Title
The Relational SNARC: Spatial Representation of Nonsymbolic Ratios?

Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 38(0)

Authors
Matthews, Percival
Meng, Rui
Toomarian, Elizabeth Y.
et al.

Publication Date
2016

Peer reviewed

The Relational SNARC: Spatial Representation of Nonsymbolic Ratios?

Percival G. Matthews (pmatthews@wisc.edu)
Department of Educational Psychology, 1025 W. Johnson Street
Madison, WI 53706 USA

Rui Meng (rmeng5@wisc.edu)
Department of Educational Psychology, 1025 W. Johnson Street
Madison, WI 53706 USA

Elizabeth Y. Toomarian (toomarian@wisc.edu)
Department of Educational Psychology, 1025 W. Johnson Street
Madison, WI 53706 USA

Edward M. Hubbard (emhubbard@wisc.edu)
Department of Educational Psychology, 1025 W. Johnson Street
Madison, WI 53706 USA

Abstract

Recent research has highlighted the operation of a ratio processing system that represents the analog magnitudes of nonsymbolic ratios. This study investigated whether such representations would demonstrate spatial associations parallel to the SNARC (spatial numeric association of response codes) effect previously demonstrated with whole number magnitudes. Participants judged whether nonsymbolic ratio test stimuli were larger or smaller than reference stimuli using response keys located alternately either on the left or on the right side of space. Larger ratio magnitudes were associated with the right side of space and smaller magnitudes with the left. These results demonstrate that nonsymbolic ratio magnitudes – defined *relationally* by pairs of components – are characterized by a left-to-right spatial mapping. The current focus on ratio magnitudes expands our understanding of the basic human perceptual apparatus and how it might provide tools that grant intuitive access to more advanced numerical concepts beyond whole numbers.

Keywords: fractions; nonsymbolic ratios; ratio processing system; SNARC effect; mental number line; magnitude representation

Introduction

Recent years have brought increased attention to human abilities to process the magnitudes of numerical fractions (Bailey, Hoard, Nugent, & Geary, 2012; Kallai & Tzelgov, 2009; Siegler et al., 2012). Although understanding this ability has practical importance and may inform the design principles for promoting school learning, there are compelling reasons to investigate the basic science behind processing fraction magnitudes as well. In particular, learning more about the representation of fraction magnitudes has implications for our understanding of how humans gain access to numerical concepts. Moreover, this line of research may provide insight into the ways humans perceive magnitudes more generally.

Much work on the psychophysics of numerical perception focuses on the ways we process whole numbers and their nonsymbolic analogs, such as the number of dots in an array or the magnitudes of whole number symbols (e.g., Kaufman, Lord, Reese, & Volkmann, 1949; Moyer & Landauer, 1967; Stevens, 1957). Indeed, seminal works have demonstrated deep parallels between the ways humans perceive the magnitudes of whole number values and their analogs in nonsymbolic numerosities. Perhaps most notable among these findings was Moyer and Landauer's (1967) discovery that, when participants were asked to compare numerals, they first converted numerals to analog magnitudes and compared them "in much the same way that comparisons are made between physical stimuli such as loudness or length of line." (p. 1520). Many have taken these parallels to suggest that the ability to process nonsymbolic numerosities may be a key foundational ability that helps ground whole number concepts (e.g., Feigenson, Dehaene, & Spelke, 2004; Nieder & Dehaene, 2009; Piazza, 2010).

Several recent studies began to suggest that humans are also capable of processing the nonsymbolic analogs of fractions values (Chesney & Matthews, 2013; Jacob, Vallentin, & Nieder, 2012; Matthews, Lewis, & Hubbard, 2015). These studies extend the psychophysics of perception to the study of nonsymbolically instantiated ratio magnitudes. What is interesting about these magnitudes is that they do not correspond to individual stimuli like a single dot array or a single line segment; instead, they are compound stimuli whose magnitudes are determined relationally. For instance, in Figure 1, panel (a) represents the same ratio as panel (b), despite the fact that each of its components is larger than those of ratio (b). Similarly, it is a smaller ratio than that in panel (c), despite the fact that its components are larger when considered individually. Focusing on the perception of nonsymbolic ratio magnitudes stands to expand our understanding of our basic perceptual apparatus and how it might provide tools that

