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In a very interesting paper,(1) Balachandran, Freund and Schumacher pointed out that as of 1963 the asymptotic behavior in t (in momentum transfer) for the nucleon electromagnetic form factors led to a sum rule which, in current terminology, is really a superconvergence relation.(2) To satisfy the superconvergence relation for the isovector form factor (F_2^V) a new vector meson (called ρ') was postulated. The isoscalar form factor (F_2^S) was treated in the same spirit, although the experimental data was not so convincing for this case. It was pointed out in Ref. (1) that "present experimental evidence does not exclude stronger asymptotic conditions on the nucleon electromagnetic form factors." However, present experimental data(3a,b) suggest even stronger asymptotic conditions, and it seems appropriate to re-investigate this matter to find new sum rules which lead to stronger and more interesting implications.

The purpose of this note is threefold.

1) First, certain important assumptions are made in Ref. (1) that have not been clearly emphasized. Part of our goal will be to make clear what these assumptions are and how they limit the strength of conclusions drawn.

2) Second, we wish to recast the usual language for fitting form-factor data by a pole approximation into a form that reveals more clearly the connection with superconvergence relations. In the usual language one asks the following question: If the form factor has a $t \to -\infty$ behavior of the form $1/|t|^n$, how many simple poles are needed to approximate the right-hand cut? The answer is (n), if there are appropriate connections between pole positions and residues.

3) Finally, we wish to point out that if [see Ref. (1) for notation] the Pauli form factors $(F_2^{V,S})$ for the nucleon satisfy the asymptotic constraint $|t|^{n+1} F_2^{V,S}(t) \longrightarrow 0$ [for $t \to -\infty$ and for all (n)] there must exist an infinite number of vector mesons with different masses and with all other quantum numbers the same.[4] The motivation for the above asymptotic constraint can be found in Ref. (3a). It is made plausible there that the form-factor data can be fitted with an exponential form of the type $\exp[-(-t)^{\frac{1}{2}}]$ (for $t < 0$). This is just the form that appears in Stack's[3c] potential model for hadron

form factors, where the hadron is taken to be "infinitely composite." Furthermore, as pointed out by Martin,[3d] a faster drop-off for $t < 0$ can not be tolerated from an S-Matrix point of view.

To establish the above property we assume the Pauli form factors $[F_2^{V,S}(t)]$ satisfy the following unsubtracted dispersion relation (we suppress superscripts and subscripts from now on):

$$F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\operatorname{Im} F(t')}{t' - t} , \tag{1}$$

where $t_0 = (2m_\pi)^2$ for F_2^V and $t_0 = (3m_\pi)^2$ for F_2^S. We now impose the asymptotic constraint

$$|t|^{n+1} F(t) \xrightarrow{t \to -\infty} 0 \quad \text{(for all } n \geqslant 0 \text{ and integral).} \tag{2}$$

We wish to consider now the minimum conditions which will permit us to write a dispersion relation for the function $t^{n+1} F(t) = G(t)$, i.e.,

$$G(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt \frac{\operatorname{Im} G(t')}{t' - t} . \tag{3}$$

There is a theorem, proved by Sugawara and Kanazawa (SK),[5] which is of help to us. In our present context, this theorem is stated as follows: If a function $f(z)$ is analytic everywhere, except for a right-hand cut on the real z-axis and is bounded at infinity by a finite but arbitrary power of z, and $f(z) \longrightarrow$ definite limits (including infinity) as $z \to \infty \pm i\epsilon$ ($\epsilon > 0$ and infinitesimal), then $f(z)$ has the same asymptotic behavior along any ray in the cut plane]. If we use this theorem the constraint[6]

$$t^{n+1} F(t) \xrightarrow{t \to +\infty \pm i\epsilon} 0 \quad \text{(all } n \geqslant 0 \text{ and integral)} \tag{4}$$

follows from (2) if (a) $F(t)$ has the form

$$F(t) \xrightarrow[t \to -\infty]{} \exp[-(-t)^{\frac{1}{2}-\gamma}], \tag{5}$$

where $0 < \gamma < \frac{1}{2}$, and (b) it does not have an infinite number of oscillations as $t \to -\infty$. These two conditions guarantee (2) and do not violate the Martin bound[3] and we shall assume them in what follows. If we now evaluate (3) at $t = 0$, we are led to the following set of sum rules (which must be satisfied simultaneously):

$$\frac{1}{\pi} \int_{t_0}^{\infty} dt' \, (t')^n \, \operatorname{Im} F(t') = 0 \quad \text{(all } n \leqslant N). \tag{6}$$

For $n \leqslant N = 0$, (6) reduces to the sum rule reported in Ref. (1) and first written down by Sachs.[8] We shall now calculate $\operatorname{Im} F(t)$ for $t > t_0$, using unitarity. If we separate the contributions from particles explicitly we end up with the exact form

$$[F(t) \sim \langle N\bar{N} \mid \gamma \rangle],$$

$$\mathrm{Im}\, F(t) = \sum_i \pi\gamma_i\, \delta(t - m_i^2) + \pi\, g(t). \tag{7}$$

