Title
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Publication Date
1990-05-01

LBL-28857
Preprint

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Submitted to Zeitschrift F. Physik A

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May 1990

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

LBL-28857

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May 25, 1990

Abstract

We show that heavy quark spectroscopy and QCD sum rules constrain Nielsen-Olesen vortices to be thin and weakly interacting. This model of QCD strings may help to understand how independent string fragmentation models can apply to high energy nuclear collisions.

*This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, by the Feodor Lynen program of the Alexander von Humboldt-Stiftung, and by a Fulbright grant from Spanish Ministry of Education and Science.

One of the current phenomenological puzzles is why *independent* string fragmentation models such as in refs. [1,2] describe so well a wide variety of new data [3] hadron-nucleus and nucleus-nucleus collisions. The independence of multi-string fragmentation is certainly a strong model assumption, which should break down when the string density exceed some critical value ρ_c. The Glauber nuclear geometry used in such models leads to a string density per unit transverse area $\rho_s \approx 2r_0\rho_0(A^{1/3}+B^{1/3})$ for central $A+B$ collisions. With $2r_0\rho_0 \approx 0.36\,\mathrm{fm}^{-2}$, ρ_s increases from $\sim 0.7/\mathrm{fm}^2$ in pp to $\sim 2.5, 3.0, 4.3\,\mathrm{fm}^2$ in central pPb, OPb, and $PbPb$ collisions respectively. Nonlinear modifications of multi-string fragmentation should occur when $\rho_s > \rho_c$. One example of possible non-linear phenomena is color rope formation [4,5]. If Q strings overlap, the effective string tension could increase to $\kappa_Q \sim 2Q\kappa_0$, leading to larger $<p_T>$ and higher heavy flavor abundancies [5].

A rough estimate of the critical string density for the breakdown of independent fragmentation is given by $\rho_c = 1/(\pi r_{rms}^2)$, where r_{rms}^2 is the root mean square transverse radius of the string. In the MIT Bag model of color flux tubes [6] the Bag constant, B, and coupling, α_s, determine both the string tension, t, and r_{rms}^2 as

$$t = (32\pi\alpha_s B/3)^{1/2} \qquad , \qquad r_{rms}^2 = (2\alpha_s/3\pi B)^{1/2} \quad . \tag{1}$$

Therefore

$$\rho_c \approx (3B/2\pi\alpha_s)^{1/2} \quad . \tag{2}$$

The original MIT parameters [6] $B^{1/4} = 0.145\,\mathrm{GeV}$, $\alpha_s = 2.2$ give $t = 0.915\,\mathrm{GeV/fm}$, $r_{rms} = 1.125\,\mathrm{fm}$, and therefore suggest a very small $\rho_c \approx 0.25/\mathrm{fm}^2$. Independent fragmentation should then be invalid already in pp collisions. Of course, with those parameters even the independent nuclear shell model would be expected to fail because the proton bag is so huge. On the other hand, a considerably different set of parameters was deduced by Hasenfratz et. al. [7] by demanding that the MIT flux tube potential reproduce heavy quark spectroscopy. They found $B^{1/4} = 0.235\,\mathrm{GeV}$, $\alpha_s = 0.385$. These parameters lead to $t = 1\,\mathrm{GeV/fm}$, $r_{rms} = 0.45$ fm, and thus to a much higher critical density, $\rho_c \approx 1.6/\,\mathrm{fm}^2$. Such a small value of the string radius is also indicated by the transverse momentum and rapidity distributions resulting from pair creation in a flux tube as shown in ref.

[8]. In fact recent QCD Lattice calculations are also consistent with such a small string radius [9].

