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Berkeley and Livermore, California

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ABSTRACT

Approximately 10^8 neutrons per discharge have been observed from linear deuterium pinches. These neutrons originated uniformly and simultaneously along a filament at the center of the discharge tube, but measurements of their energies showed that they were produced by a small group of axially accelerated deuterons and, therefore, were not of thermonuclear origin. It is tentatively proposed that the deuterons are accelerated by axial electric fields created by the growth of $m = 0$ ("sausage-type") instabilities.

I. INTRODUCTION

Work with the Pinch Effect at the Berkeley Radiation Laboratory started in April 1948 with equipment originally intended as a focusing device for the cyclotron beam.[1] The constriction of the gaseous discharge with higher currents was noted and the importance of a symmetrical current return path determined. When the Sherwood Project was started, the information obtained from this experiment was applied to the problem of producing controlled thermonuclear energy.

Several inert gases were used in the earliest studies of the linear pinch for the Sherwood Project, with particular emphasis upon helium because of its low atomic weight and the usefulness of the 4686 Å ionized helium line for diagnostic purposes. However, it was felt from the beginning that the detection of neutrons from D-D reactions would be a very powerful tool for optimizing the operating conditions, so as soon as there was spectroscopic evidence that energies of several hundred electron volts could be produced in helium discharges the gas was switched to deuterium. Neutrons were quickly detected and the average number of neutrons per discharge was increased rapidly.

A number of experiments were performed which gave the origin of the neutrons in space and time, and the effects of variations in circuit parameters, gas pressure, and impurities. All of the measurements seemed consistent with a thermonuclear reaction except that the neutron yield, which eventually reached 10^8 per discharge, was very much higher than had been expected from simple calculations.

In the end we were able to show by measurement of the neutron energies that the reactions were not of thermonuclear origin but rather were produced in the plasma by deuterons which were accelerated along the axis of the discharge tube to energies up to 2 x 10^5 electron volts (several times the voltage across the tube), probably by very strong electric fields produced transiently by m = 0 (sausage-type) instabilities.[2]

II. THEORY

The simple theory of the dynamic pinch is now rather widely understood[3-10] so in this section only a brief summary of some of the ideas will be given. If a longitudinal electric field is suddenly applied to a cylinder of ionized low density gas, a longitudinal current flows in a thin layer at the surface of the plasma and the column collapses radially under the magnetic pressure. A shock wave preceding the current sheath goes through the axis and reverses the direction of the current sheath causing the column to expand. The plasma may expand and contract several times before instabilities destroy the ordered motion. The theory of this model has been developed by Marshall Rosenbluth.[10]

The circuit is shown schematically in Fig. 1. In the experiments described here the inductance external to the pinch tube is much larger than the inductance of the pinch during most of the time of interest, and consequently the current through the plasma is $I = I_o \sin \omega t = \omega C V_o \sin \omega t$, where

ω = 2π (frequency of the current oscillation)
C = capacitance of the condenser bank
V_o = initial voltage of the condenser bank.

The equation of motion of the contracting pinch is described in detail in Appendix I – based upon the assumption of snow-plow hydrodynamics and a thin boundary layer current. The time rate of change of the momentum equals the force.

$$-\frac{d}{dt}\left(M\frac{dr}{dt}\right) = \frac{B^2}{8\pi} = \frac{I^2}{8\pi\, 25\, r^2} \qquad (1)$$

where M is the accumulated mass per square centimeter of the contracting pinch surface.

$$M = \frac{\rho}{2\pi r}\int_R^r 2\pi r\, dr \qquad (2)$$

$$= \frac{\rho}{2r}\left(R^2 - r^2\right)$$

ρ = initial density in g/cm^2
r = pinch radius
R = initial radius.

