was implemented [148]. In this simulation, the CFRP strips with a thickness of 1.2 mm, a width of 100 and 50 mm, and Young's modulus of 165,000 N/mm² were modeled. The strips were then pulled together with a series of contracting cables that spanned between sliding anchor points along the edges of the strips. The goal of this simulation was to determine a deformed geometry that couples multiple strips together in a predefined connection logic, while simultaneously giving each strip the possibility to rotate freely into the one equilibrium shape that results in the least amount of stress within the given boundary conditions. As a result of this process, each connection detail between the strips had a unique geometry with locally specific clamping angles between the strips. The colors in the image

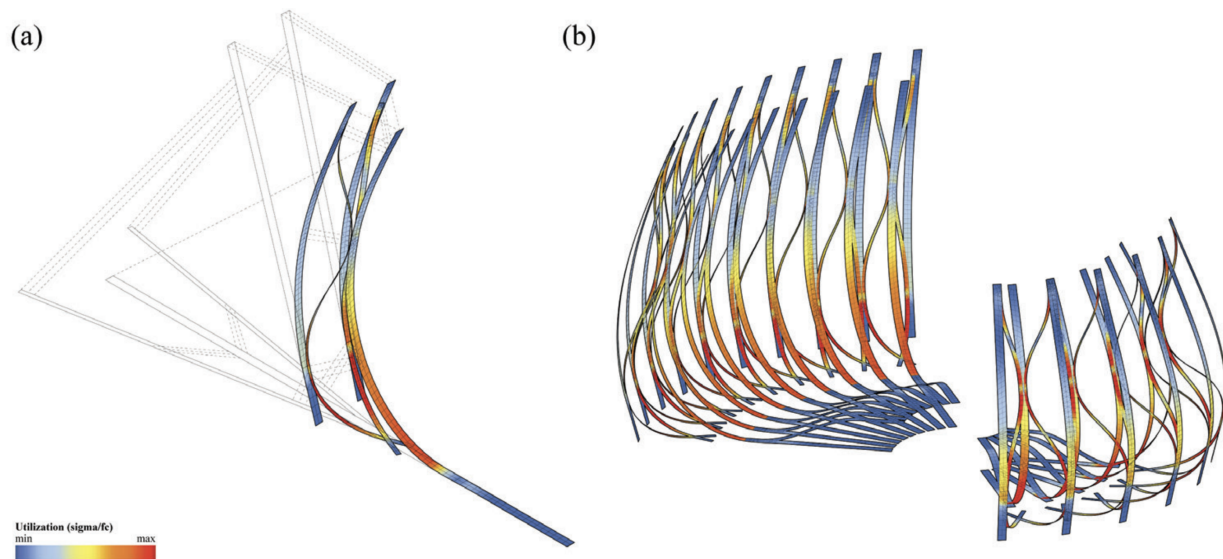


Figure 3.5: By using finite element models, the prototype's exact geometry is determined (a) and analyzed the stresses in the elastically-bent carbon fiber reinforced strips (b); areas in red indicate zones of higher stress due to bending and/or torsion. The strips were modeled with a thickness of 1.2 mm, a width of 100 and 50 mm, and Young's modulus of 165,000 N/ mm².

represent the locally varying stress conditions caused by the bending and torsion of the thin CFRP strips. These stresses and the changes in geometry were monitored closely and checked for their compatibility with the material limitations.

Prototyping Results

To test the practical feasibility of the cactus-inspired design principles and derived form-finding methods, a prototype was conceptualized and built at full scale. The final design was a 6-meter-tall structure that acted both as a space-defining architectural installation as well as a seating opportunity during various events. The entire structure was made from 50 and 100 mm wide CFRP strips with 1.2 mm thickness and sheets of 19 mm and 6 mm thick ACX plywood. The shape of the pavilion is a seamless transition between a dense horizontal bundle of strips that acts as a bench and a vertical lattice network that rises fan-like to the sky. Similar to the biological role model, each strip alone was far too soft to carry bigger compression loads or to reach any notable height. However, when bent and coupled together, the flexible strips supported each other and formed a very lightweight yet rigid system. In order to further stabilize the interlaced lamellar structure, the outermost strips were built as a sandwich structure, in which two strips of CFRP were glued together with a 20 mm thick styrofoam core. While all other parts in this pavilion were flat-produced and elastically bent into shape, these strips were laminated on a curved formwork that precisely followed the shape determined from the digital form-finding. To anchor the temporary structure and to secure it safely

to the ground, the bench of the pavilion was made of heavier ACX plywood sheets with a thickness of 19 mm. As illustrated in Figure 3.6a, its design was based on a radial waffle structure with vertical and horizontal blades that were CNC routed and could be interlocked with each other easily. Despite the use of a heavier foundation, the prototype was extremely light and weighed only 238 kg. Of this total weight, the CFRP structure only accounted for 60 kg. This result was achieved through cross-bracing two types of CFRP strips. The 100 mm wide strips carried the vertical loads and had a total length of 238 m, while the 50 mm strips had a total length of 140 m and were used to cross-brace the structure. Since all strips were fabricated by means of a pultrusion process, and thus composed of only a single layer of lateral fibers, they were particularly prone to fracture and needed to be protected against exceedingly high torsional loads and potential local damages. It was therefore important to handle the strips with great care and to follow an assembly sequence that would not deform the strips beyond their material limitations. Furthermore, it was crucial to develop a connection detail that allowed for quick and easy construction without any cutting, drilling, or bolting that would cause cracking in the strips. In order to address these challenges, the connection details were built by laser-cutting thin sheets of 6 mm plywood, as shown in Figure 3.6b. Each connection detail featured a specific slit layout that was custom-made to the required target angle of each node and shaped in such a way that it would be easy to insert the strips from one direction. As shown in Figure 3.7, this detail simplified the construction process significantly and enabled the students to fabricate and erect the entire pavilion, shown in Figure 3.8, within just two days and to disassemble it after the exhibition in only one day. Finally, 90% of the strips were preserved during the construction process, and to be able to salvage the precious material and reuse it for a follow-up prototype.

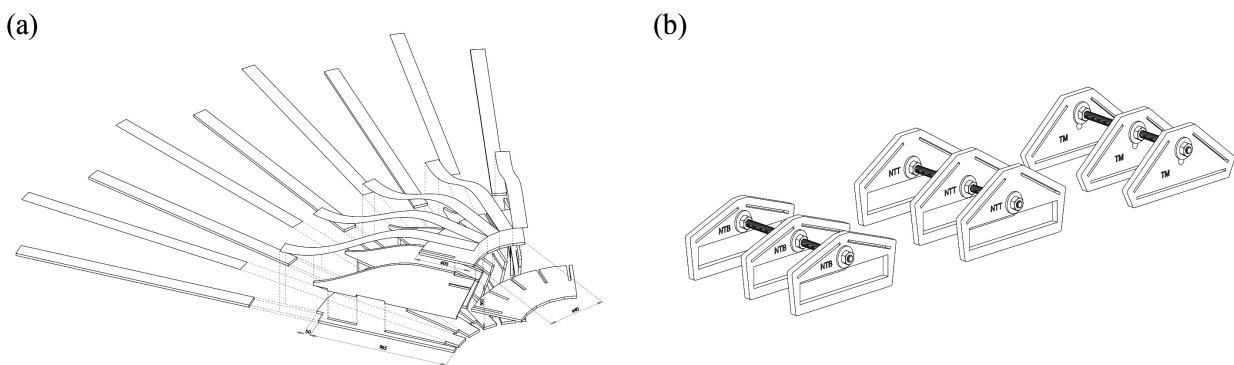


Figure 3.6: (a) Bench construction and (b) bespoke connection details with custom angles at different locations of the structure.

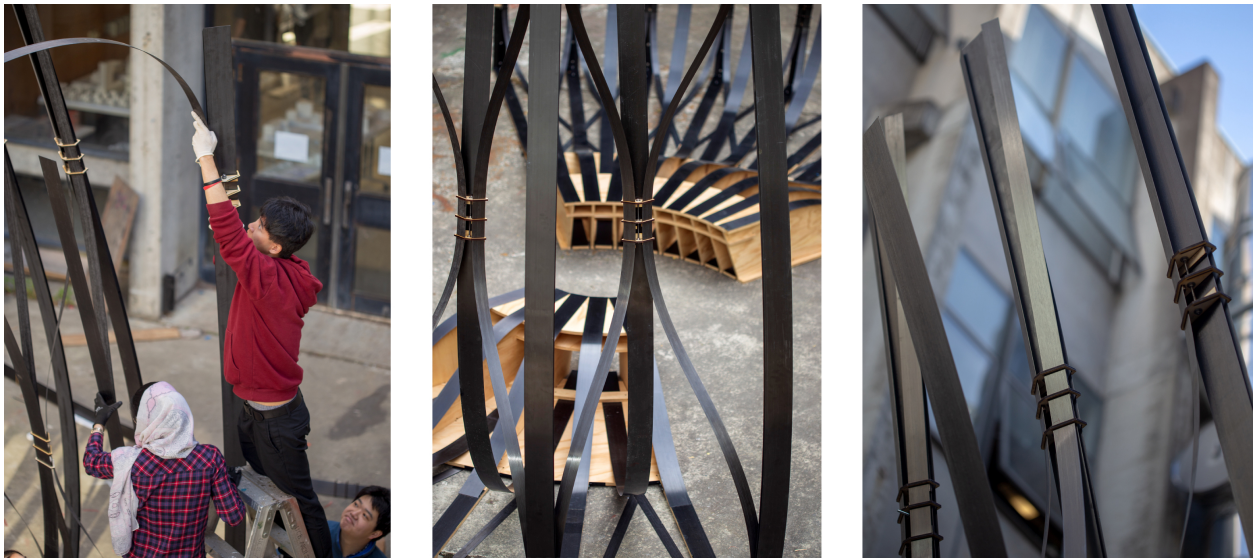


Figure 3.7: Within two days, the 6m-tall research pavilion was fabricated and built.



Figure 3.8: Studio One Research Pavilion on display in the courtyard of the College of Environmental Design (CED) during the annual exhibition at the University of California, Berkeley.

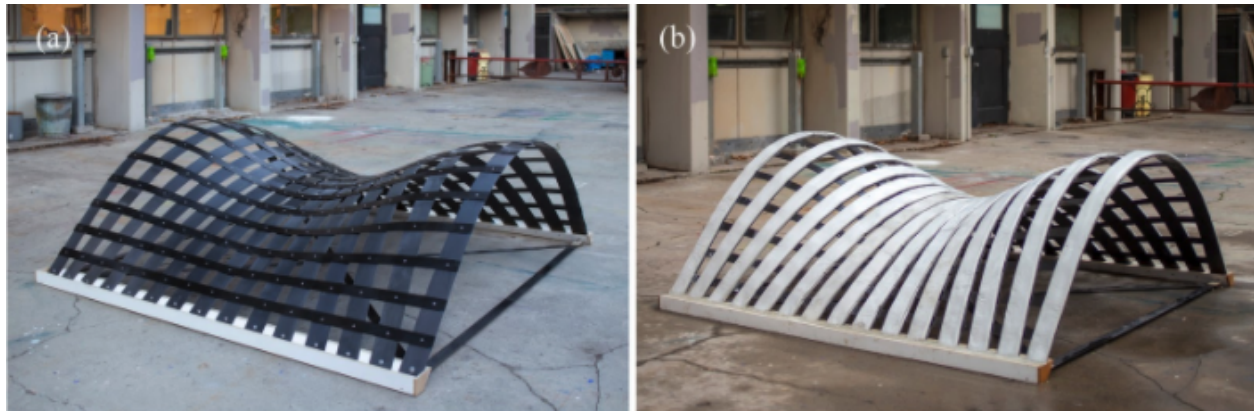


Figure 3.9: (a) Full-scale CFRP formwork (b) and finished carbon/concrete hybrid shell.

3.2 Hybrid Gridshell Prototype

The goal of the second feasibility study was to determine whether bending-active structures can be used as lost formwork for the construction of concrete gridshells. To investigate this hypothesis, a prototype of the proposed system was built as a proof of concept. It provided the opportunity to conduct a series of experiments in order to understand the possibilities and limitations of this construction method. Special emphasis was placed on learning more about key design constraints and incorporating them into a workflow as part of a performance-based generative design system. The prototype itself is a bending-active concrete gridshell structure fabricated using strips of pultruded unidirectional carbon fiber reinforced polymers (CFRP) as a lost formwork, seen in Figure 3.9. The following chapter will describe in more detail the material limitations and the resulting geometrical constraints that were key to the design of the prototype. This is followed by a discussion of the fabrication process and post-analysis of the built prototype.