The problem of string structure can also be addressed based on the Abelian-Higgs model of confinement. Nielsen and Olesen proposed long ago [10] that vortex solutions in that model may provide an magnetic analog to QCD strings. In that model confinement of colour charges is assumed to be due to a dual Meissner effect [10]-[14]. The structure of vortices in that model has been studied numerically in refs. [15] and [16]. Unlike the MIT model, the Abelian-Higgs Lagrangian contains three free parameters that may be taken as B, α_s, and t, and its vortices are characterized by to two intrinsic length scales, the London penetration length, λ_L and the Higgs Compton wavelength, $1/m_H$. By fixing the three parameters to the Hasenfratz values, Alcock et al. [15] found an approximate relation between $\lambda_L \approx 1/m_H$ and that such a vortex has $r_{rms} \approx 0.45\,\text{fm}$, surprisingly close to the MIT tube value for the same parameters in spite of the very different structure of such vortices. However, the relation $\lambda_L \approx 1/m_H$ has another potentially important consequence that is often overlooked. There exists a theorem [17,18] that the interaction energy between vortices with $\lambda_L = 1/m_H$ vanishes identically for any separation between them! On the other hand, conventional type II vortices with $\lambda_L > 1/m_H$ have repulsive interactions, while for type I vortices with $\lambda_L < 1/m_H$ have attractive interactions.

The above results suggest that not only could independent fragmentation work because the strings are thin but also because they are weakly interacting.

In the present note we extend the previous work on Nielsen-Olesen vortex structure by deriving a rigorous identity for r^2_{rms} and incorporating constraints imposed by QCD sum rules. We show that the r^2_{rms} of MIT flux tubes and Nielsen-Olesen vortices are identically equal when the three parameters of the Abelian-Higgs model are fixed by the two of the MIT model. Therefore, the approximate numerical result obtained variationally in [15] is shown to be an equality. We furthermore generalize the analysis by allowing B to vary independently from t and α_s in the range constrained by QCD sum rules. This provides a new determination of the range of possible vortex structures as determined by $\lambda_L m_H$. We find that r_{rms} varies by no more than 10% from the Hasenfratz flux tube value, and that

$0.9 < \lambda_L m_H < 2.0$ is consistent with the possibilty that QCD strings could be only weakly interacting. We interpret these results as providing further phenomenological support for the applicability of independent multi-string models to describe at least the initial phase of particle production in high energy nuclear collisions.

Nielson-Olesen vortices are classical solutions of the Abelian Higgs model as specified by [10]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \left|(i\partial_\mu - qA_\mu)\phi\right|^2 - \frac{h}{4}\left(\phi_V^2 - |\phi|^2\right)^2 \quad . \tag{3}$$

Its free parameters are the Higgs electric charge q, the value ϕ_V of the Higgs field in the vacuum and h, the coupling constant of of the Higgs self-interaction. In the ground state the Higgs field acquires a nonvanishing vacuum expectation value with $\langle|\phi|\rangle = \phi_V$. This leads to a mass term for the gauge bosons with $m_A = \sqrt{2}q\phi_V$, which is the inverse of the London penetration length λ_L. In the ground state, Higgs excitations have a mass, $m_H = h^{1/2}\phi_V$.

The effective potential between opposite magnetic monopoles of magnetic charge, Q_M, in the ground state is Coulombic at short distances and linear at large:

$$V(R) \approx Q_M^2/(4\pi R) + tR \quad . \tag{4}$$

Used as a model of the effective heavy quark-antiquark potential, Q_M is related to α_s via $Q_M^2/4\pi = 4\alpha_s/3$. On the other hand, due to the monopole quantization condition Q_M is related to the Higgs electric charge q by the relation $Q_M = 2\pi N/q$ with N being an integer. Assuming the $N = 1$ monopole potential to model the elemetary quark potential, the effective strong coupling constant is related to q via

$$\alpha_s = 3\pi/(4q^2) \quad . \tag{5}$$

The string tension in this model is computed by constructing static axial symmetric solutions of the form

$$\vec{A} = \sqrt{2}\phi_V a(\rho)\vec{e}_\varphi \quad , \quad \phi = \phi_V \eta(\rho) e^{in\varphi} \tag{6}$$

where n is an integer and the dimensionless transverse coordinate is given by $\rho = \sqrt{2} q \phi_V |\vec{x}_\perp|$. The field equations in these units are

$$
\begin{aligned}
-\eta'' - \frac{1}{\rho}\eta' + \frac{1}{\rho^2}\xi^2\eta + \frac{\lambda}{2}\eta(\eta^2 - 1) &= 0 \quad , \\
-\xi'' + \frac{1}{\rho}\xi' + \eta^2\xi &= 0 \quad ,
\end{aligned}
\tag{7}
$$

