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Observed strategies in the freehand drawing of complex hierarchical diagrams

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Abstract

Chunk decomposition and assembly strategies have been found in the drawing of complex hierarchical diagrams (specifically AVOW diagrams). Analysis of 40 diagrams produced by five participants provided evidence for the strategies based on the duration of pauses between drawn elements. The strategies were initially discovered using a new visualization technique developed to allow the detailed examination of the sequential order of diagram drawing in conjunction with information about the durations of pauses associated with drawn elements.

Keywords: Drawing; chunks; pause analysis; Graphical Protocol Analysis; AVOW diagrams; adaptive strategies.

Introduction

How do people draw diagrams? Understanding the nature of graphic production processes has potentially important theoretical implications for Cognitive Science and practical applications in related disciplines, because drawing is a challenging cognitive activity that involves many processes spanning a wide range of functions of the cognitive architecture. Some notable previous studies include Goel's (1995) investigations of the critical role of sketching in design problem solving, at high level, and Van Sommers (1984) classic work and Akin's (1986) studies on elementary processes of drawing, at a low level. From a developmental perspective, Karmiloff-Smith (1990) investigated children's increasing flexibility in their use of schemes with age as they drew imaginary objects.

However, there has been relatively little research on the processes of drawing at an intermediate cognitive level that examines what strategies are used when drawing complex diagrams. For example, given a diagram with many elements and a rich structure, what factors actually govern the sequencing and timing of the production of those elements? Cheng, McFadzean & Copeland (2001) studied the role of chunks in copying simple geometric figures and found that three hierarchical levels could be distinguished by analyzing the pauses between drawn elements. Obaidallah and Cheng (2009) studied the strategies used in the drawing of the Rey-Osterrieth complex figure, as used in the psychometric test of memory, which is composed of 56 lines that are organized into distinct patterns. Also using the analysis of pauses between drawn elements, they found that when the figure is reproduced from memory drawers tended to process the elements in a hierarchical fashion. The production of individual lines conforms to the distinctive patterns and at a

higher level these patterns tend to be drawn together in groups common across the drawers.

We have used the terms *Graphical Protocol Analysis*, GPA, (Cheng & Rojas-Anaya, 2006, 2007) for the technique of recording, analyzing and interpreting chunking based behaviours in graphical production, which we take to include drawing, writing natural language and also writing formal notations (e.g., mathematical equations). With GPA, the latency or pause just preceding the execution of a drawing action – making a mark – reflects the amount of cognition required and as such reveals boundaries between the processing of successive chunks, because the addition retrieval and planning operations required to start the production of a new chunk that are not presented for elements within a chunk. As extensively exploited in many classical studies in cognitive science, pauses with particularly long durations will be taken to reveal the presences of transitions between chunks.

One possible reason for the scarcity of studies on production strategies in drawing is the lack of effective techniques to combine information about the sequence of drawn elements with data on the duration of pauses, so that the presence and role of chunks can be related to the overt drawing behaviours. Thus, one of the two general aims of the paper is to introduce a technique to integrate these sources of data in the form of a readily interpretable visualization of graphical protocols.

The other aim of this paper is to examine the nature of the strategies that are used in drawing to produce complex hierarchically structured diagrams. Such diagrams will naturally be treated as chunks on more than one level, so how are those chunks structures processed in order to generate a coherent linear sequence of drawing actions? Diagrams are, of course, a diverse class of cognitive artifacts, so the choice of target stimuli is large. However, the adopted class of diagrams must meet two competing demands: (i) have a sufficiently complex hierarchical structure to permit the manifestation of nontrivial drawing strategies; (ii) be sufficiently simple and coherent that participants can feasibly learn them in the context of an experiment and that drawings of them can be readily analyzed. A class of hierarchical complex diagrams that meets these demands are *AVOW diagrams* invented by Cheng (2002) to the model of the behaviour of basic electrical circuits. AVOW diagrams represent individual circuit components, networks of components and whole circuits, respectively, as rectangles, small sets of rectangles and complete groups of rectangles.

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Fig 1a shows a simple electrical circuit and Fig 1b is an AVOW diagram for that circuit. We call a single AVOW rectangle a *box*. AVOW diagrams for more complex circuits include: (Fig 1c) two resistors in series; (d) two resistors in parallel; (e) two resistors in parallel that are together in series with another; (f) two in series that are in parallel with one other.