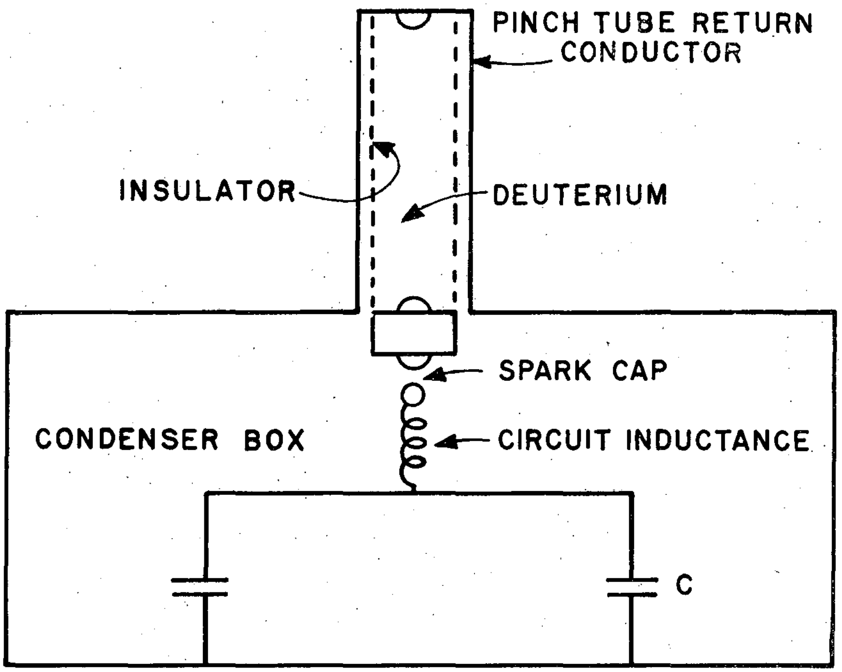

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Fig. 1. Pinch circuit schematic.

When the resulting equation is integrated from the initial radius to a small final radius like 1/10 R, the time becomes

$$t_P = 1.39\, R \sqrt[4]{\frac{100\, \pi\rho}{I_o^2\, \omega^2}} \text{ seconds} \tag{3}$$

which is in good agreement with experiments using deuterium pressures of 50 microns to 5 mm. of Hg.

Typical data taken on each pulse were the current through the pinch tube, the voltage across the tube and the neutron flux, all as functions of time (Fig. 2). The voltage signal is equal to (d/dt)(LI), except for a small resistive drop, and the trace of Fig. 2 may be interpreted as follows:

A. Full condenser voltage is across the pinch tube.

B. The gas is ionized and the voltage is $L\dot{I} + IR$.

C. and D. The current sheath is moving toward the center of the tube and the increase in voltage is given principally by $I\dot{L}$.

E. The shock wave which has preceded the boundary of the plasma has gone through the axis of the tube and reversed the direction of motion of the current sheath changing the sign of the $I\dot{L}$ term.

F. $I\dot{L}$ is still negative but the outward speed of the current sheath is decreasing.

G. The sheath has made its maximum radial excursion and is about to move toward the axis once more.

H. thru L. are the records of three more compressions. In some cases the peak voltages are many times the voltage originally applied to the tube. At times later than L (or sometimes K) the voltage trace consists of a very high frequency noise signal lasting for approximately 10^{-7} sec after which all detail disappears. Either instabilities fill the whole tube with plasma at this time or the insulator becomes conducting. In either case the interior of the pinch tube is isolated from the external circuits.

An estimate of the yield from a possible thermonuclear reaction is given in the Appendix, where it is shown that a negligible number of reactions is expected.

