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Author
Clarke, John.

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LOW FREQUENCY MEASUREMENTS WITH JOSEPHSON DEVICES

John Clarke*
Department of Physics, University of California;
Inorganic Materials Research Division,
Lawrence Berkeley Laboratory,
Berkeley, California, 94720;
and Cavendish Laboratory, Cambridge, England.

This paper briefly surveys the application of Josephson devices to the measurement of low frequency magnetic fields and voltages. The various types of Josephson junctions are described. The sensitivities of DC and RF SQUIDS are compared, and the noise limits of these devices discussed. The inherent noise of a device with inductance 10^{-9}H is expected to be of order $10^{-6}\phi_o/\sqrt{Hz}$, whereas the observed noise limit is typically $10^{-3} - 10^{-4}\phi_o/\sqrt{Hz}$, implying that noise in the room temperature electronics and /or environmental noise are the limiting factors. The magnetic field sensitivity of SQUIDS may be improved by the use of a flux transformer to perhaps $10^{-14}T/\sqrt{Hz}$. Transformer coupling also greatly extends the range of SQUIDS used as voltmeters: in the He^4 temperature range, it is possible for the measurement to be Johnson noise limited in circuits whose resistance is $\lesssim 1\Omega$.

Introduction

It is now almost ten years since the publication of the article by B.D. Josephson[1] entitled "Possible New Effects in Superconductive Tunnelling", which was published on July 1, 1962. Among other effects, Josephson predicted that it should be possible to pass a supercurrent between two superconductors which are "weakly coupled", for example, by a tunnelling barrier. This tunnelling process is in fact an example of the macroscopic quantum nature of superconductivity; another example is flux quantization[2,3], the quantization of the magnetic flux threading a superconducting ring in units of the flux quantum $\phi_o = h/2e \simeq 2 \times 10^{-15}$Wb. Josephson's original paper stimulated intensive research, both experimental and theoretical, and the field of "Josephson Tunnelling" rapidly developed. The fundamental aspects of the subject have been thoroughly investigated and are by now well understood; most of the current emphasis is upon applications. The applications have been in three main areas: the measurement of low frequency magnetic fields and voltages; the detection of high frequency electromagnetic radiation; and the measurement of the fundamental constant ratio e/h and the maintaining of the standard volt. The last two topics are reviewed elsewhere in these proceedings by S. Shapiro and D.B. Sullivan; this paper is concerned with the first topic. We shall first briefly review the various types of Josephson junction that have been made and studied, and then describe and compare the two types of basic interferometer, the DC SQUID and the RF SQUID. The noise limits of these devices will be discussed. The practical application of the devices to magnetometry and voltage measurement is outlined; and the limitations of these devices discussed.

Types of junction

We shall understand the term "Josephson junction" to include any weakly-coupled pair of superconductors, no matter what the detailed transport mechanism of the junction may be. The only important property here is that the junction be able to carry a supercurrent up to a well defined maximum value known as the critical current, i_c, which is typically a few μ A to a few mA. For currents higher than i_c, a voltage exists across the junction.

The "classic" type of junction in which Josephson tunnelling was first observed by Anderson and Rowell,[4] is the evaporated thin film tunnel junction. A narrow film of superconductor is evaporated on to a substrate and oxidized by one of several techniques[5] to a depth of 10 - 20 Å. A second strip of superconductor evaporated across the first completes the junction. Early tunnel junctions of this kind were not very stable or reproducible, and other types of junction were subsequently developed and used in device work. However, in the last two years or so, a number of authors[6] have successfully prepared Nb-NbOx-Pb junctions, in which the niobium film is oxidized by a glow discharge. It appears that this type of junction can be produced with predictable characteristics, and that its thermal cycling and storage properties are excellent. The critical current of a Nb-NbOx-Pb junction is almost independent of temperature below 4.2 K, a considerable advantage in device applications. Furthermore, tunnel junctions seem to be less prone to damage from electrical transients than other types. The only drawback to the tunnel junction is its high capacitance (typically $10^{-10} - 10^{-9}$ F) which prevents its use in high frequency work. However, it may well be that the tunnel junctions will finally emerge as the most reliable type of Josephson junction.