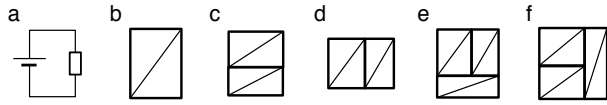


Fig 1. An electric circuit (a) and some AVOW diagrams

Although not important for the present paper, the height, width, area and slope of the diagonal of any individual box, or composite of rectangles, respectively represents the voltage, current, power and resistance of components or networks. AVOW diagrams substantially improve the ease of learning about basic electricity (Cheng, 2003; Cheng & Shipstone, 2003). In the experiment, participants were shown the standard drawing of electrical circuits consisting of rectangular icons for resistors connected together by lines representing wires. Their task was to draw AVOW diagrams to solve problems about the operation of the circuits.

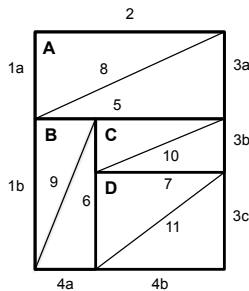


Fig 2. One AVOW diagram: two drawing strategies

So, theoretically, what are the possible strategies that may be used to draw AVOW diagrams from given circuit diagrams? Whilst learning about AVOW diagrams participants gradually acquire graphical chunks consisting of simple groups of AVOW boxes that correspond to common networks (Cheng, 2002). Figs 1b-d are examples of patterns that learners easily acquire as chunks (Cheng, 2002). Thus, drawing a complex AVOW diagram is likely to be organized around the production of AVOW boxes in such groups. There are, at least, two ways in which a diagram with multiple AVOW boxes may be drawn, which proceed in opposite “directions”: (a) a *decomposition* strategy in which an overall diagram is broken down into chunks and then sub-chunks; (b) an *assembly* strategy in which chunks are built in turn from sub-chunks in an incremental fashion. These strategies have been informally observed in previous studies with AVOW diagrams.

To make this more concrete, consider how the two approaches could be used to draw Fig 2, which consists of four AVOW boxes. Boxes C and D constitutes a chunk for two resistors in series, which together many make up a higher level chunk of B in parallel with [C+D], that in turn is part

of the overall diagram comprising A in series with B+[C+D]. Under the decomposition strategy the rectangle for the overall diagram is drawn, lines 1a-1b-2-3a-3b-3c-4a-4b. Then this overall rectangle is subdivided at the top level by drawing line 5, which is followed by a subdivision at the next lowest lever with line 6. The last subdivision by line 7 makes its splits the lowest level. Finally, each of the diagonals for the boxes are drawn in turn, lines 8, 9, 10 and 11, repeating the order of the chunks.

Under the assembly strategy the drawing progresses by drawing individual AVOW boxes in turn, for example starting with lines 1a, 2, 3a, 5 and 8. The next to be drawn builds on to the first: lines 1b, 6, 4a and 9. This is followed by lines 7, 3b and 10 for the penultimate box and lines 4b, 3c and 11 finally.

The diagonals in the boxes are typically seen as secondary to the perimeter of the rectangles, which define the size and shape of the boxes, so two alternative strategies may be applied to draw them. In the *one-at-a-time* approach whenever a box is completed its diagonal is immediately drawn before continuing with the verticals or diagonals of other boxes. Clearly, in these cases the diagonal should be considered as part of the lowest level chunks that are individual AVOW boxes. In the *all-together* approach all the diagonals are filled together after the pattern for a particular chunk is completed. It is tempting to consider such groups of diagonals as constituting some form of sub-chunk, merely because they are all drawn together, but this is a supposition to be tested.

In order to test whether these strategies exist, two forms of related evidence will be used. First, the order of production of sequences of lines will be closely examined in relation to the pauses in graphical production. This will be achieved using the new visualization technique that is to be introduced here. Second, four predications (P1-P4) have been formulated that relate to differences in the duration of pauses at particular points in the drawing of complex AVOW diagrams, on the basis that long pauses are indicative of the processes occurring at the boundary between chunks.

P1: For the decomposition strategy it is predicted that the pauses for the drawing of the first line of the overall rectangle for a group of AVOW boxes will be longer than the pauses for the subdivisions of the rectangle, because of the extra processes are involved in the initiation of the new chunk.