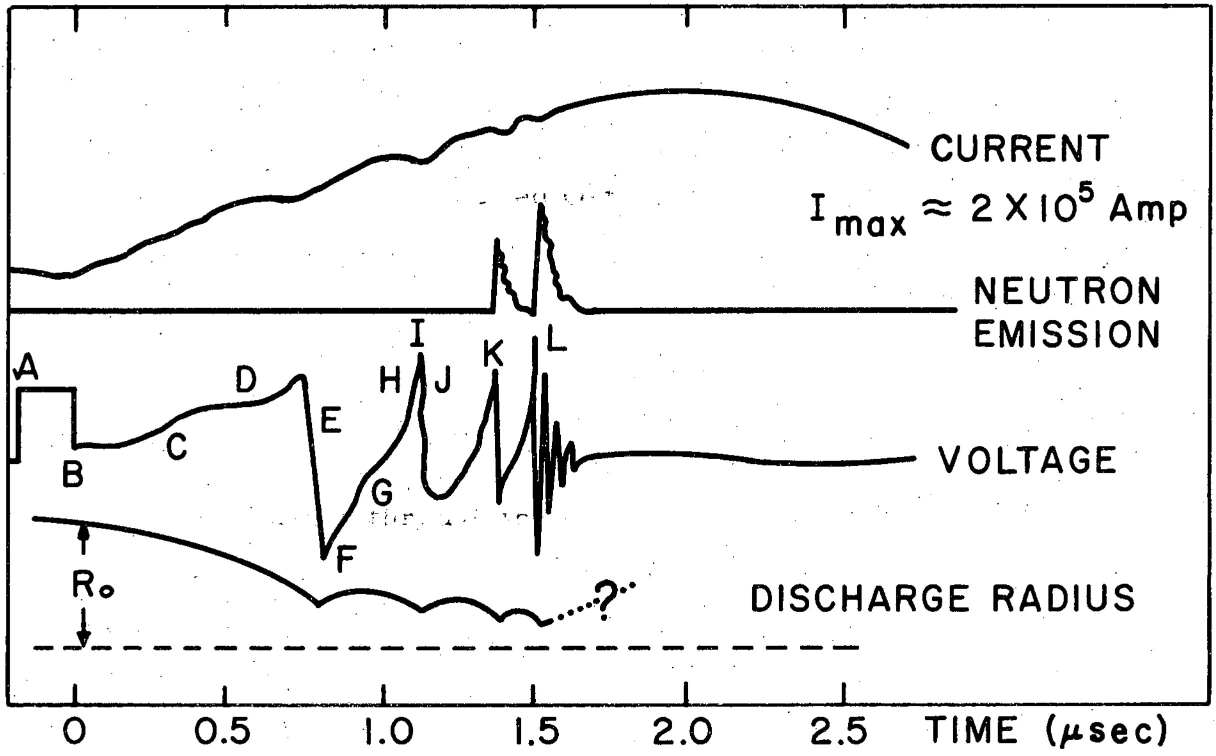

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Fig. 2. Pinch signals.

III. EXPERIMENTAL PROCEDURE

A. Apparatus

The pinch circuit was designed to have minimum inductance external to the pinch tube so that a large fraction of the condenser voltage would appear across the tube and the current would rise as rapidly as possible. Two exceptions should be noted: firstly, only easily available condensers were used in order that there should be no delays in replacing defective units and, secondly, the pinch tube was mounted outside of the condenser box (increasing the inductance somewhat) so that measuring equipment could be placed easily near the regions of interest. The condenser bank contained 25 Cornell-Dubilier 0.5 μf, 50 kv condensers and together with the spark gap had an inductance of 2×10^{-7} henrys.

Approximately thirty pinch tubes were built in connection with the experiments described in this paper. Figure 3 is an exploded view of a typical early tube showing the bolted O-ring seal system, aluminum electrodes, gas connections, the copper return conductor and cooling water connections. Also shown are the 7.5 cm x 45 cm quartz insulator and a polyethylene sleeve which sealed in the cooling water.

Figure 4 is a cross-sectional view of another tube of somewhat different construction.

The voltage across the tube was monitored by using the cooling water as a voltage divider, and the current was obtained from non-inductive shunts. Pure gases were flushed through the tubes continuously to reduce the contaminant problem.

Pyrex tubing was used originally but an examination of the voltage signals showed that the plasma radius remained constant for about one-half microsecond longer than expected, indicating that a considerable amount of material was being evolved from the surface. When the pyrex was changed to quartz the current sheath was observed to leave the wall almost immediately and much more detail was visible on the voltage trace, Fig. 5. With other parameters unchanged, one hundred times as many neutrons were produced with a quartz insulator as with one made of pyrex.