The point contact junction[7] consisting of a sharpened point, usually of Nb, pressed against a block, also usually of Nb, has been very widely used in devices. Two variants have been used: in one the pressure between point and block is adjustable when the device is in the cryostat, while in the other, the point is adjusted before it is cooled down.[8-10] The point contact suffers to some extent from mechanical instability, and its characteristics can be modified considerably by electrical transients. The critical current usually increases appreciably as the temperature is lowered from 4.2 K to 1 K, a fact which is something of a disadvantage in device work. Nevertheless, the point can readily be readjusted and is easily made; this type of junction is probably the most popular at present.

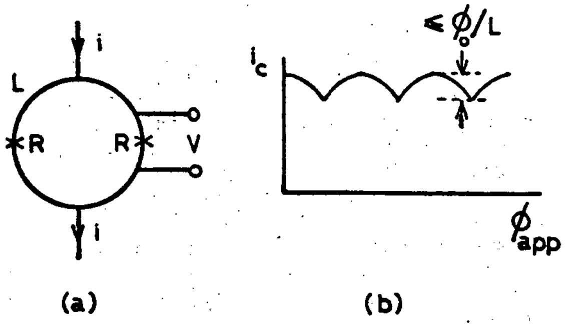
The Anderson-Dayem bridge[11] consists of a thin film containing a narrow constriction. The bridge is difficult to make with a sufficiently small critical current, and the critical current is strongly temperature dependent. Because of its very small dimensions, the constriction is rather susceptible to burn-out by electrical transients. However, bridges made by Considori et al.[12] from $NbSe_2$ show considerable promise, as they are relatively easy to fabricate and much more robust than evaporated bridges. A variant of the bridge has been used very successfully by Mercereau and his co-workers[13], who overlay the constrictions with a thin film of normal metal. The critical current of a bridge of given dimensions is thereby appreciably lowered, and is also much less temperature dependent.

Finally, a useful type of junction has been the SLUG, which consists of a bead of Pb-Sn solder (a superconductor) frozen around a piece of niobium wire[14]. This junction is reasonably robust and recyclable, and is certainly the easiest of all to fabricate. It also has the advantage of being self-screening against changes in external magnetic fields.

All of the above junctions have been successfully used in instrumentation by various groups. Long-life of any of these devices demands care in handling, avoidance of thermal shock, and protection from water vapour. However, the respective users of these devices have reported that with appropriate precautions a given device may be used for scores of experiments without significant deterioration in its characteristics or performance.

Double junction quantum interference devices (DC SQUIDS)

The early instruments utilized a superconducting ring containing two junctions, as shown in Fig.1(a). Jaklevic, Lambe, Silver, and Mercereau[15] showed that a magnetic field applied to the ring caused the critical current of the two junctions to oscillate, the period being the flux quantum ϕ_o (Fig. 1(b)). The device is often known as the DC SQUID (Superconducting Quantum Interference Device). A number of groups have successfully used this configuration as the basis of a magnetometer, almost always using point contact junctions (an exception being the SLUG). Highly instrumented versions have been produced.[16]



(a) (b)

Fig. 1 (a) Configuration of DC SQUID. The ring inductance is L, and each junction has a resistance R.

(b) Modulation of critical current by externally applied flux.

A typical DC SQUID has a ring area $\sim 10^{-6}$ m^2, an inductance $L \sim 10^{-9}$ H, a junction resistance $R \sim 10 - 20\ \Omega$ and a critical current $i_c \sim$ 5-50 μA. The critical current is measured continuously, by means of simple room-temperature electronics, and the change arising from a change in applied flux of perhaps $10^{-3}\ \phi_o$ in a 1 Hz bandwidth can be readily detected, corresponding to a magnetic field sensitivity of $\sim 10^{-12} T/\sqrt{Hz}$.

It is interesting to consider the maximum voltage output available from a DC SQUID. The modulation depth of the critical current δi_c (the decrease in i_c when the applied flux is changed from $n\phi_o$ to $(n + \frac{1}{2})\ \phi_o$) is less than or equal to ϕ_o/L, where L is the inductance of the ring[17]. The equality holds for identical junctions in the limit $Li_c/\phi_o \gg 1$. If the junctions are biased at a constant current ($> i_c$), the resultant change in voltage across them is just $R\delta i_c/2$, which has a maximum value

$$\delta V_{max} \simeq R\phi_o/2L\,. \qquad (1)$$

For $R \sim 10\Omega$, and $L \sim 10^{-9}$ H, we find $\delta V_{max} \sim 10^{-5}$V. In practice, the modulation depth may be somewhat less because the limit $Li_c/\phi_o \gg 1$ does not usually apply; $\delta V \sim 10^{-6}$V might be a more realistic figure. The smallest resolvable fraction of a flux quantum is therefore set by the smallest detectable change in δV, which appears to be of order $10^{-9} V/\sqrt{Hz}$ for state-of-the-art room temperature pre-amplifiers, matched to the junction resistance with a suitable transformer. This resolution corresponds to $10^{-3}\ \phi_o/\sqrt{Hz}$.