Material and Geometry

For the design of the prototype, several design objectives were considered, aiming at balancing the limitations of the materials used and the resulting constraints in terms of the gridshell's geometry. For the construction of the formwork, the CFRP strips from the previous project were harvested and reused. These are carbon fiber reinforced polymer (CFRP) strips produced by the pultrusion process with a thickness of 1.2 mm and a width of 5 and 10 centimeters, respectively. As mentioned in the previous section, the material tests conducted by the team showed that the CFRP strips have a minimum bending radius of 40 cm and a maximum twist angle of 30° for a 5 cm wide and 100 cm long specimen. In addition, the manufacturer's data sheet stated that the Young's modulus of the strips was 165,000 N/mm². To achieve a gridshell design with beneficial structural performance, the team chose a geometry with high synclastic and anticlastic double curvature. The team hypothesized that if the proposed design technique can be implemented for such a challenging double-curvature

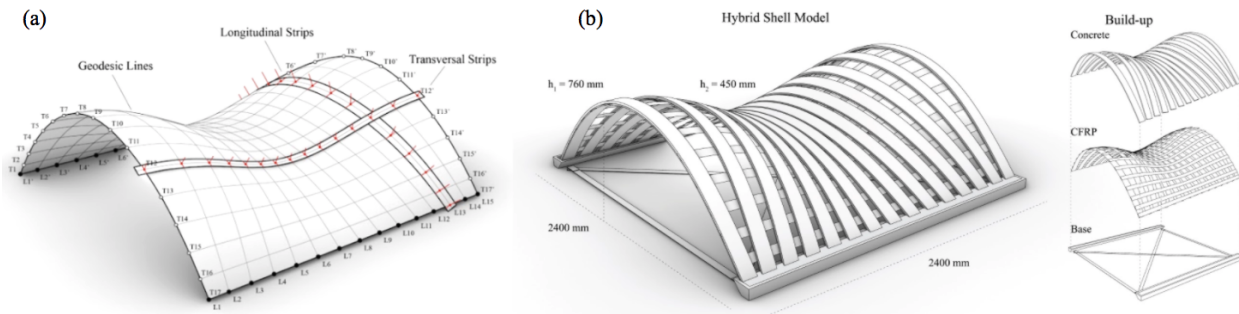


Figure 3.10: a) Generation of developable strips b) model of the gridshell with the base frame, strips of CFRP, and concrete.

geometry, it can most likely be integrated into any other free-form gridshell geometry with less extreme curvature. For the design of the basic geometry of the gridshell, it was important to respect the material limits of the CFRP strips and not exceed their bending and twisting capacity. The result was a gridshell of 2.4 m x 2.4 m with a mid-span height of 76 cm, as shown in Figure 3.10. It was designed using the NURBS modeling software Rhinoceros. Figure 3.10a shows the control logic of the strip layout. Each strip follows a geodesic curve that represents the shortest path from one edge point to the opposite edge point on the target surface. The method of geodesic curves is used to ensure that the strips, although different in length, are all straight even when unrolled. The curves are then extruded to 10 cm or 5 cm using a locally changing tangent plane of the surface, Figure 3.10b. The formwork consists of a total of 32 pultruded CFRP strips, 15 in the longitudinal direction and 17 in the transverse direction.

Fabrication

The gridshell was built at full scale to test the feasibility of the proposed technique and to study the material behavior in detail. It was constructed in two days by a small team of UC Berkeley students. To support the gridshell, a base frame was designed, using CFRP strips (Figure 3.11a). A total of 82 m long CFRP strips were used to produce the grid. For the concrete layer above (Figure 3.11b), a fast-setting, high-strength, low-shrinkage premixed mortar was used. The mortar has 28 days mean compressive strength of 48.3 N/mm² according to ASTM Procedure C109, which is tested on 50 mm cube specimens. The mortar set in 15-35 minutes and 60 minutes after the final set developed a mean compressive strength of 17.2 N/mm². To increase the concrete's tensile capacity for this project, a grid of glass fiber reinforced polymer composites (GFRP) was embedded in the concrete, shown in Figure 3.11b. Nails were glued to the carbon fiber strip formwork at 10 cm intervals to improve the mechanical bond between the carbon fiber and the applied concrete layer. This allows better transmission of shear forces between the two materials. To place the concrete on the bending-active CFRP formwork, the concrete strips were poured one by one flat on the ground, wrapped in plastic film, lifted and draped over the formwork when the concrete was not yet fully

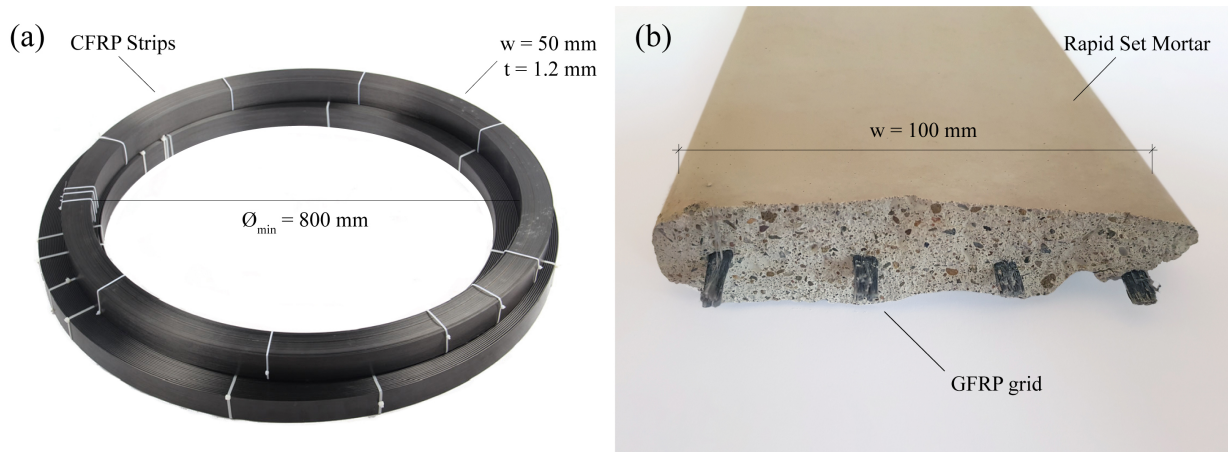


Figure 3.11: The main materials in this project were (a) strips of fiber-reinforced polymer composites (CFRP) and (b) concrete strips made from a premix fast-setting, high-strength mortar. (Figure 3.11a with the permission of Shanghai Horse Construction)



Figure 3.12: Sequential loading of the lost formwork with concrete strips.

set and still malleable, as shown in Figure 3.12. This step-by-step construction started with the longitudinal strip in the center and then added strips to the left and right to avoid uneven weight distribution and thus undesirable deflections of the formwork. Once the concrete strips were draped over and pressed against the CFRP formwork, the concrete began to harden and bond, creating a hybrid material cross-section that significantly increased the load-bearing capacity of the otherwise quite flexible formwork. As the strips hardened they improved the load-bearing capacity of the otherwise flexible formwork significantly. This sequential curing process took about 8 hours.

Loading and Analysis

To get an initial sense of the load-bearing capacity of the hybrid gridshell, a rudimentary test was conducted in which the structure was subjected to the weight of several students who stepped onto the gridshell one after the other. The gridshell was able to handle this loading scenario remarkably well, supporting the weight of 8 people. Each person stepped onto the gridshell after the other



Figure 3.13: Ultimate loading test reveals the shell's resilient behavior.

(Figure 3.13). The first three people stood on the central arch in such a way that they roughly represented a distributed load. It was observed that this loading of the central arch had almost no effect on the adjacent arches. This may be due to the soft transverse strips that connect the longitudinal strips. Finally, after 8 people roughly spread out on the gridshell, it failed. Prior to failure, the gridshell was able to withstand a vertical load of about 5.6 kN (8 people, each exerting an average of about 0.7 kN vertical load on the structure), which was twice the load capacity predicted by the digital simulations and therefore a surprising result [124]. At the moment of failure, the support of the center arch slid horizontally, resulting in a forced change of geometry, thus amplifying the bending moment in the arch. Due to this altered support condition, the gridshell lost its advantageous shell effect and temporarily behaved more like a curved beam, which is a less efficient structural system. This led to the failure of the central arch and the collapse of the entire gridshell. When the gridshell collapsed, the failure of the concrete strips was brittle, but the broken pieces stayed together because the concrete was reinforced with a GFRP mesh inside. After removing the broken concrete pieces from the carbon strips, it was interesting to observe the remarkable resilience of the bending-active formwork, as the formwork immediately bounced back to its original geometry and regained its initial shape, as shown in Figure 3.13.

Conclusion

From the two feasibility studies performed, the following general conclusion can be drawn with respect to bending-active structures:

- Aesthetic and design freedom: Bending-active structures allow the creation of unique geometry and visually attractive architectural forms, which is considered difficult to achieve

with conventional construction methods. Fluid and organic shapes created by these types of structures can allow designers, architects, and engineers to produce innovative and iconic designs.

- **Form-finding and digital tools:** The use of advanced digital tools and form-finding techniques has enabled the creation of feasible designs for bending-active structures that take into account material constraints and high bending stresses during the construction process or in the final state of the structure. This highlights the role of computational design methods in exploring complex and adaptable structures.
- **Lightweight architecture and construction:** The two feasibility studies carried out demonstrated the great potential and feasibility of using flexurally active structures in architecture and construction. They showed that the application of the bending-active approach with CFRP strips as material can lead to suitable structures for covering large spans by benefiting from their low self-weight and high stiffness, temporary structure due to their easy transport and assembly, and earthquake-prone regions by leveraging their low self-weight and potentially forgiving response to deflection.
- **Construction efficiency:** The demonstrators showed high efficiency and simplicity in the use of bending-active structures as a construction method. By using prefabricated, initially flat, flexible elements, rapid assembly and disassembly could be achieved, resulting in reduced construction time and labor costs. In addition, the potential of reuse of the CFRP strips for different projects, coupled with the ease of decomposition of the structure into different material groups, make this method not only cost-efficient but also compatible with the principles of the circular economy and demonstrate commitment to environmental sustainability.
- **Understanding material properties:** Both studies emphasize the need to properly understand the properties and limitations of the flexible material, in the case of this dissertation CFRP, when working with bending-active structures. The findings from the initial digital and physical material experiments were used to guide the design and manufacturing process of the two prototypes. Due to the unidirectional nature of the CFRP strips, careful handling of the CFRP strips was required to avoid damaging them. However, more research is needed to understand the interaction of CFRP and concrete in hybrid material cross-sections. Therefore, the next chapter will focus on this very issue.
- **Geometry and stiffness:** The experiments have shown that increased stiffness can be achieved through highly curved geometry and appropriate connections between the individual components, even when highly flexible materials and elements such as millimeter-thin CFRP strips are used. Additional design strategies, such as cross-bracing or creating cross-sections from multiple materials, can also help make the structure more resistant to loads from different directions and out-of-plane buckling. Thanks to these approaches, bending-active structures can obtain higher stiffness and also withstand additional loads, thereby serving as formwork.

Further investigations are required to gain a deeper understanding of the impact of geometrical parameters on the performance of the hybrid gridshell, which will be the focal point of discussion in Chapter 5.

In summary, the results of the feasibility studies carried out have helped to increase knowledge in the growing field of bending-active structures. They have further validated the possibility of practical implementation of these structures on a larger scale and real-world applications.

Chapter 4

Material experiments

4.1 Introduction

The feasibility studies presented in Chapter 3 show that a hybrid gridshell with a multi-material cross-section can be created by using a stay-in-place bending-active CFRP formwork, with concrete forming the top layer and CFRP as the bottom layer and serving as a tensile reinforcement for the concrete. In this concept, the goal of the multi-material composite is to leverage the strength of each material to improve the overall performance of the hybrid gridshell, making it lighter and stronger. A number of studies have already investigated the bending behavior of different concrete mixes and cross-section designs, the effect of various amounts of internal reinforcement, the type and amount of external reinforcement, the percentage of pre-stressing, and debonding mechanisms of fiber-reinforced polymer (FRP) layers, and different configurations of external reinforcement [151]–[157]. However, there are not too many studies that focus on the flexural stiffness of hybrid concrete-CFRP structures, especially in the context of using this combination of materials for hybrid gridshells based on bending-active formwork. Therefore, for this hybrid system to be used effectively, a comprehensive understanding of the material's behavior and its limitations is required. In addition to mechanical strength, the modulus of elasticity is one of the most important properties in the design of structures, as it controls stiffness and thus deformation and suitability for use. Consequently, the investigations in this chapter aim to perform a preliminary experimental investigation of the material properties of this hybrid CFRP-concrete cross-section and to provide some initial design guidelines for its implementation. For this purpose, a series of four-point bending tests were carried out, which are explained in more detail in the following sections.

4.2 Methodology

A comprehensive testing program is developed to assess several critical aspects related to the addition of the CFRP layer to concrete. Our primary objectives are as follows:

i) evaluating the efficiency of CFRP integration: Hybrid specimens of concrete and CFRP layer are tested to gain information about the impact of CFRP on the structural performance of the concrete layer. In addition, for meaningful comparison purposes, the concrete beams, with the same geometrical properties of the hybrid CFRP-concrete specimens are tested. They serve as the baseline for evaluating the improvement introduced by the addition of the CFRP layer.

ii) investigating the role of the connection method between CFRP and concrete: The CFRP-concrete specimens are categorized into two groups: a) the concrete and CFRP layers are connected through nails b) the connection between the concrete and CFRP layer is achieved using epoxy glue. This would enable the analysis of the influence of the connection method (full or partial bonding) between the layers.

iii) assessing the impact of internal reinforcement in comparison to the external CFRP reinforcement: for this purpose, all specimen categories are tested both with and without internal mesh reinforcement to further study the effectiveness of the internal vs. external reinforcement methods in the hybrid species.