P2: Under the assembly strategy it is predicted that the pause before beginning to draw a sequence of AVOW boxes will be longer when there are more AVOW boxes in the chunk. Specifically, the pause for a chunk consisting of a single AVOW box (Fig 1b) will be shorter than for chunks consisting of two or more purely in series or in parallel (e.g., Fig 1c or 1d).

P3: With the assembly strategy it is predicted that for a chunk consisting of two or more boxes, the pause before the first box will be longer than the second and subsequent boxes, because extra processes are involved in the initiation

of the new chunk that are not present for later boxes. Note the similarity to P3 to P1.

P4: This prediction concerns the drawing of immediately consecutive diagonals under the all-together approach. P4a: If they are being treated as a single chunk, then the pause before drawing first diagonal should be longer than the other diagonals in the sequence. P4b: Also, if they are being treated as a single chunk, then the pause for the first diagonal in a set of more than one should be longer the pause for the first diagonal in general. The reasoning behind this pair of predictions resembles that for P3 and P2, respectively, but at the level of diagonals rather than boxes.

To test the strategies, drawings of AVOW diagrams were obtained for another experiment that studied learning with AVOW diagrams over many sessions. Computer logs of participants drawings were taken from the last of six hour-long sessions, in which participants had practiced drawing many AVOW diagrams of increasing complexity in order to learn about electric circuits. The two tasks selected for analysis here involved drawing AVOW diagrams to model circuits with different configurations. The general details of the overarching learning experiment are not important to the graphical protocol analyses conducted here.

Method

Participants

Four women and four men took part in the overarching learning experiment, but three were eliminated from this experiment because they did not successfully draw diagrams for all of the tasks selected for the present analysis. All were University of Sussex students with little prior knowledge of basic electricity. They were paid for participation and tested individually.

Materials

The target circuit diagrams were presented in booklets, which sat on a small easel to the side of a Wacom Intuos3 graphics tablet. For each drawing a sheet of plain paper was clipped to the tablet and the drawings were made using a Wacom inking pen. A Java programme specially written in our lab for GPA studies captured the pen movements, including the positions and times of all touches and lifts of the pen to and from the paper. The temporal accuracy of the logging was better than 1 ms.

For the purpose of the present experiment two test items, here labeled TA and TB, were selected from the last session of the overarching learning experiment, in which the greatest number of participants had successfully completed drawing solutions. Task TA involved a circuit with five resistors and the problem of modelling circuit behaviour in normal operation and three fault conditions. Task TB involved three resistors with two switches and the problem of modelling circuit behaviour in all four combinations of switch positions. As four diagrams were produced for both TA and TB, and five participants took part, a data set of 40 diagrams was available for analysis. (For reference, in the overarch-

ing learning experiment TA was task 2 and TB was task 3, in session 6).

Procedure

In the overarching learning experiment the participants worked through explanations and examples of graded topics in electricity. For each topic they drew AVOW diagrams as solutions to exercises in which conventional circuit diagrams were presented with different values of resistors or varying positions of switches. Immediately following each exercise they were given feedback in the form of drawings of correct AVOW diagrams. With each exercise they were instructed to first workout their solutions by sketching whatever AVOW diagrams they wished and once they were confident in their answer to draw their complete solution diagram neatly on a fresh sheet of paper. Just the final solutions diagrams were selected for analysis here, because participants would have been producing familiar diagrams from memory and so will have structures of chunks that are both active and relatively stable.

In order to generate readily interpretable graphical protocols participants were trained to draw straight lines by always lifting the pen off the paper whenever they wished to change direction. That participants quickly adapted to this approach was clear for three reasons: (a) they needed just a few practice items to become comfortable with this style of drawing; (b) except once at the beginning of each session they needed no further prompts to use the approach; (c) there are few cases of failures to lift the pen at the end of lines, where they just produced L shape lines. Thus, it is unlikely this style of drawing will have impacted on the chunk level drawing strategies in which we are interested.

Visualization for graphical protocol analysis

In order to easily see the order of line drawing and to relate these to pause durations a new graphical protocol visualization has been developed. Fig 3 and 4 show examples of the visualization (the letters and numbers have been added as a coordinate reference system). The thicker grey lines are strokes drawn by the participants; or more precisely a line connecting the end points of those strokes. The continuous *trajectory curve* runs between successive lines, intersecting each of them at a point near their beginning (one quarter of the way along): it begins at A4 in Fig 3 and A3 in Fig 4. The curve reveals both the order of production of the lines and the direction in which each is drawn. The small circles at the intersection of the lines and the trajectory curve have four sizes which represent the duration of pauses for the associated line: from small to large these are ≤ 1 , ≤ 2 , ≤ 5 , > 5 s. For example, in Fig 3 the point at D1 is actually 5,940 ms, E14 is 1,233 ms and L13 is 657 ms. Additional symbols drawn over these points, such as the 'X' at A4 in Fig 3 or the square at H7 in Fig 4, will be explained below.