The absolute number of neutrons per pulse was obtained with a long boron counter and calibrated Po-Be source, but for day-to-day use a lead-shielded, europium-activated lithium iodide crystal and photomultiplier surrounded by 5 cm of paraffin was more convenient. The timing of the neutron bursts was obtained from proton recoils in small volumes of plastic scintillator. Several plastic scintillators were used to compare the timing and relative numbers of neutrons from different parts of a discharge. The spatial origin of the neutrons was determined with plastic scintillators which were either imbedded in blocks of paraffin containing channels of various sizes or hidden in the shadow of a scattering material. Neutron and gamma ray energies were measured with nuclear emulsion.

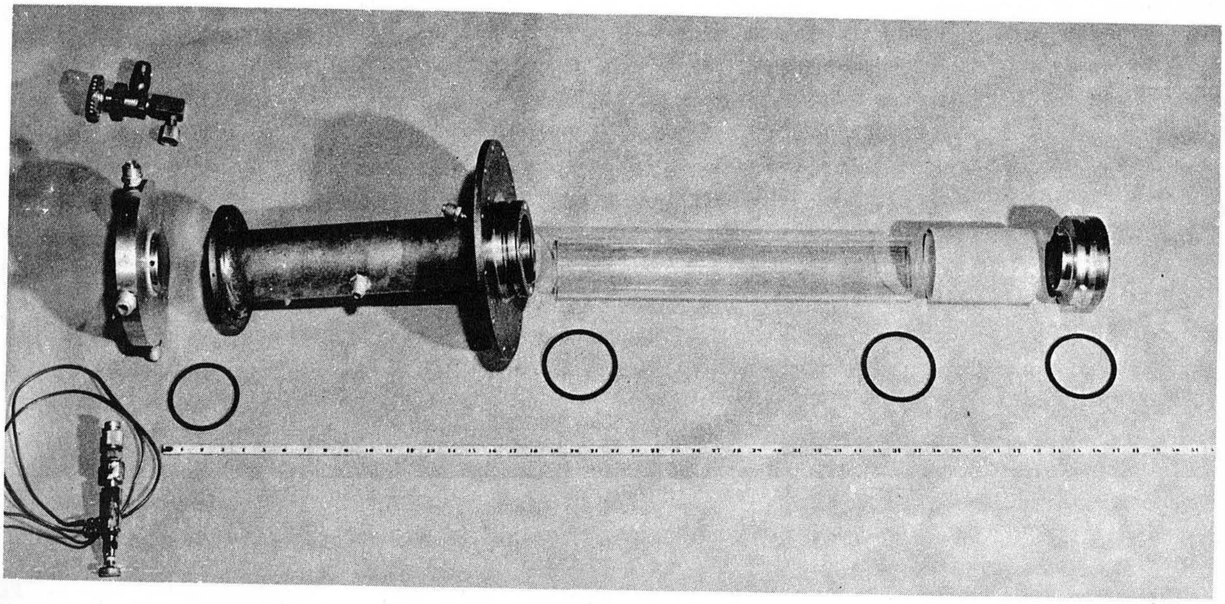

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Fig. 3. Exploded view of a pinch tube.

