# Analytical Modeling of Link and Path Dynamics and Their Implications on Packet Length in MANETs

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## Abstract

We present an analytical framework and statistical models to accurately characterize the lifetime of a wireless link and multihop paths in mobile ad hoc networks (MANET). We show that the lifetimes of links and paths can be computed through a two-state Markov model and that the analytical solution follows closely the results obtained through discrete-event simulations for two mobility models, namely, random direction and random waypoint mobility models. We apply these models to study practical implications of link lifetime on routing protocols. We compute optimal packet lengths as a function of mobility, and show that significant throughput improvements can be attained by adapting information packet lengths to the mobility of nodes in a MANET.

## 1. INTRODUCTION

The communication protocols of mobile ad hoc networks (MANET) must cope with frequent changes in topology due to node mobility and the characteristics of radio channels. From the standpoint of medium access control (MAC) and routing, node mobility and changes in the state of radio channels translate into changes in the state of the wireless links established among nodes, where typically a wireless link is assumed to exist when two nodes are able to decode each other's transmissions.

The motivation for this paper is that, while the behavior of wireless links is critical to the performance of MAC and routing protocols operating in a MANET, no analytical model exists today that accurately characterizes the lifetime of wireless links, and the paths they form from sources to destinations, as a function of node mobility. As a result, the performance of MAC and routing protocols in MANETs have been analyzed through simulations, and analytical modeling of channel access and routing protocols for MANETs have not accounted for the temporal nature of MANET links and paths. For example, the few analytical models that have been developed for channel access protocols operating in multihop ad hoc networks have either assumed static topologies (e.g., [1]) or focused on the immediate neighborhood of a node, such that nodes remain neighbors for the duration of their exchanges (e.g., [2]). Similarly, most studies of routing-protocol performance have relied exclusively on simulations, or had to use limited models of link availability (e.g., [3]) to address the dynamics of paths impacting routing protocols (e.g., [4]).

This paper provides the most accurate analytical model of link and path behavior in MANETs to date, and characterizes the behavior of links and paths as a function of node mobility. The importance of this model is twofold. First, it enables the investigation of many questions regarding fundamental tradeoffs in throughput, delay and storage requirements in MANETs, as well as the relationship between many crosslayer-design choices (e.g., information packet length) and network dynamics (e.g., how long links last in a MANET). Second, it enables the development of new analytical models for channel access, clustering and routing schemes by allowing such models to use link lifetime expressions that are accurate with respect to simulations based on widely-used mobility models.

Recently, Samar and Wicker [5, 6] pioneered the work of analytical evaluation of link dynamics. They further provided good insights on the importance of an analytical formulation of link dynamics to the optimization of the protocol design. However, Samar and Wicker assume that communicating nodes maintain constant speed and direction in order to evaluate the distribution of link lifetime. This simplification overlooks the case in which either of the communicating nodes changes its speed or direction while the nodes are in transmission range of each other. As a result, the results predicted by Samar and Wicker's model could deviate from reality greatly, being overly conservative and underestimating the distribution of link lifetime [5, 6], especially when the ratio $R/v$ between the radius of the communication range $R$ to the node speed $v$ becomes large, such that nodes are likely to change their velocity and direction during an exchange.

The contribution of the paper is to provide a two-state Markov model that better describes the mobility behaviors for communicating nodes. Section 2. describes the network and mobility models used to characterize link and path behavior. Section 3. describes the proposed analytical framework and presents our results on link lifetime, and Section 4. extends these results to path dynamics. Our approach is based on a two-state Markovian model that reflects the movements of nodes inside the circle of transmission range and builds an analytical framework to accurately evaluate the distribution of link lifetime.

Our model subsumes the model of Samar and Wicker [5, 6] as a special case, and provides a more accurate characterization of the statistics of link lifetime. Section 5. illustrates the accuracy of our analytical model by comparing the analytical results against simulations based on the random direction mobility model (RDMM) and the random waypoint mobility (RWP) model.

Sections 6. illustrate how our model can be applied to practical problems in MANETs, where our analytical framework is applied to optimal segmentation (information packet length) of information streams. Our results reveal that packet lengths should be designed to be linearly proportional to the ratio $R/v$, and show that the optimal packet length for a given $K$-hop path should be designed to be $R/(vK)$.

## 2. SYSTEM MODEL

We consider a square network consistent with several prior analytical models of MANETs [7, 8, 9]. The entire network is of size $L \times L$ and there are $n$ nodes initially randomly deployed in the square network.

Nodes are mobile and initially equally distributed over the network. The movement of each node is unrestricted, i.e, the trajectories of nodes can be anywhere in the network. The model of node mobility falls into the general category of random trip mobility model [10], where nodes' movement can be described by a continuous-time stochastic process and the movement of nodes can be divided into a chain of trips.

Communication between nodes is allowed only when the distance between the two communicating nodes is less than $R$ and can be performed reliably. The communication between any two nodes within that communication circle satisfies the minimum SINR (signal to interference plus noise ratio) requirement with certain outage probability in the wireless fading environment.

A typical communication session between two nodes involves several control and data packet transmissions. Depending on the protocol, nodes may be required to transmit beacons to their neighbors to synchronize their clocks for a variety of reasons (e.g., power management, frequency hopping). Nodes can find out about each other's presence by means of such beacons, or by the reception of other types of signaling packets (e.g., HELLO messages). Once a transmitter knows about the existence of a receiver, it can send data packets, which are typically acknowledged one by one, and the MAC protocol attempts to reduce or avoid those cases in which more than one transmitter sends data packets around a given receiver, which typically causes the loss of all such packets at the receiver. To simplify our modeling of link lifetimes, we assume that the proper mechanisms are in place for neighboring nodes to find each other, and that all transmissions of data packets are successful as long as they do not last beyond the lifetime of the wireless link between transmitter and receiver. Relaxing this simplifying assumption is the subject of future work, as it involves the modeling of explicit medium access control schemes (e.g., [1]).