The graphical protocol visualization was implemented using the standard graph plotting capabilities of Microsoft Excel.

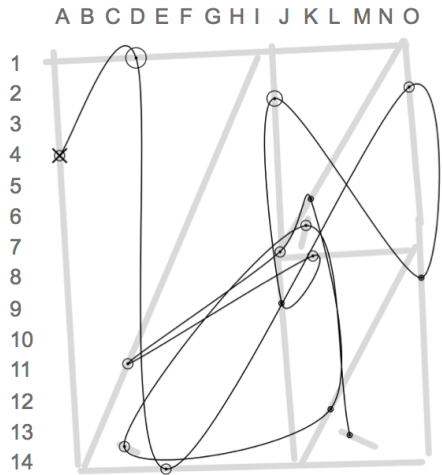


Fig 3. Protocol exhibiting the decomposition strategy

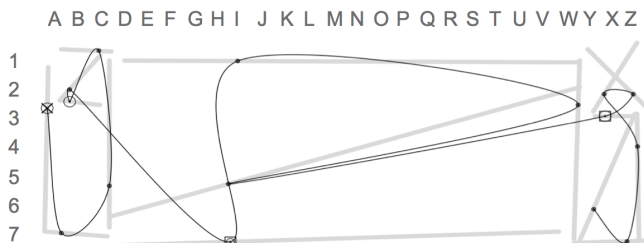


Fig 4. Protocol exhibiting both strategies

Results

To examine the strategies all the logs for the two tasks of the six participants were presented using the new visualization. By inspection it was found that the drawing of all boxes could be exclusively classified into either the decomposition or the assembly strategies, as described using Fig 2. For example, for one selected diagram of participant P3 in the solution to TB, Fig 3, starts with the drawing of an overall bounding box (A4–D1–E14–O3–O8), followed by the subdivision into two halves (J9–I3), and then the right half was subdivided (K7), which is clearly a decomposition approach. Following the subdivisions the three diagonals were completed (C11–K6–K13), and unusually this participant then labeled the boxes (D13–K7–L13). Fig 4 shows one of the diagrams that participant P6 drew for TB that uses both the decomposition and assembly strategies in a single drawing. Initially, the outline of an encompassing box is drawn to the left (A3–A7–C6–C1), that was then subdivided (B3), before one diagonal is drawn (B2); the other was forgotten. This is followed by a single large box (H7–I1–W3) with its diagonal filled immediately (H5), which is a use of the assembly strategy building onto the previously drawn pair of boxes. The pair of boxes on the right is also drawn by assembly, because the upper boxes is drawn fully (X3–Z2–X2) before the lower box (Z4–Z7–Y6). The descending diagonal at Z2 is one of the rare cases where the participant neglected to lift the pen from the paper at a change of direction, so that the line actually represents two line segments for the top and right side of the top box.

Similar inspection of all the other 38 diagrams showed that all the groupings of boxes are readily classified in terms of the two strategies.

Table 1. Means of the trial median pause for various measures of the two strategies and for diagonals

Measure	Pause (ms)	Predictions and cases
a) Start diagram	2,450	
Decomposition		
b) Begin a group	1,928	P1; b>c
c) Subdivision	1,400	5/7
Assembly		
d) All boxes	4,380	
e) Single box	10,800	P2; e<f
f) 1st box in sequence of 2+ boxes	4,490	3/7
g) 1st box in sequence (any length)	4,370	P3; g>h
h) Boxes except 1st in sequence	763	7/7
Diagonals		
i) All diagonals	629	
j) 1st diagonal in a set of any size	654	
k) Diagonals except the 1st in a set	690	
l) First diagonals in a set of 2+ diagonals	690	