In addition to physical experiments, an analytical study of the proposed hybrid cross-section was carried out to compare the results with the experimental results and to confirm the reliability of the obtained experimental data.

Experimental Study

A total of eighteen specimens were fabricated and divided into six groups (3 specimens each), including: (i) concrete (C), (ii) concrete with internal GFRP mesh (C Mesh), (iii) concrete-CFRP bonded together by nail connectors (C-CFRP Nail), (iv) concrete (with internal GFRP mesh) and external CFRP, interconnected by nail connectors (C-CFRP Nail Mesh), (v) concrete-CFRP interconnected by epoxy resin (C-CFRP Epoxy), and (vi) concrete (with internal GFRP mesh) and external CFRP interconnected by epoxy (C-CFRP Epoxy Mesh). Finally, three concrete cylinders were fabricated on which compression tests were performed.

Test Specimens and Material

All beams were 30.4 cm long, 5.08 cm wide, and 1.4 cm deep (referring to the concrete layer). The concrete cylinders were 20.32 cm high and 10.16 cm in diameter. Figure 4.1 shows the dimensions and connection details of the tested beams in this study. All beam and cylinder specimens were cast from the same batch of Sikacrete®-752 3D concrete with a water-cement ratio of 17% and cured in the fog chamber for 28 days. In addition, nine of the beams were reinforced with an embedded GFRP mesh at the midplane of their cross-sectional height. Six of the eighteen test specimens mechanically connected the CFRP strip to the concrete through nails. This was achieved by attaching the nails to the CFRP strips with epoxy resin and later pouring the concrete on top. The 10 nails are distributed in two parallel rows along the length of the beam, spaced 6 cm apart and 3.2 cm from the short edge. The CFRP strip used for 12 of the specimens has a thickness of 0.2 cm.

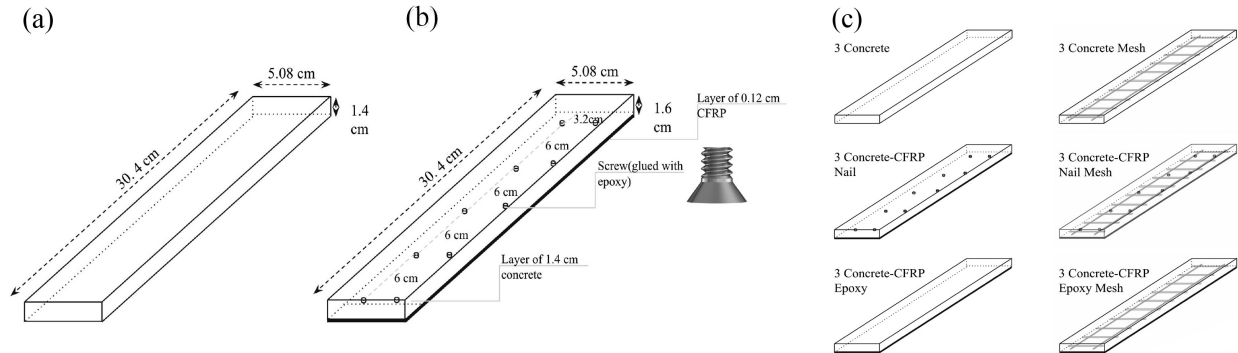


Figure 4.1: (a) Dimensions of specimens without CFRP (b) Specimens with CFRP and nail connector arrangement for C-CFRP Nail beams (c) Configuration details of the individual beam categories.

Compression Test

28 days after casting, three concrete cylinders were tested for compression using the ASTM C39 procedure [158] and the Forney Compression Machine (700K capacity), as shown in Figure 4.2.

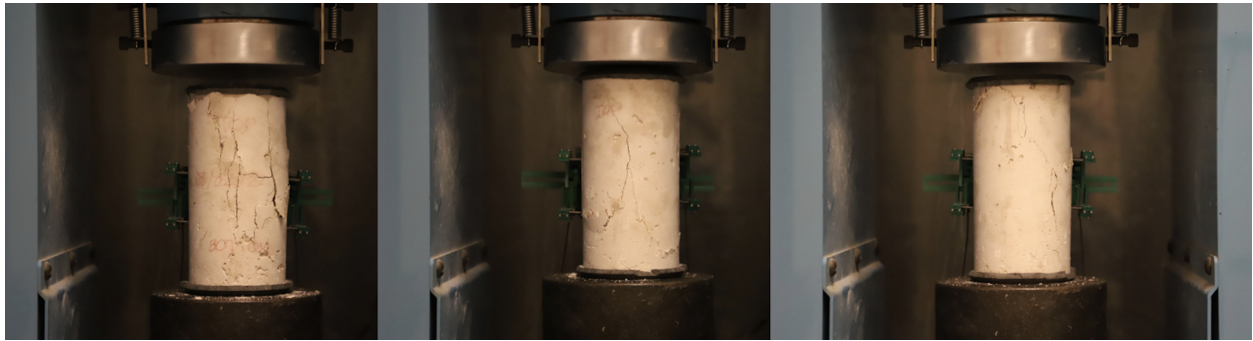


Figure 4.2: Three concrete cylinder specimens after performing the compression test.

To determine the compressive strength of the concrete, Equation 4.1 [159] and the failure loads obtained from the compression tests are used.

$$f_c = \frac{F}{A_c} \quad (4.1)$$

In the equation, F represents the failure load in Newtons (N) and A_c represents the area of the specimen on which the load was applied in square millimeters (mm^2). The mean strength value, f_c , of the concrete cylinder obtained was 31.55 MPa (N/mm^2).

Young's Modulus Calculation from the Compression Test

This section focuses on studying the modulus of elasticity of the specimens. The static pressure modulus of elasticity is the ratio between a compressive stress difference and the corresponding so-called elastic deformation per unit length [160]. The modulus of elasticity is calculated using the stresses (σ_o and σ_i) and strains (ϵ_o and ϵ_i) at points $0.33f_c$ and 0.5 N/mm^2 , respectively, and the associated measuring length l_o of the extensometer (used for calculating the strain values from the displacement measurements), from Equation 4.2:

The mean value for the concrete's longitudinal modulus of elasticity, E , was $33,718 \text{ N/mm}^2$.

$$E = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_o - \sigma_i}{\epsilon_o - \epsilon_i} \quad (4.2)$$

where:

E : Modulus of elasticity [N/mm^2]

σ_o : Test stress at $0.33f_c$ [N/mm^2]

σ_i : Test stress at 0.5 N/mm^2

ϵ_o : Strain measured at σ_o

ϵ_i : Strain measured at the stress of 0.5 N/mm^2

Four-point Bending Test

To perform the four-point bending test, the specimens were externally instrumented and loaded. Two vertical loads were applied symmetrically with respect to the mid-span of the beams, creating a constant bending moment region of 7.62 cm . All the specimens were simply supported and had an effective span of 22.86 cm . Figure 4.3 shows the experimental setup in detail, along with the dimensions. The applied load and vertical displacements were monitored throughout the tests. Displacement was measured at mid-span using linear variable differential transformers (LVDTs). The applied two concentrated loads to the beam were monitored using load cells. All data obtained from the load cells and the LVDTs were recorded using a data acquisition system.

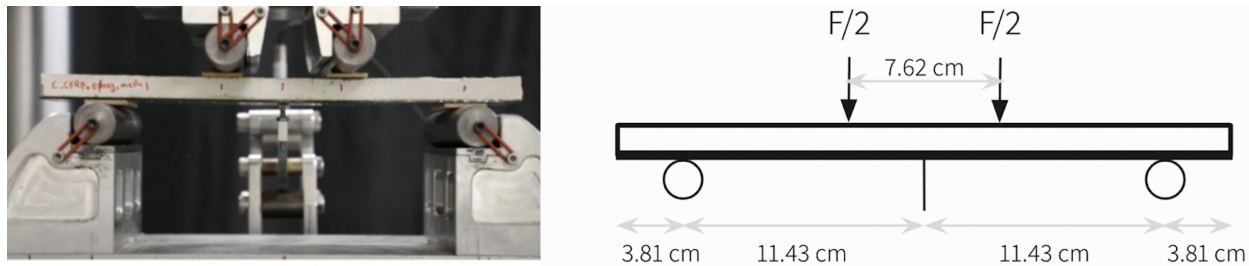


Figure 4.3: Left: Experimental setup Right: Dimensions, loading, and support conditions.

Analytical Study

The transformed properties of the hybrid cross-section are calculated via the modular ratio n , which is described as shown below and used to obtain the transformed concrete section from the composite section (Equations 4.3-4.9). This value determines the reduced cross-section values of the lower (CFRP) layer. The original C-CFRP and transformed calculated concrete cross-sections are shown in Figure 4.4.

$$n = \frac{E_{CFRP}}{E_c} \quad (4.3)$$

$$A_{CFRP,tr} = A_{CFRP} \cdot n \quad (4.4)$$

$$I_{CFRP,tr} = I_{CFRP} \cdot n \quad (4.5)$$

$$A_i = A_{CFRP,tr} + A_C \quad (4.6)$$

$$Z_i = \frac{A_{CFRP,tr} \cdot Z_{CFRP,tr} + A_C \cdot Z_C}{A_i} \quad (4.7)$$

$$I_i = I_{CFRP,tr} + A_{CFRP,tr} \cdot \Delta Z_{CFRP,tr}^2 + I_C + A_C \cdot \Delta Z_C^2 \quad (4.8)$$

$$W_i = \frac{I_i}{Z_i} \quad (4.9)$$

With

ΔZ_* : Distance of partial center of gravity from the total center of gravity

* $_{CFRP}$, * $_C$: CFRP layer, Concrete Layer

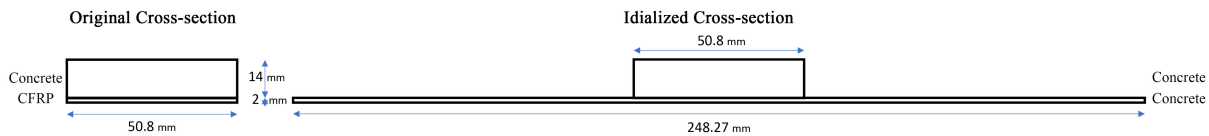


Figure 4.4: Left: Original concrete-CFRP cross-section. Right: Transformed concrete cross-section.

The cracking moment is calculated using the mean concrete tensile strength (f_{ctm}). This is according to Equations 4.10 and 4.11, calculated from the concrete's compressive strength. The calculated transformed concrete cross-section and the cracking moment, force, and associated displacement are presented in Tables 4.1 and 4.2.

$$f_{ctm} = 0.3 \cdot (f_{ck})^{\frac{2}{3}} \text{ [N/mm}^2\text{]} \quad (4.10)$$

$$M_{cr} = W_i \cdot f_{ct,eff} \quad (4.11)$$

Table 4.1: Transformed cross-section properties.

n	Z_i (mm)	I_i (mm⁴)	f_{ck} (N/mm²)	f_{ctm} (N/mm²)	W_i (mm³)	M_{cr} (Nmm)
4.9	5.7	30495	32	3	2964	8879

Table 4.2: Analytical calculation of P_{cr} and Δ_{cr} associated with the transformed cross-section.

I (mm⁴)	E (N/mm²)	L (mm)	a (mm)	M_{cr} (Nmm)	P_{max} (N)	Δ_{max} (mm)
30,495	33,718	229	76	8,879	117	0.024

4.3 Discussion

Figure 4.5 shows the load-displacement diagrams for each of the specimens in the four-point bending test. The plots represent the sum of the loads from the two load cells versus the measured deflection of the beam at mid-span. Figure 4.6 shows the load-deflection diagram of the C-CFRP Epoxy and C-CFRP Epoxy Mesh specimens compared to the calculated stiffnesses up to the cracking moment M_{cr} . Figure 4.7 shows the test setup for the four-point bending test and the appearance of the specimens after testing.

In Figure 4.5a, which shows the concrete-only specimens, i.e. control specimens, an almost linear behavior is observed until the ultimate load is reached, after which the load suddenly drops to zero since the specimens are brittle and have no reinforcement. The calculated modulus of elasticity of these concrete beams from the four-point bending test (32,875 N/mm²) is compatible with the calculated modulus of elasticity from the compression test of the concrete cylinder (33,718 N/mm²). The maximum load capacity of specimens C1 and C2 is lower than that of specimen C3 because they had some preliminary damages at the edges of their cross-sections.

For the concrete mesh specimens (Figure 4.5b), after the crack initiation at about 200-267 N, some strength gain is observed due to the internal GFRP mesh reinforcement, which also increases the ultimate load capacity by up to 30%.

