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Author
Gaillard, M.K.

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M.K. Gaillard

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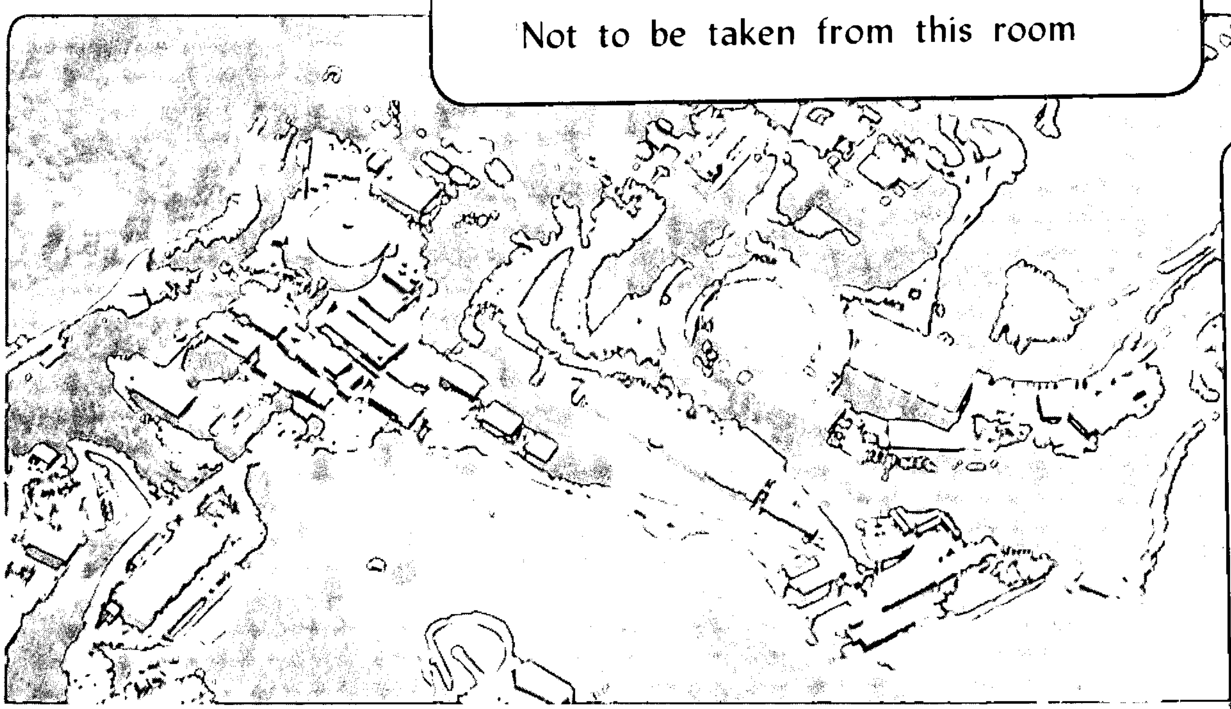

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MODULAR INVARIANT GAUGINO CONDENSATION*

Mary K. Gaillard

Department of Physics, University of California
and
Physics Division, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley, California 94720

Abstract

The construction of effective supergravity lagrangians for gaugino condensation is reviewed and recent results are presented that are consistent with modular invariance and yield a positive definite potential of the no-scale type. Possible implications for phenomenology are briefly discussed.

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INTRODUCTION

Attempts to make the connection between superstrings and observed particle physics must be able to account for the origin of supersymmetry (SUSY) breaking. In this context, the SUSY mass gap, which recent LEP data[1] suggest lies at about a TeV, in turn governs the scale of electroweak symmetry breaking, namely the value of the Higgs *vev* :

$$v = \frac{m_{\rm Higgs}}{\sqrt{2\lambda_{\rm Higgs}}} \simeq \frac{1}{4} TeV. \qquad (1)$$