Predictions

As all the approaches to the drawing of the diagrams fit the two strategies it is possible address the predictions made above that concern the durations of pauses in relations to how the diagrams might be chunked. Thus, a coding scheme was devised to classify the circumstance under which each line was drawn. These included: (a) the start of a new diagram – shown by an ‘X’ in the protocol visualizations (e.g., Fig 4, A3); (b) the start of group boxes under the decomposition strategy along with the number of boxes created by the subdivisions; (c) the start of a sequence of boxes under the assembly strategy – shown by a square in the visualization (e.g., Fig 4, H7 and X3) – along with the number of boxes in the sequence; (d) the start of a set of one or more consecutive diagonals, along with the number of them. The start of a set of one or more consecutive labels was also coded but as there were few of these and they as they occurred as the last elements to be drawn in a diagram, they are omitted from the analysis. The codes were applied to the logs by two coders and there was precise agreement on all of the codes for the lines for 37 of the diagrams, with the three disagreements resolved through discussion. This level of agreement is not surprising as all the drawing conformed to just the two strategies under consideration.

From the coded logs the pause value measures needed to test the predictions were computed. For each trial – comprising the four diagrams for one task – the medians of the relevant pauses were computed for each participant. Medians were used as pause data in GPA data is skewed. Aggregation was performed the level of trials rather than diagrams so there were sufficient data points for the computation of the median for each of the measures. Table 1 gives measures and trends, where comparisons between measures are made, as well as the number of individual trials support-

ing the given trend. The latter is given in relation to the total number of trials for which data was available. This is less than 8 because the strategies were not exhibited in every trial. In making comparisons between pairs of values, it should be noted that the time for the execution of elementary operations is typically considered to be of the order of 100 ms. Therefore, any differences between values that are several times this magnitude are likely to be meaningful, because they are likely to be due to differences at the level sets of operations. If a difference is less than 100 ms it is likely to be due to individual differences in speeds of processing rather than variations in the sequences of operations that underpin different sub-tasks.

Prediction P1 is supported to the extent that the time to draw the perimeter rectangle of a group of AWO boxes is substantially greater than the pause for sub-divisions; Table 1 lines (b) and (c). Of the seven trials exhibiting the decomposition strategy five had trends in the expected directions.

Prediction P2 is not supported as the time to begin drawing a single box alone is substantially longer than the time to begin drawing the first box in a sequence with more than one box; Table 1, f and g. However, there are nearly equal numbers of trials that have trends that are or are not in the expected direction.

Prediction P3 is supported strongly as the pause for the first box in a sequence is longer than for the remaining boxes in the sequence; Table 1g & 1h. This trend is as expected for all seven trials for which the assembly strategy was exhibited. In Fig 4, the occurrence of large circles (long pauses) associated with the square symbols (beginning of assemblies) at H7 and X3 with the presences of small circles elsewhere illustrates this pattern of data.

Prediction P4 is concerned with possible patterns of pauses if the drawing of sets of consecutive diagonals would be treated as if they were chunks. The specific prediction are actually irrelevant because the pause values for all the conditions, Table 1, j and l, have pause values that are essentially the same; the maximum difference between any of the measures is 60 ms, which is lower than the execution time of elementary operations. Further, the absolute value of the pauses for drawing a diagonal is relatively small, which suggests that relatively simple sequences of operations underpins their production.

Comparing the values of the decomposition operations with the assembly operations, Table 1b-c versus 1d-h, an interesting pattern may be noted. To start a diagram by decomposition takes about 2 s whereas to start an assembly takes more than double the time at over 4 s, but continuing the subdivisions of a decomposition take about 1.5 s, which is twice the time for continuing with the subsequent assembly of boxes at about $\frac{3}{4}$ s.

Discussion

Drawing is an interesting phenomenon for study in cognitive science as the graphical production of a drawing engages many cognitive processes and potentially places large

demands on the cognitive system. This will likely require sophisticated strategic adaptations to manage the diverse loads on the system. Previous work has been just begun to examine the underlying cognitive processes of drawing. The findings here provide some further insight into nature of the strategies used in the drawing of complex hierarchical diagrams. Previously, Obaidallah and Cheng (2009) showed that the patterns of lines in the Rey-Osterrieth figure were encoded as chunks and that the reproduction of the diagram exploited those chunks in a decomposition approach that operated at three levels. The results here extend those findings in various ways.