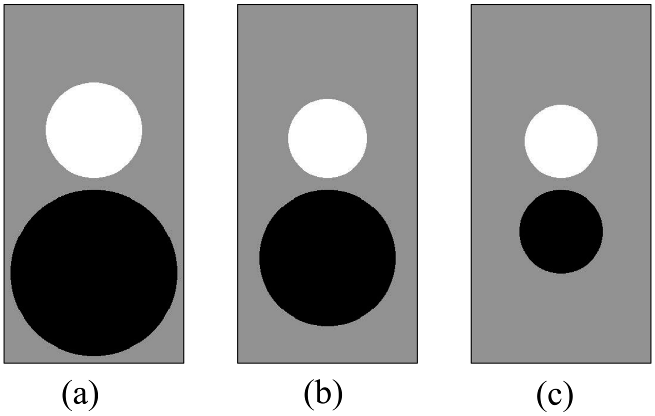

Figure 1. Sample circle stimuli; panels (a) and (b) are two possible instantiations of the magnitude 1/3; panel (c) represents the magnitude 7/9.

grant intuitive access to more advanced numerical concepts beyond whole numbers.

To date, studies have suggested that the nonsymbolic ratio processing is operative among nonhuman primates (Jacob et al., 2012), human infants (McCrink & Wynn, 2007), elementary school age children (Boyer, Levine, & Huttenlocher, 2008), typically developing adults (Chesney & Matthews, 2013), and individuals with limited number vocabularies and formal arithmetic skills (McCrink, Spelke, Dehaene, & Pica, 2013). Moreover, the ability correlates with symbolic math abilities in ways that are distinct from the approximate number system or ANS (Matthews et al., 2015, see also Hansen et al., 2015). Despite the above evidence demonstrating that humans can process nonsymbolic ratio magnitudes, much remains unknown about the nature these abilities.

In order to better understand whether non-symbolic ratios – these visuospatial fraction analogs – are processed similarly to whole number magnitudes, we tested whether they elicited spatial mappings. Many have argued for the association of number and space (e.g., Hubbard, Piazza, Pinel, & Dehaene, 2005; Newcombe, Levine, & Mix, 2015), with research on the SNARC (spatial numeric association of response codes) effect as the most well-known example (Dehaene, Bossini, & Giraux, 1993). The SNARC effect – an association between left responses and small numerical quantities and between right responses and large numerical quantities – is considered to be an indication that numbers are represented on a mental number line, spatially oriented from left to right with increasing magnitude (Dehaene et al., 1993). Indeed, in an oft-cited piece, Walsh (2003) predicted that the SNARC effect is only a special case of a SQUARC (spatial quantity of response codes) whereby all quantities represented in analog form are mapped to space.

Whether the analog perception of nonsymbolic ratios is cast in terms of number specifically or in terms of magnitude more generally, we hypothesize that such magnitudes should be associated with space, resulting in a SNARC or SQUARC effect. For convenience sake, we will simply refer to any such findings with the current stimuli as a SNARC effect. With the present study, we sought to elicit the SNARC using novel ratio stimuli.

Method

Participants

37 undergraduate students at a major Midwestern university participated for course credit (13 males; 30 right handed; ages 18 – 22).

Materials and Design

All stimuli were presented on 22” monitors using the MATLAB Psychophysics Toolbox Version 3 (PTB-3) (Brainard, 1997; Pelli, 1997; Kleiner et al., 2007). We constructed three different types of nonsymbolic ratio comparison tasks: nonsymbolic circle ratios, nonsymbolic line ratios, and nonsymbolic dot ratios (see Figure 2 for sample ratio formats). In each, participants were asked to decide whether a test ratio was larger or smaller than a reference ratio. For some symbolic SNARC tasks (e.g., Dehaene, Dupoux & Mehler, 1990; Gevers, Verguts, Reynvoet, Caessens & Fias, 2006; Santens & Gevers, 2008), stimuli are compared to a memorized standard located halfway between the extremes of the stimulus set. However, there is no nonsymbolic standard that could be referenced in memory without a label to serve as a perfect analog for this task. Thus, we used a nonsymbolic standard presented before each test stimulus in place of a memorized standard in the current paradigm. The set of test stimuli were used to instantiate magnitudes corresponding to the 26 irreducible, single-digit proper fraction values except ½, which was at the midpoint of the stimulus range and was reserved as the reference value for comparison (drawn from the symbolic fractions list used in Toomarian and Hubbard, under review). Because the stimuli were nonsymbolic, the “irreducible” and “single-digit” qualifiers do not strictly apply in the current case. The sizes of the ratio components used to instantiate those values varied from trial to trial, and participants were asked to focus on the overall ratio each time. The reference ratio was fixed to ½, and the sizes of the components used to instantiate it also varied from trial to trial. We randomly varied components as a control to reduce reliance on the magnitudes of individual components and possible pattern-recognition strategies. These tasks assessed the ability to discriminate between nonsymbolic ratio values composed of circle areas, line segments, or dot arrays.