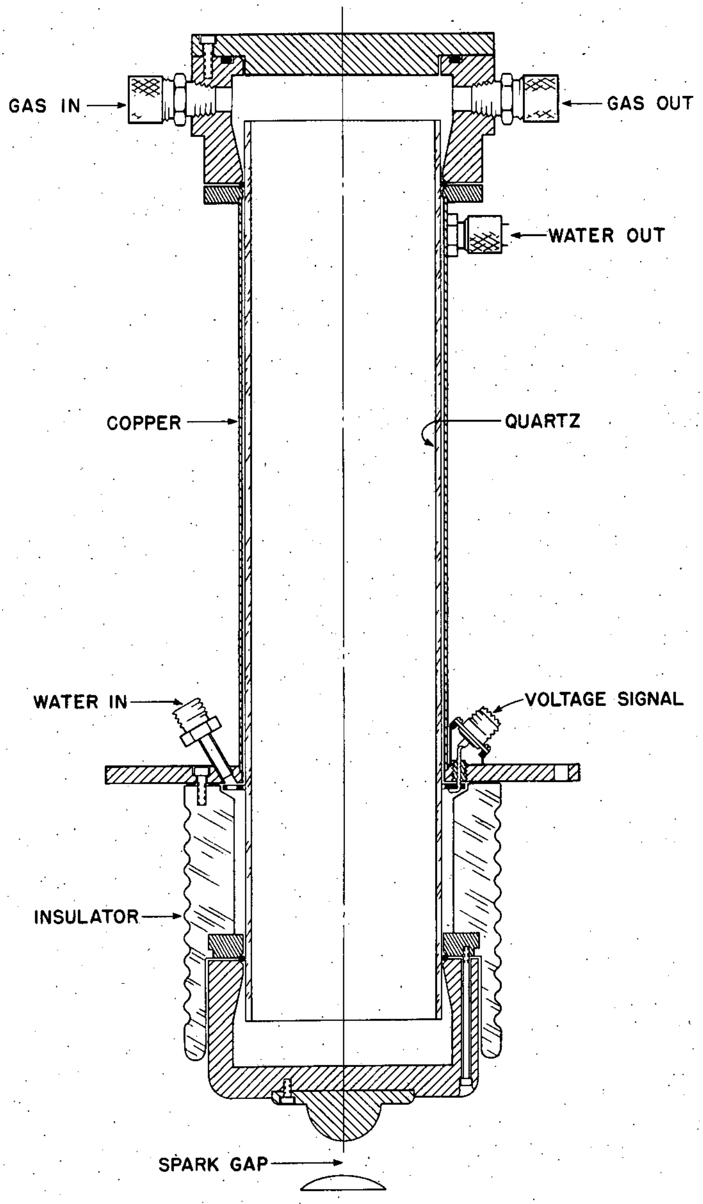

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Fig. 4. Cross section of a pinch tube.

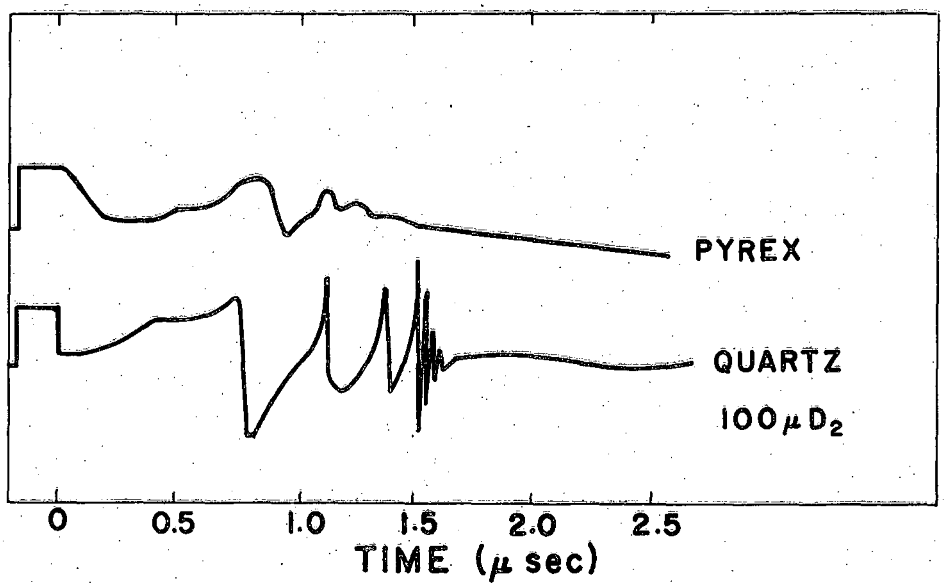

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Fig. 5. Voltage across a pinch in pyrex and quartz insulator tubes.

IV. NEUTRON MEASUREMENTS PERTAINING TO THE PRODUCTION MECHANISM

This material is presented in a more or less chronological fashion. The early experiments seemed to be reasonably consistent with a thermonuclear origin of the neutrons, whereas the later experiments conclusively excluded this possibility.