The following mathematical notations are used throughout the paper. $p(\cdot)$ stands for the probability density function (PDF), $p(\cdot|\cdot)$ denotes the conditional probability density function and $F(\cdot)$ is the complementary cumulative distribution function (CCDF).

## 3. LINK LIFETIME

A bidirectional link exists between two nodes if they are within communication range of each other. In this paper, we do not consider unidirectional links, given that the vast majority of channel access and routing protocols use only bidirectional links for their operation. Hence, we will refer to bidirectional links simply as links for the rest of this paper.

The wireless link between nodes $m_a$ and $m_b$ is broken when the distance between nodes $m_a$ and $m_b$ is greater than $R$, their transmission range and their distance increases. When a data packet starts at time $t_2$, the positions of node $m_b$ could be anywhere inside the communication circle defined by the transmission range of $m_a$.

Let $B$ ($bits/s$) be the transmission rate of a data packet, $L_p$ be the length of the data packet, and $t_2 + T_L$ denote the moment that node $m_b$ is moving out of the communication circle. A data packet can be successfully transferred only if nodes $m_a$ and $m_b$ stay within their communication range during the whole communication session of the data packet, that is,

$$L_p/B \quad \leq \quad T_L \tag{1}$$

where $T_L$ is the link lifetime (LLT) denoting the maximum possible data transfer duration. Statistically, $T_L$ specify the distribution of residence time that measures the duration of the time, for node $m_b$, starting from a random point inside the communication circle with equal probability, to continuously stay inside the communication circle before finally moving out of it. Furthermore, its CCDF is denoted by $F_L(t)$

$$F_L(t) = P(T_L \geq t) \tag{2}$$

The link outage probability $P_{L_p}$ associated with a particular packet length $L_p$ can be evaluated as

$$P_{L_p} = P(T_L < \frac{L_p}{B}) = 1 - F_L(\frac{L_p}{B}) \tag{3}$$

### 3.1. Distribution of Relative Velocity

Fig. 1 shows the transmission zone of a node (say node $m_a$) which is a circle of radius R centered at the node. The figure shows another node (say node $m_b$) starting DATA communication with node $m_a$ at time $t_2$. As shown in the left side of the figure, at time $t_2$, node $m_a$ is moving at speed $v_a$ of direction $\theta_a$ while node $m_b$ moves at speed $v_b$ and direction $\theta_b$.

Alternatively, if we consider node $m_a$ as static, node $m_b$ is then moving at their *relative speed* $v_r$ and direction $\theta_r$. An example of resulting trajectories of node $m_b$ moving at relative

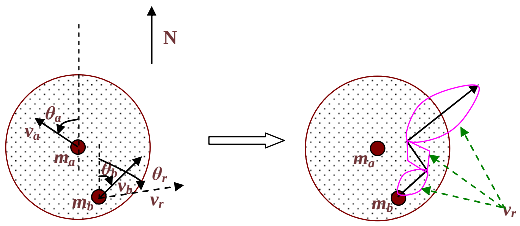

**Figure 1.** Graphical Illustration of Relative Velocity

velocity is given in the right side of Fig. 1. With the assumption that both $\theta_a$ and $\theta_b$ are uniformly distributed within $[0,2\pi)$, it can be concluded that composite direction $\theta_r = \theta_b - \theta_a$ is also uniformly distributed within $[0,2\pi)$. In this case, the relative speed $v_r$ can be expressed as

$$v_r = \sqrt{v_a^2 + v_b^2 - 2v_a v_b \cos\theta_r} \tag{4}$$

Conditioning on $v_a$ and $v_b$ and noting the symmetric property of $\theta_r$, the distribution of $v_r$ can be computed as

$$p(v_r) = E_{\{v_a,v_b\}}(p(v_r|v_a,v_b)) \tag{5}$$

$$\begin{aligned} p(v_r|v_a,v_b) &= p(\theta_r)|\frac{d\theta_r}{dv_r}| \\ &= \frac{1}{\pi}|\frac{d}{dv_r}(arccos(\frac{v_a^2+v_b^2-v_r^2}{2v_a v_b}))| \\ &= \begin{cases} g(v_r,v_a,v_b), & |v_a-v_b| \le v_r \le v_a+v_b \\ 0, & others \end{cases} \end{aligned} \tag{6}$$

where $g(x,y,z) = \frac{2}{\pi}\frac{x}{\sqrt{2(x^2y^2+x^2z^2+y^2z^2)-x^4-y^4-z^4}}$.

In particular, if both nodes move at the same speed $v = v_a = v_b$, we will have

$$p(v_r|v) = \begin{cases} \frac{2}{\pi}\frac{1}{\sqrt{4v^2-v_r^2}}, & v_r \in [0,2v) \\ 0, & others \end{cases} \tag{7}$$

### 3.2. Distribution of Link Lifetime (LLT)

The essence of modeling link dynamics in MANETs consists of evaluating the distribution of LLT, because it reflects the link dynamics resulting from the motions of nodes. LLT measures the duration of time for a node to continuously stay inside the communication range of another node. In our model, this range is a circle.

Clearly, different mobility models and parameters lead to different LLT distributions, and the main challenge in modeling LLT consist of making the problem tractable and relevant. We know that the relative movement of nodes consists of a sequence of mobility trips, derived from the chain of mobility trips of the two communicating nodes. Let $A_s$ be the starting point of current mobility trip and the end point of the current trip is denoted by $A_d$, and $A_d$ may be anywhere in the cell, i.e., inside or out of the communication circle. In the case that $A_d$ is located inside the communication circle, it serves as the starting point (i.e., new $A_s$) for the next trip and the whole process is repeated. In the evaluation of LLT, the repeating procedure ends when the final $A_d$ is outside of the communication circle.