Circle Ratios Each stimulus was composed of a pair of circles, with one serving as a figurative ‘numerator’ and another as the ‘denominator’. Numerator circles were white and denominator circles were black, both presented on a gray background, similar to the stimuli used by Matthews et al. (2015). Ratio size was defined as the area of the white circle up top to the area of the black circle on bottom. The summed areas of the numerator circle and the denominator circle were constrained such that it occupied between 20-40% of an invisible rectangle of 600 pixels high and 300 pixels wide.

Line Ratios Each stimulus was composed of a pair of lines. Shorter white lines served as numerators, and longer black lines served as denominators for all stimuli. They were both presented on a gray background. The distance between two lines was 50 pixels, and the width of both lines was 20 pixels. The summed lengths of the numerator and denominator lines were constrained such that they occupied 30 – 60% of the height of a 600 x 300 pixel frame. Additionally, the vertical position of the shorter line varied randomly.

Dot Ratios Each stimulus was composed of a pair of dot arrays arranged to form a nonsymbolic ratio. The numerators of the nonsymbolic numerosity arrays were composed of black dots on a white background, and the denominators were composed of white dots on a black background. Both were presented in a 600 x 300 pixel frame on a gray background. Individual dot size varied between 8 and 12 pixels in diameter. To discourage participants from counting or using computational procedures to estimate the ratios, the summed numerosity of numerator dots and denominator dots fluctuated from 150 to 200. The smallest numerosity displayed in any given array was 15. Dots were randomly and evenly distributed in each array.

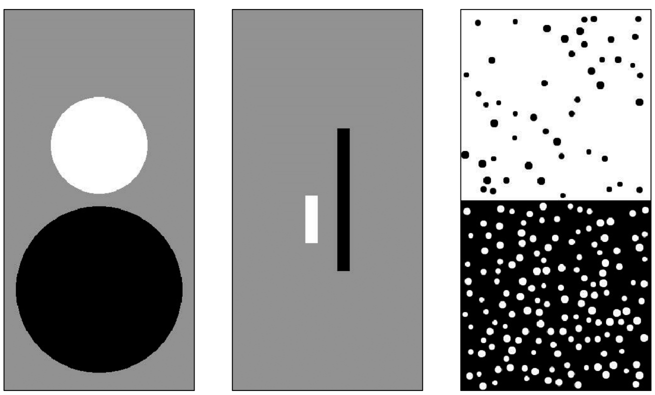

Figure 2: Sample circle, line, and dot ratio stimuli; all represent fraction magnitude of 1/3.

Procedure

Participants were presented with three blocks of comparison trials. Each block was composed wholly of trials of a particular stimulus type. Presentation order of circle and line blocks were counterbalanced across participants, and dot block was always presented last. Participants first saw instructions, then received eight practice trials, and then performed the formal experimental trials. Accuracy feedback was provided for practice trials but not for test trials. Each participant completed all tasks in one hour-long session.

Participants were instructed to press the space bar to initiate each trial. Each trial began with a fixation cross presented in the center of the screen for 500 ms, immediately followed by an 800 ms presentation of the reference ratio stimulus (various instantiations of ½). The reference stimulus was followed by another fixation cross for 500 ms, followed by the test ratio stimulus which remained on screen until participants submitted a response or timed out at 3000 ms (see Figure 3).

