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On constitutive relations for a rod-based model of a pneu-net bending actuator

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1	On Constitutive Relations for a Rod-Based Model of a Pneu-Net		
2	Bending Actuator		
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#### 7 Abstract

The recent surge of interest in soft robotics has led to interesting designs and fabrication of flexible actuators composed of soft matter. Modeling these actuators to obtain quantitative estimates of their dynamics is challenging. In the present paper, a rod-based model for a popular pneumatically activated soft robot arm is developed. The model is based on Euler's theory of the elastica and is arguably the simplest possible model. Through a synthesis of experiment and theory, we find that the constitutive relations needed to accurately capture the deformation of the arm differ considerably from the simple classical relation that the bending moment is linearly proportional to a change in curvature. The present paper also provides a framework to evaluate whether future soft robot actuator designs can be captured using simple models.

<sup>8</sup> Keywords: Soft robots, Euler's elastica, Rod theories, Pneumatic actuation, Pneu-Net actuator

#### 9 1. Introduction

The design of pneumatically actuated flexible arms have been championed by several research 10 groups for the past two decades. The most notable proponents are Koichi Suzumori and his col-11 leagues at Okayama University [1, 2, 3, 4] and, more recently, George Whitesides and his colleagues 12 at Harvard University [5, 6]. The latter group merged pneumatic artificial muscle technologies with 13 emerging paradigms in soft lithography and microfluidics to produce new classes of soft biologically-14 inspired robots. Of particular relevance to the present paper is the so-called pneu-net architecture 15 in which soft silicone elastomer is embedded with an array of connected air pockets that can cause 16 each limb to bend when inflated. Modeling these flexible devices is challenging and, apart from a 17 handful of works including [7, 8, 9], is dominated by finite element models that capture the coupling 18 between the state of pressure in the air chambers of the arm and the resulting overall deformation. 19

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<sup>20</sup> While the results produced by finite element models are interesting and compelling, they are dif-

<sup>21</sup> ficult to use to generate tractable dynamic models for the arms. Developing models of the latter

<sup>22</sup> type are desirable for the development of control algorithms and improved understanding of the design parameters for soft robots.

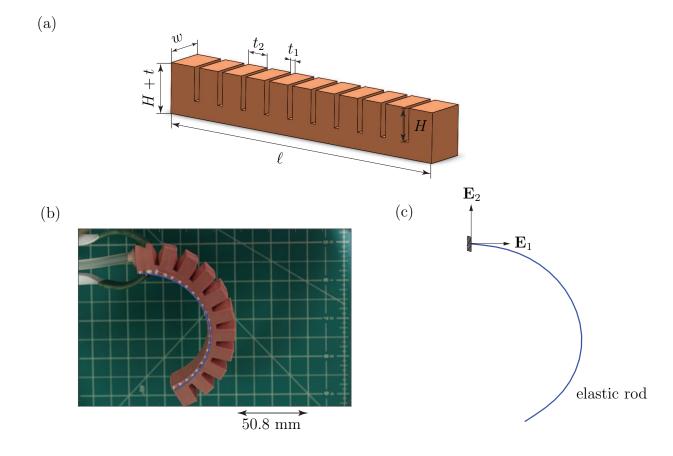


Figure 1: The pneumatically actuated soft robot limb. (a) Schematic of the actuator with the labeling of its dimensions; (b) the actuator which is clamped at one end and free at the other subject to an air pressure of 31 kPa; (c) the elastica model for the deformed arm. The dimensions of the arm featured in (a) and (b) and throughout this paper are w = 15 mm, H = 12 mm, t = 3 mm,  $t_1 = 2 \text{ mm}$ ,  $t_2 = 8 \text{ mm}$ , and  $\ell = 112 \text{ mm}$ .

23

The present paper seeks to examine the efficacy of using a simple rod-based model to predict 24 the dynamics of a pneumatically actuated flexible arm shown in Figure 1. The design of the 25 actuator can be found on the popular online resource [10] and the arm also features in several 26 recent articles [5, 6]. We seek to develop a rod-based model for this actuator. The development 27 has two experimental stages. In the first series of experiments, one end of the arm is clamped and 28 the curvature of the rod as a function of pressure is measured. This data is then used to determine 29 the constitutive relations for a rod-based model of the arm which is terminally loaded at the free 30 end. The complexity of the resulting constitutive relations is surprising (see Eqn. (10) below). The 31 series of tests that we perform to determine the constitutive relations are simple and can be used 32 to examine future designs of soft robot arms with a goal of producing designs that are easier to 33 model using a rod theory. Our work is closely related to the modeling work of Majidi et al. [7] 34

 $_{35}$  however our model and the particular soft robot arm design considered are different and, partially

 $_{36}$  as a result, we find constitutive equations that are dramatically different from those presented by

37 these authors.