where the prime denotes the derivative with respect to ρ, and $\xi(\rho) \equiv n - \rho a(\rho)$. The energy density corresponding to such a solution is given by $\mathcal{H} = 4q^2\phi_V^4\tilde{\mathcal{H}}$, where

$$
\tilde{\mathcal{H}} = \frac{1}{2}\eta'^2 + \frac{1}{2\rho^2}\xi'^2 + \frac{1}{2\rho^2}\xi^2\eta^2 + \frac{\lambda}{8}(\eta^2 - 1)^2 \quad , \tag{8}
$$

and $\lambda \equiv h/2q^2 = m_H^2/m_A^2$. For the string tension to remain finite, the boundary conditions

$$
\begin{aligned}
a &\to 0 \quad \text{and} \quad \eta \to 0 \qquad \text{for} \quad \rho \to 0 \\
\xi &\to 0 \quad \text{and} \quad \eta \to 1 \qquad \text{for} \quad \rho \to \infty
\end{aligned}
\tag{9}
$$

have to be fulfilled. The second equation implies that total magnetic flux is equal to $2\pi n/q$. The string tension t is then given by

$$
t = 2\phi_V^2 t_0 = 2\phi_V^2 \int 2\pi d\rho \rho \tilde{\mathcal{H}} \quad . \tag{10}
$$

In general the field equations have to be solved numerically. The result for $\lambda = 1$ is shown in fig. 1. However, we now show that for $\lambda = 1$ not only the string tension but also second moment of the energy distribution can be determined analytically. As shown in [17,18] a lower bound of the energy per unit length is achieved if

$$
\eta' = \frac{1}{\rho}\xi\eta \quad , \qquad \frac{1}{\rho}\xi' = \frac{1}{2}(\eta^2 - 1) \quad . \tag{11}
$$

In that case the fields automatically satisfy (7). With (11) the dimensionless string tension, t_0, can be integrated by parts, yielding

$$
t_0 = 2\pi \int \rho d\rho \left\{ \frac{1}{\rho^2}\xi'^2 + \frac{1}{\rho^2}\eta^2\xi^2 \right\} = -\lim_{\rho \to 0} \frac{2\pi\xi\xi'}{\rho} = \pi n \tag{12}
$$

due to the eqs. (7) and (11). In fact, it was proven in [18] that this linear proportionality between the magnetic flux and the energy holds for the solution of lowest energy in the general case of n parallel flux tubes at arbitrary separations (given by the zeros of the Higgs field). Therefore $\lambda = 1$ vortices do not interact. The axially symmetric case corresponds to n elementary vortices superimposed at the origin.

The ground state energy density in this model is simply given by $B = h\phi_V^4/4$. Noting that for $\lambda = 1$ vortices, $h = 2q^2 = 3\pi/(2\alpha_s)$, the string tension is then given by

$$t = 2\pi n \phi_V^2 = n(32\pi\alpha_s B/3)^{1/2} \quad , \tag{13}$$

which is identical to the MIT model for $n = 1$ [15,16].

Remarkably, the second moment of the vortex energy distribution can also be computed in the case $\lambda = 1$. Using the eqs. (7), (9) and eq. (11) again, we find that the dimensionless second moment is given by

$$t_0 r_0^2 = 2\pi \int \rho d\rho \, (\xi'^2 + \eta^2\xi^2) = -4\pi \int d\rho \, \xi\xi' = 2\pi n^2 \quad . \tag{14}$$

The mean square radius is therefore given by

$$r_{rms}^2 = r_0^2/(2q^2\phi_V^2) = n^2(2\alpha_s/3\pi B)^{1/2} \quad , \tag{15}$$

which is also identical to the MIT model for $n = 1$. This property of Nielsen-Olesen vortices seems to have gone unnoticed before. However, this identity holds only for the second moment of the energy distribution. As the energy extends further out in the Abelian Higgs model its higher moments are larger than in the MIT-bag model, whereas the first moment is smaller. For the MIT bag model one finds $<r>/r_{rms} = 0.9428$, whereas the Abelian Higgs model gives 0.863, independent of the choice of α_s and t.