After viewing both the reference and the test ratios, participants were asked to decide if the second ratio was larger or smaller than the first, which was always equal to ½. Participants made decisions via button press on a standard keyboard (19 mm horizontal center-to-center distance), with either the left ("D" key) or right ("K" key) buttons indicating that the second ratio was larger. Response side was counterbalanced both within blocks of corresponding stimuli and across participants, meaning that half of the participants pressed "K" for larger with right hand and "D" for smaller with left hand first, while the other half participants pressed "D" for larger with left hand and "K" for smaller with right hand first. Each ratio value appeared eight times for circle and line stimuli, but appeared only four times for dot stimuli. There were 208 total trials for each circle and line stimuli type, and 104 trials for dot stimuli type, resulting a total of 520 trials.

Note that our primary interest was in finding the SNARC for ratios instantiated by continuous quantities (circles and lines) because – unlike dot arrays – individual circles and lines do not map to any particular number. We therefore felt that ratios of continuous nonsymbolic quantities provided a more stringent test for ratio processing than dots. Given time constraints for each experimental session, we only conducted a relatively impoverished test of the dot comparison condition: dot comparisons always came last, and stimuli were only seen half as often as the line and circle stimuli.

For line stimuli, participants were told to estimate "the ratios between line lengths." For circle stimuli, participants were specifically told to estimate "the ratios between circle *areas*, or how much room each circle takes up on the screen." They were also told stimuli would flash too briefly to measure or to use calculations, so they should "just try to feel out the ratio instead of applying a formula." These instructions parallel protocols we have used elsewhere (Chesney & Matthews, 2013; Matthews et al., 2015), and participants quickly understood the tasks.

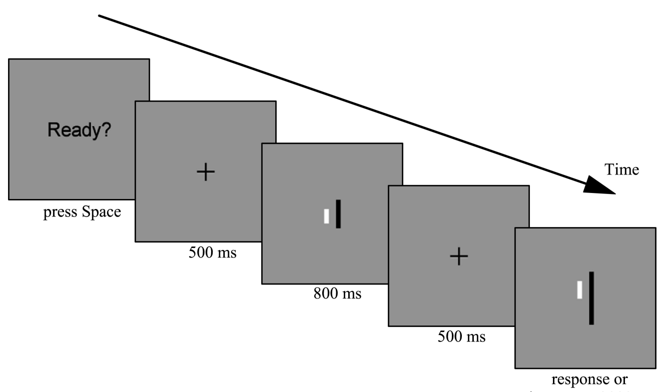

Figure 3. A sample line ratio comparison trial.

Results

Prior to analysis, we first excluded trials with reaction times below 250 ms or that were more than 3 standard deviations faster or slower than a participant's mean reaction time for that task. Nine participants were excluded from the analyses because of very low accuracy (below 60%) in the experimental task. For remaining participants (n=28), average accuracy across three tasks was: 87.31% (circle ratios), 88.70% (line ratios), and 85.58% (dot ratios). Only RTs for accurate responses were used for SNARC analyses,

To test for the presence of the SNARC effect, mean reaction times for correct responses made with the left hand were subtracted from right hand mean reaction times for each fraction magnitude across all participants. This measure, dRT, is positive when a participant responds faster with the left hand for a particular stimulus and negative if the participant responds faster with the right hand. A classical SNARC effect is noted when dRTs are more positive for small numbers and more negative for large numbers, yielding a negative slope when dRT is regressed against stimulus magnitude. However, it is often the case that dRTs for small numbers are not actually positively valued (e.g., Dehaene et al., 1993; Fias, Brysbaert, Geypens, & d'Ydewalle, 1996) because intercept is confounded with handedness effects (i.e., right hand responses are typically faster in right handed participants). Thus, slope is generally taken as the key measure of the SNARC effect.

We ran an omnibus regression on mean dRTs for against ratio magnitudes, including stimulus type and stimulus type X magnitude interactions in the model (line stimuli served as the baseline, with dummy codes for circle and dot stimuli). There was a significant regression slope based on magnitude b = -66.57, p = .03. However, there was no significant main effect for stimulus type, nor was there a significant type X magnitude interaction. This indicates that the SNARC effect for all three types of stimuli were statistically indistinguishable, suggesting that the effects on dRT really did correspond to the ratio magnitudes presented independently of format.