A. Yield

Neutron yields as functions of deuterium pressure and condenser voltage for a 45 cm long, 5.5 cm diameter pinch tube are shown in Fig. 6. The yield was predicted to increase with increasing voltage (see Appendix) and the decrease at high voltages is not understood. A theory that this effect was caused by improper ionization at large field strengths has been disproved. There are indications that the insulating tube may break down before $m = 0$ instabilities (which are now thought to be the origin of the D-D reactions) have grown to large amplitudes.

B. The Time of Neutron Emission

The maximum neutron emission was usually at the time of the third contraction of the plasma, although some tubes gave sizeable yields on the second bounce (Fig. 2). The shape of the curve is interesting in that it rises to the maximum rate of neutron emission in about 10^{-8} seconds and then falls to zero in approximately 2×10^{-7} seconds. The emission at the time of maximum contraction and maximum energy in the pinch is consistent with thermonuclear origin, but the sudden rise in output probably can be explained only by invoking some sort of instability. This will be discussed in Section V.

C. Quenching by Impurities and Axial Magnetic Field

Impurities would lower the neutron output in two ways if the plasma existed for a sufficient length of time: firstly, by reducing the ion energies by the amount required to ionize the impurities, and secondly, by lowering the electron temperature by bremsstrahlung. The second effect is unimportant in these experiments because there is insufficient time for appreciable energy exchange between ions and electrons.

The first experimental evidence concerning impurities was the one-hundred-fold increase in neutron yield when the insulator was changed from pyrex to quartz. Contaminant gases were then deliberately introduced and the observed variation of yield with contaminant concentration is shown in Fig. 7.

The yield also decreased greatly when weak axial magnetic fields were applied to the tube (Fig. 8). It will be shown later that this behavior is difficult to explain in a manner compatible with a true thermonuclear reaction unless the compressions are extremely high.

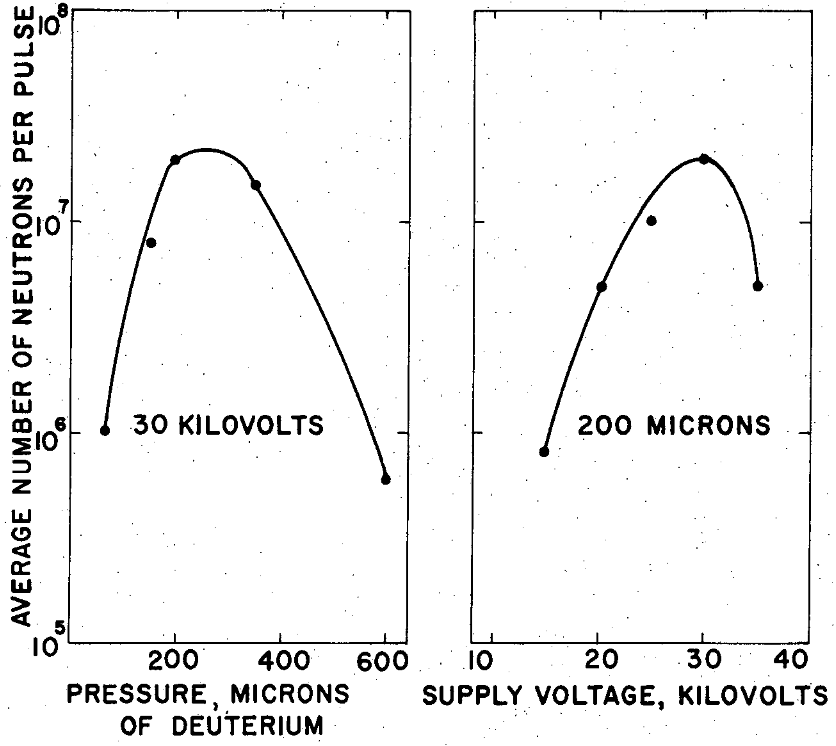

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Fig. 6. Neutron yields vs. voltage and pressure in 5 cm x 45 cm quartz tube.