#### 38 2. Methods

We use the popular design of a pneu-net actuated soft robot limb shown in Figure 1. Details on the fabrication of this device can be found at [10]. In our case, the limb is composed of silicone rubber ADDV M 4601 (2-part silicone rubber, parts A & B) purchased from Wacker Chemie AG and manufactured using a 3D printer at the Institute for Machine Elements, Engineering Design and Manufacturing at the Technische Universität Bergakademie Freiberg in Germany. The chambers on the upper surface of the actuator can be filled with air and, by controlling the pressure, the arm can be deformed. Examples of this situation are presented in Figure 2(b).

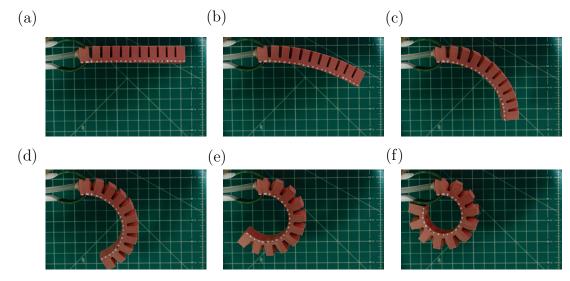


Figure 2: Deformed states of the soft robot arm for various values of the pressurization p. a) Pressure p = 0, b) Pressure p = 5 kPa, c) Pressure p = 17.2 kPa, d) Pressure p = 30.8 kPa, e) Pressure p = 45.3 kPa, and f) Pressure p = 57.4 kPa.

As shown in Figure 2, one measure of the characterization of the deformation of the arm is to measure the deformed shape of a material line embedded on the bottom surface of the arm. Clearly, as the pressure increases, the curvature of the material line increases.

It is possible to estimate the curvature using standard numerical techniques from the shape of 49 the material line. To this end, a series of white dots (optical targets) with a distance of 5 mm 50 are painted along the lower part of the soft robot arm. Then the arm is clamped on one side and 51 horizontally positioned. Due to the large flexibility of the soft actuator, an out-of-plane deformation 52 is inevitable. However, because this deformation is small compared to the bending deformation, 53 we neglected it for the subsequent analysis. During experiments, air was pressured into the arm 54 and its deformed shape was digitally recorded. The amount of air was gradually increased by 2 55 milliliters and the corresponding pressure was measured with a pressure gauge PCE-P50. 56

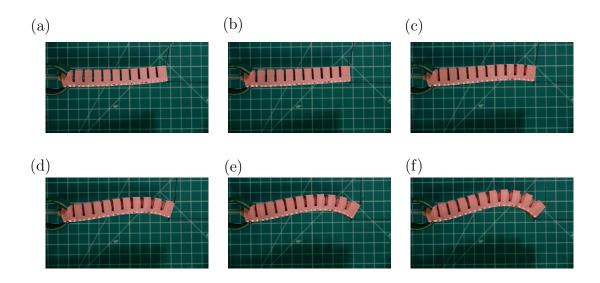


Figure 3: Deformed states of the terminally loaded soft robot arm for various values of the pressurization p. (a) Pressure p = 0 kPa, (b) Pressure p = 5 kPa, (c) Pressure p = 17.2 kPa, (d) Pressure p = 31 kPa, (e) Pressure p = 44.9 kPa, and (f) Pressure p = 58.2 kPa.