Single junction quantum interferometers (RF SQUIDS)

One of the greatest drawbacks of the point-contact DC SQUID was the problem of mechanical stability in a structure involving two contacts. At least partly as a consequence, work over the last five years has concentrated on the single junction interferometer[18] or RF SQUID, which consists of a single junction mounted on a superconducting ring. One approach has been to mount a point contact on a ring machined from a single piece of niobium, the whole structure being very rigid [8,10]. Alternative and equally successful devices have been made by evaporating a thin superconducting ring around a 1 - 2 mm diameter quartz rod, and making a single weak link in the ring[13, 19]. The critical current is measured by an rf technique shown schematically in Fig. 2. The RF SQUID is coupled to the coil (L_T) of a tank circuit, driven at its resonant frequency $\omega/2\pi$, typically 10 - 30 MHz. The voltage developed across the tank circuit is amplified and rectified. The amplitude of the output voltage is an oscillatory function of the flux applied to the RF SQUID, the period again being ϕ_o. Sensitivities approaching $10^{-4}\ \phi_o$ can be achieved in a 1 Hz bandwidth, corresponding to a magnetic field resolution of nearly $10^{-13}T/\sqrt{Hz}$.

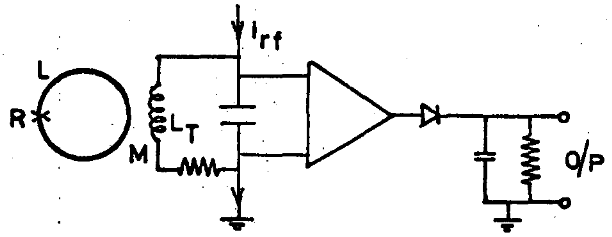


Fig. 2 Principle of operation of the RF SOUID.

It is interesting to compare the voltage output of the RF SQUID with that of the DC SQUID. It may be shown[8] that the voltage modulation across the tank circuit is

$$\delta V \sim \omega\phi_o L_T/2M , \qquad (2)$$

where $M^2 = K^2LL_T$. Eq. (2) is valid provided that the dissipation in the SOUID is greater than or comparable with the dissipation in the tank circuit: this criterion corresponds to $K^2Q \geq 1$, where Q is the quality factor of the tank circuit. Now the maximum angular frequency at which the SQUID may be operated is $\omega_{max} \sim R/L$, and the maximum value of δV is therefore

$$\delta V_{max} \simeq \frac{R\phi_o}{2L}\left(\frac{L_T}{M}\right) . \qquad (3)$$

The factor $R\phi_o/2L$ represents the voltage change across the junction, and is the same as for the DC SOUID. However, since R is typically 100Ω (somewhat higher than for a DC SQUID), and $L \sim 10^{-9}$ H, V_{max} is obtained at a frequency of 10^{11} Hz. Most RF SQUIDS are at present operated at frequencies three to four orders of magnitude lower than this maximum frequency, and the voltage modulation depth is correspondingly much lower. It is the transformer voltage gain, represented by the factor L_T/M, which boosts the signal up to a high level and produces the extremely high sensitivity. For the device of Zimmerman et al[8], the appropriate values $\omega/2\pi$ = 30 mHz, $L_T \sim 10^{-7}$ H, $M \sim 10^{-9}$ H inserted in Eq. (2) yield $\delta V \sim 2 \times 10^{-5}$ V. Hence a voltage resolution of $10^{-9}\ V/\sqrt{Hz}$ at the preamplifier corresponds to a flux resolution of $\sim 10^{-4}\phi_o/\sqrt{Hz}$. Operation of the device at higher frequencies should improve the sensitivity: for example, Zimmerman and Frederick[20] have operated an RF SOUID at 300 mHz, and observed the expected order of magnitude increase in signal to noise ratio.