A popular candidate mechanism for SUSY breaking is gaugino condensation in a "hidden" (*i.e.*, with only gravitational strength couplings to "observed" matter) SUSY Yang-Mills sector of the effective supergravity theory in four dimensions. According to this scenario, the asymptotically free and infrared enslaved SUSY Yang-Mills theory becomes confined at some scale Λ_c where gaugino condensation occurs:

$$< \bar{\lambda}\lambda >_{hid} \sim \Lambda_c^3. \qquad (2)$$

This gaugino condensate can trigger "local" SUSY breaking in the sense that the gravitino acquires a mass: $m_{\tilde{G}} \neq 0$. This symmetry breaking should be communicated to the observable sector, via radiative corrections, in the form of a SUSY mass gap, *i.e.*, "global" SUSY breaking. It is the task of the theory to predict the correct scale for the SUSY mass gap, in particular the fact that it is very small in comparison with the fundamental scale of the theory–namely, the string tension which is comparable to the square of the reduced Planck mass: $M_{Pl} = (8\pi G_N)^{-\frac{1}{2}} \simeq 1.8 \times 10^{18} GeV$. A possibly important ingredient in understanding this hierarchy of scales is the fact that in many string compactifications the effective low energy theory possesses classical nonlinear symmetries[2–4] that help to suppress the communication of SUSY breaking from the hidden sector to the observable sector.

The point of view presented in this talk is based on work in collaboration with Pierre Binétruy. A crucial feature of our approach is that we demand that the effective theory have vanishing vacuum energy in the approximation that we are working in. The point is that whatever unknown mechanism one might appeal to in order to suppress the cosmological constant can also affect the other parameters of the theory. A different approach has been considered by other authors[5–7].

I will first describe the construction[3,4] of the effective superpotential for gaugino condensation for a prototype effective supergravity theory from superstrings, following the Veneziano-Yankielowicz[8] analysis of SUSY QCD and the generalization of their result by Taylor[9] to the supergravity case, in which the gauge coupling is determined by the *vev* of the dilaton. I will mention generalizations to more realistic models and comment on the phenomenology of the effective theory. I will then show how these results must be modified[5,6] so as to restore modular invariance or space-time duality, that is, invariance under inversion of the radius of compactification: $R \to R^{-1}$. This modification can be interpreted as a threshold correction[10] arising from the integration over heavy string modes.[11] The resulting effective theory has an unbounded potential. I will show how this disaster can be averted by a reinterpretation[12] of the results, and briefly comment on the prospects for phenomenology.

Closely related talks were given at this conference by M. Cvetič, J.-P. Derendinger, J. Louis and T. Taylor.

GAUGINO CONDENSATION IN SUPERGRAVITY

In the Kähler covariant superfield formulation[13] of supergravity, the lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{pot} + \mathcal{L}_{YM}. \qquad (3)$$

The first term

$$\mathcal{L}_E = -3\int d^4\Theta \mathcal{E}\mathcal{R} + h.c. \qquad (4)$$

is the generalized Einstein term. It contains the pure supergravity part as well as the (noncanonical, *i.e.*, including derivative couplings) kinetic energy terms for the chiral supermultiplets. The second term:

$$\mathcal{L}_{pot} = \int d^2\Theta \mathcal{E} e^{K(\Phi,\bar\Phi)/2} W(\Phi) + h.c., \qquad (5)$$

contains the Yukawa couplings and the scalar potential, and the third term

$$\mathcal{L}_{YM} = \frac{1}{4}\int d^2\Theta \mathcal{E} f(\Phi) W^a_\alpha W^\alpha_a + h.c. \qquad (6)$$

is the Yang-Mills lagrangian. The expansion of the above expressions in terms of component fields includes derivatives that are covariant with respect to general coordinate, gauge and Kähler transformations. A Kähler transformation is a redefinition of the Kähler potential $K(\Phi,\bar\Phi) = K(\Phi,\bar\Phi)^\dagger$ and of the superpotential $W(\Phi) = \overline{W}(\bar\Phi)^\dagger$ by a holomorphic function $F(\Phi) = \bar F(\bar\Phi)^\dagger$ of the chiral supermultiplets $\Phi = (\varphi, \chi)$:

$$K \to K' = K + F + \overline{F}, \quad W \to W' = e^{-F} W. \qquad (7)$$

Since this transformation changes $e^{K/2}W$ by a phase that can be compensated by a phase transformation of the integration variable Θ, the theory defined above is classically invariant[14,13] under Kähler transformations provided one transforms the superfields $\mathcal{R}$ and W^a_α by a compensating phase; for example the Yang-Mills superfield transforms as:

$$W^a_\alpha \to e^{-i\mathrm{Im}F/2} W^a_\alpha. \qquad (8)$$

This last transformation, which implies a chiral rotation on the left-handed gaugino field λ^a_L:

$$\lambda^a_\alpha \to e^{-i\mathrm{Im}F/2}\lambda^a_\alpha, \qquad (9)$$

is anomalous at the quantum level, a point that will be important in the discussion below. (Here a is a gauge index and α is a Dirac index.).

The theory is completely specified by the field content, the gauge group and the three functions K, W and f of the chiral superfields. One can fix the "Kähler gauge" by a specific choice of the function F. In particular, choosing $F = \ln W$ casts the lagrangian in a form[14] that depends on only two functions of the chiral superfields, f and $\mathcal{G} = K + \ln|W|^2$.

To construct an effective potential for gaugino condensation we introduce a composite superfield operator[8] U as an interpolating field for the Yang-Mills composite operator:

$$\frac{1}{4} W^a_\alpha W^\alpha_a \Rightarrow U = e^{K/2}\widetilde{W}(H). \qquad (10)$$

Here $H = h + \tilde\Theta_R \chi^H_L + \cdots$ is an ordinary chiral supermultiplet of zero Kähler weight, that represents the lightest bound state of the confined SUSY Yang-Mills sector, just as in low energy QCD the pion is an interpolating field for the composite quark operator: $\bar{q}(1+i\gamma_5)q \Rightarrow \sigma + i\vec{\pi}\cdot\vec{\tau}$. Kähler invariance requires

$$\widetilde{W}(H) \to e^{-F}\widetilde{W}(H) \tag{11}$$

under (7). A key element in the identification (10) is the fact that the Yang-Mills chiral multiplet W_α has a different Kähler weight[13,3,6] from that of ordinary chiral multiples of weight zero.

I will first consider a prototype[15] supergravity model from superstrings, with just one modulus T and one matter generation. The functions K, W and f are given in terms of the superfields $\Phi = \{\Phi^i, S, T\}$ by

$$f = S, \tag{12a}$$

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}-|\Phi|^2), \quad |\Phi|^2 = \sum_i \Phi^i\bar{\Phi}^{\bar{\imath}}, \tag{12b}$$

$$W(\Phi) = c_{ijk}\Phi^i\Phi^j\Phi^k + \tilde{c}. \tag{12c}$$

The last term in the superpotential W parameterizes a possible additional source of nonperturbative SUSY breaking, for example, a nonvanishing (quantized) *vev* for the antisymmetric tensor field strength H_{LMN}, of 10-dimensional supergravity:

$$\tilde{c} \propto \int dV^{lmn} < H_{lmn} > = 2\pi n \neq 0, \quad l,m,n = 4,\ldots,9,$$
$$H_{LMN} = \nabla_L B_{MN}, \quad L,M,N = 0,\ldots,9,. \tag{13}$$

The gauge coupling constant at the GUT scale is determined by the *vev* of the dilaton field: $S = s + \bar{\Theta}_R\chi_L^S + \cdots$,

$$< \mathrm{Re}s > = g^{-2}, \tag{14}$$

and the scales of the theory are determined by the *vev* of the Kähler potential:

$$\frac{\Lambda_{GUT}}{M_{Pl}} = \frac{1}{RM_{Pl}} = (2g)^{\frac{2}{3}} < e^{K/6} > \approx < (\mathrm{Re}s\mathrm{Re}t)^{-\frac{1}{2}} > \tag{15}$$

(assuming $< |\varphi^i|^2 > \ll < \mathrm{Re}t >$).