For analyzing the digitized images of the deformed arm, we first performed a correction of the 57 lens distortion and then loaded the images in MATLAB. The image processing toolbox provides a 58 convert-function from RGB to gray scale, which is used to specify a color spectrum to detect the 59 white targets. By converting the image to black and white, only areas in the defined color spectrum 60 remained white, while the surroundings were black. Before locating the white areas, small holes are 61 closed and objects smaller then a defined threshold are deleted in the digital image. We then used 62 the MATLAB image processing toolbox to export the position of the center points of the optical 63 targets, and size of each area, the length of the smallest and largest axis, and its orientation. These 64 values are saved and used to prescribe a corresponding set of points on the soft robot arm and to 65 delete other objects with a similar color spectrum. This process is executed for each image. 66

For dimensionless values, the length  $\ell$  of the arm is extracted using the end points of the initial position (pressure 0 kPa) and used as a scaling factor. Because the painted dots on the robot arm are not perfectly aligned along the axis of the actuator, a Gaussian process regression is used to smooth the measured center points [11]. Additionally, the coordinates of the first target point are shifted to the origin.

The curvature  $\hat{\kappa}$  of the space curve defined by the targets is determined from the smoothed deformation by the general description for a plane curve defined in Cartesian coordinates,  $\mathbf{r} = \mathbf{r}_4 \quad x\mathbf{E}_1 + y\mathbf{E}_2$ ,

$$\hat{\kappa} = \frac{x'y'' - x''y'}{(x'^2 + y'^2)^{3/2}},\tag{1}$$

<sup>75</sup> here the prime denotes derivative with respect to the arc-length parameter s. As shall be discussed <sup>76</sup> later in more detail, at the conclusion of the first set of experiments, the intrinsic curvature,  $\kappa_0$ <sup>77</sup> as a function of the pressure p and arc-length parameter s can be found and, in the second set of <sup>78</sup> experiments, the curvature  $\kappa$  in the deformed configuration can be determined for loads superposed <sup>79</sup> on the pressurized state.

To determine the constitutive relations for the bending moment as a function of the change in 80 curvature, we next adapted the first experiments by loading the actuator at its end (see Figure 3). 81 To achieve this, a pair of strings were fixed on the end and connected to two spring dynamometers 82 to measure the applied forces in  $\mathbf{E}_1$  and  $\mathbf{E}_2$  direction. In the initial state (pressure 0 kPa) only a 83 pre-load  $F_A = 0.25$  N was applied along the  $\mathbf{E}_1$  direction. As the forces on the strings were changed, 84 the deformation of the end of the arm prevented the strings from aligning with the respective  $\mathbf{E}_1$ 85 and  $\mathbf{E}_2$  directions. Consequently, the angles subtended by the strings were also recorded so that 86 the resultant force acting on the arm could be computed. 87

#### <sup>88</sup> 3. A Model Based on Rod Theory

While it is possible to use some nonlinear rod theories, such as those developed by Green, 89 Naghdi and their coworkers [12, 13, 14, 15], which accommodate cross-sectional deformation and 90 extension, here we seek the simplest possible nonlinear rod theory due to Euler in 1744 [16, 17]. In 91 this theory, a material curve of the arm is identified with the centerline of the rod and the position 92 of a material point of the arm is identified with the position vector  $\mathbf{r}$  of a material point, labelled 93  $\xi$ , of the centerline of the rod. Referring to Figures 1 and 4, the material curve of the arm is the 94 set of points colored in white on the lower half of the arm. The length  $\ell$  of the material curve of 95 the elastica is identified with the undeformed length of the material curve of the arm. 96

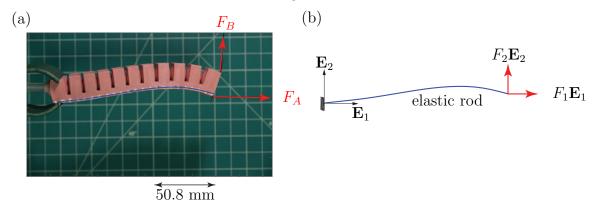


Figure 4: The actuator which is clamped at one end and loaded with a force  $\mathbf{F} = 0.175\mathbf{E}_1 + 0.07\mathbf{E}_2$  Newtons at the other subject to an air pressure of 31 kPa. In (a) the deformed actuator is shown and in (b) an elastica model for this actuator is presented.