Figure 4.5c shows that the C-CFRP Nail beams experience an initial steep strength drop related to the delamination of the CFRP layer and a second drop due to the brittle failure of the concrete layer.

Figure 4.5d shows the C-CFRP Nail Mesh beams initially experiencing delamination of the CFRP layer, followed by cracking within the concrete layer and some strength gain due to the internal GFRP mesh reinforcement.

Figures 4.5e and 4.5f show that the initial stiffness of the C-CFRP Epoxy specimens with and without internal reinforcement decreases sharply as a result of the delamination of the CFRP layer and subsequent failure of the concrete layer. However, the ultimate strength of these two groups are much higher than the other 4 groups (about 300% higher, load capacity of 1.2 kN compared to only 300 N).

In the C-CFRP Epoxy Mesh category, one specimen was completely broken when removed from the casting formwork, while another specimen showed minor cracking. The calculated flexural stiffness (EI) values for C-CFRP Epoxy Mesh specimens 1 and 2 were determined to be about 1,350,000 (kN.mm²) and 202,000 (kN.mm²), respectively. However, due to the large difference between the results of these two specimens and to ensure the validity and generality of the results, it is planned to repeat a four-point bending test on this particular category of specimens. This set of specimens displays a significant improvement in their ultimate load capacity compared to the concrete control beams.

The analytical calculations for the C-CFRP Epoxy specimens with and without reinforcement mesh suggest that the first crack should occur at approximately 516 N, as shown in Figure 6. However, the onset of cracking took place at about 800 N, possibly due to the epoxy preventing crack development up to a higher load.

Finally, the flexural stiffness (EI) of each specimen is calculated using Equation 4.12, and the average flexural stiffness of the three specimens in each category is shown in Table 4.3.

$$EI = \frac{Fa(3L^2 - 4a^2)}{24\Delta_{\max}} \quad (4.12)$$

Where:

EI = flexural stiffness (N.mm²)

P = total point load (N)

a = distance from support to the point load (mm)

L = length of the span under consideration (mm)

$\Delta_{\max}$ = mid-span deflection (mm)

Table 4.3: Average flexural stiffness (EI) of each category of beams.

Specimen Category	C	C Mesh	C-CFRP Nail	C-CFRP Nail Mesh	C-CFRP Epoxy
Average EI (N.mm ²) (uncracked)	570,051,659	582,398,188	611,749,422	648,362,988	632,864,071

4.4 Summary and Conclusion

This study investigates the flexural behavior of eighteen beam specimens, six of which were reinforced with CFRP strips via epoxy adhesive or nail shear connectors. The test variables studied

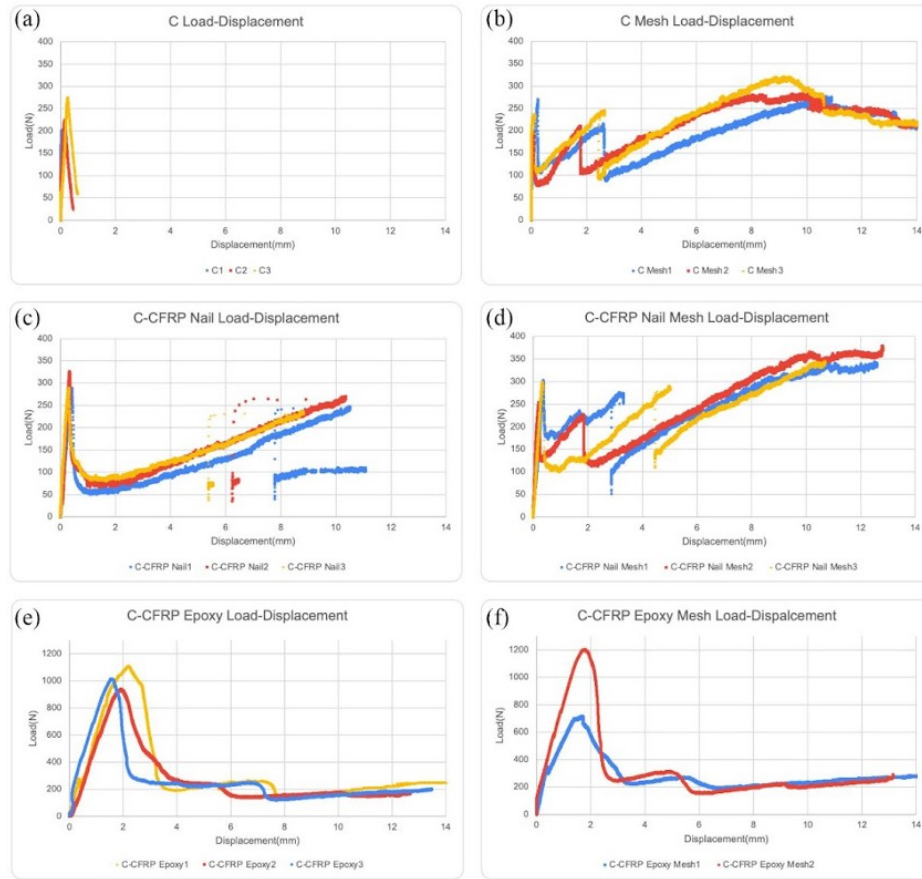


Figure 4.5: Load-deflection relationships of each category of the specimens under the four-point bending test: (a) Concrete (b) Concrete Mesh (c) Concrete-CFRP Nail (d) Concrete-CFRP Nail Mesh (e) Concrete-CFRP Epoxy (f) Concrete-CFRP Epoxy Mesh.

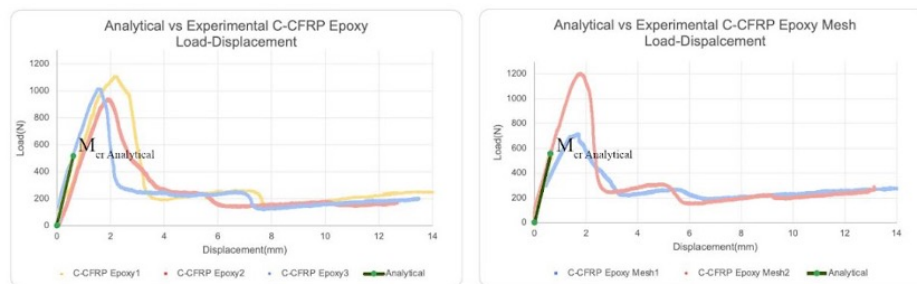


Figure 4.6: Analytical M_{cr} for hybrid C-CFRP cross-section vs. experimental load deflection of C-CFRP Epoxy with and without mesh.



Figure 4.7: Beams after performing the four-point bending test (each row, from left to right): (a) C 1-3 (b) C Mesh 1-3 (c) C-CFRP Nail 1-3 (d) C-CFRP Mesh Nail 1-3 (e) C-CFRP Epoxy 1-3 (f) C-CFRP Epoxy Mesh 1-3.

in the experimental program were the internal GFRP mesh reinforcement, the external CFRP layer, and the type of connection between the external CFRP and the concrete. All specimens were tested under four-point bending, and three concrete control specimens were used for the comparative strength. In addition to the experimental tests, simplified transformed section analyses were conducted. Based on the experimental and analytical studies, the following observations and conclusions were made:

- All hybrid beams failed due to delamination of the CFRP strips, confirming the importance of proper shear connections between the two layers.
- For the hybrid specimens with nails, it is observed that the composite effect does not fully develop, which may be due to the weak connection between the nails and the CFRP layer and the small force transfer area between the nails and the CFRP layer.
- For the hybrid specimens with epoxy resin, the composite effect has a significant influence on the performance of the cross-sections. These specimens have a higher ultimate flexural capacity (up to 320% more compared to the control beams), and a significant decrease in flexural stiffness is observed after delamination of the CFRP layer.
- For the specimens without CFRP layer, the internal mesh reinforcement contributes to the increase of the ultimate flexural capacity of the specimens (to about 30% of the original crack load of the specimen). However, for hybrid cross-sections, the ultimate load-bearing capacity depends more on the composite effect of the hybrid cross-section rather than on the presence of the internal mesh.
- In the case of hybrid cross-sections, the addition of the mesh also helps to restore the stiffness lost due to cracking in the concrete. It can also be seen that the delamination of the CFRP layer greatly reduces the stiffness of the cross-section.
- The first crack in most specimens occurs at a force of about 220-300 N (about 25% of the ultimate load of the specimens with epoxy). For the C-CFRP Epoxy specimens, the experimental cracking moment is higher than the calculated cracking moment, which could be attributed to the crack-preventing effect of the epoxy.

Based on the results of this study, for achieving higher performance, it is suggested to employ the full bonding method, potentially with higher strength epoxy, provided that the construction technique considerations allow this method. Finally, additional research would be required to study more effective shear connection methods, for cases where there is a requirement for simpler separation of materials to promote the circular economy or when construction methods impose certain limitations.

Chapter 5

Parameter Sensitivity Analysis

5.1 Introduction

In addition to the material composition and cross-sectional shapes discussed in the previous chapter, the global geometry of shells and gridshells plays a crucial role in various performance factors such as their structural stiffness and material utilization. Designing shell geometries, however, is a particularly difficult task. This chapter therefore focuses on the exploration of the effects of different geometrical parameters for hybrid gridshells on their local and global structural behavior. By systematically varying key parameters and evaluating their influence on the performance of the gridshell, valuable insights will be gained that can be incorporated into the design process and optimize the overall efficiency of the structure.

Among the parameters used to generate the gridshells on different target surfaces are the span-to-rise ratio, the support conditions, and the grid density pattern. The shapes created are then loaded and evaluated according to their global stiffness, maximum deflection, and maximum utilization. Finally, based on this analysis, the results are summarized and recommendations are made for the development of future gridshells.

5.2 Methodology

To conduct this investigation, this study used a step-by-step workflow, as shown in Figure 5.1. First, a set of target surfaces was created on which the corresponding gridshells were generated. Both the geometry of the target surface and the geometrical properties of the gridshells depended on very few parameters and were set to generate a large pool of structures to be investigated. Subsequently, a structural analysis was carried out to determine the respective load-bearing capacity of the individual gridshells. Finally, the test results were compared in order to discuss more general trends and conclusions.

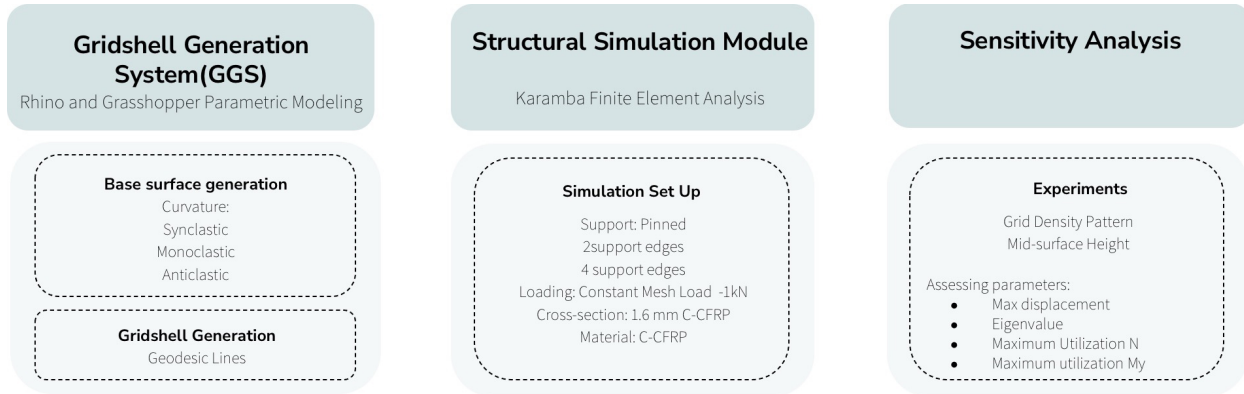


Figure 5.1: Step-by-step workflow.

Gridshell Generation System (GGS)

To quickly investigate different gridshell settings, a Gridshell Generation System (GGS) was developed, which is a parametric model in the Rhinoceros/ Grasshopper 3D modeling software. The GGS consists of two modules: (i) The first generates an 8m x 8m square target surface as a base. (ii) The second module generates a gridshell on this surface, which can be controlled in its density and cross-sectional profiles. In the following, each module is described in more detail:

Base Surface Generation

As shown in Figure 5.1, the workflow with the GGS starts with the creation of a Non-Uniform Rational B-Splines (NURBS) base surface (S_B) using four boundary edges and a freely adjustable midpoint of the surface. The boundary edges are splines derived from the corner points of an invisible rectangle, set by a pair of length and width values (w, l), as well as freely movable center points controlled via (x_i, y_i, z_i) coordinates. The midpoint of the surface is controlled in its position via (x_0, y_0, z_0) coordinates. The parametric model of the base surface (S_B) has therefore a total of 17 adjustable values. This setup allows the base surface to take on various shapes and cover a wide range of Gaussian curvature. The Gaussian curvature of the surface at a specific point is defined as the product of two principal curvatures and can be positive, negative, or zero. A positive Gaussian curvature means the resultant surface is synclastic or dome-shaped. A negative one means the surface is anticlastic or saddle-shaped. If the Gaussian curvature is zero, the surface has a single curvature (vault-shaped) and is monoclastic [161]. In the context of this study, all three conditions are considered for the generation of the gridshells.