As illustrated in Fig. 2, the procedure for evaluating the LLT can be modeled as a two-state Markovian process. The residence state $S_0$ represents the scenario where the end point $A_d$ of current trip is located inside the communication circle, while the departing state $S_1$ refers to the complementary scenario where $A_d$ will be outside of communication circle. Compared to the model by Samar and Wicker [5, 6], in which only the last scenario (i.e., state $S_1$) is considered, the two-state Markovian model reflects the motion of nodes more accurately, which leads to better results in evaluating link dynamics.

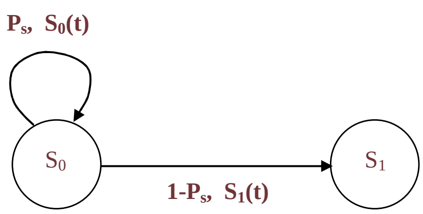


**Figure 2.** Two-state Markovian model for LLT evaluation

Let $P_s$ be the *residence probability*, which denotes the probability that $A_d$ is located inside the communication circle. The PDF $S_0(t)$ specifies the distribution of sojourn time of mobility epochs when a node stays in state $S_0$. Correspondingly, the PDF $S_1(t)$ is used to measure the distribution of departing times when nodes move out of communication circles and switch to the state $S_1$.

Before eventually moving out of the communication circle, i.e., being switched to the departing state $S_1$, nodes may stay at the residence state $S_0$ multiple times. Let $N_i$ be the integer variable counting the number of times for a node to remain in state $S_0$, and $\{S_{0,0},\ldots,S_{0,N_i-1}\}$ be the associated random variables that specify the duration of time of trips for each return.

Clearly, $\{S_{0,0},\ldots,S_{0,N_i-1}\}$ are random variables of the same distribution but correlated. However, to make our problem more tractable, we assume that $\{S_{0,0},\ldots,S_{0,N_i-1}\}$ are statistically i.i.d random variables of distribution $S_0(t)$. Our simplifying assumption makes the final result slightly deviated from the real situation when the residence probability becomes larger. However, as we will see later, our model still provides a good approximation, even with a large residence probability.

We define $S_1$ as the random variable measuring the departing time of distribution $S_1(t)$. Simply, one can evaluate conditional link life time $T_L(N_i)$ and $P(N_i = K)$ as

$$T_L(N_i) = \sum_{i=0}^{N_i-1} S_{0,i} + S_1, \tag{8}$$

$$P(N_i = K) = P_s^K. \tag{9}$$

The characteristic function $U_{T_L}(\theta)$ for the LLT $T_L$ can now be evaluated as

$$\begin{aligned} U_{T_L}(\theta) &= E(e^{j\theta T_L}) \\ &= \sum_{k=0}^{\infty} E(e^{j\theta(\sum_{i=0}^{k-1} S_{0,i}+S_1)})P(N_i = k) \\ &= \sum_{k=0}^{\infty} U_1(\theta)U_0(\theta)^k P_s^k \\ &= \frac{U_1(\theta)}{1-U_0(\theta)P_s} \end{aligned} \quad (10)$$

where $U_0(\theta)$ and $U_1(\theta)$ are the characteristic functions of $S_0(t)$ and $S_1(t)$ respectively.

When the communication circle is small with respect to the network size and nodes' speed, $A_d$ will be mostly located out of the communication circle of $A_s$. Consequently, one will have $P_s \ll 1$. Given that $U_0(\theta)$ is the characteristic function of $S_0(t)$, one has $|U_0(\theta)| \leq 1$. Finally, it is clear that $U_0(\theta)P_s \ll 1$. Therefore, Eq. (10) can be approximated as

$$U_{T_L}(\theta) \approx U_1(\theta) \quad (11)$$

For clarity, we call Eq. (10) the Exact LLT (ES-LLT), which is based on the two-state Markovian model. The approximation in Eq. (11) is called Approximated LLT (AS-LLT), and it reflects the scenario considered by Samar and Wicker [5, 6]. As we will see later, for random direction mobility model (RDMM), the analytical expression of AS-LLT is the same to the expression in [5, 6], except for a normalization factor.

## 3.3. Practical Implications

It is clear that the two-phase Markov model is a general model able to evaluate other networks with the two building blocks $S_0(t)$ and $S_1(t)$ adapted for the specific network and mobility models, including but not restricted to the random trip mobility model.

However, in some practical scenarios, the analytical formulations of $S_0(t)$ and $S_1(t)$ might not be available. Under such circumstances, one can collect a few trace data to obtain $S_0(t)$ and $S_1(t)$ and still give an accurate estimate of the overall link lifetime. By doing so, it can greatly reduce the amount of empirical data necessary to accurately estimate link lifetime. Furthermore, one can also obtain analytical formulations by curve-fitting empirical data and incorporate these formulations to Markov model for an analytical study of the mobility characteristics.

## 3.4. Link Lifetime in The Random Direction Mobility Model

The random direction mobility model (RDMM) is an important mobility model for MANETs. It improves on the random waypoint mobility (RWP) model on the stationary uniform nodal distribution, and has been widely adopted [11, 12, 13, 14, 15]. However, the analysis on the characteristic of link lifetime of RDMM is quite limited. In the section, we will supplement a deeper understanding of RDMM, by providing the analytical expression of characterizing its link lifetime.