Whereas the MIT-bag model contains only two parameters B and α_s, the Abelian Higgs model has three parameters ϕ_V, q, and h. Thus, if we fix the electric charge q by the magnetic monopole interaction strength and ϕ_V by the string tension t, we are still free to vary λ. In this way we can accomodate larger values for the vacuum energy B, as suggested by the QCD sum rules [19,20], without changing t or α_s. However, with increasing λ the string tension t also increases. Readjusting ϕ_V in order to keep t constant somewhat reduces B, with the result that it grows slower than linearly with λ. The solid line in figure 2 shows the bag constant $\overline{B}$ normalized as

$$\overline{B} \equiv B/B_{MIT} = \frac{32\pi\alpha_s}{3t^2}\,B = \lambda\,\frac{\pi^2}{t_0^2} \quad . \tag{16}$$

This quantity remains invariant for different choices of α_s and t and is exactly one for any parametrization within the MIT-bag model. The dashed curve in fig. 2 represents the scaled root mean square radius, given by

$$\overline{r} \equiv r_{rms}/r_{MIT} = \sqrt{\frac{3tr_{rms}^2}{8\alpha_s}} = \sqrt{\frac{t_0 r_0^2}{2\pi}} \quad , \tag{17}$$

which is also independent of α_s and t. ($\overline{B}$ and $\overline{r}$ are independent of α_s and t in other models as well, as for example in the Friedberg-Lee model). As one can see the radius decreases very slowly with increasing λ.

Varying B independently from α_s and t is interesting insofar as QCD sum rules favour a rather larger value for the bag constant. According to Reinders et. al. [20] the charmonium spectrum can be best described by assuming the gluon condensate strength $<0|\alpha_s F^2/\pi|0>$ to be in the range $(360 \pm 20\,\mathrm{MeV})^4$. This is related to the gluonic vacuum energy density by

$$\varepsilon_{vac}^G = \frac{\beta(g)}{8g} <0|F^2|0> \approx -\frac{11-\frac{2}{3}N_F}{32} <0|\frac{\alpha_s}{\pi}\,F^2|0> \quad , \tag{18}$$

where β is the Callan Symanzik function and N_F the number of flavours. On the other hand, the quark contribution is only of the order $\epsilon_{vac}^q \approx (150\,\mathrm{MeV})^4$, which changes $B^{1/4}$ only by a few percent. With $N_F = 4$ the resulting values for ε_{vac}^G are between $(229\,\mathrm{MeV})^4$ and $(271\,\mathrm{MeV})^4$. Within the Abelian Higgs model these values can be reproduced by $0.9 < \lambda < 2.0$, assuming $\alpha_s = 0.385$ and $t = 0.2\,\mathrm{GeV}^2$.

In this respect the Abelian Higgs model differs considerably from the Friedberg-Lee model [21], where for given α_s and t the volume energy must be chosen smaller than the corresponding MIT value. In contrast to the electric flux in the Friedberg-Lee model, the magnetic flux in the Abelian Higgs model can penetrate into the nonperturbative vacuum. For a given α_s, i. e. for a given total flux, that means the region where the Higgs condensate is expelled can be made smaller in the Abelian Higgs model, leading to a lower energy for a given volume energy density B. In turn this implies that one has to choose a higher volume energy in order to reproduce a given string tension. Thus the fact that B can be made rather large is closely related to the fact that the vector boson mass remains finite in the condensed Higgs phase. Unfortunately this also implies that the vector bosons, which are a dual description of gluons, are not absolutely confined. Expressing m_A in physical units,

$$m_A = \left(\frac{6\pi B}{\alpha_s \lambda}\right)^{1/4} = \sqrt{\frac{3\pi t}{4\alpha_s t_0}} \quad , \tag{19}$$

one finds that the mass of the dual gluons is rather low. For $t = 0.2\,\mathrm{GeV}^2$ and $\alpha_s = 0.385$ one finds $m_A = 624\,\mathrm{MeV}$ for $\lambda = 1$ and $m_A = 580\,\mathrm{MeV}$ for $\lambda = 2$. This may present a problem if one wants to use the Abelian Higgs model for dynamical calculations, because dual gluons can propagate into the nonperturbative vacuum if their energy is larger than m_A.