<sup>97</sup> We assume that the centerline of the elastica is inextensible. In this instance, the coordinate  $\xi$ <sup>98</sup> can be identified with the arc-length parameter *s* of the centerline. The unit tangent vector to the <sup>99</sup> centerline can be parameterized by an angle  $\theta$ ,

$$\frac{\partial \mathbf{r}}{\partial s} = \cos(\theta) \mathbf{E}_1 + \sin(\theta) \mathbf{E}_2,\tag{2}$$

and the signed curvature of the centerline can then be defined using  $\theta$ :

$$\kappa = \frac{\partial \theta}{\partial s}.\tag{3}$$

We shall assume that the centerline of the elastica has an intrinsic curvature  $\kappa_0$  for the unloaded but pressurized actuator. This curvature will be identified as a function of the pressure p in the sequel. The bending moment M in the elastica is assumed to be a function of the difference in the current (loaded actuator) and intrinsic curvature:  $M = \mathcal{M}(\kappa - \kappa_0)$ . In the vast majority of works, the function  $\mathcal{M}$  is assumed to be linear:  $M = EI(\kappa - \kappa_0)$  where EI is known as the bending stiffness or flexural rigidity.

We shall assume that the end of the elastica at s = 0 is fixed at the origin while the other end is loaded by a terminal force **F**. The deformed static shape,  $\mathbf{r} = x\mathbf{E}_1 + y\mathbf{E}_2$ , of the elastica is determined by a set of boundary conditions, constitutive equations for M, and a pair of balance laws:

$$\begin{aligned} x(s) &= \int_{0}^{s} \cos(\theta(u)) \, du, \\ y(s) &= \int_{0}^{s} \sin(\theta(u)) \, du, \\ \mathbf{n}(s=\ell) &= \mathbf{F}, \\ M &= \mathcal{M}\left(\frac{\partial \theta}{\partial s} - \kappa_{0}\right), \\ \frac{\partial \mathbf{n}}{\partial s} + \rho \mathbf{f} &= \mathbf{0}, \\ \frac{\partial M}{\partial s} &= -(\cos(\theta)\mathbf{E}_{2} - \sin(\theta)\mathbf{E}_{1}) \cdot \mathbf{n}. \end{aligned}$$
(4)

Here,  $\mathbf{n} = n_1 \mathbf{E}_1 + n_2 \mathbf{E}_2$  is known as the contact force and  $\rho \mathbf{f}$  is the assigned body force. For the applications in the sequel, we assume the deformation of the rod is planar. Consequently, we set  $\rho \mathbf{f} = \mathbf{0}$  and conclude that  $\mathbf{n}$  is constant throughout the rod and can be determined from the boundary conditions. The function  $\mathcal{M}$  remains to be prescribed. In the sequel, we use the weight of the arm mg to non-dimensionalize the forces, the quantity  $mg\ell$  to non-dimensionalize moments, and the length  $\ell$  to non-dimensionalize s, x, and y.

#### 117 4. Modeling the Pressurized Arm

In the first set of experiments, the arm is clamped at one end and the pressure is increased from 0 through a discrete set of values (see Figures 2 and 5). The deformed shape (x(s), y(s)) of the material curve on the arm is digitized and recorded. As shown in Figure 5(a), the deformed shape of the curve can then be produced and the curvature  $\kappa = \kappa_{p_0}$  computed using (1). The resulting curvature depends on the pressure p, is non-uniformly distributed along the curve, and can be used to define the function  $\kappa_{p_0}$ :

$$\kappa_{p_0} = \kappa_{p_0}(p, s). \tag{5}$$

<sup>124</sup> By way of illustration, 10 distinct examples of the function  $\kappa_{p_0}(\cdot, s)$  are shown in Figure 5(b). <sup>125</sup> We shall see later that, for a given s,  $\kappa_{p_0}$  can sometimes be approximated by a linear function of <sup>126</sup> pressure.

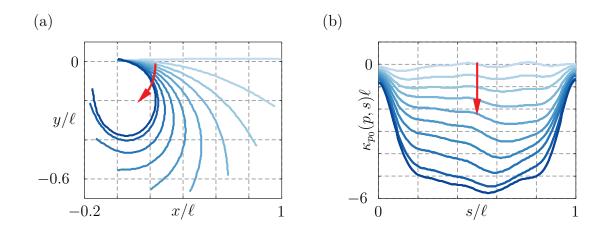


Figure 5: Measured (a) deformed shapes of the rod and (b) curvature  $\kappa_0 = \kappa_{p_0}(p, s)$  for increasing values of the pressure. The pressure p in these figures takes the values 0, 5, 10.9, 17.2, 23.8, 30.8, 38.3, 45.3, 51, and 55 kPa and the arrows indicate the direction of increasing p.