For $\tilde{c} = 0$, the supergravity theory defined above is classically invariant[2,3] under the nonlinear $SL(2,\mathcal{R}) \otimes U(1)_R$ transformations:

$$T \to T' = \frac{aT - ib}{icT + d}, \quad \Phi^i \to \Phi^{i'} = \frac{e^{i\beta}\Phi^i}{icT+d}, \quad S \to S' = S,$$

$$ad - bc = 1, \quad a,b,c,d,\beta \text{ real}, \tag{16}$$

where $U(1)_R$, with parameter β, is the usual R-symmetry of SUSY theories. Eq.(16) effects a Kähler transformation (7) with

$$F = 3\ln(icT + d) - 3i\beta, \tag{17}$$

under which the full lagrangian is invariant provided the gaugino fields undergo the chiral transformation (9). In addition to the chiral anomaly associated with (9), the transformations (17) include an anomalous conformal transformation, namely a scaling of the effective cut-off Λ_{GUT}, (15), of the theory:

$$\Lambda_{GUT} \to e^{\mathrm{Re}F/3}\Lambda_{GUT}, \tag{18}$$

For the theory defined above, the effective lagrangian for gaugino condensation is given by[9,3]

$$\mathcal{L}_{pot}^{eff} \equiv \int d^2\Theta\mathcal{E}e^{K/2}W(H,S)$$
$$= \int d^2\Theta\mathcal{E}U2b_0\lambda\ln(H/\mu) + h.c. = \int d^2\Theta\mathcal{E}e^{K/2}\widetilde{W}(H)2b_0\lambda\ln(H/\mu) + h.c.$$
$$= \int d^2\Theta\mathcal{E}e^{K/2}2b_0\lambda e^{-3S/2b_0}H^3 + h.c., \tag{19}$$

where b_0 determines the β-function for the confined Yang-Mills theory:

$$\frac{\partial g}{\partial \ln\mu} = -b_0^3,$$

and λ and μ are constants of order unity. The H-superfield kinetic energy term is determined by the Kähler potential[3,4]:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}-|\Phi|^2 - |H|^2). \tag{20}$$

Under a Kähler transformation (7,11) with $H \to e^{-F/3}H$, the lagrangian (19) undergoes the shift ($T = t + \bar{\Theta}_R\chi_L^T + \cdots$)

$$\delta\mathcal{L}_{pot}^{eff} = -\frac{2b_0}{3}\int d^2\Theta\mathcal{E}F(T)U + h.c.$$
$$= \sqrt{-\det g}\frac{b_0}{3}(\mathrm{Re}F(t)F^{\mu\nu}F_{\mu\nu} + \mathrm{Im}F(t)\tilde{F}^{\mu\nu}F_{\mu\nu} + \cdots), \tag{21}$$

which correctly reproduces the known variations under the trace and conformal anomalies.[8]

In addition to the exact classical $SL(2,\mathcal{R}) \otimes U(1)_R$ symmetry there are several approximate symmetries that are correctly embedded in the construction (19). In particular there is a nonanomalous[16,9] $U(1)_R$ symmetry, under which $S \to S + 2b_0 i\beta$, that is exact up to other quantum corrections, for example, those from the (weakly coupled) observable sector. Other approximate symmetries include[4] (in appropriate limits) both anomalous and nonanomalous conformal transformations $x \to \lambda x$, and the full isometry group of the Kähler metric.