In RDMM, node movement is independently and identically distributed (iid) and can be described by a continuous-time stochastic process. The continuous movement of nodes is divided into mobility epochs during which a node moves at constant velocity, i.e., fixed speed and direction. But the speed and direction varies from epoch to epoch. The time duration of epochs is denoted by a random variable $\tau$, assumed to be exponentially distributed with parameter $\lambda_m$. Its complementary cumulative distribution function (CCDF) $F_m(\tau)$ [13] can be written as.

$$F_m(\tau) = exp(-\lambda_m \tau) \quad (12)$$

The direction during each epoch is assumed to be uniformly distributed over $[0, 2\pi)$ and the speed of each epoch is uniformly distributed over $[v_{min}, v_{max}]$, where $v_{min}, v_{max}$ specify the minimum and maximum speed of nodes respectively. Speed, direction and epoch time are mutually uncorrelated and independent over epochs. The stationary node distributions of the location and direction have been shown to be uniform [16].

To evaluate the LLT $T_L$, we need to evaluate $P_s$, $S_0(t)$, and $S_1(t)$.Let $z_d$ denote the least distance to be traveled by node to move out of the communication circle, starting from the position $A_s$ with the direction and speed $v$ being kept unchanged. A graphical illustration of $z_d$ is presented in Fig. 3. The probability $P_s$ can now be evaluated through $z_d$ as

$$P_s = E_{z_d}(P_s(z_d)) = \int_{z_d} P_s(z_d)p(z_d)dz_d \quad (13)$$

$$\begin{aligned} P_s(z_d) &= \int_{v_r} P(\tau \leq \frac{z_d}{v_r})p(v_r)dv_r \\ &= \int_{v_r} (1 - F_m(\frac{z_d}{v_r}))p(v_r)dv_r \\ &= \int_{v_r} (1 - exp(-2\lambda_m z_d/v_r))p(v_r)dv_r \end{aligned} \quad (14)$$

where $P_s(z_d)$ is the conditional probability of $P_s$ on $z_d$. $p(z_d)$ is PDF of $z_d$ and the evaluation of $z_d$ directly follows from [17] being calculated as

$$p(z_d) = \begin{cases} \frac{2}{\pi R^2}\sqrt{R^2 - (\frac{z_d}{2})^2}, & \text{for } 0 \leq z_d \leq 2R \\ 0, & \text{elsewhere} \end{cases} \quad (15)$$

where $R$ specifies the radius of the communication circle.

$S_0(t)$ is the PDF of the time duration for nodes to return to the state $S_0$. Conditioning on $z_d$ and assuming that the starting time is at time 0, $S(t)$ is the probability of the node $m_b$ changing its

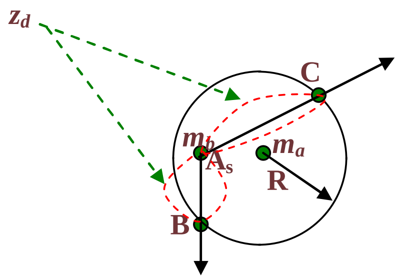


**Figure 3.** Graphical Illustration of $z_d$.

relative velocity at time $t$ on condition that $A_d$ is located inside the communication circle. We now compute $S_0(t)$ as below

$$S_0(t) = E_{z_d}(S_0(t|z_d)) \tag{16}$$

$$\begin{aligned} S_0(t|z_d) &= \frac{1}{P_s} P(t = \tau, z_d \geq v_r \tau | z_d) \\ &= \frac{1}{P_s} 2\lambda_m e^{-2\lambda_m t} \int_0^{\min\{V_m, \frac{z_d}{t}\}} p(v_r) dv_r \end{aligned} \tag{17}$$

where $S_0(t|z_d)$is the conditional PDF on $z_d$ and $V_m$ is the maximum speed of $v_r$.

$S_1(t)$ can be evaluated in much the same way as we have done for $S_0(t)$. Conditioning on $z_d$ and assuming that the starting time is at time 0, $S_1(t)$ is simply the probability of the node $m_b$ moving out of the communication circle at time $t$ with relative velocity being kept constant. Similar to the above case, we have

$$S_1(t) = E_{z_d}(S_1(t|z_d)) \tag{18}$$

$$\begin{aligned} S_1(t|z_d) &= \frac{1}{1-P_s} P(t = \frac{z_d}{v_r}, z_d \leq v_r \tau | z_d) \\ &= \frac{1}{1-P_s} P(\tau \geq t) p(v_r = \frac{z_d}{t}) |\frac{d}{dt}(\frac{z_d}{t})| \\ &= \frac{1}{1-P_s} exp(-2\lambda_m t) p_{v_r}(\frac{z_d}{t}) \frac{z_d}{t^2} \end{aligned} \tag{19}$$

where $S_1(t|z_d)$ is the conditional PDF on $z_d$ using Jacobian transformation. An alterative way to evaluate $S_1(t)$ is as follows:

Let us define $v_{s_1}$ to be the conditional relative velocity associated with state $S_1$ such as $p(v_{s_1}) = p(v_r|S_1)$ and it should be noted that the distribution of $v_{s_1}$ can be greatly different from the distribution of $p(v_r)$. We can then compute $S_1(t)$ as

$$S_1(t) = E_{v_{s_1}}(S_1(t|v_{s_1})) \tag{20}$$

$$\begin{aligned} S_1(t|v_{s_1}) &= \frac{1}{1-P_s} P(t = \frac{z_d}{v_{s_1}} \mid z_d \leq v_{s_1}\tau) \\ &= \frac{1}{1-P_s} P(\tau \geq t) p(z_d = v_{s_1} t) \frac{d}{dt}(v_{s_1} t) \\ &= \begin{cases} \frac{4e^{-2\lambda_m t}}{\pi(1-P_s)} \frac{v_{s_1}}{2R} \sqrt{1-(\frac{v_{s_1} t}{2R})^2}, & 0 \leq t \leq \frac{2R}{v_{s_1}} \\ 0, & \text{elsewhere} \end{cases} \end{aligned} \tag{21}$$

where $S_1(t|v_{s_1})$ is the conditional PDF of $S_1(t)$ on $v_{s_1}$. A detailed examination of Eq. (20) reveals that it shares the same core analytical expression of link lifetime distribution of Eq. (15) in [6], with the only exception that a normalization factor $e^{-2\lambda_m t}/(1-P_s)$ accounts for the probability of nodes leaving for state $S_1$. It implies that AS-LLT formula, solely relying on $S_1(t)$, gives the same link lifetime distribution as in [6].