For ease of presentation, we depict the results of separate regressions for each format in Figures 4, 5, & 6. The slopes for each was significant ($b_{circles}$ = -59.84, *p* = .02; b_{lines} = -66.57, *p* = 0.04; b_{dots} = -68.33, *p* = 0.04). These figures illustrate that all three types of nonsymbolic ratio stimuli elicited robust spatial associations – classical SNARC effects.

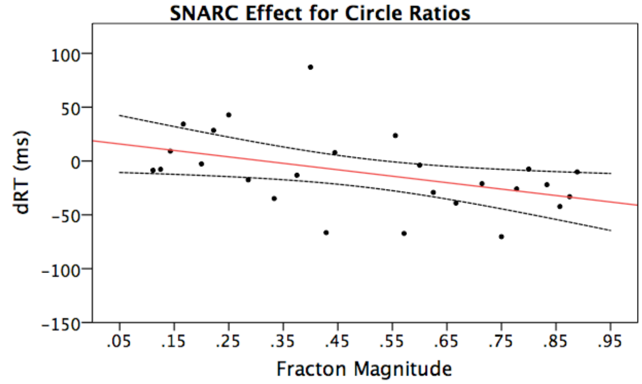

Figure 4: dRT as a function of fraction magnitudes for circle ratio stimuli.

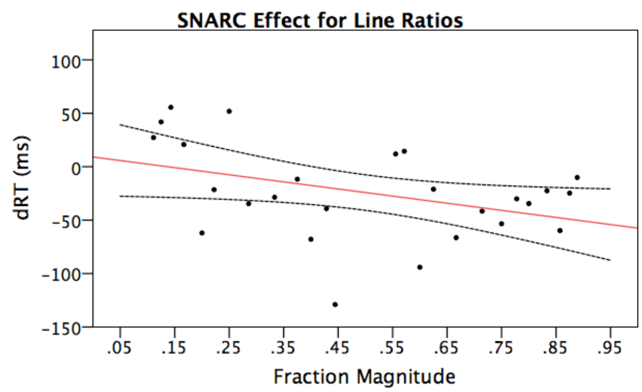

Figure 5: dRT as a function of fraction magnitudes for line ratio stimuli.

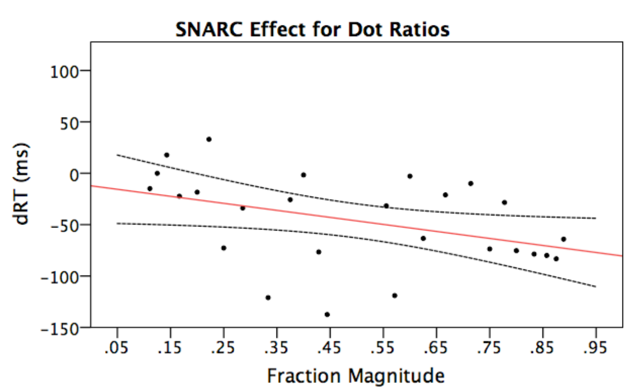

Figure 6: dRT as a function of fraction magnitudes for dot ratio stimuli.

Discussion

The current experiments demonstrated the existence of spatial associations with magnitudes instantiated using three different types of nonsymbolic ratios (i.e., dot arrays, line pairs, and circle pairs). These effects paralleled the classical SNARC, but differed in two important ways: First, the stimuli were nonsymbolic, instead of symbolic. Second, the stimuli were ratios, so the magnitudes were defined relationally as opposed to corresponding to the absolute magnitude of any individual component. Several aspects of the current experiment make important contributions to our understanding of human magnitude processing.

First, the current study provides further evidence for the existence of a ratio processing system and extends our understanding of its operation. Participants performed accurately on a perceptually based task comparing ratio magnitudes, replicating recent work demonstrating human sensitivity to nonsymbolic ratio magnitudes (e.g., Jacob et al., 2012; Matthews & Chesney, 2015; Matthews et al., 2015; McCrink et al., 2013; Meert, Grégoire, Seron, & Noël, 2012).