Gridshell Generation

After creating the base surface, the second module of the GGS can be used to generate a bending-active gridshell on this target geometry. This gridshell is composed of a network of continuous strips spanning in warp and weft directions. Since these strips were originally straight and could

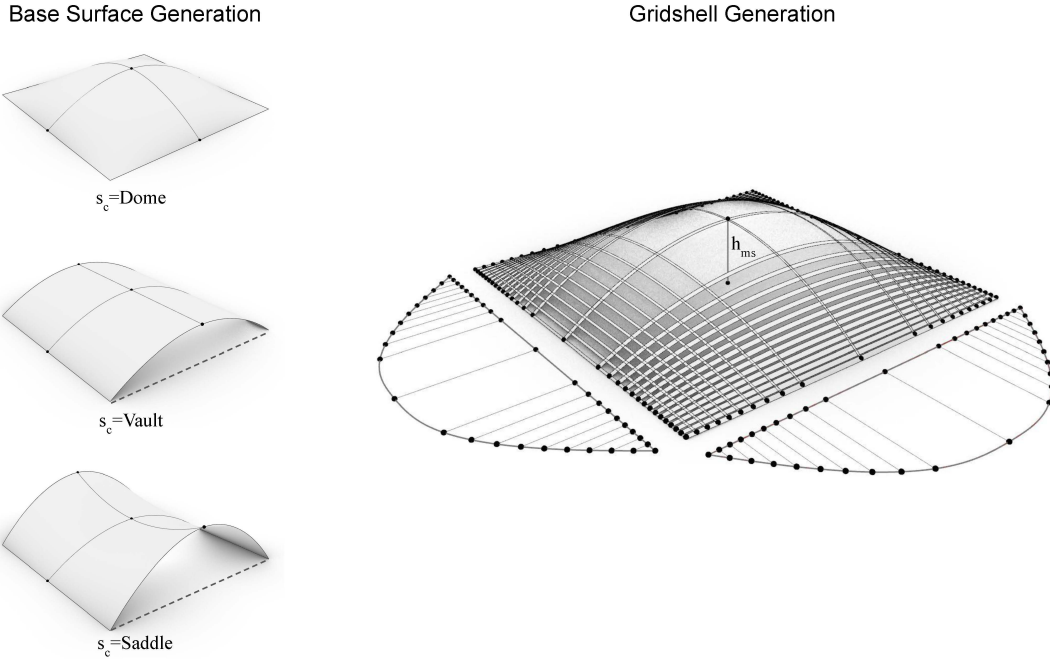


Figure 5.2: Key geometric modeling parameters used for base surface and gridshell generation.

only obtain their curved geometry by bending and twisting, the strip network must follow geodesic lines on the base surface. Finally, the pattern of the gridshell can be fine-tuned further by adjusting the number of strips in each direction, their cross-sectional profile and material composition, and the density of strips along the edges of the base surface, as shown in Figure 5.2.

Key Geometric Parameters

While the Gridshell Generation System (GGS) allows for the rapid creation of a wide variety of structures, the main objective of this particular study is to investigate key geometric parameters and understand their impact on the structural performance of the gridshells. Special focus is therefore placed on (i) the grid density pattern, (ii) the mid-surface height (span-to-rise ratio), (iii) the number of supported edges, and (iv) the surface curvature, as shown in Table 5.1. The individual parameters are defined as follows:

- The grid density pattern (p_{gd}), is a sliding value between 0 and 1, where $p_{gd} = 0$ corresponds to a denser grid pattern in the center of the base surface S_B , and $p_{gd} = 1$ corresponds to a denser grid pattern towards the edges. Therefore, ($p_{gd} = 0.5$) represents an evenly distributed grid pattern over the entire surface.
- The mid-surface height (h_{ms}) is the height value of S_B in its centroid location.

- The number of supported edges (n_{se}) refers to the structural support of the gridshell. It can either be along two opposite edges of S_B ($n_{se} = 2$), or along all four edges of S_B ($n_{se} = 4$).
- The surface curvature (S_c) of the studied base surfaces and resulting gridshells can either be described as synclastic ($S_c = dome$), monoclastic ($S_c = vault$), or anticlastic ($S_c = saddle$).

Table 5.1: Model parameters and their step sizes.

Model Parameters	Range	Step Size
Grid density pattern (p_{gd})	0 to 1	0.05
Mid-surface height (h_{ms})	0 m to 3 m	0.25
Number of support edges (n_{se})	2 edge and 4 edge support	N/A
Surface curvature (s_c)	dome, vault, saddle	N/A

Structural Analysis and Evaluation Criteria

After creating an extensive pool of 192 test specimens, the next objective of this study is to evaluate each of the gridshells performance based on their structural capacity. The implementation involved using a finite element analysis (FEA) method provided by the Karamba 3D plugin for the Rhinoceros/ Grasshopper environment[162]. This plugin allows the study of beam or shell structures under different loads and boundary conditions while taking full advantage of the parametric modeling capabilities of the Grasshopper environment. For further investigation, each gridshell is converted into a model consisting of individual beam elements by segmenting each strip at the intersection points with the other strips. Then, in addition to the self-weight, a 1kN/m² mesh load is applied by projecting it onto the model in the negative z-direction. Applying the load in this way instead of, for example, line loads, has the advantage that the tributary area for each beam element is determined automatically, even if the spacing of the strips in the grid layout changes. As for the boundary conditions, the strips in the gridshell are pinned at their ends by the restraint T_x , T_y , T_z , and R_x . Based on the respective experiment, this setting either applies to all strips in the gridshell, resulting in four structurally supported edges, or it is only activated for strips in one direction, which means that the gridshell is only supported at two opposite edges. The values used to define the hybrid material properties of the concrete and carbon fiber reinforced polymer (C-CFRP) cross-section are shown in Table 5.2. After specifying the beam elements, loads, support conditions, and materials, each of the generated gridshells is analyzed for its maximum displacement as an indicator of overall stiffness and the first eigenvalue as an indicator of global behavior, and maximum internal forces of the members.

Table 5.2: Material properties of the C-CFRP hybrid material.

Material	Thickness (cm)	Width (cm)	Young's Modulus Direction 1 (E_1 - kN/cm ²)	Young's Modulus Direction 2 (E_2 - kN/cm ²)
C-CFRP	1.6	10	3834	2671.4

Experiments

The following two experiments will address the question of how the grid density pattern and the mid-surface surface height affect gridshells with different curvatures and support conditions. Each aspect is examined separately and described in more detail below.

Grid Density Pattern

In the first experiment, the mid-surface height of the gridshells is kept constant at 1.5 m. The examined geometric variable is the grid density pattern defined by the pattern distribution parameter (p_{gd}). It starts at 0 and is gradually increased to 1 with a step size of 0.05. Figures 5.3 and 5.4 illustrate the maximum displacements of selected gridshells with two and four supported edges as boundary conditions. Shown from left to right are gridshells with increasing grid density pattern ($p_{gd} = 0, 0.5, 1$), while the gridshells shown from top to bottom vary in curvature ($s_c = \text{dome, vault, saddle}$). Figure 5.5 and 5.6 presents the first global eigenmode analysis for the same selected gridshells.

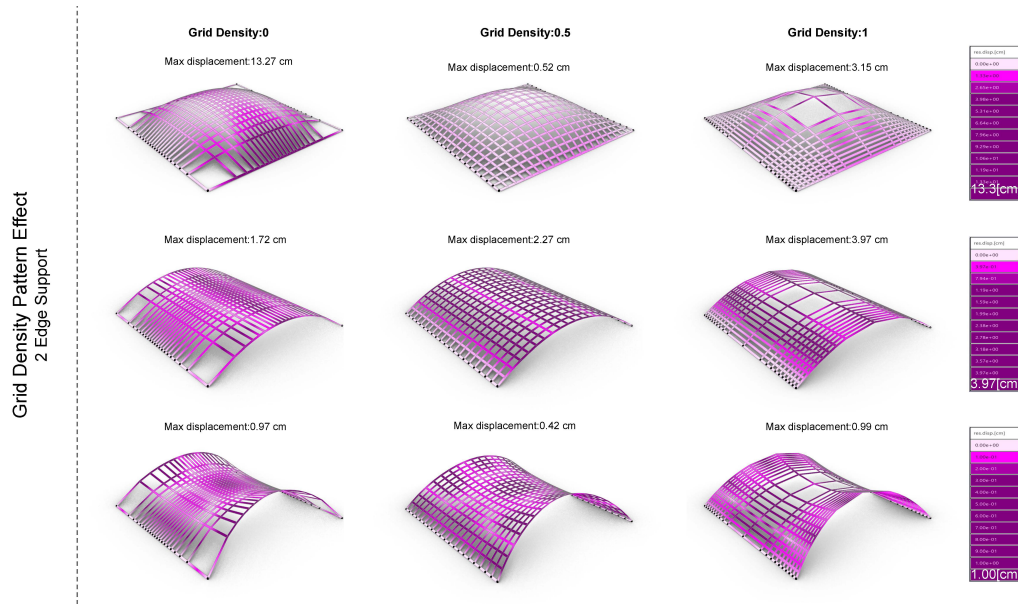


Figure 5.3: Effects of grid density pattern on the maximum displacement for two support edges.

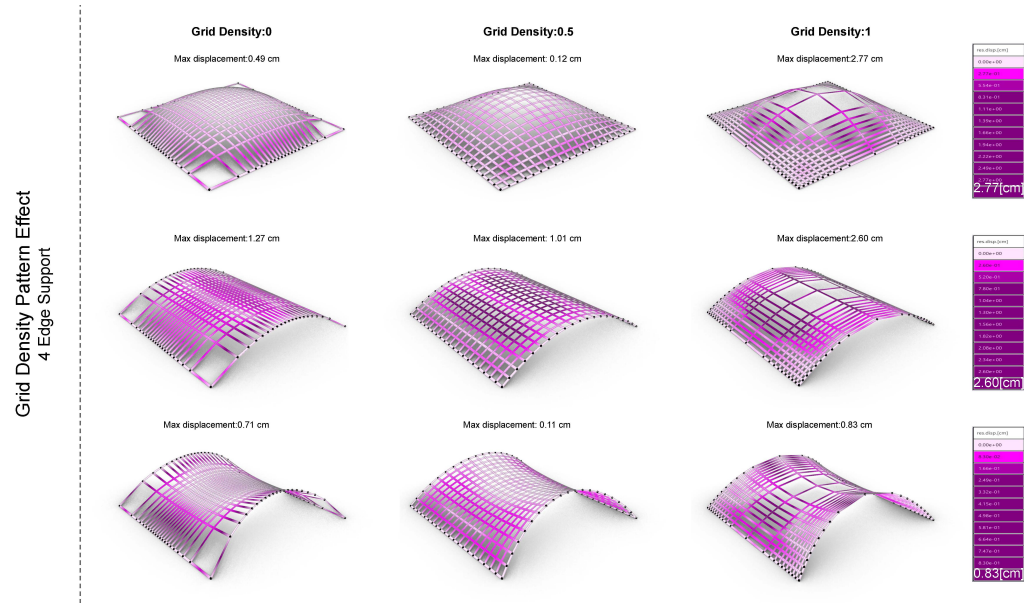


Figure 5.4: Effects of grid density pattern on the maximum displacement for four support edges.

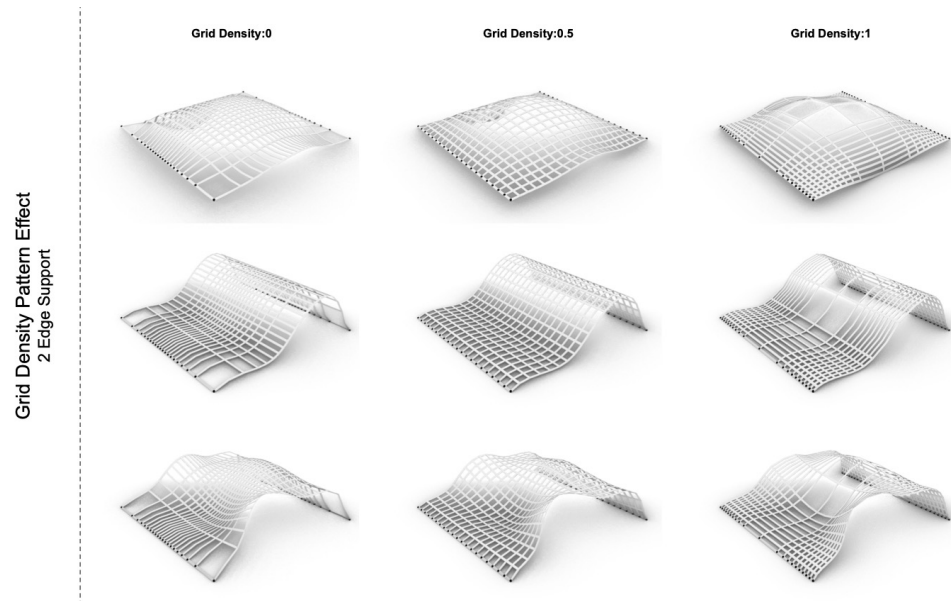


Figure 5.5: Effects of grid density pattern on first global eigenmode for two support edges.