## 4. PATH LIFETIME IN MANETS

We have examined the dynamics of link lifetime for a point-to-point link. However, for most cases in MANETs, a packet need to be forwarded by several intermediate nodes before finally reaching the destination. The source node, intermediate nodes and destination node collectively form a multi-hop path for the packet. Clearly, path dynamics is also an essential metric for protocol design and optimization. Han et al. show [18, 19] that asymptotically, path dynamics will converge to be exponentially distributed. The statement works well when a path involves a significant number of hops but not for paths with small to moderate number of hops. In this section, we will extend the proposed analytical framework to evaluate path dynamics with small to moderate number of hops, assuming that each link along the path behaves independently of others. In reality, adjacent links may have some correlation which is difficult to account for. The model of dependent links requires a number of conditional probability distribution and a solution may not be feasible. As to be observed, the independence assumption greatly simplifies the analysis but still provides good approximations.

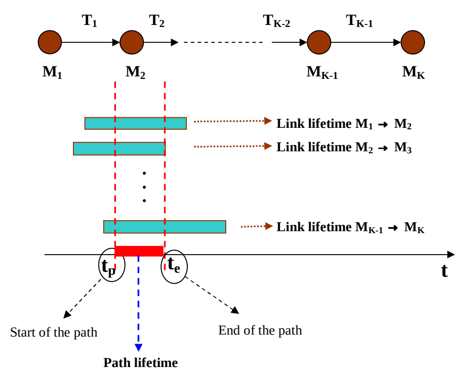


**Figure 4.** Path structure.

As illustrated in Fig. 4, a packet from the source node $M_1$ needs to follow the directional links $\{T_1 \to T_2 \to \ldots \to T_{K-1}\}$ to reach the destination node $M_K$. Successful delivery of the packet requires that none of these links on the path breaks during packet transmission. When either of them breaks, the path will no longer exist and path discovery process needs to be reinitiated to find an alternative path. In other words, lifetime $T_P(K)$ of the $(K-1)$-hop path is the minimum lifetime of these directional links and can be written as

$$T_P(K) = \min\{T_1, \ldots, T_{K-1}\}, \tag{22}$$

where $T_i$ for $1 \leq i \leq K-1$ is the link lifetime. Since links are assumed to operate independently with i.i.d motion[1], their lifetime also follows the same statistical distribution as $T_L$. However, when the source node initiates a data transfer to the destination node, links may have been in existence for some time; therefore, as Figure 4 illustrates, the link lifetime $T_i, i \in \{1, \ldots, K-1\}$ of the directional link on the data path should be the *residual lifetime* of the link, i.e,

$$T_i = T_L(\varepsilon_i), i \in \{1, \ldots, K-1\} \tag{23}$$

where $\varepsilon_i \geq 0$ is a random variable representing the elapsed time of the link $M_i \to M_{i+1}$ before the data path started and clearly, $T_L = T_L(0)$.

From Section 3., we know that the evaluation of $T_L(\varepsilon_i)$ depends on a set of three parameters, i.e., the spatial distribution of nodes at time $\varepsilon_i$, the distribution of speed $v_r(\varepsilon_i)$ at time $\varepsilon_i$, and the residual change time distribution $\tau(\varepsilon_i)$ at $\varepsilon_i$. At time 0 and $\varepsilon_i$, nodes are expected to follow the same stationary distribution and therefore resemble each other. Similarly, it can be expected that the speed distribution of $v_r$ will be also the same. Therefore, we expect that the distribution of $\tau(\varepsilon_i)$ and $\tau(0)$ will resemble each other. In particular for RDMM model, we know that the distribution of $\tau(0)$ for the RDMM model is exponentially distributed and can be characterized by a Poisson process. Referring to memoryless property of the exponential distribution, the distribution of $\tau(\varepsilon_i)$ and $\tau(0)$ will exactly resemble each other. Finally, we conclude that the distribution of $T_i$ will resemble the distribution of $T_L = T_L(0)$.

Summarizing the above discussion, the CCDF $F_P(K,t)$ of the lifetime for a $(K-1)$-hop path can be computed as

$$F_P(K,t) = F_L^{K-1}(t). \tag{24}$$

# 5. MODEL VALIDATION

## 5.1. Simulation Setup

In the simulation, there are a total of 100 nodes randomly placed in a $1000m \times 1000m$ square cell. Each node has the same transmit power and two profiles of the radio transmission range are chosen for simulation. Both are within the coverage of IEEE 802.11 PHY layer and they are $\{200m, 100m\}$. After initial placement, nodes keep moving continuously according to the RDMM model. The mobility parameter $\lambda_m$ is the same as the one in [20] ($\lambda_m = 4$), indicating that, on average, nodes change their velocity at every $\frac{1}{4}$ hour. Furthermore, we assume that every node is moving at the same constant speed and only its direction is changed according to the RDMM model. The simulation with variable speeds can be obtained by averaging the results from every speed with respect to the distribution of speed $v$. However, it should be noted that the relative speed between nodes are not constant and its statistics are derived in Section 3.1.. Three different speeds are simulated $v \in \{1, 10, 20\}(m/s)$, from pedestrian speed to normal vehicle speed. Combining the power profile and velocity profile, six different scenarios are simulated $\{I : (200m, 1m/s); II : (100m, 1m/s); III : (200m, 10m/s); IV : (100m, 10m/s); V : (200m, 20m/s); VI : (100m, 20m/s)\}$.