By writing (19) in the form

$$\mathcal{L}_{pot}^{eff} = \int d^2\Theta \mathcal{E} U[S + \frac{2b_0}{3}\ln(4Ug^2(S)/\Lambda_{GUT}^3(\Phi)\lambda\mu^3)] + h.c., \qquad (22)$$

we obtain a direct physical interpretation of the result. Solving the effective theory for the condensate *vev* yields:

$$< H >= h_0 = \mu e^{-1/3}, \quad \text{or} \quad < \bar{\lambda}\lambda >_{hid} = 4 < U >= \frac{\lambda h_0^3}{g^2}\Lambda_c^3, \qquad (23)$$

where I used (14), (15) and the renormalization group relation $\Lambda_c = e^{-\frac{1}{2b_0 g^2}}\Lambda_{GUT}$. Then the factor

$$< 2b_0 \ln(H/\mu) >= \frac{1}{g^2}\left[1 + \frac{2b_0 g^2}{3}\ln(4 < U > g^2/\Lambda_{GUT}^3\lambda\mu^3)\right]$$
$$= \frac{1}{g^2}\left[1 + \frac{2b_0 g^2}{3}\ln(4e^{-1}\Lambda_c^3/\Lambda_{GUT}^3)\right] \qquad (24)$$

includes the one-loop Yang-Mills field wave function renormalization from the compactification scale to the condensation scale, up to finite corrections. The results (23) and (24), and in particular the factor g^{-2} in (23), coincide[4,6] precisely with the results of instanton calculations[17] in SUSY Yang-Mills theories.

It is straightforward to generalize the above formalism to the case of several gaugino condensates[18,12] by the replacement in (19)

$$\widetilde{W}(H,S) \to \sum_a \widetilde{W}_a(H_a,S),$$

and to multi-(moduli + generation) models. For example, for orbifold compactification with three moduli and three matter generations in the untwisted sector[19], one obtains the result (19) with the Kähler potential (20) replaced by[12] (here α is a generation index)

$$K_{eff} = -\ln(S+\bar{S}) - 3\ln\left(\prod_{\alpha=1}^{3}[\bar{T}_\alpha + T_\alpha - |\Phi_\alpha|^2]^{\frac{1}{3}} - |H|^2\right) + \text{twisted sector terms}.$$

PHENOMENOLOGY

If we fix H at its ground state value (23), we obtain an effective theory for Φ^i, S, T and the observable-sector Yang-Mills fields that is defined by $(12a,b)$ and the superpotential

$$W(\Phi) = c_{ijk}\Phi^i\Phi^j\Phi^k + \tilde{c} + \tilde{h}e^{-3S/2b_0}, \quad \tilde{h} = -\frac{2b_0}{3}\lambda\mu^3 e^{-1}, \qquad (25)$$

which is precisely the effective theory of obtained by Dine *et al.*[16] using arguments based on the nonanomalous $U(1)_R$ symmetry. As explained in Refs. 3,4, this truncation is exact at the classical level for the theory defined by (19), (20). In general one would have to include all tree diagrams with internal H-lines, but these vanish at the ground state of the theory so defined. (Note that this holds only if there is a stable minimum of the full theory.)

The effective theory defined by (25) has a positive semi-definite potential which vanishes at the minimum. If $\tilde{c} = 0$, the vacuum energy is minimized for $\tilde{h} = 0$ ($< H >= 0$) or $< s > \to \infty$ ($g = 0$), that is, condensation does not occur and supersymmetry remains unbroken. For $\tilde{c} \neq 0$ the effective theory has the following properties at the classical level[16] and at the one-loop[20] level: the cosmological constant vanishes, the gravitino mass $m_{\tilde{G}}$ can be non-vanishing, so that local supersymmetry is broken, in which case the vacuum is degenerate, and there is no manifestation of SUSY breaking in the observable sector. Nonrenormalization theorems for supergravity, together with the classical $SL(2,\mathcal{R}) \otimes U(1)_R$ symmetry, indicate[4] that these results will persist to all orders of the effective theory defined by (25).