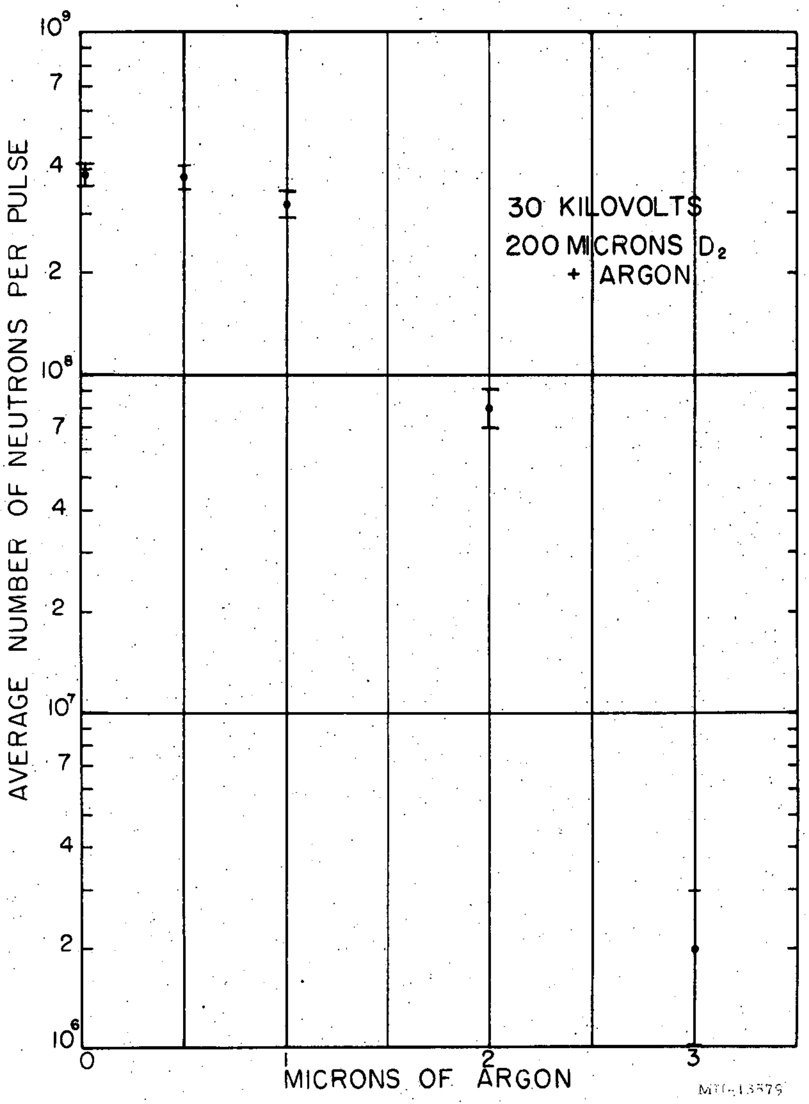

Fig. 7. Neutron yield from 5 cm x 90 cm quartz tube with 200 μ D_2 + an argon partial pressure.