Nodes are randomly activated for data transmission. The traffic of activated nodes are supplied from a CBR source with a packet rate $0.5p/s$. Given that the choice of specific MAC layer and routing protocol may affect the results, we assume perfect MAC and routing, rendering zero delays or losses due to such functionality, enabling the simulation to capture statistics solely due to mobility.

## 5.2. Accuracy of Models

**Table 1.** Residence Probability $P_s$.

| **Radius (m)** ($R$) | **Speed** $v$ $(m/s)$ | | |
|---|---|---|---|
| | $v = 1$ | $v = 10$ | $v = 20$ |
| $R = 100$ | $P_s = 0.194$ | 0.033 | 0.018 |
| $R = 200$ | $P_s = 0.3072$ | 0.058 | 0.033 |

Table 1 describes the residence probability $P_s$ for all six scenarios. It can be observed that as shown in Eq. (16) and (18), the characteristics of mobility are governed by the relative radius (ReR) $\frac{R}{v}$, the ratio between the radius $R$ of communication circle and speed $v$. Among the six different scenarios, there are five different ReR values $\{5, 10, 20, 100, 200\}$ since the IV and V scenarios are of the same ReR and are expected to exhibit similar results, as will be seen from simulation results. As shown in Table 1, the residence probability increases with ReR, indicating that it is more likely for nodes with larger ReR to stay inside the communication circle.

Fig. 5 presents the results for link lifetime ES-LLT and AS-LLT predicted by our analytical model and obtained by simulations. The results clearly confirm that the two-state Markovian model is a powerful tool to model link dynamics of link lifetime distribution as a function of node mobility. It can be also observed that the ES-LLT formula, obtained from the Markovian model, shows good match with the simulations in all scenarios. On the other hand, the AS-LLT formula with the simplified assumptions corresponding to the model by Samar and Wicker [5, 6] gives good approximations to the simulations only for small values of ReR ($\frac{R}{v}$) and greatly deviates from the simulations when ReR becomes large, i.e., larger residence probability $P_s$ and larger possibility for nodes to stay inside communication circle.

As stated in section 3.3., in some practical scenarios, the analytical formulations of $S_0(t)$ and $S_1(t)$ might need to be obtained from empirical data to characterize the overall link lifetime. Fig. 6 presents such a result, where a total of 10000 trace datas are generated from random waypoint (RWP) model to evaluate $S_0(t)$ and $S_1(t)$, respectively.

There is no analytical formulations of $S_0(t)$ and $S_1(t)$ for RWP, because we do not have the closed-form CCDF for RWP similar to Eq.(12). However, the two-phase Markov model can still be applied by using empirical simulated data to estimate link lifetime. The results clearly confirm the accuracy, effectiveness and generarity of Markov model to analyze more practical mobility models.

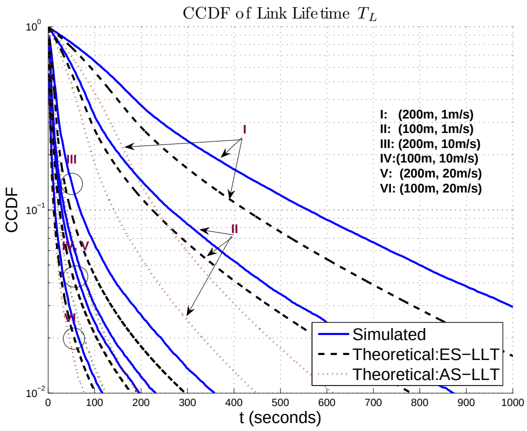


**Figure 5.** Link Lifetime $T_L$ (RDMM): Simulated, ES-LLT(Markovian), AS-LLT.

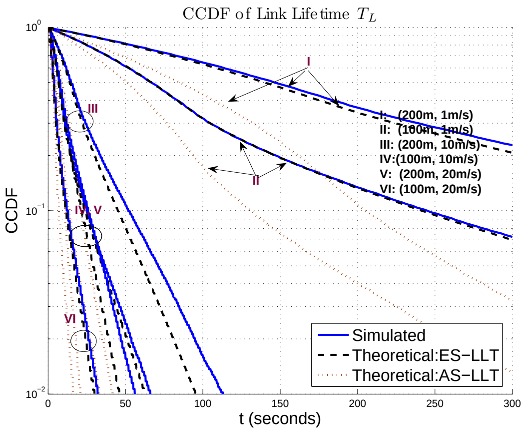


**Figure 6.** Link Lifetime $T_L$ (RWP): Simulated, ES-LLT(Markovian), AS-LLT.

Figs. 7 and 8 present the results of path lifetime. It can be observed that path lifetime can be well modeled by the proposed Markovian model, while slightly affected by the independence assumption.

In summary, the Markovian model (ES-LLT formula) is a much more accurate model than the AS-LLT formula [5, 6]

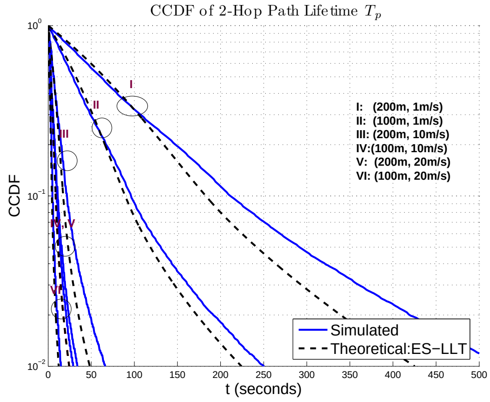


**Figure 7.** Simulation: 2-Hop Path Lifetime.

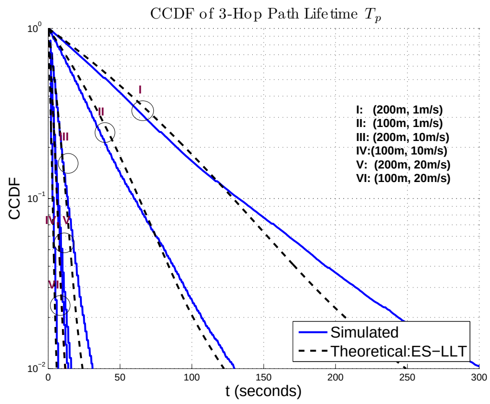


**Figure 8.** Simulation: 3-Hop Path Lifetime.

for all ranges of ReR and shows good approximations to all simulations, in contrast to the AS-LLT formula that gives good approximation only when ReR is relatively small.