Including loop correction from the H-sector, one finds[3] that masses are generated for the gauginos of the observable sector that are of order

$$m_{\tilde{g}} \sim \frac{m_{\tilde{G}} m_H^2 \Lambda_c^2}{(4\pi M_{Pl})^4} < 4 \times 10^{-15} M_{Pl} \sim 7TeV,$$

$$\text{for } m_{\tilde{G}} < m_H \sim \Lambda_c < 10^{-2} M_{Pl}, \tag{26}$$

where m_H is the mass of the H-supermultiplet. The factor $(4\pi)^{-4}$ appears in (26) because the effect arises first at two-loop order in the effective theory, the factor $m_{\tilde{G}}$ is the necessary signal of SUSY breaking, the factor m_H^2 is the signal of the anomalous breaking of $SU(2,\mathcal{R}) \otimes U(1)_R$, and Λ_c^2 is the effective cut-off. This last factor arises essentially for dimensional reasons: the couplings responsible for transmitting the knowledge of symmetry breaking to the observable sector are nonrenormalizable interactions with dimensionful coupling constants proportional to M_{Pl}^{-2}. Gauge nonsinglet scalar masses masses are protected[21] to one further loop order by the Heisenberg symmetry[22] of the Kähler potential (12b):

$$\delta\Phi^i = \alpha^i, \quad \delta T = \bar{\alpha}_i \Phi^i, \quad \delta K = 0. \tag{27}$$

Note that the ground state equations give

$$m_{\tilde{G}} = < e^{K/2} W > \approx \frac{\lambda \mu^3 \Lambda_c^3}{2eg^4 M_{Pl}^2}, \quad \mu\Lambda_c \sim \left(\frac{\tilde{c}e}{2\lambda}\right)^{\frac{1}{3}} \Lambda_{GUT}, \tag{28}$$

so it is not possible to generate a hierarchy of more than a few orders of magnitude between $m_{\tilde{G}}$ and Λ_{GUT} if $\tilde{c}$ is quantized as in (13). However this initial small hierarchy is enough to generate a viable gauge hierarchy if observable SUSY breaking is sufficiently suppressed, as in (26), relative to local SUSY breaking. For example, recent LEP data[1] suggest $\Lambda_{GUT} \sim 10^{16} GeV$, $g^{-2} \sim 2$, so for a hidden E_8 gauge group ($b_0 = .56$) we get $\Lambda_c \sim .6\Lambda_{GUT} \sim 3 \times 10^{-3} M_{Pl}$.

RESTORATION OF MODULAR INVARIANCE

In the formalism presented above, the continuous classical symmetry $SL(2,\mathcal{R})$ is broken by anomalies at the quantum level. However the discrete subgroup $SL(2,\mathcal{Z})$ [a,b,c,d integers in (16)] of $SL(2,\mathcal{R})$ is known[23] to be an exact symmetry to all orders in string perturbation theory. This so-called "modular invariance" is restored by adopting, instead of (19), the effective lagrangian[6]

$$\mathcal{L}_{pot}^{eff} = \int d^2\Theta \mathcal{E} e^{K/2} 2b_0 \lambda e^{-3S/2b_0} H^3 \ln(H\eta^2(T)/\mu) + h.c., \tag{29}$$

where

$$\eta(T) = e^{-\pi T/12} \prod_{m=1}^{\infty} (1 - e^{-2m\pi T}) \tag{30}$$

is the Dedekind η-function. This is the unique function of the chiral superfields that has the required analyticity and $SL(2,\mathcal{Z})$ transformation properties[6]. This additional contribution to the Yang-Mills wave function renormalization can be understood[24] as arising from finite threshold corrections[10,11] to the leading log approximation that arise from heavy string mode loops, and is closely related to the anomalous quantum correction due to the (nonrenormalizable) coupling of the Kähler connection to the axial $U(1)_R$ current[4,25,24]. The result (29) has been generalized to the cases of several gaugino condensates[18,12] and of several moduli.[12]

The effective scalar potential[5,6] for the theory defined by (29) is unbounded from below. Specifically, the potential takes the form

$$V = e^K \left[\sum_i |V_i|^2 + X(t,\bar{t}) \left|\frac{\partial W}{\partial t}\right|^2\right], \tag{31}$$

where the function $X(t,\bar{t})$ is negative for Re$t \leq 1.9$. Therefore the potential is unbounded in the direction $< e^K > \propto$ Re$s^{-1} \to \infty$ ($g \to \infty$). On the other hand, the term $|\partial W/\partial t|^2$ that drives the potential negative is proportional to b_0^2, *i.e.*, is of two loop order. Since the construction (29) is based on one-loop results, this term is unreliable, and any effective theory that coincides with the one defined by (29) in order b_0 is equally valid.