Noise limitations

Despite their extreme sensitivity, the present quantum interferometers are not limited by inherent noise, but rather by amplifier or environmental noise. If we assume that a junction in a non-zero voltage regime generates Johnson noise corresponding to its resistance R, the noise current generated in an RF SOUID in a resistive state is given by

$$(\overline{i_N^2})^{\frac{1}{2}} = (4kTB/R)^{\frac{1}{2}}, \qquad (4)$$

where B is the system bandwidth, set by a time-constant in the external electronics. If we take $T \sim 4K$, $B \sim 1Hz$, $R \sim 100\Omega$, we find $(\overline{i_N^2})^{\frac{1}{2}} \sim 10^{-12}A$. In an inductance of $10^{-9}H$, this current noise corresponds to a flux noise $\sim 10^{-6}\phi_o$. The inherent noise is therefore two orders of magnitude below the observed noise level. In the same way, the inherent noise of DC SQUIDS is two or even three orders of magnitude below the measured noise. In both cases, the noise at the input of the room-temperature preamplifier limits the sensitivity. Attempts are being made to boost the output from the quantum devices. In the case of the RF SQUID, this end may be achieved by working at higher frequencies, as mentioned earlier. Clarke and Paterson[21] have used an asymmetric DC SQUID in which the i_c versus ϕ_{app} pattern is heavily skewed, the slope $\delta i_c/\delta\phi_{app}$ being increased by more than two orders of magnitude. It seems that sensitivities approaching $10^{-6}\phi_o$ may ultimately be achieved, provided that other forms of inherent junction noise, such as 1/f noise, do not become important.

In all applications, very careful screening of the device against external noise is essential. The superconducting circuitry is enclosed in a superconducting can, unless external fields are to be measured, and the cryostat surrounded by at least one μ-metal can. All leads entering the cryostat should be filtered to minimize rf pick-up. Many workers have also obtained better sensitivities by placing their apparatus in a shielded room; this proceedure does not seem to be essential however, provided the remaining shielding and filtering are adequate.

Application of SQUIDS to magnetometry

Either kind of SQUID may be used as a magnetometer. In many applications, the field is coupled to the SQUID by means of a superconducting flux transformer. The SQUID is usually used as a null detector in a negative feedback circuit which is also coupled magnetically to the SQUID (see Fig. 3). The current fed back, I, is proportional to the applied field H_o, and is often measured in a bandwidth B which is set by a time-constant outside the feedback circuit.

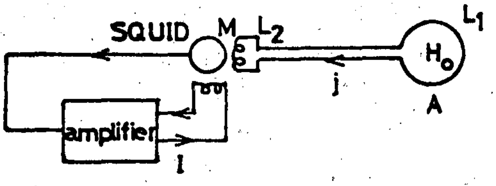


Fig. 3 Flux transformer-coupled magnetometer. The SQUID is used as a null-detector in a negative feedback circuit. H_o is the applied magnetic field.

Let the root-mean square flux noise of the SQUID be $\alpha\phi_o B^{\frac{1}{2}}$, where α is $10^{-3} - 10^{-4}$, and B the limiting bandwidth. This noise is equivalent to a current noise in the superconducting flux transformer $\alpha\phi_o B^{\frac{1}{2}}/M$, where M is the mutual inductance of the secondary (L_2) and the SQUID. The field H_o applied to the pick up loop (L_1) of area A generates a current in the transformer $j = H_o A/(L_1 + L_2)$, neglecting stray inductances. The field resolution δH_o of the device is found by equating $\delta H_o A/(L_1 + L_2)$ to the current noise:

$$\delta H_o = \left(\frac{L_1 + L_2}{AM}\right) \alpha\phi_o B^{\frac{1}{2}}. \tag{5}$$

If $L_1 \gg L_2$, $(L_1 + L_2)/A \simeq L_1/A$ is inversely proportional to the radius of the pick-up loop, and in principle, arbitrarily high field sensitivites may be obtained by making the pick-up loops sufficiently large. In practice, a resolution of $10^{-14} T/\sqrt{Hz}$ is achievable.

Magnetic susceptibilities may be determined by observing the change in flux in the pick-up loop when a sample is inserted into the pick-up loop in the presence of a magnetic field[19].

Use of SQUIDS as voltmeters

SQUIDS have been widely used as voltmeters. The voltmeter commonly consists of a SQUID coupled to a superconducting coil of N turns and inductance L_v, which is in series with a resistance R_s. (See Fig. 4). The unknown voltage, V_o, is applied as shown. The SQUID is used as a null detector, as in the magnetometer, and current I is fed back into the series resistor R_s, so that $V_o = IR_s$. The open-loop time-constant of the circuit is L_v/R_v, where

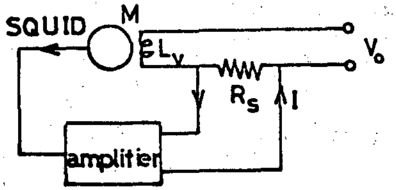


Fig. 4 SQUID used as voltmeter. The SQUID is used as a null detector in a negative feedback circuit.