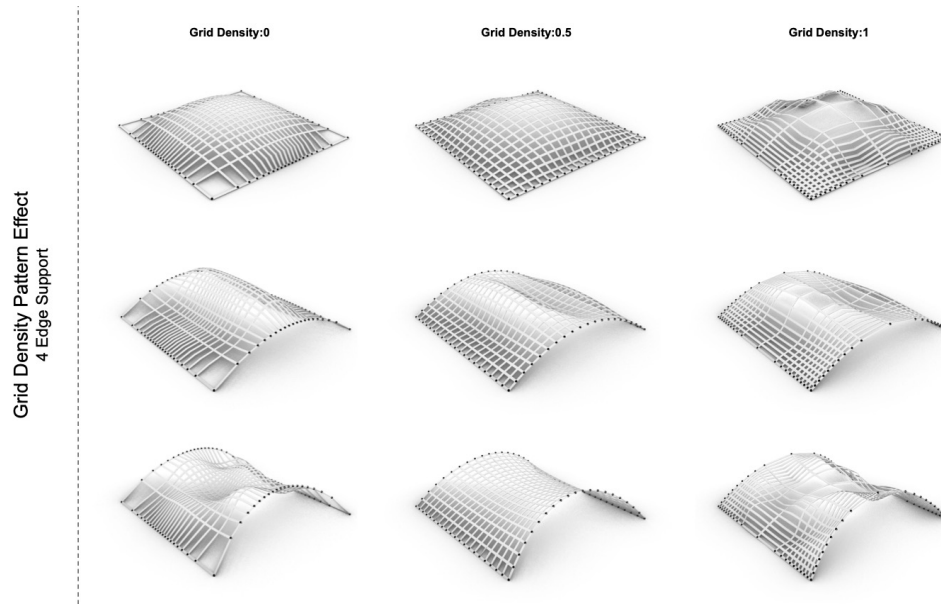


Figure 5.6: Effects of grid density pattern on first global eigenmode for four support edges.

Mid-surface Height

For the second experiment, the grid density pattern remains constant with a uniform distribution in warp and weft direction of 0.5. The geometric variable studied here is the mid-surface height (h_{ms}), which ranges from 0 to 3 meters and is increased with a step size of 0.25 m. Figures 5.7-5.10 show the maximum displacement as well as eigenmode behavior for selected gridshells with two and four edge supports.

5.3 Results

Grid Density Pattern

The results of the first experiment, investigating the impact that changes in the grid density pattern have on different gridshells can be summarized as follows:

Two Edge Supports ($n_{se} = 2$)

For gridshells with two supported edges, the collected data in Figure 5.11 reveal the following trends. When the variable for the grid density pattern is increased, which means that the pattern changes from being denser mid-surface to becoming denser towards the edges, the tests show that the first global eigenvalue and therefore the global stiffness increases for all the surface curvature types (Figure 5.11b). Furthermore, as seen in Figure 5.11a, when increasing the grid density pattern, a decreasing trend of maximum displacement is observed for the dome category and an increasing

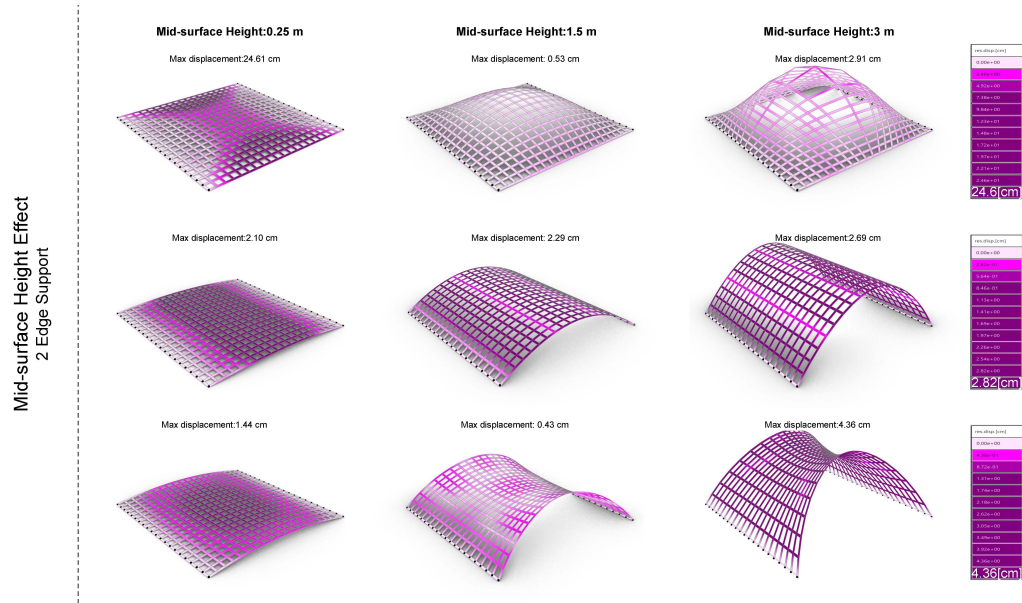


Figure 5.7: Effects of mid-surface height on the maximum displacement for two support edges.

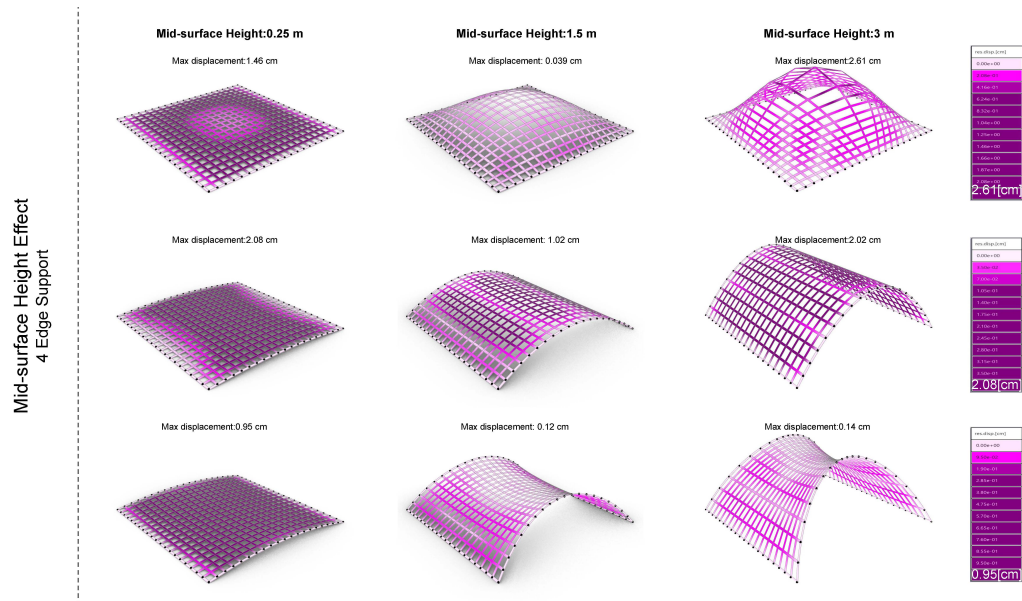


Figure 5.8: Effects of mid-surface height on the maximum displacement for four support edges.

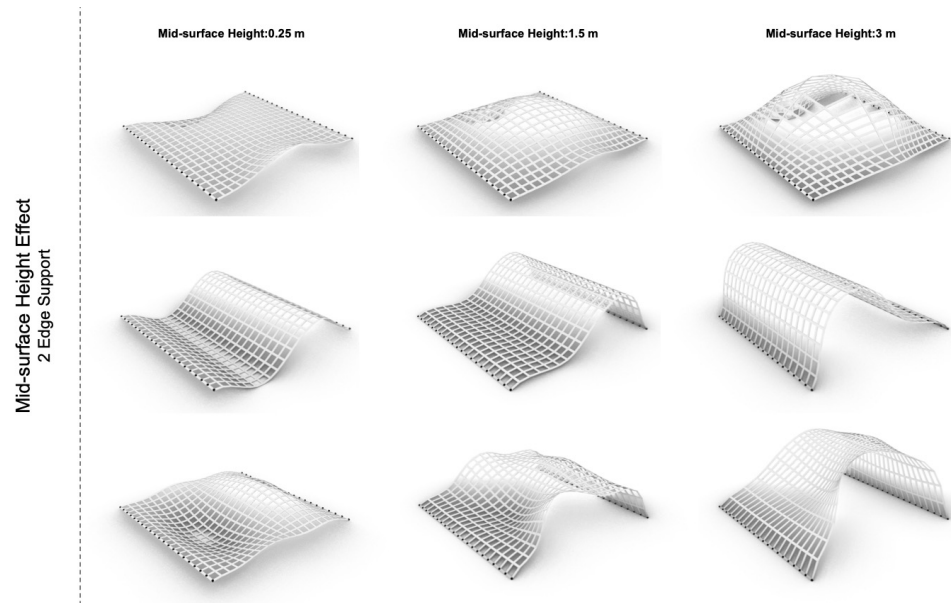


Figure 5.9: Effects of mid-surface height on eigenmode for two support edges.

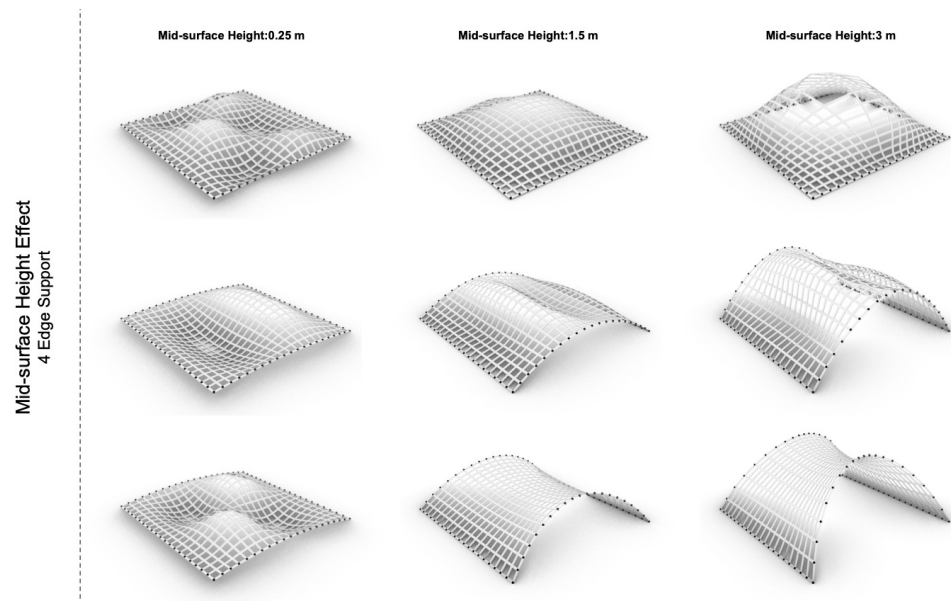


Figure 5.10: Effects of mid-surface height on eigenmode for four support edges.

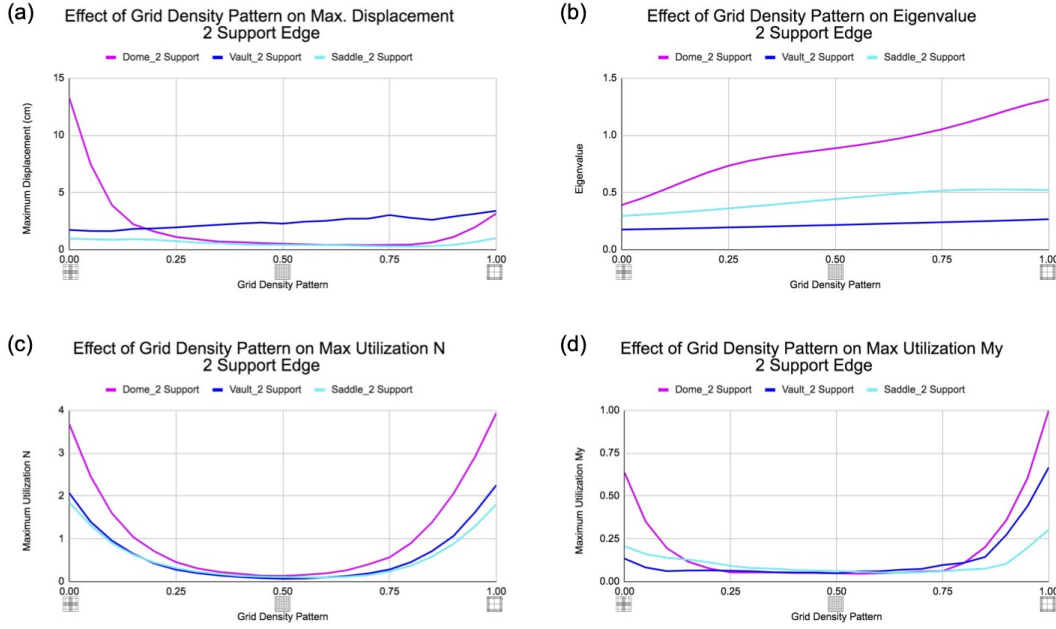


Figure 5.11: Effects of grid density pattern on gridshells with two edge supports, on (a) maximum displacement, (b) eigenvalue, (c) maximum utilization N, and (d) maximum utilization M_y .

trend for the vault. The behavior of the domes could be explained by the fact that for the dome with 2 edge supports, the maximum displacement occurs at the unsupported edges, so increasing the density of the grid toward these edges helps stiffen the edges, decreasing the maximum displacement. With respect to global stiffness, Figure 5.11b represents that the dome category has the highest first global eigenvalue, followed by the saddle, and finally the vault category. However, from the point of view of local stability as in Figure 5.11a, the saddle has the lowest maximum displacement, followed by the dome (from after $p_{gd} = 0.2$), and then the vault. Looking at the effect of the grid density pattern in the graph of the maximum utilization N in Figure 5.11c, a symmetrical behavior with a symmetry axis of $p_{gd} = 0.5$ can be observed for all surface curvature types. This implies that by increasing the surface density either towards the edges or towards the center of the surface, the maximum utilization N of the beam elements increases. Furthermore, the saddle and vault categories have lower maximum utilization N than the dome category. Finally, as can be noted from Figure 5.11d, by increasing the grid density pattern, a decreasing behavior for the maximum utilization M_y which is followed by a sharp increase, at about $p_{gd} = 0.75$, is noted in all surface curvature.