<sup>127</sup> To model the first set of experiments, we assume the elastica is clamped at s = 0 and is unloaded <sup>128</sup> at  $s = \ell$ :  $\mathbf{n} = \mathbf{F} = \mathbf{0}$ . We prescribe the intrinsic curvature  $\kappa_0$  of the arm using the function  $\kappa_{p_0}$ :

$$\kappa_0 = \kappa_{p_0}(p, s). \tag{6}$$

<sup>129</sup> Assuming a constitutive relation

$$M = \mathcal{M}_1 \left( \kappa - \kappa_{p_0} \right) \text{ where } \mathcal{M}_1(0) = 0 \tag{7}$$

and  $\mathcal{M}_1$  is otherwise an arbitrary differentiable function, we find that the bending moment in the elastica is zero provided  $\kappa = \kappa_{p_0}$ . Consequently, the balance laws (4)<sub>5.6</sub> are trivially satisfied.

#### 132 5. Modeling the Terminally Loaded Arm

<sup>133</sup> We now consider an extension to the previous experiment where terminal forces are applied to <sup>134</sup> the end  $s = \ell$  of the rod. An example of this situation is shown in Figure 4. As the pressure p is <sup>135</sup> varied, the arm deforms and, as recorded in Table 1, the terminal loads  $\mathbf{F}_A$  and  $\mathbf{F}_B$  also change. <sup>136</sup> Discretizing the material curve on the arm, the deformation of this curve can be recorded and <sup>137</sup> the curvature  $\kappa$  computed (see Figures 3 and 6). In addition, the angles needed to relate the <sup>138</sup> measured forces  $\mathbf{F}_A$  and  $\mathbf{F}_B$  to the components  $F_1$  and  $F_2$  of the resultant force  $\mathbf{F} = F_1\mathbf{E}_1 + F_2\mathbf{E}_2$ <sup>139</sup> are determined.

We can use the results of the earlier experiment to compute  $\kappa_0 = \kappa_{p_0}(p, s)$  induced by the pressure p. Thus for each given  $p, \kappa - \kappa_0$  for the configurations shown in Figure 6 can be computed. Simply assuming that  $M = EI(\kappa - \kappa_0)$  where EI is determined from the geometry of the arm and the elastic modulus of the silicon leads to results that are inadequate. Consequently, an alternative approach was used to prescribe the constitutive relations. Using Eqns. (4)<sub>1,2,5,6</sub>, we note that the

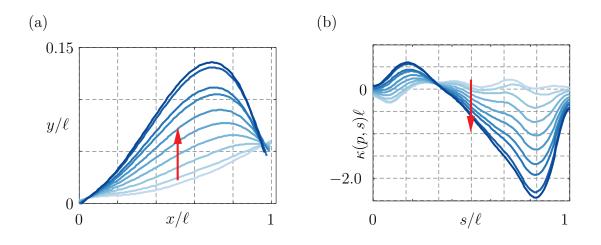


Figure 6: Measured (a) deformed shapes of the rod and (b) curvature  $\kappa$  for increasing values of the pressure and various terminal loadings. The pressure p in these figures takes the values 0, 5, 10.9, 17.2, 23.8, 30.8, 38.3, 45.3, 51, and 55 kPa and the arrows indicate the direction of increasing p.

bending moment M(s) in the elastica can be expressed as a function of the terminal load:

$$M(s) - M(\ell) = \int_{s}^{\ell} \mathbf{F} \cdot (\cos(\theta(u))\mathbf{E}_{2} - \sin(\theta(u))\mathbf{E}_{1}) du$$
$$= (x(\ell) - x(s)) F_{2} - (y(\ell) - y(s)) F_{1}.$$
(8)

Motivated by the above identity, we define an estimate  $M_{\text{est}}(s)$  for M(s) based on measurements of the terminal load **F** and the deformed shape **r** of the material curve:

$$M_{\text{est}}(s) = (x(\ell) - x(s)) F_2 - (y(\ell) - y(s)) F_1 + M(\ell).$$
(9)

We can then examine how  $M_{\text{est}}(s)$  varies along the length of the rod and in particular how it varies with  $\kappa - \kappa_0 = \kappa - \kappa_{p_0}(p, s)$ . These results are shown in Figure 7. Clearly, the moment is no longer a simple, classic, linear function of the curvature difference:  $M \neq EI(\kappa - \kappa_0)$ .