This in itself represents a contribution to the basic science of human magnitude representation. The nonsymbolic aspects of the current experiment stand alongside other research demonstrating that the basic operation of the SNARC effect applies to nonsymbolic magnitudes. For instance, Holmes & Lourenco (2011) found similar spatial associations using the valence of facial expressions of emotion, and Rusconi, Kwan, Giordano, Umiltà, and Butterworth (2006) demonstrated spatial associations with musical pitch. All together these works seem consistent with speculation that the SNARC effect may be a more general association between quantities and space – a SQUARC.

Here we underscore the fact that the use of ratio magnitudes in this study represents a major departure from previous studies involving automatic associations of space and magnitude (be they symbolic or nonsymbolic). Previous SNARC/SQUARC studies have involved quantities with stand-alone magnitudes: the size of a number (Dehaene et al., 1993), the angriness of an expression (Holmes & Lourenco, 2011), or the "height" of a musical pitch (Rusconi et al., 2006). In the present case, the magnitudes involved emerged from the relations between two components, each of which had its own magnitude. That is, even though each circle in a ratio stimulus had its own magnitude, the effects found here reflected a magnitude *determined by the relation between the two* – one that can be expressed by multiple components of different sizes so long as the pairs maintain the same ratio value. The current findings suggest that this emergent relational magnitude is automatically mapped to space as well.

These results may also have implications for how we think about the human number sense from an applied perspective. Unlike research involving faces and musical pitch, the ratio stimuli used in the current experiments are fraction analogs. Each is a concrete instantiation of a fraction value, and prior work has shown that human adults can make the mapping between these nonsymbolic stimuli and symbolic numbers (Matthews & Chesney, 2015). Moreover, Matthews et al. (2015) have shown that college student's ability to compare nonsymbolic ratios predicts symbolic fraction and algebra skills. Thus, Lewis et al. (2015) have essentially argued that nonsymbolic ratios are protonumerical and that ratio processing abilities may be an early-emerging tool that provides intuitive access to fractions concepts. The existence of a SNARC for these fraction analogs adds to the potential of this speculative account.

In summary, the current study contributes another advancement in a recent string of studies shedding light on the impressive extent of human ratio processing abilities (Boyer & Levine, 2012; Jacob et al., 2012; Matthews & Chesney, 2015; Matthews et al., 2015; McCrink & Wynn, 2007; Möhring, Newcombe, Levine, & Frick, 2015). Each step foregrounding the perception of ratio magnitudes expands our understanding of the basic human perceptual apparatus. This in turn informs theory about the foundational inputs upon which concepts can be built.

Acknowledgments

This research was supported in part by NSF Grant DRL-1420211 to Percival Matthews and Edward Hubbard.

References

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, *113*(3), 447–455. http://doi.org/10.1016/j.jecp.2012.06.004

Boyer, T. W., & Levine, S. C. (2012). Child proportional scaling: Is 1/3 = 2/6 = 3/9 = 4/12? *Journal of Experimental Child Psychology*, *111*(3), 516–533. http://doi.org/10.1016/j.jecp.2011.11.001

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, *44*(5), 1478–1490. http://doi.org/10.1037/a0013110

Chesney, D. L., & Matthews, P. G. (2013). Knowledge on the line: Manipulating beliefs about the magnitudes of symbolic numbers affects the linearity of line estimation tasks. *Psychonomic Bulletin & Review*, *20*(6), 1146–1153. http://doi.org/10.3758/s13423-013-0446-8

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, *122*(3), 371–396. http://doi.org/10.1037/0096-3445.122.3.371

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(3), 626.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314. http://doi.org/10.1016/j.tics.2004.05.002

Fias, W., Brysbaert, M., Geypens, F., & d'Ydewalle, G. (1996). The importance of magnitude information in numerical processing: Evidence from the SNARC effect. *Mathematical cognition*, *2*(1), 95-110.Hansen, N., Jordan, N. C., Siegler, R. S., Fernandez, E., Gersten, R., Fuchs, L., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development, 35*, 34-49. doi:10.1016/j.cogdev.2015.02.011

Gevers, W., Verguts, T., Reynvoet, B., Caessens, B., & Fias, W. (2006). Numbers and space: a computational model of the SNARC effect. *Journal of Experimental Psychology: Human Perception and Performance*, *32*(1), 32.