Four Edge Supports ($n_{se} = 4$)

Figure 5.12 shows the behavior of gridshells with four supported edges. Here, with an increasing grid density pattern, from Figure 5.12b, it can be observed that the eigenvalues and therefore the global stiffness become higher in all categories (except in the saddle category after $p_{gd} = 0.95$ and

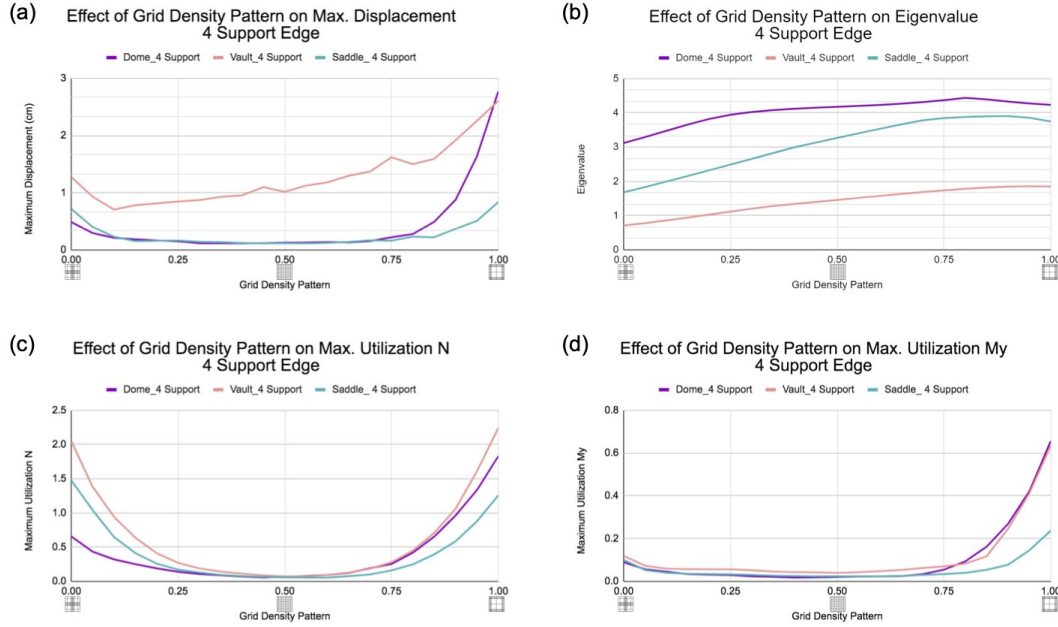


Figure 5.12: Effects of grid density pattern on gridshells with four edge supports, on (a) maximum displacement, (b) eigenvalue, (c) maximum utilization N, and (d) maximum utilization M_y .

in the dome category after $p_{gd} = 0.85$). The dome and the saddle categories have higher global stability, as can be seen from their higher first global eigenvalue. In addition, shown in Figure 5.12a, an increase in the pattern distribution variable results in an initial decrease in the maximum displacement in all three categories of surface curvature. This initial decrease lasts until $p_{gd} = 0.4$ for the dome geometry, until $p_{gd} = 0.1$ for the vault geometry and up to $p_{gd} = 0.55$ for the saddle geometry. Overall, the saddle and dome categories have lower maximum displacement compared to the vault category. This is also almost consistent with the global stability trend for these categories. Figure 5.12c represents that from $p_{gd} = 0$ to $p_{gd} = 0.5$, the dome has a lower maximum utilization N compared to the saddle, and after that, the dome gains a higher maximum utilization N than the saddle. It can also be observed that the vault category has the highest maximum utilization N. For the maximum utilization M_y , from Figure 5.12d, it can be stated that as the grid density pattern increases, similar to the two edge supports condition, there is first a decrease in the maximum utilization M_y and then this value increases from about $p_{gd} = 0.7$ for the dome and saddle category, and $p_{gd} = 0.6$ for the vault category. Secondly, the saddle has almost the lowest value for the maximum utilization M_y when the grid density pattern is changed.

Mid-surface Height

The results of the second experiment, which examined the effects of changes to the mid-surface height, can be summarized as follows:

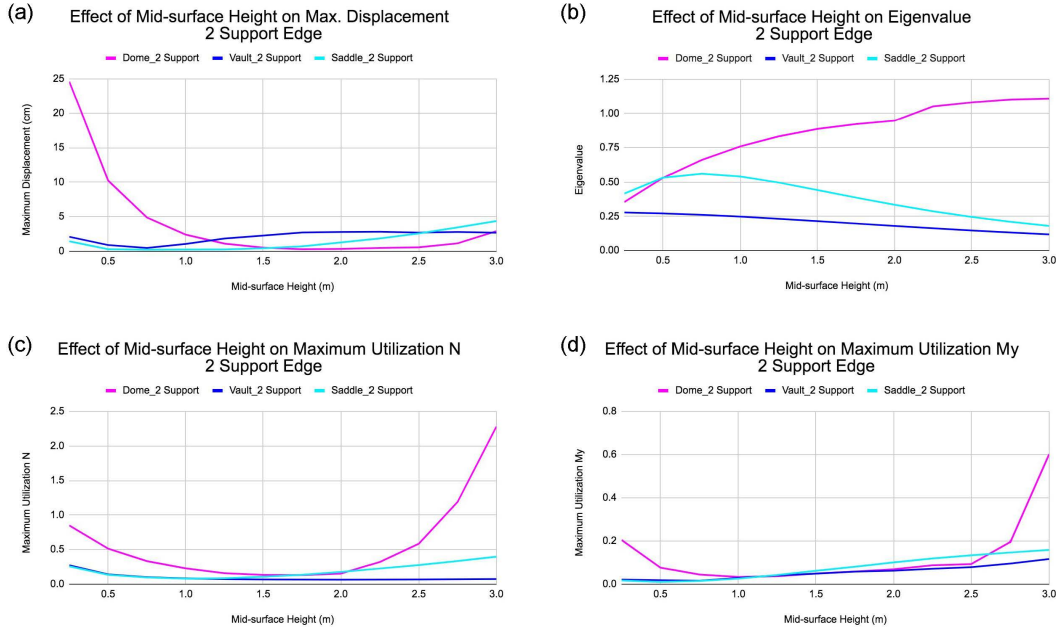


Figure 5.13: Effects of mid-surface height on gridshells with two edge supports, on (a) maximum displacement, (b) eigenvalue, (c) maximum utilization N , and (d) maximum utilization M_y .

Two Edge Supports ($n_{se} = 2$)

Figure 5.13 shows the behavior of gridshells with two edge supports. As seen in Figure 5.13b in the dome category, the increase in the height of the mid-surface results in an increase in the eigenvalue and therefore global stiffness. However, for the vault and saddle categories, almost the opposite trend is observed (for the saddle, the eigenvalue decreases after $h_{ms} = 0.75m$). This is consistent with the maximum displacement pattern of the categories as seen in Figure 5.13a: the higher the mid-surface height becomes, the larger the maximum displacement for the vault and saddle (after $h_{ms} = 0.5m$), and the smaller it gets for the dome category. When increasing the mid-surface height, an initial decrease followed by an increase can be observed in Figure 5.13c, for all the surface curvature types with respect to their maximum utilization N . This decrease lasts up to $h_{ms} = 1.75m$ for the dome category, $h=2$ for the vault category, and $h_{ms} = 1m$ for the saddle category. As can be seen from Figure 5.13d, with the increase of mid-surface height, the maximum utilization M_y for the vault and saddle categories increases. Generally, the lowest utilization N can be observed for gridshells with a mid-surface height between $1 - 2m$.

Four Edge Supports ($n_{se} = 4$)

For all geometric categories with four supporting edges, as seen in Figure 5.14b, the first global eigenvalue rises with the increase of mid-surface height (except for the saddle category, where the increase continues up to $h_{ms} = 0.75m$). The dome category has a higher first global eigenvalue

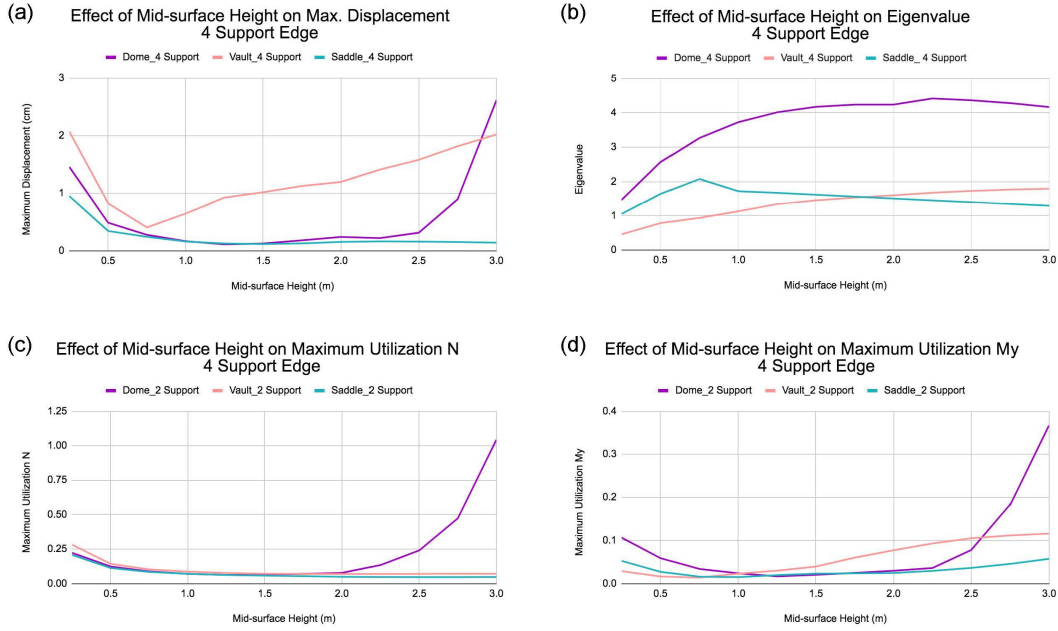


Figure 5.14: Effects of mid-surface height on gridshells with four edge supports, on (a) maximum displacement, (b) eigenvalue, (c) maximum utilization N, and (d) maximum utilization M_y .

than the vault and saddle category. The saddle category has a higher first global eigenvalue than the vault category up to $h_{ms} = 1.75m$. The local stability behavior from Figure 5.14a shows that the saddle category has almost the lowest maximum displacement, followed by domes and vaults. With the increase in the mid-surface height, a strong decrease in the maximum displacement values for the category of vaults can be observed. This decrease of the maximum displacement continues up to $h_{ms} = 0.75m$ for the category of vaults. In Figure 5.14c, increasing the mid-surface height decreases the maximum utilization N for the saddle and vault category. It also decreases this value for the dome category up to $h_{ms} = 1.25m$ and after that the maximum utilization N increases. In addition, all three surface curvature categories have around the same maximum utilization N up to $h_{ms} = 1.25m$. As demonstrated in Figure 5.14d, for the maximum utilization M_y , when increasing the mid-surface height, all the categories observe an initial decrease accompanied by an increase later.

5.4 Conclusion

This chapter introduces a Gridshell Generation System (GGS), a novel tool to rapidly generate gridshells on varying target surfaces. The parametric nature of this tool allows the gridshells to be freely adjusted in terms of grid density pattern, mid-surface height, and the number of edge supports. This freedom allows to investigate the influence of those geometric factors on the structural performance and material utilization of the gridshells in more detail, with mainly expected but also

somewhat surprising results.