We therefore reinterpret the previous results as follows. We define the effective theory for gaugino condensation by the lagrangian[12]:

$$\mathcal{L}_{pot}^{eff} = \int d^2\Theta \mathcal{E} SU + h.c. = \int d^2\Theta \mathcal{E} S e^{K/2} \lambda e^{-3S/2b_0} H^3 + h.c., \tag{32a}$$

$$K = -\ln(S + \bar{S})$$
$$-3\ln(T + \bar{T} - |\Phi|^2 - |H|^2[\tfrac{5}{3} - \tfrac{1}{3} b_0 f(S,\bar{S}) \ln(H\eta^2(T)/\mu) + \text{h.c.}]). \tag{32b}$$

If we take $f(S,\bar{S}) = 2S^{-1}$, then we can simply interpret the "new" chiral superfield H of (32) as related by the "old" H of (29) by a wave function renormalization, *i.e.*,

$$H_{new} = H_{old} \left[1 + \frac{2b_0}{3S} \ln(e^{-S/2b_0} H\eta^2(T)/\mu)\right] + O(b_0^2). \tag{33}$$

In other words, the composite superfield $U_{new} = e^{K/2} \lambda e^{-3S/2b_0} H_{new}^3$ is related to the old one by a (field dependent) renormalization. Note that without the $O(b_0^2)$

corrections, (33) is just a holomorphic chiral field redefinition that cannot change the theory. The $O(b_0^2)$ terms in fact contain the nonholomorphic pieces implicit in the redefinition from (29) to (32). If instead we take $f(S,\bar{S}) = 4(S+\bar{S})^{-1}$, the form of the superpotential (32b) agrees with one-loop corrections[26] to the Kähler potential that would arise from the self-interactions of H via the tree superpotential defined by (32a). In either case the theory defined by (32) is identical to the one defined by (29) and (20) to first order in the loop expansion parameter b_0. Specifically, the lagrangian (32) has the correct conformal anomaly in order b_0, and the correct chiral anomaly to all orders, provided the (anomalous) transformation properties of the "renormalized" fields H_{new}, W^a_{new} are defined in terms of the "old" fields with canonical transformation properties, via the appropriate functional relation, such as (33).

At its classical level, the theory defined by (32) has once again a vanishing cosmological constant and (for $\tilde{c} \neq 0$) a degenerate vacuum with local SUSY breaking ($m_{\tilde{G}} \neq 0$) possible, and again no SUSY breaking appears in the observable sector at the classical level of this effective theory. The vanishing of the cosmological constant is assured because the derivatives of the generalized Kähler potential $\mathcal{G} = K + \ln|W^2|$ satisfy the "no-scale" condition[12]

$$\mathcal{G}_a \mathcal{G}_{\bar{b}} M^{a\bar{b}} = 3, \qquad (34)$$

where $\mathcal{G}_{a\bar{b}}$ is some submatrix of the Kähler metric $\mathcal{G}_{i\bar{j}}$ and $M^{a\bar{b}}$ is its inverse:

$$\mathcal{G}_{c\bar{b}} M^{a\bar{b}} = \delta^a_c. \qquad (35)$$

For example, for the 1×1 submatrix $\mathcal{G}_{t\bar{t}}$, (34) reduces to

$$\mathcal{G}_t \mathcal{G}_{\bar{t}} = 3\mathcal{G}_{t\bar{t}}, \qquad (36)$$

which is a differential equation that can be integrated to give

$$\mathcal{G}(T,\bar{T},Z,\bar{Z}) = -3\ln[f(T,Z,\bar{Z}) + f(\bar{T},\bar{Z},Z) + g(Z,\bar{Z})], \quad Z \neq T. \qquad (37)$$

For an $n\times n$ submatrix in (34), the vacuum is degenerate in n complex directions, since there are only $N-n$ complex vacuum conditions, where N is the total number of chiral supermultiplets.