<sup>151</sup> We henceforth assume that  $M_{\text{est}}(\kappa - \kappa_0)$  can be approximated by a pair of linear functions:

$$M_{\rm est} (\kappa - \kappa_0) = \begin{cases} M_{O_1} + mg\ell^2 \alpha_1 (\kappa - \kappa_0) & s \in [0, \ell_1), \\ M_{O_2} + mg\ell^2 \alpha_2 (\kappa - \kappa_0) & s \in [\ell_1, \ell], \end{cases}$$
(10)

where  $M_{O_{1,2}}$  and the dimensionless flexural rigidities  $\alpha_{1,2} = \alpha_{1,2}(p)$  are piecewise constants which are pressure dependent. A representative example of such a prescription can be seen in Figure 8(a) where  $\alpha_1 = -0.029$  and  $M_{O_2} = -0.012 \ mg\ell$ . For a given p, we can use our knowledge of  $\kappa_{p_0}(p, s)$ to determine  $\kappa_0$ . Then, using knowledge of  $\mathbf{F}$ , the deformed shape of the rod can be determined using Eqn. (4) with M given by Eqn. (10) to derive the flexural rigidity as a function of p (see Figure 8(b)).

#### 158 6. Results

With known of the pressure-dependent parameters  $\ell_1 = \ell_1(p)$ ,  $\alpha_1 = \alpha_1(p)$ ,  $\alpha_2 = \alpha_2(p)$ , and  $\kappa_0 = \kappa_{p_0}(p, s)$ , we are able to capture the deformation of the soft actuator for varying pressure and

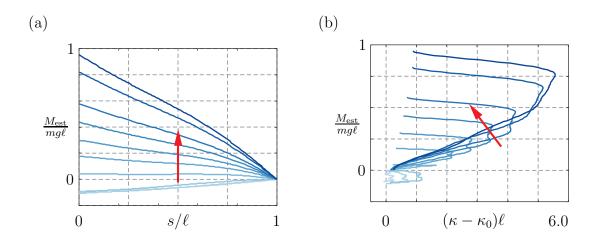


Figure 7: Measured (a) dimensionless bending moment  $\frac{M_{est}(s)}{mg\ell}$  of the rod computed using Eqn. (9) and (b) the bending moment  $\frac{M_{est}(\kappa-\kappa_0)}{mg\ell}$  as a function of  $\kappa - \kappa_0$  for increasing values of the pressure and various terminal loadings. The pressure p and terminal load **F** for these figures takes the values shown in Table 1 and with increasing values of p indicated by the arrows.

<sup>161</sup> boundary conditions using the rod model. For a fixed end and a defined position of the other end, <sup>162</sup> the deformation and resultant forces were calculated for various pressures. As can be seen from <sup>163</sup> Figure 9(a,b), the predicted values of  $\kappa$  and **r** are very good. However, the calculated end-loads <sup>164</sup> only agree for  $F_2\mathbf{E}_2$  and are overestimated otherwise. The decreasing forces along the axis of the <sup>165</sup> soft actuator during experiments may be a result of the extensibility of the actuator (which we <sup>166</sup> have not modeled).

We note that of all the parameters we varied, such as the axial force component  $F_1$ , the length  $\ell_1$  of section  $s \in [0, \ell_1)$ , and the flexural rigidities  $\alpha_1$  and  $\alpha_2$ , the results for the deformed centerline are most sensitive to changes in  $\ell_1$  (see Figure 10). In this figure, the deformed shape **r** depends on the end loads, and the constitutive parameters. What is clearly visible from Figure 10(a,c) is that the overall shape of the actuator does not change its characteristic features with variations of  $F_1$  and  $\alpha_{1,2}$ . A variation of  $\ell_1$ , by way of contrast, results in a dramatic change to the slope at the fixed end. Later on, we shall observe that the value of  $\ell_1$  increases with p.

Based on the results present, we can state that the elastic rod model is capable of predicting the deformed state of a pressurized soft actuator. However, the fidelity of the predictions depends on an accurate value of the parameter  $\ell_1$ .

#### 177 7. Concluding Remarks

In this paper we have measured the intrinsic curvature  $\kappa_0 = \kappa_{p_0}(p, s)$  produced by an air pressure in a soft actuator. Despite the simplicity of the boundary conditions, the resulting curvature field  $\kappa_{p_0}(p, s)$  is non-uniform and depends on the pressure p in a non-trivial manner. With the help of these results, we showed that the bending moment in a rod-based model for the actuator can be approximated by a pair of piecewise linear functions. The resulting model can then be used to predict the deformed shape of the arm subject to terminal loading of the type that would feature in applications.