First, the decomposition strategy in drawing complex diagrams is a general phenomenon to the extent that it is exhibited in quite a different class of diagram, which has recursively nested chunks on multiples levels. Under this strategy an overall framework is first produced and is successively divided into parts that match the structure of simple chunks. Second, an alternative assembly strategy was observed that constructs a diagram in an incremental piecewise fashion by adding new components on to previously drawn parts of the diagram, which operates in a working outwards manner rather than the working inwards approach of the decomposition strategy. Third, it was found that these strategies are not used exclusively when drawing a particular diagram but may occur within the same diagram, as occurred in Fig 4. The conditions under which the alternatives are selected for use would be interesting to investigate in order to understand whether it is contingences related to the physical process of drawing, or the familiarity and ease of processing certain chunks, or a subtle combination of the two, that determines strategy choice. Fourth, the differences in the durations to starting and continuing the execution of each strategy (Table 1 e-f vs. g-h) suggests that the costs of performing the strategies may be differently distributed over the process of drawing a whole diagram. Notwithstanding the previous point, this is another factor that is likely to influence the selection of drawing strategies.

These findings should be treated with caution as 3 participants were excluded, because they did not produce a complete set of diagrams. This may have introduced a bias, with the sample favouring participants who are more competent at drawing or better learners.

The introduction of the new visualization (Fig 3 and 4) supported the ready interpretation of the strategies in the graphical protocols and the clear determination that the decomposition and assembly strategies were being used, and exclusively so. A particular feature of the GPA visualization is the introduction of a trajectory curve intersecting the drawn lines, which appears to be an effective way to show the sequential order and direction of the drawn lines. In our previous experiments with visualization designs, software generated lines joined the end of one drawn line to the beginning of the next, but this tends to make the display too clutter for easy interpretation. The trajectory curves also appears easier to use than animations of the drawing with which we and others (e.g., *Eye and Pen*, [2024](http://</p></div><div data-bbox=)

www.eyepandpen.net) have experimented, because the curves not only provide local spatial and temporal detail but also give an overview of the whole course of drawing.

Chunks clearly have a core role in the functioning of both strategies. The support for the first and third predictions provides some evidence for this view. Both predictions are based on the idea that the time to start the production of the first element in a chunk will be longer than subsequent elements, because there will be initial recall and planning actions associated with the first element that do not have to be repeated for later elements. This is consistent finding across many different tasks, such as writing memorized sentences (van Genuchten & Cheng, 2010). It would be surprising if it did not apply to drawing also. However, the absence of support for prediction P2, on first sight, is contrary to this expectation, in that it was found that the pause before the production of a single box was longer than that for the production of a chunk comprising multiple boxes. There are at least two possible explanations for this seemingly anomalous result. First, the longer pauses for a single box may be because single boxes may have occurred more frequently at the beginning of a new diagram, which will have additional process not found in the continuation of a diagram, such as decisions about the location and size of the first box. Second, the coding of the protocols assumed that the only chunks possessed by the participants were simple sequences of boxes purely in series or parallel; patterns c and d in Fig 1. However, if participants possessed chunks such as Fig 1e and f, then the drawing of many solo boxes would actually be the initiation of the drawing of a more complex chunk comprised of three boxes. From the experiment in Cheng (2002) it seems likely that by the sixth session in the present overarching learning experiment the participants would possess such chunks. Therefore, the elevated time for a chunk of one box probably has been erroneously misclassified and should be associated with complex chunks comprising multiple boxes.

Prediction P4 is interesting because it assumes that sets of diagonals might be processed as chunks and so would have patterns of pauses that reflect those in predictions P1 to P3, but at a lower level. However, from the uniformity of the pauses for different set sizes and sequential positions it must be inferred that sets of diagonals were not being treated as chunks. Rather it appears that participants might simply have been using a simple strategy of spotting and filling in empty gaps in the AVOW diagrams. This is consistent with the duration of the pauses, which would sufficient for a set of operations such as: visually search for a gap, initiate a goal to fill the gap, move the pen to target area, prepare to make a stroke. The occurrence such a sequence of actions is not in itself surprising, but it makes an instructive point that care must be taken when analyzing the process involved in the drawing complex hierarchical diagrams: one should expect diverse strategies, some of which may be based on the chunking of the diagrammatic components, including the decomposition and assembly strategies, but others may

simpler approaches that directly exploit the structure of the external graphical environment.

To conclude, it should be noted that the findings here are based on just one class of complex diagrams. The observed strategies may not be manifest in other types of diagrams, especially so if they are less hierarchical in nature, and other strategies are likely to exist for different classes of diagram.

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