PHENOMENOLOGY REDUX

Consider quite generally a superpotential of the form

$$W = W(\Phi^i) + \sum_a W_a(H_a, S) + \tilde{c}, \qquad (38)$$

where I have allowed for the possibility of a nonperturbative source of SUSY breaking, such as (13), which also breaks modular invariance. Take as Kähler potential

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T} - \sum_i B_i|\Phi^i|^2 - \sum_a B_a|H_a|^2), \qquad (39)$$

where B_i, B_a are modular invariant, field dependent wave function normalization factors as in (32b). Solving the ground state conditions gives $< V >= 0$, and gaugino condensation is possible for $\tilde{c} = 0$ if there is more than one condensate[27,18], provided[12] that the β-functions of the factor gauge gauge groups do not all have the same sign. (Alternatively, there could be a cancellation in the vacuum energy between gaugino condensates and the *vev* of a gauge nonsinglet scalar potential.[28]) However the ground state conditions for the various fields may be summed to give[12]

$$< W >=< \tilde{c}[1 + \sum_i \beta_i|\varphi^i|^2 + \sum_a \beta_a|h_a|^2] >, \qquad (40)$$

where β_i, β_a are related to the β-functions appearing in B_i, B_a, so the gravitino mass (28) vanishes if $\tilde{c} = 0$ and local SUSY breaking does not occur.

Local SUSY breaking is again possible for $\tilde{c} \neq 0$ and, as for the effective theory studied previously, the observable SUSY mass gap vanishes at the "classical" level of the effective theory. The analysis[29] at the one loop level of this effective theory is considerably more complicated, and one can expect some qualitative differences from the model studied previously. Although degeneracy in the T-axion directions is lifted at one loop, these directions remain nearly flat for large radii. Writing

$$\eta(T) = e^{-\pi T/12}\{1 + O[\delta(T)]\}, \quad < \delta(T) > \approx (2\times 10^{-3})^{<\mathrm{Re}t>} + O(\delta^2), \qquad (41)$$

the potential is flat in the T-axion direction in the limit of vanishing δ. In the same approximation CP-violation is absent (in contrast to the effective theory studied previously where it was set to zero by hand[20]). Since $SL(2,\mathcal{R})$ is explicitly broken by the threshold corrections, *i.e.*, by $\eta(T)$, one expects that observable SUSY breaking will be generated at the one-loop level of this effective theory. However, in the limit $\delta(T) \to 0$, there is a residual symmetry–namely the diagonal of $U(1)_R$ and the Peccei-Quinn $U(1)$ subgroup ($T \to T + i\gamma$) of $SL(2,\mathcal{R})$–that may help to suppress these effects. Note that LEP data[1] suggest $<\mathrm{Re}T>\sim 5 \times 10^3$ for some "average" compactification radius. Finally the presence of the η-function breaks the Heisenberg invariance (27) of the Kähler potential, which also served to protect gauge nonsinglet scalar masses. However the more realistic case of three moduli (and three matter generations) has a higher degree of classical degeneracy, which can also play a role in suppressing these masses at the one loop level if the minimum does not lie at the symmetric point where the radii $< t_\alpha >$ are all equal. Finally, since the calculation[11] of the threshold corrections neglects any possible φ^i-dependence, and since with the reinterpretation (32) there is no holomorphicity restriction, it is conceivable that the correct φ-dependence could restore an invariance similar to (27). (Recall that the radii of compactification, as determined by taking the 10-dimensional field theory limit of the untwisted sector are $R_\alpha =< t_\alpha - \frac{1}{2}|\varphi_\alpha|^2 >$.) In conclusion, it is not implausible that a viable hierarchy my emerge in this effective theory, but further investigation is needed.

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720