Holmes, K. J., & Lourenco, S. F. (2011). Common spatial organization of number and emotional expression: A mental magnitude line. *Brain and Cognition*, *77*(2), 315–323. http://doi.org/10.1016/j.bandc.2011.07.002

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, *6*(6), 435–448. http://doi.org/10.1038/nrn1684

Jacob, S. N., Vallentin, D., & Nieder, A. (2012). Relating magnitudes: the brain's code for proportions. *Trends in Cognitive Sciences*, *16*(3), 157–166. http://doi.org/10.1016/j.tics.2012.02.002

Kallai, A. Y., & Tzelgov, J. (2009). A generalized fraction: An entity smaller than one on the mental number line. *Journal of Experimental Psychology: Human Perception and Performance*, *35*(6), 1845–1864. http://doi.org/10.1037/a0016892

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology*, *62*(4), 498–525.

Lewis, M.R., Matthews, P. G., & Hubbard, E. M. (2015). The non-symbolic foundations of fraction understanding. In D. B. Berch, D. C. Geary, & K. Mann Koepke (Eds.), *Development of mathematical cognition: Neural substrates and genetic influences*. San Diego, CA: Academic Press.

Matthews, P. G., & Chesney, D. L. (2015). Fractions as percepts? Exploring cross-format distance effects for fractional magnitudes. *Cognitive Psychology*, *78*, 28–56. http://doi.org/10.1016/j.cogpsych.2015.01.006

Matthews, P. G., Lewis, M. R., & Hubbard, E. M. (2015). Individual Differences in Nonsymbolic Ratio Processing Predict Symbolic Math Performance. *Psychological Science*, 0956797615617799. http://doi.org/10.1177/0956797615617799

McCrink, K., Spelke, E. S., Dehaene, S., & Pica, P. (2013). Non-symbolic halving in an Amazonian indigene group. *Developmental Science*, *16*(3), 451–462. http://doi.org/10.1111/desc.12037

McCrink, K., & Wynn, K. (2007). Ratio Abstraction by 6-Month-Old Infants. *Psychological Science*, *18*(8), 740–745. http://doi.org/10.1111/j.1467-9280.2007.01969.x

Meert, G., Grégoire, J., Seron, X., & Noël, M.-P. (2012). The mental representation of the magnitude of symbolic and nonsymbolic ratios in adults. *The Quarterly Journal of Experimental Psychology*, *65*(4), 702–724. http://doi.org/10.1080/17470218.2011.632485

Möhring, W., Newcombe, N. S., Levine, S. C., & Frick, A. (2015). Spatial Proportional Reasoning is Associated with Formal Knowledge about Fractions. *Journal of Cognition and Development*. http://doi.org/10.1080/15248372.2014.996289

Moyer, R. S., & Landauer, T. K. (1967). Time required for Judgements of Numerical Inequality. *, Published Online: 30 September 1967; | doi:10.1038/2151519a0*, *215*(5109), 1519–1520. http://doi.org/10.1038/2151519a0

Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: the intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, *6*(6), 491–505. http://doi.org/10.1002/wcs.1369

Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annual Review of Neuroscience*, *32*, 185–208. http://doi.org/10.1146/annurev.neuro.051508.135550

Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, *14*(12), 542–551. http://doi.org/10.1016/j.tics.2010.09.008

Rusconi, E., Kwan, B., Giordano, B. L., Umiltà, C., & Butterworth, B. (2006). Spatial representation of pitch height: the SMARC effect. *Cognition*, *99*(2), 113–129. http://doi.org/10.1016/j.cognition.2005.01.004

Santens, S., & Gevers, W. (2008). The SNARC effect does not imply a mental number line. *Cognition*, *108*(1), 263-270.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early Predictors of High School Mathematics Achievement. *Psychological Science*, *23*(7), 691–697. http://doi.org/10.1177/0956797612440101

Stevens, S. S. (1957). On the psychophysical law. *Psychological Review*, *64*(3), 153–181. http://doi.org/10.1037/h0046162

Toomarian, E.Y. & Hubbard, E.M. Stimulus Characteristics and Strategic Variability Modulate Spatial Representations of Fractions. Manuscript submitted for publication.

Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, *7*(11), 483–488. http://doi.org/10.1016/j.tics.2003.09.002