Then

$$F(t) = \sum_i \frac{\gamma_i}{m_i^2 - t} + R(t), \tag{8}$$

where[9]

$$R(t) = \int_{t_0}^{\infty} dt' \, \frac{g(t')}{t' - t} \approx \sum_{n=0}^{\infty} \frac{R^{(n)}}{t^{n+1}} \tag{9}$$

and $R^{(n)} = - \int_{t_0}^{\infty} (t')^n \; g(t')\, dt'$ are just the moments of $R(t)$. What is usually done now is to make the "pole approximation" to $F(t)$. This amounts to setting $R^{(n)} = 0$ for all (n). The sum rule (6) then becomes

$$\sum_i (m_i^2)^n \; \gamma_i + R^{(n)} = 0 \equiv \sum_i (m_i^2)^n \, \gamma_i \, . \tag{10}$$

To satisfy (10) for all $n \leqslant N$ (a positive integer) an infinite number of vector mesons is needed. It is instructive to see how this works. Consider e.g. $n \leqslant N = 0$. We need at least 2 vector mesons for a nontrivial result and this was the important observation made in Ref. (1). For $n \leqslant N = 1$ we must satisfy 2 sum rules simultaneously, i.e.,

$$\left.\begin{aligned} &\frac{1}{\pi} \int_{t_0}^{\infty} dt' \;\; \text{Im}\, F(t') \;=\; 0 \qquad &(a) \\ &\frac{1}{\pi} \int_{t_0}^{\infty} dt' \; t' \; \text{Im}\, F(t') \;=\; 0. \qquad &(b) \end{aligned}\right\} \tag{11}$$

For $i = 1,2$ (10) reads

$$\left.\begin{aligned} &\gamma_1 + \gamma_2 = 0 \qquad &(a) \\ &m_1^2\, \gamma_1 + m_2^2\, \gamma_2 = 0 \qquad &(b) \end{aligned}\right\} \tag{12}$$

Now use (12a) in (12b) and get $m_1 = m_2$. To satisfy (11) in a nontrivial way we need at least 3 distinct vector mesons. Now consider the case $n \leqslant N$ = a "very large" positive integer (i.e., $N \to \infty$) and keep 3 particles. If m_1 is the largest mass it will turn out that $\gamma_1 = 0$, and the problem reduces to a previous special case. We can continue in this way to get the desired result, i.e., to satisfy (10) for $n \leqslant N$, $N + 2$ vector mesons are needed.

Finally we wish to use the work in Ref. (1) to illustrate how the full power of unitarity may or may not imply that ρ' exists. This means we shall consider (10) with $n = 0$ in the form

$$\sum_i \gamma_i + R^{(0)} = 0. \tag{13}$$

For the case F_2^V, $\sum_i \gamma_i = \gamma_\rho$ and we have the relation

$$\gamma_\rho = - R^{(0)} . \qquad (14)$$

Equation (14) insures $F(t) \xrightarrow{t \to -\infty} 1/t^2$ without postulating ρ'. It is clear that (8) along with (14) can be used to fit the isovector form-factor data; we need only choose an appropriate phenomenological form for $g(t)$. However, if we assume there exists no relation of type (14) this necessarily implies ρ' exists.

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2. V. de Alfaro, S. Fubini, G. Rosetti, and G. Furlan, Phys. Letters, 21, 576 (1966); L. D. Solov'ev, Soviet JNP 3, 131 (1966).

3. a) T. T. Wu and C. N. Yang, Phys. Rev., 137, 708B (1965);
   b) M. Goitein, J. R. Dunning, and R. Wilson, Phys. Rev. Letters 18 1018 (1967);
   c) J. D. Stack, "Rapidly Decreasing Form Factors and Infinitely Composite Particles" (University of Illinois preprint);
   d) A. Martin, Il Nuovo Cimento, 37, 671 (1965).

4. This conclusion follows unambiguously only if there are no detailed cancellations between simple pole terms and continuum contributions from the unitarity relation. This point will be discussed in more detail at the end of this work.

5. M. Sugawara and A. Kanazawa, Phys. Rev., 123, 1895 (1961). As an example of a function $[g(t)]$ that has a power bound for $t \to -\infty$ but does not satisfy the condition $g(t) \longrightarrow$ definite limits $t \to \infty \pm i\epsilon$ consider

$$g(t) = e^{-(-t)^{\frac{1}{2}}} + 1/(-t)^n .$$

As $t \to \infty \pm i\epsilon$, $g(t)$ oscillates so that the SK theorem cannot be used.

6. In Ref. (1) it was assumed that a dispersion relation for $t\,F(t)$ can be written when $|t|\;F(t) \xrightarrow{t \to -\infty} 1/|t|^{\gamma}$ $(\gamma > 0)$ and $F(t)$ itself satisfies an unsubtracted dispersion relation. We see that this is not enough information to use the SK theorem. In particular, we also need to know that $t\;F(t) \xrightarrow{t \to \infty \pm i\epsilon}$ definite limits.

7. We note here that the assumed nonoscillatory fall-off for $F(t)$ $(t < 0)$ is suggested by the data.[3a,b]

8. R. G. Sachs, Phys. Rev. 126, 2256 (1962).

9. What we really mean in Eq. (9) is $\int_{t_0}^{\infty} = \int_{t_0}^{L} + \int_{L}^{\infty}$.

   Now choose (L) large enough so that $\int_{L}^{\infty} \approx 0$. Then for $t > L$ Eq. (9) is defined by a converging power series.