# 6. PACKET LENGTHS AND AND THEIR OPTIMIZATION

## 6.1. Link Lifetime and Packet Length

In wireless communication, source information stream usually needs to be segmented into a sequence of fixed-length information blocks for transmission. These information blocks will be further processed (e.g. channel encoding) to fit into various transmission schemes.

Given that nodes move in a MANET, the data transfer can be temporarily broken if any link on the path to the destination is broken. An alternative path may not be available immediately, and significant delay can be incurred in repairing a route. Within the context of MANETs, it is important to use packet lengths that maximize the end-to-end throughput. If a data-packet length is too long, frequent link breaks can lead to significant packet dropout during the transfer. On the other hand, if data packet length is too short, the packet-header overhead and channel access overhead can reduce the effective throughput significantly. Hence, a judicious choice of packet lengths as a function of link dynamics can be of great importance in maximizing throughput in MANETs. However, this problem has been overlooked, because its solution requires knowledge of statistics of link lifetime. With the computed CCDF in Section 3., we are now able to provide segmentation schemes optimized on various systematic constraints.

When the length of packets is constant, it is natural to ask what the optimal packet length would be. For every packet length $L_p$, we know that there is an associated link outage probability $P_{L_p}$ specifying the probability of link breach during packet transfer. Every dropped packet during link outage needs to be retransmitted and therefore reduces the effective throughput. The optimal packet length is chosen such that the total throughput is maximized.

One approach is to simply choose the maximum possible packet length $L_0$ that satisfies a pre-defined link outage probability requirement. We term this strategy as link outage priority design (LOPD) and it can be described as

$$L_0 = max_{L_p}\ P_{L_p} \leq \omega_p \tag{25}$$

where $\omega_p$ is a constant to specify the link dropout probability requirement.

Alternatively, we can use a cost function $C(L_p, P_{L_p})$ that incorporates the negative effect from the packet retransmission into evaluating the effective throughput $T(L_p)$ for a specific packet length $L_p$. It is worthy of noting that the cost function $C(L_p, P_{L_p})$ could be a systematic constraint from upper layer to consider the negative effects from delay and packet retransmissions etc. Further optimizing the effective throughput $T(L_p)$ gives the optimal packet length $L_0$. Consequently, the strategy is termed as link throughput priority design (LTPD).

In the LTPD design, when the packet length is $L_p$, we can describe the effective throughput $T(L_p)$ function as

$$T(L_p) = (1 - P_{L_p}) \cdot L_p - C(L_p, P_{L_p}) \cdot P_{L_p} \cdot L_p \tag{26}$$

The optimal packet length $L_0$ will be the one that maximizes the effective throughput

$$L_0 = max_{L_p}\ T(L_p) \tag{27}$$

Normally, $P_{L_p}$ is a monotonically decreasing function w.r.t. packet length. When the cost function is chosen to be a constant penalty value, i.e., $C(L_p, P_{L_p}) = C$, by taking the derivative with respect to $L_p$, the optimal packet length $L_0$ is the value satisfying

$$1 - (1+C)P_{L_0} = (1+C)L_0\ \frac{dP_{L_p}}{dL_p}\bigg|_{L_p = L_0} \tag{28}$$

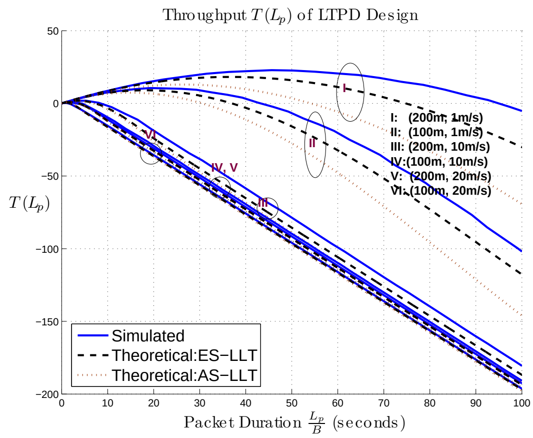


**Figure 9.** LTPD Design.

In Fig. 9, we exploit the application of link lifetime distribution to the optimization of packet-length design using the same examples of the previous section. For illustration purpose, the cost function for our example of LTPD design is chosen as a constant penalty value 2 (i.e., $C(L_p, P_{L_p}) = 2$). However, it should be noted that the practical cost function can be much more complicated and determined by upper layer for a cross-layer optimization solution. Computing the optimum choice for $C(L_p, P_{L_p})$ is beyond the scope of this paper. The effective throughput $T(L_p)$ is computed for every $L_p$ and drawn for all three methods: Simulated, ES-LLT (Markovian model) and AS-LLT. As expected, ES-LLT approximates the simulation very well, while AS-LLT tends to conservatively underestimate the effective throughput for larger ReR. In addition, all curves of the effective throughput (either Simulated, ES-LLT or AS-LLT formula) are convex functions with numerical solution readily available.