In summary, we found that static Abelian Higgs vortices are thin ($r_{rms} \approx 0.45\,\mathrm{fm}$) and interact only weakly ($\lambda \approx 1$) within the range of vacuum energy densities consistent with QCD sum rules. This model therefore provides a picture in which independent multi-string fragmentation can hold for complex nuclear collisions in spite of the high string densities. This does not imply the absence of final state interactions, but rather that the initial hadronization phase may more plausibly be described by models such as LUND or DPM.

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Figure Captions

Figure 1: The vortex solution for $\lambda = 1$. The solid line represents the Higgs field, the dashed line the magnetic flux, the dashed-dotted the energy density and the dotted line the energy density weighed by r. All quantities are plotted in arbitrary units.

Figure 2: The volume energy $\overline{B} \equiv B/B_{MIT}$ and the radius $\overline{r} \equiv r_{rms}/r_{MIT}$ for different values of λ. $\overline{B}$ and $\overline{r}$ are independent of the specific choice of α_s and t.

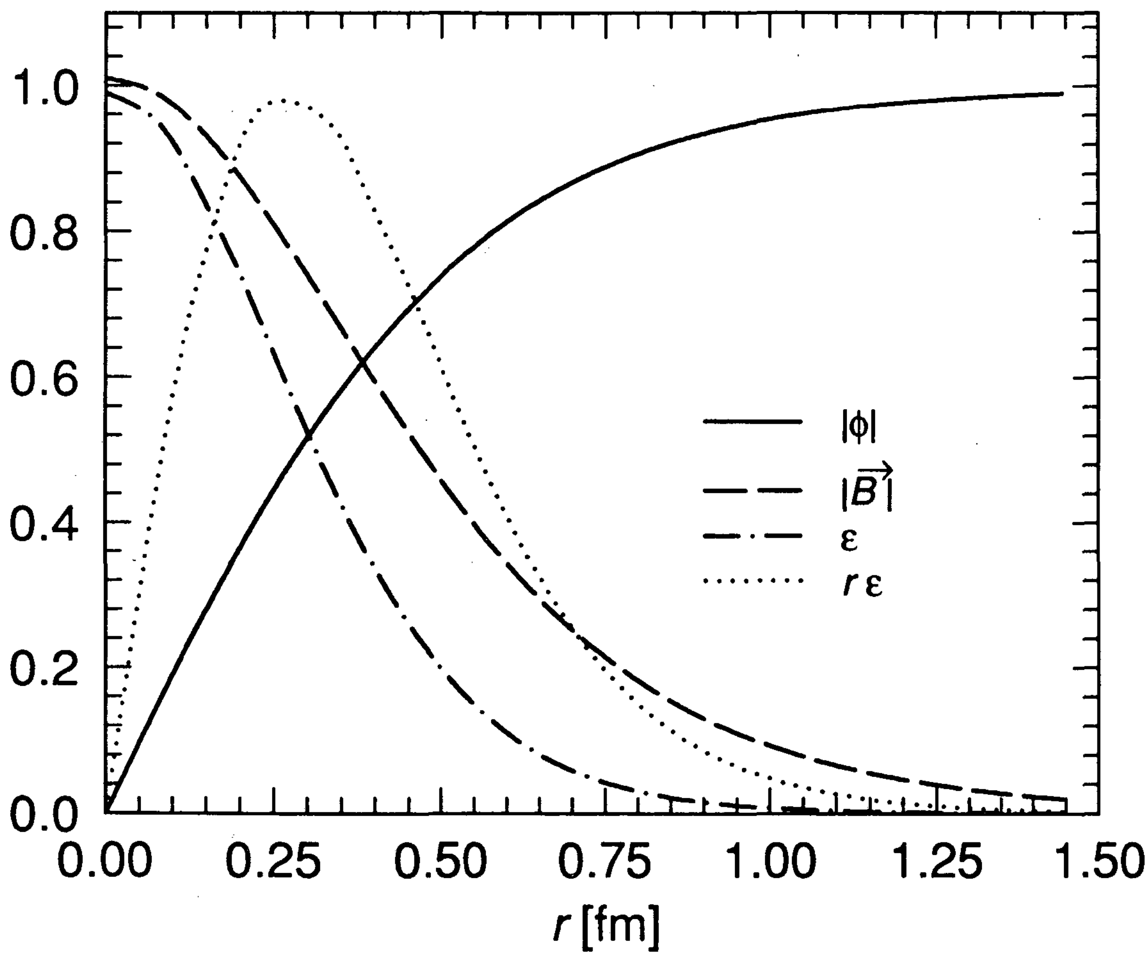

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Figure 1

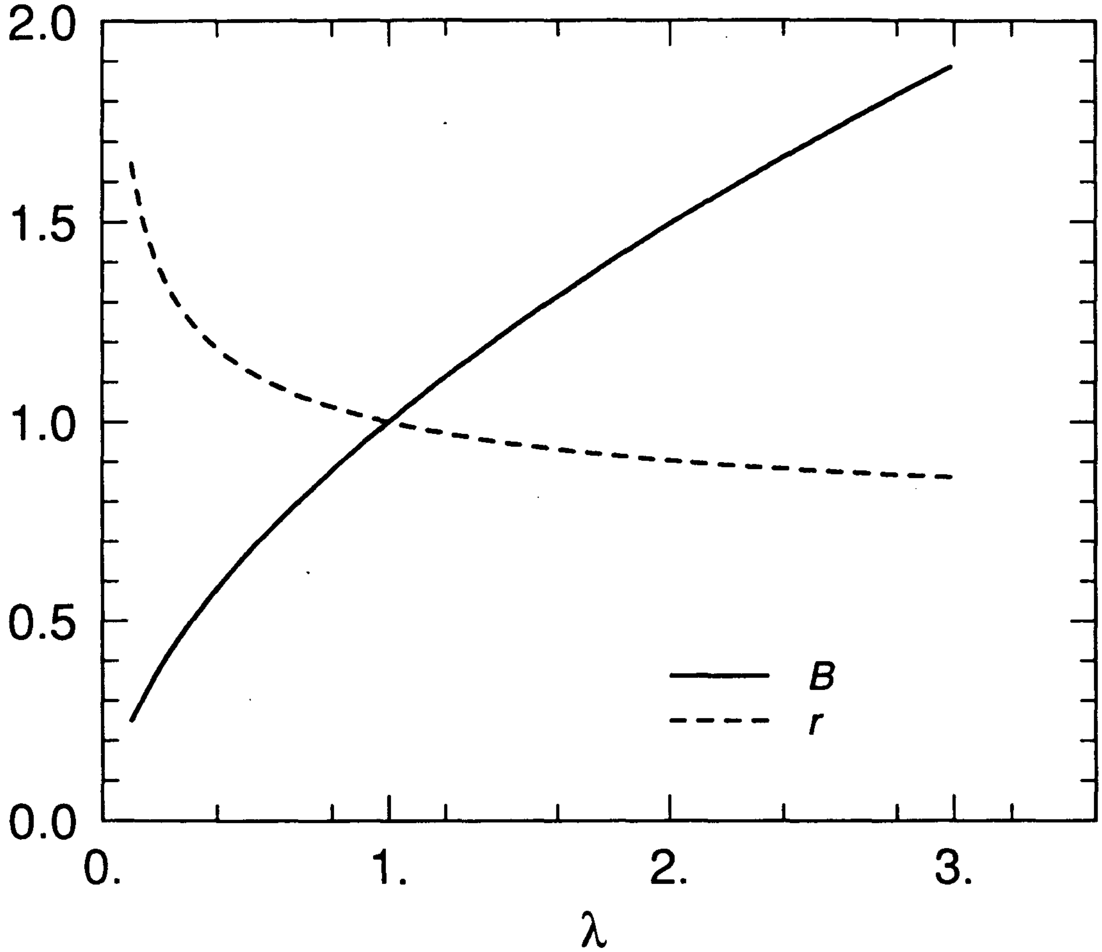

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Figure 2

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