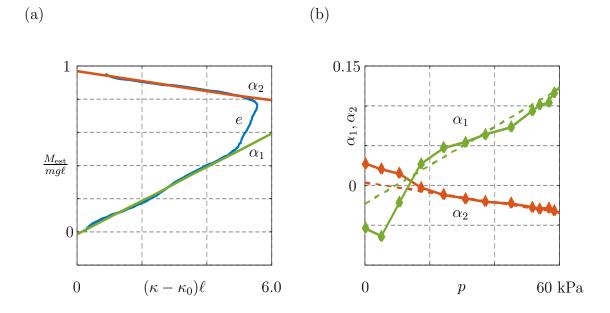


Figure 8: Measured (a) bending moment  $M_{est}(s)$  of the rod as a function of  $(\kappa - \kappa_0)\ell$  for p = 53.8 kPa using Eqn. (9), where e denotes the measured value, and (b) the dimensionless flexural rigidities  $\alpha_1$  and  $\alpha_2$  (cf. Eqn. (10)) as functions of the pressure p. Here, the dashed lines correspond to the linear regression of  $\alpha_{1,2}$ .

It is clearly of interest to compare our findings to the modeling predictions of Majidi et al. [7] who found that the flexural rigidity is an affine function of pressure and  $\kappa_0$  is a linear function of *p* in the limiting case where *p* is small. Referring to dimensions given in Figure 1, and Eqns. (5) and (10), we recall, from [7, Eqns. (4) & (9)], that

$$\kappa(p,s) = \left(\frac{H^2 t_1 w}{2D(t_1 + t_2)}\right) p, \qquad M = \left(D + \left(\frac{H^3 t_1 w}{4(t_1 + t_2)}\right) p\right) \left(\kappa - \kappa(p,s)\right), \tag{11}$$

where the flexural rigidity  $D = \frac{Et^3w}{12}$ . While the design considered in the present paper is different 189 from the one considered in [7], we find that several of its characteristics are similar. For example, 190 referring to Figure 11(a,b) for a given location s along the arm, the curvature  $\kappa_0$  and  $\kappa - \kappa_0$ 191 can be closely approximated as a linear function of p. Even for the rigidities  $\alpha_{1,2}$  we deduce 192 from our experiments, a linear relation of p can be approximated for high values (see Figure 8(b)). 193 However, the most significant novel feature of our constitutive relations Eqn. (10) is the non-uniform 194 characteristics of the intrinsic curvature in s (i.e.,  $\kappa_{p_0}(p, s)$ ). This in turn leads to the non-classical 195 constitutive relation EI(s) with sections with different flexural rigidities. These two features are 196 unique to our results and to the best of our knowledge have not been described previously in the 197 literature on soft robot arms. 198

<sup>199</sup> Modeling the actuator as simple inextensible uniform, albeit, nonlinear rod which exhibits planar <sup>200</sup> deformations is clearly a very coarse model. Such a model cannot capture subtle features of the <sup>201</sup> actuator such as its extensibility or the warping of the cross sections. We could consider the elastica <sup>202</sup> as a low-order member of a hierarchy of rod theories. Then, in principal, by modeling the actuator <sup>203</sup> using a directed rod theory that captures warping and extensibility, we should be able to develop

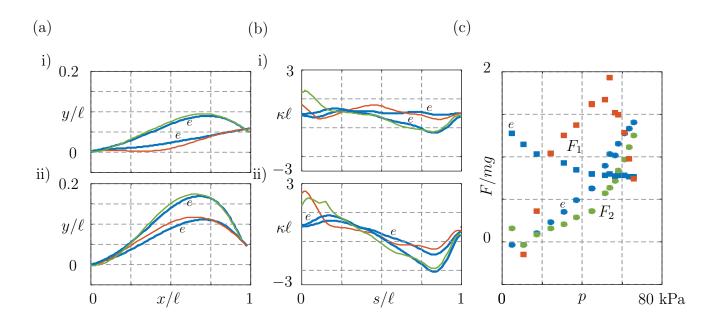


Figure 9: Measured and predicted (a) shapes of the centerline of the rod (using Eqn. (4) and (b) values of the according curvature i) for  $\ell_1 = 0.1$ , p = 5 and 31 kPa, and ii) for  $\ell_1 = 0.15$ , p = 44.9 and 63.6 kPa, and (c) measured and predicted force components  $F_{1,2}$  where p takes the values shown in Table 1. Measured values are displayed in blue and labeled with e.