From both experiments, it can be concluded that the gridshells with all four edges supported have higher stiffness and generally lower displacements than their counterparts with only two edge supports. This can be explained by the fact that gridshells with four edge supports can transfer the load in two directions and thus show a more effective load-bearing behavior.

Based on the first experiment, which focused on studying the grid density pattern, it can be concluded that the more uniformly the grid density is distributed, the higher the stiffness, the lower the deflections, and the more balanced the use of the elements. High performance is reached by gridshells with a grid density between 0.25-0.75, and the almost best results for a perfectly evenly distributed density of $p_{gd} = 0.5m$. This can be explained by the fact that with an evenly distributed grid, the individual beam elements are almost of equal length and thus have comparatively similar load-bearing behavior. Local bending phenomena are therefore less prominent and the gridshell carries the load more uniformly with the same load on each element. For gridshells with a too uneven distribution of strips, the effects of local instabilities become more pronounced as the length of the individual elements increases.

Finally, from the second experiment, which studied the influence of the mid-surface height, it can be stated that the best balance between stiffness and utilization is achieved for gridshells with a medium rise of 1-2 meters. For gridshells with a span of 8 m, this relates to a span-to-rise ratio of 8:1 or 4:1 respectively. Within this range, the axial compression forces in the gridshells are the more dominant factor compared to the bending moments resulting from global or local buckling phenomena.

Chapter 6

Conclusion and Future Outlook

6.1 Bending-active Structures as Lost Formwork

The main contribution of this dissertation is the introduction of a novel approach to the design and fabrication of hybrid gridshells using a lost formwork made from elastically bent carbon fiber reinforced polymer strips. The key findings from Chapter 3 show that utilizing bending-active structures as lost formwork for concrete gridshells can combine the high load-bearing capacity and compressive strength of concrete with the high tensile strength of carbon and result in a system that can be easily constructed. As discussed in this chapter, bending-active gridshells, when used without the additional concrete layer, are subject to certain structural limitations when exposed to additional loads. In comparison, concrete shell structures, in general, offer structural advantages and are considered material-saving systems. However, they require a large amount of auxiliary scaffolding and must be reinforced with additional material to accommodate tension forces. Therefore, as presented in this dissertation, using a bending-active formwork in the construction of concrete gridshells can combine the aforementioned advantages of both systems while helping to overcome the disadvantages of the other. The method facilitates the construction of very light structures, highlighting the potential applicability of this method in earthquake-prone regions. Furthermore, the results of the basic loading of a hybrid gridshell created in this way showed the high strength of the system in relation to its very low weight. Additionally, the implementation of this concept, by introducing openings has the potential of addressing the long-lasting issue of bringing daylighting to larger-span and deep concrete shells. One of the most important lessons learned from fabricating the gridshell prototype and studying its failure mode is the increased awareness of the importance of stabilizing the supports when a load is applied. The careful design and construction of these structural supports, which must not be allowed to slip under load, is critical for the optimal performance of the gridshell.

In summary, using the bending-active formwork technique allows the hybrid gridshell to be produced without the need for additional scaffolding. In direct comparison with the conventional construction of concrete shells, this method stands out for its fast assembly speed and significantly lower material consumption, as no complex, costly, and excessive auxiliary scaffolding is required.

Such a construction method can therefore play an important role in achieving the goal of saving materials, reducing CO_2 emissions, and lowering labor costs. By facilitating the design and construction process of hybrid gridshell systems, it is possible to increase the use of such efficient structural systems in various building applications, while reducing concrete consumption in architecture and lowering the associated carbon footprint due to the high material efficiency of this building system.

6.2 Hybrid Cross-section in Bending-active Structures

Chapter 4, which investigated the material behavior of hybrid concrete-carbon cross-sections, confirms the hypothesis that the lost CFRP formwork can act as the external tensile reinforcement for the concrete layer, resulting in a very thin, yet strong hybrid gridshell design. It highlights the synergy achieved by combining the high tensile strength of the bending-active carbon fiber formwork with the high compressive strength of the concrete layer. By combining the two materials to form a hybrid gridshell, the load-bearing capacity and stiffness of the structure are significantly increased, enabling it to span long distances and support additional live and dead loads such as cladding, wind, and snow loads with minimal material input. More specifically, the tests have shown that the developed concrete-CFRP hybrid cross-section increases the ultimate flexural capacity of the hybrid cross-sections connected with nails by about 50% and that of the cross-sections connected with epoxy resin by about 300%.

However, due to the low cross-sectional height of the specimens, it was found that the introduction of even the smallest cracks can significantly reduce the overall performance of the cross-sections. This underlines the importance of good manufacturing quality and handling of the specimens both during and after fabrication.

For all hybrid cross-sections, the experiment revealed a consistent failure pattern: in all specimens, the CFRP layer debonded first, followed by the destruction of the concrete layer. Based on this observation, it is strongly recommended that future work further investigate the possibilities of using higher-strength epoxy resins to improve the bond and load transfer between the two materials in the hybrid cross-section.

The insights and measurements gained from the material experiments can help designers and engineers make better-informed decisions and incorporate this knowledge in the early stages of the design process to avoid later changes to the structure, which are usually not so easily taken into account.

6.3 Geometrical Impact

Another important contribution of this dissertation is the development of the Grid Generation System (GGS) and thereby the presentation of a workflow that makes the exploration of gridshell designs through the adjustment of a few parameters fast and easy. Using the GGS, the effect of various geometric properties, such as the mid-surface height, the grid density pattern, the number of supporting edges, and the surface curvature, on the performance of the hybrid gridshell can be

studied in more detail and incorporated into future design decisions. The study conducted in Chapter 5 showed that gridshells with all four edges supported have higher stiffness and lower deformation than their two-edge supported counterparts. This improved performance is most likely due to the more effective, two-directional load transfer in the four-edge-supported gridshells. From the investigation of the grid density patterns, gridshells with a more evenly distributed density exhibited higher stiffness, reduced deflections, and more balanced element utilization. Reasonable results were achieved with either a moderately denser grid towards the edges or a moderately denser grid towards the middle of the shell's surface, with an almost ideal performance observed for perfectly even density distribution. This observation could be due to the fact that in uneven distribution of the grid density pattern, the long individual element leads to local failures and therefore reduces the general performance of the gridshell. The experiments showed that when evaluating the effects of grid density pattern, it is critical to consider both the local and global performance of the gridshells while limiting the length of the elements. This is especially important when the grid density pattern is changed while the number of strips remains constant. As a few of the elements could become too long, leading to local failure and reducing structural performance primarily attributed to this small subset of elements.

Moreover, the study on the mid-surface height shows that a span-to-mid-surface height ratio of 8:1 or 4:1 for such hybrid gridshells leads to a better balance between the stiffness of the gridshell and its utilization. This underlines the importance of optimizing the mid-surface height of these gridshells as one of the first factors to look at for better performance. The Gridshell Generation System and the information gained from the analysis allow architects and engineers to gain a deeper understanding of the relationship between form and structural performance, and to strike a harmonious balance between aesthetics and functionality when designing hybrid gridshells.

6.4 Outlook and Future Work

The framework proposed in this dissertation can lead to economic and sustainable benefits in the design and construction of free-form concrete gridshells. With a high potential for material-efficient and structurally advantageous geometries, such systems would allow designers to simultaneously achieve aesthetic explorations and performance optimizations. Finally, the proposed framework may allow designers and engineers to make informed critical decisions in the early stages of design, avoiding costly trial-and-error cycles in the construction process.

Finally, based on the findings and observations made in this dissertation, potential opportunities and directions for continuing this area of research are identified:

Material Studies

A possible next step is to conduct additional digital studies to validate and refine the results of the physical experiments. It will also allow to validate and compare the results obtained from our experiments through multiple avenues. Conducting digital studies also provides an opportunity to extend the results of our physical experiments. A wide range of concrete layer thicknesses and

their influence on the structural performance of the cross-section can be effectively studied and analyzed through digital simulations. These simulations provide a cost-effective and efficient means of investigating numerous scenarios and allow us to gain a better understanding of the behavior and response of the hybrid structure under various conditions.

Wood-concrete Hybrid Material

To further improve the sustainability aspect of this proposed structural system, the use of wood as an alternative to CFRP in the hybrid concrete-CFRP cross-section is a possible avenue that could also be explored. This research would focus on investigating the material properties of wood 3D-printed concrete cross-sections and then comparing their structural performance with CFRP 3D-printed concrete counterparts. By conducting such a comparative study, one can gain valuable insight into the extent to which wood-based hybrids can compete with or even surpass CFRP in terms of their structural capabilities.

Geometry

The studies conducted in Chapter 5 are just the beginning, as more attributes can be explored such as the number of strips and grid type. Furthermore, Understanding how diagonal gridshells compare to parallel gridshells and exploring the impact of limited individual element lengths on performance are exciting and valuable avenues to be investigated. For instance, the number of strips and grid orientation(diagonal vs. perpendicular), are instances that demand further exploration. Additionally, in order to gain a better understanding of continued explorations can focus on limiting the individual elements' length to avoid the local phenomena affecting the results of the performance of the gridshells. Using the collected data and knowledge through these sensitivity studies could lead to a more efficient optimization process especially when dealing with multi-objective optimization of the hybrid gridshell designs.

Fabrication

Future research could investigate whether the lost CFRP formwork is sufficiently strong to support the weight of multiple concrete layers, as this may be desirable for some applications. However, during the construction process, the formwork is subjected to significantly higher loads with each new layer. Future studies could therefore aim to evaluate whether the sequence or location of concrete placement, or placement in relation to curing time, could potentially influence the system and mitigate the load applied to the CFRP formwork and consider stiffness variations during construction.

In addition, when the concrete layer is applied to the CFRP formwork, casting in one direction (warp or weft) introduces a height difference that must be compensated for when casting in the other direction. To solve this problem, the details of the warp and weft design must be carefully examined or alternative approaches must be explored, such as optimizing the material deposition sequence to mitigate this problem. Addressing the above challenges represents one possible measure for

advancing hybrid gridshell construction. With thoughtful details and possible improvements in the geometric stiffness and material properties of the formwork, these obstacles could be overcome and the full potential of this method could be realized.

3D Printing Concrete on the Bending-active Formwork

Another step to deepen this research, which the team has already started, is the idea of applying the concrete to the bending-active formwork via a robotic 3D printing platform. This approach would need to combine parametric modeling, 3D scanning, automatic print path adjustment, and the use of a multi-axis robotic printing system. It offers the potential for building affordable, lightweight, fast, and sustainable roof structures in a very efficient and economical manner. The unique advantage of this technique is that the printing layers can directly follow the arch-shaped curved formwork, which in turn acts as a lightweight and material-saving auxiliary structure and allows printing over large spans. The use of a robot to 3D print the concrete onto the formwork also enables fine-tuning of the printing paths and better distribution of the concrete weight over the entire formwork. An automated workflow such as this could eliminate the limitations imposed by current architectural-scale 3D printing practices, which involve slicing geometries horizontally and printing flat layers, making it difficult to fabricate structures with longer spans and requiring an unsustainable amount of additional support material. An automated workflow such as this could eliminate the limitations imposed by current architectural-scale 3D printing practices, which involve slicing geometries horizontally and printing flat layers, making it difficult to fabricate structures with longer spans and requiring an unsustainable amount of additional support material.

Multi-objective Design Optimization

Future work could also take up the idea of multi-objective design optimization and incorporate other performance matrices and criteria. Digital form-finding processes, for example, could be motivated by improving hybrid gridshells not only in terms of structural performance, but also with respect to aspects such as material consumption, embedded CO₂ emissions, or daylighting and energy performance. In this context, for example, the natural lighting of the interior of the gridshell or human comfort levels could become additional design drivers.

Daylighting, in particular, can have a positive impact on reducing the energy consumption of buildings covered with a concrete shell, as illumination is responsible for 15% of the energy consumption of buildings worldwide [163]. However, with traditional continuous shell construction, which contains few or no openings, daylight is blocked from reaching the central parts of the enclosed space beneath the shell structure, resulting in large, under-lit, and uncomfortable areas. However, incorporating openings for natural lighting into continuous concrete shells involves tradeoffs, as they can affect the structural performance of such shells if not used properly. Hybrid gridshells, based on a bending-active structure, offer exciting opportunities to develop strategies for leveraging daylight in architecture. By harnessing the potential of natural sunlight with hybrid gridshells, architects can create sustainable, visually appealing, and occupant-friendly spaces. Therefore, the development of tools and design methods that can help explore different design

alternatives, evaluate tradeoffs in the design of gridshells that more readily allow openings and skylights, and provide informative, performance-based analysis in the early stages of the design and construction process can have a significant impact on the use of such structures.

Future studies mentioned above, along with similar research avenues that focus on resolving design and construction challenges of concrete shells have the potential to pave the path toward enabling widespread adoption of these highly efficient architectural structures. This will contribute to bringing us closer to the overarching sustainability goals within the construction industry.

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