$R_v = R_s$ + source resistance is the total circuit resistance, and we assume L_v dominates the stray inductance. This time-constant is reduced by a factor equal to the loop gain in the feedback mode. The bandwidth B with which I is measured is usually (but not necessarily) restricted by a time-constant outside the feedback circuit.

The noise limitations of this voltmeter can be estimated as follows[10, 22-24]. Suppose that the SQUID has a mean square flux noise $\alpha^2\phi_o^2 B$, where, as before, $\alpha \simeq 10^{-3} - 10^{-4}$ and B is the limiting bandwidth. The corresponding mean square current noise referred to the voltmeter circuit (Fig. 4) is $\alpha^2\phi_o^2 B/M^2$, where M is the mutual inductance of L_v and the SQUID. The Johnson current noise generated in the circuit is $4kTB/R_v$. If we assume that no environmental noise is picked up by the circuit, the voltage measurement will be limited by the Johnson noise developed in R_v provided that $4kTB/R_v \geqslant \alpha^2\phi_o^2 B/M^2$. This criterion may be written

$$\frac{T}{R_v} \geqslant \frac{\alpha^2\phi_o^2}{4kM^2} = \frac{\alpha^2\phi_o^2}{4kK^2 L_v L}, \tag{6}$$

where K is the coupling coefficient between L_v and the SQUID inductance L. When the inequality in Eq.(6) is satisfied,the voltmeter is "ideal"; in other words, a more sensitive voltage measurement for the given values of T and R_v is not possible. The voltmeter is ideal for a sufficiently high value of T and/or a sufficiently low value of R_v. It is clearly advantageous to make L_v as large as possible, provided that the circuit time-constant L_v/R_v does not exceed a few seconds; although negative feedback reduces the time-constant, a very large open loop time-constant can make the circuit difficult to set up. In addition, as L_v is made larger and larger, the coupling to the SQUID becomes poorer, and K decreases. If we set $\alpha \sim 10^{-4}$, $K \sim 0.5$, $L \sim 10^{-9}H$, and choose $L_v^{max} = \tau_{max} R_v$, with $\tau_{max} \sim 1$ sec, we find from Eq.(6) that the circuit is ideal provided $T \gg \alpha^2 \phi_o^2/(4kK^2 L\tau_{max}) \sim 3 \times 10^{-6}K$. This temperature thus represents the equivalent noise temperature of the voltmeter.

The largest value of R_v for which the voltmeter is still ideal is determined by practical considerations. In principle, L_v can be made very large, and the SQUID matched to correspondingly large values of R_v. Unfortunately, the coupling coefficient, K, falls off for higher L_v, so that the inductance ($\propto N^2$) increases faster than the square of the transformer current gain ($\propto K^2N^2$). Giffard et al.[10] quote an upper limit on R_v of about 1Ω at 4 K. It is important to work towards the matching of higher resistances; for example, high frequency Josephson junction detectors (see the article by S. Shapiro) usually have resistances of a few tens of ohms, and their performance might be appreciably improved if their characteristics were determined by a properly matched SQUID circuit.

The lowest value of R_v that may be used is limited by the lowest achievable stray inductance, perhaps 10^{-8}H, to about $10^{-8} - 10^{-9}\Omega$. The Johnson noise in a resistance of $10^{-8}\Omega$ at 10 mK, about $10^{-16}V/\sqrt{Hz}$, is eaily observable.[10]

Summary

Both double and single junction quantum interference devices have been developed into reliable instruments for measurements at low temperatures. The devices are usually used in a feedback mode. The single junction device is at present the more popular, and the best versions have a flux noise of $10^{-4}\phi_o/\sqrt{Hz}$. In the future, improvements on this level of one to two orders of magnitude seem likely. In magnetometry, the field sensitivity of the SQUID approaches $10^{-13}T/\sqrt{Hz}$, a figure that may be improved by at least an order of magnitude by means of a flux transformer. A transformer-coupled voltmeter has an equivalent noise temperature of a few μK. The voltmeter is ideal in the sense that its resolution is limited by Johnson noise in the circuit resistance, provided this resistance is less than about 1Ω at 4 K.

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