a more faithful model. However, this more faithful model comes at a price of added computational
and analytical complexity. Surely, one of the advantages of soft robotics is the freedom to produce
a wider variety of designs? If so, perhaps one of the design criteria could be the ease of development
of a faithful model for the actuator? We hope the methods presented in this paper provide readily
accessible tools that can be used to assess such a design criterion.

#### 209 8. Conflicts of Interest

None of the authors have a conflict of interest.

#### **9.** Acknowledgements

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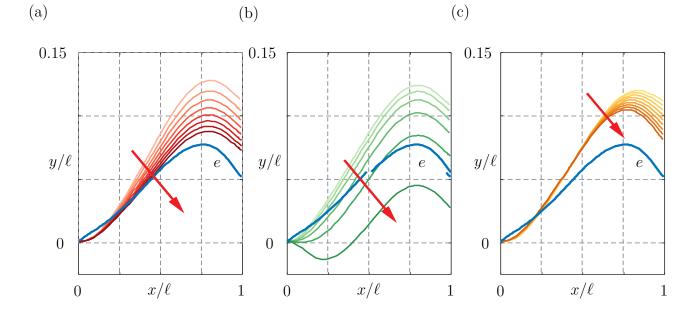


Figure 10: Measured and predicted shapes of the centerline of the rod (using Eqn. (4)) for p = 24.2 kPa and (a) for varying force components  $F_1$  in a range of  $F_1 \pm 20\%$ , (b) for varying  $\ell_1$  of  $s \in [0, \ell_1)$  with  $0.05 < \ell_1 < 0.15$ , and (c) for varying  $EI(\alpha_{1,2})$  in a range of  $\alpha_{1,2} \pm 20\%$ . Increasing values of  $F_1$ ,  $\ell_1$  and  $EI(\alpha_{1,2})$  are indicated by the arrows and e denotes the measured value.

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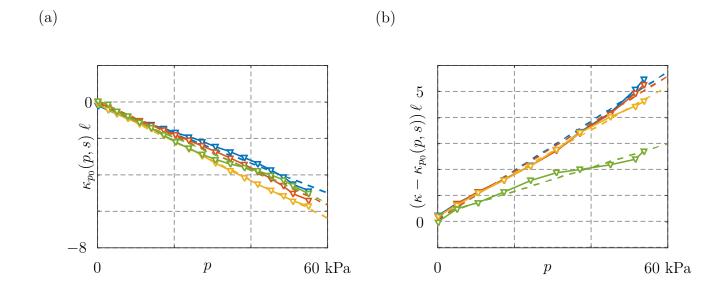


Figure 11: Experimental results for (a) the intrinsic curvature  $\kappa_0$  as a function of pressure p, and (b) the curvature difference  $\kappa - \kappa_0$  as a function of pressure p where  $\kappa_0 = \kappa_{p_0}(p, s)$  for four discrete values of s. For the data shown,  $\nabla$  corresponds to the material point  $s/\ell = 0.2$ ,  $\Box$  corresponds to the material point  $s/\ell = 0.4$ ,  $\triangle$  corresponds to the material point  $s/\ell = 0.6$ ,  $\bigcirc$  corresponds to the material point  $s/\ell = 0.8$ , and the dashed lines correspond to the linear regressions.

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## 277 Appendix

No.	Pressure (kPa)	$F_A$ Newtons	$F_B$ Newtons
1	0	0.25	0
2	5	0.25	0
3	10.5	0.225	0
4	17.2	0.2	0.025
5	24.2	0.2	0.05
6	31	0.175	0.07
7	37.1	0.16	0.1
8	44.9	0.15	0.125
9	51.5	0.15	0.175
10	53.8	0.15	0.2
11	56.6	0.15	0.2
12	58.2	0.15	0.225
13	61.3	0.15	0.25
14	63.6	0.15	0.26
15	66	0.15	0.275

Table 1: Table of values of pressure p and terminal loadings in Figure 7 and Figure 9.