The optimized solutions $\frac{L_0}{B}$ of protocol on packet design for all design methods graphically illustrated Fig. 10. In the simulation, the link outage tolerance of LOPD design is set to be

$\omega_p = 0.1$, i.e., the maximum link outage probability should be less than 10%. Two key observations should be made: (1) For both LTPD and LOPD designs, the ES-LLT (Markovian model) approaches the simulated optimal solution well, and signifies substantial improvement of throughput over the AS-LLT model ([5, 6]); and (2) LTPD design suggests a balanced design between longer packet and larger retransmission rate to offer higher throughput over LOPD design. LOPD design, on the other hand, tends to be more conservative on the throughput but characterizing less packet retransmission.

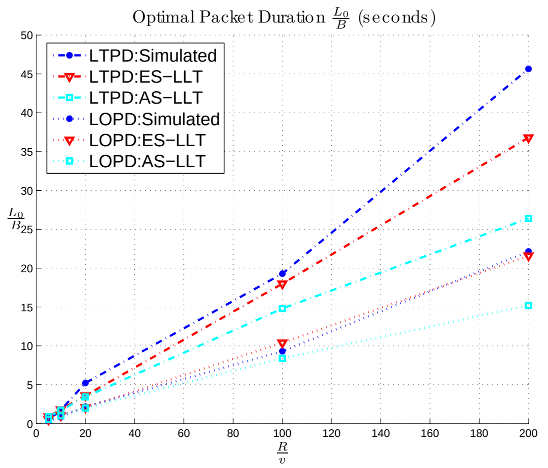


**Figure 10.** Optimal Packet Duration $\frac{L_0}{B}$.

Another important observation from Fig. 10 is that the optimal solutions, obtained from either the simulation or Markovian ES-LLT formula, exhibit linear proportion to the ReR value $\frac{R}{v}$. It suggests that mathematically, the optimal packet design should follow the rule[2]

$$\frac{L_0}{B} = \Theta(\frac{R}{v}) \quad (29)$$

## 6.2. Path Lifetime and Packet Length

We can also investigate the optimal packet length for a given path and the effect of hop count $(K-1)$ on the optimal packet length. Extending the optimal packet design example in Section 6. for a 2-hop path, the obtained results are shown below.

In Fig. 11, we only present the results following LOPD design strategy because the penalty of a path breakage is usually pretty high and a more practical design is to ensure that packet can get through the path with low outage probability. For example, in AODV protocol [21], when an existing path breaks, the source needs to flood the network to reinitiate a route to the destination. Furthermore, similar to the case of link lifetime, the linear relationship between the optimal packet length and network parameters can also be observed. Although only the results for 2-hop and 3-hop paths are shown here, we have examined cases with different hop counts (various $K$) and they all exhibit similar behaviors.

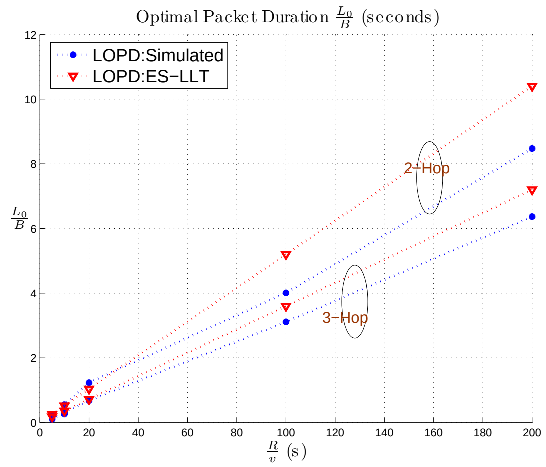


**Figure 11.** Optimal packet length for multi-hop paths.

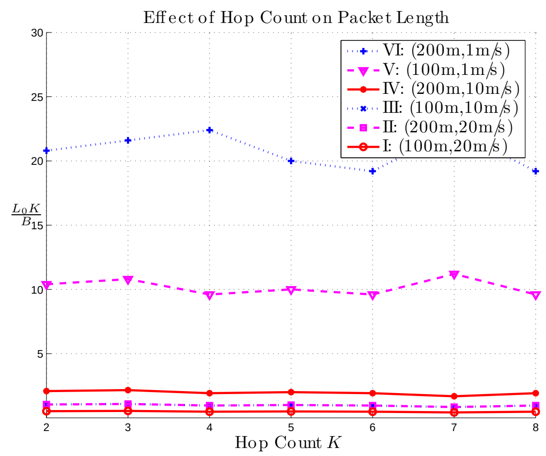


**Figure 12.** Effect of hop count on packet length.

Another aspect examined here is the effect of hop count on the choice of optimal packet length. In Fig. 12, for each $K$-hop path, the optimal packet length is chosen based on LOPD design criterion. We can see that the packet length should also be chosen such that[3]

$$\frac{L_0 K}{B} = \Theta(1). \quad (30)$$

Combining our observations from Figs. 11 and 12, we conclude that the packet length for a $K$-hop path should be designed as

$$\frac{L_0}{B} = \Theta(\frac{R}{vK}). \quad (31)$$

[2] We recall that $f(n) = \Theta(g(n))$ means there exist positive constants $c_1, c_2$ and $M$, such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ $\forall n > M$.

[3] Equivalently, we can transfer $K$ to the other side of this equation. It means that when the number of hops increases for a constant bandwidth $B$, the packet length should decrease.

## 7. CONCLUSIONS

We have presented an analytical framework for the characterization of link and path lifetimes in MANETs with unrestricted mobility. Given the existence of prior attempts to incorporate link dynamics in the modeling of routing and clustering schemes [22, 4, 23], we believe that this new framework will find widespread use by researchers interested in the analytical modeling and optimization of MAC and routing protocols in MANETs. The advantage of our framework is that it accurately describes link and path dynamics as a function of node mobility.

We illustrated how our framework can be applied by using it to address the optimization of packet lengths as a function of link and path dynamics in MANETs. The optimized solutions obtained from the proposed analytical framework show a substantial improvement on network throughput and protocol performance.

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