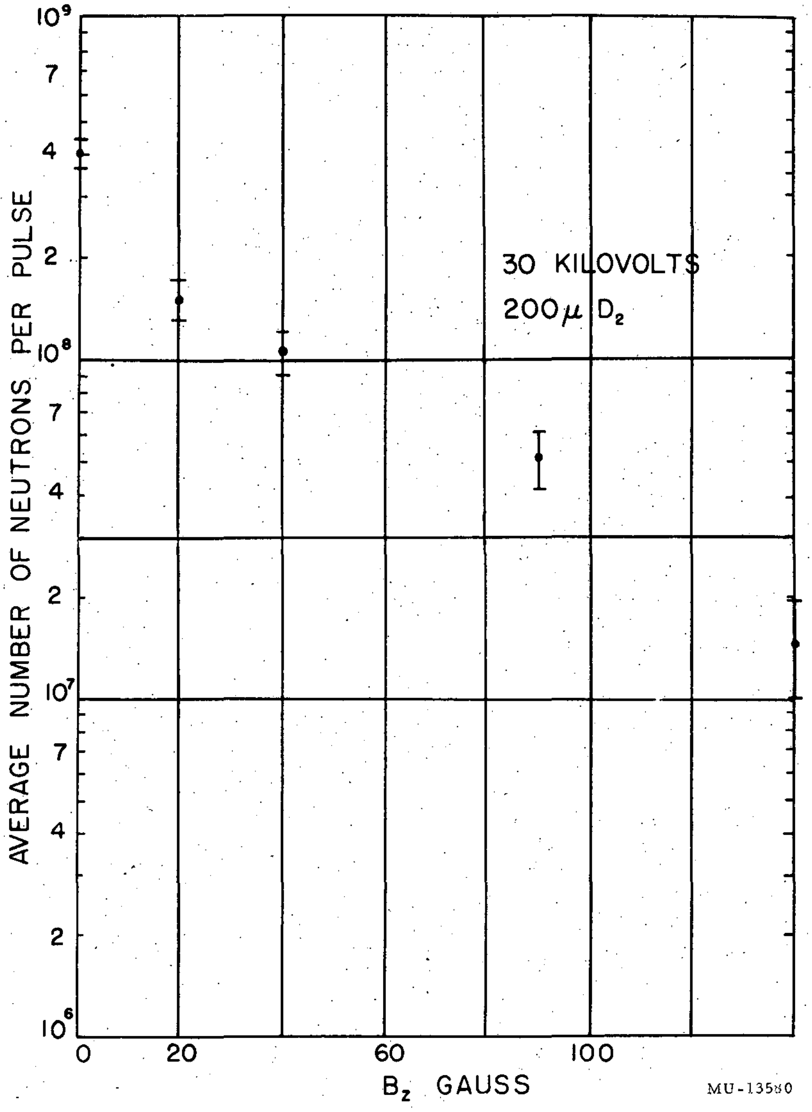

Fig. 8. Neutron yield from 5 cm x 90 cm quartz tube with 200 μ D_2 and an axial magnetic field.

D. The Spatial Origin of the Neutrons

The axial distribution.

One of the first measurements was to determine whether neutrons were produced at one of the electrodes or along the axis. Lead-shielded plastic scintillator recoil detectors in paraffin collimators were moved along the axis while the total yield was monitored with a LiI detector several feet away. In this manner it was shown the neutron production decreased slightly near the electrodes, excluding the bombardment of adsorbed deuterium as a production mechanism.

Because of the difficulty of collimating neutrons, measurements were also made on a pinch tube 90 cm long and 5.5 cm in diameter, with plastic scintillator recoil detectors shielded only against x-rays (Fig. 9).

We may assume the detector has a small effective area at a distance of 4 cm from the center line of the tube, the same when viewed from any angle. If the neutron production is uniform along the axis, the calculated relative numbers detected will be at a distance z along the axis as shown in Fig. 9. The experimental agreement is quite good.

Because it was difficult to collimate the neutrons striking the neutron recoil detector, it was not possible to determine the radial extent of the source of neutrons precisely. However, it was easy to prove that the neutrons did not originate at the walls and, in fact, came from a cylinder not greater than two centimeters in diameter. It was then possible to proceed to a detector capable of better angular resolution, namely a plastic scintillator recoil detector in the shadow of a thin wedge of scattering material (copper and wolfram were used) (Fig. 10). The data, Fig. 11, indicated neutron production within a cylinder of not more than one centimeter in diameter, but greater precision could not be achieved without additional detectors because the constricted plasma apparently was displaced as much as 0.5 cm from the geometrical axis of the pinch tube during some of the discharges.

E. The Simultaneity of Neutron Emission Along the Axis

Two recoil detectors were placed on the side of the 90 cm long tube mentioned in IV(D), one of them at a fixed position as a monitor and the other moved parallel to the axis of the tube. Although some random structure in time was observed from one discharge to the next, the onsets of neutron signals were simultaneous to within 10^{-8} seconds at all points. The uniformity in intensity and simultaneity along the axis disposed of the possibility that deuterons accelerated in a high electric gradient near an electrode might react with deuterons along the axis and thus be responsible for the observed neutron production.

F. Neutron Energy Measurements

Although the rapid quenching of the neutron production by small percentages of impurities seemed to favor a thermonuclear reaction, the

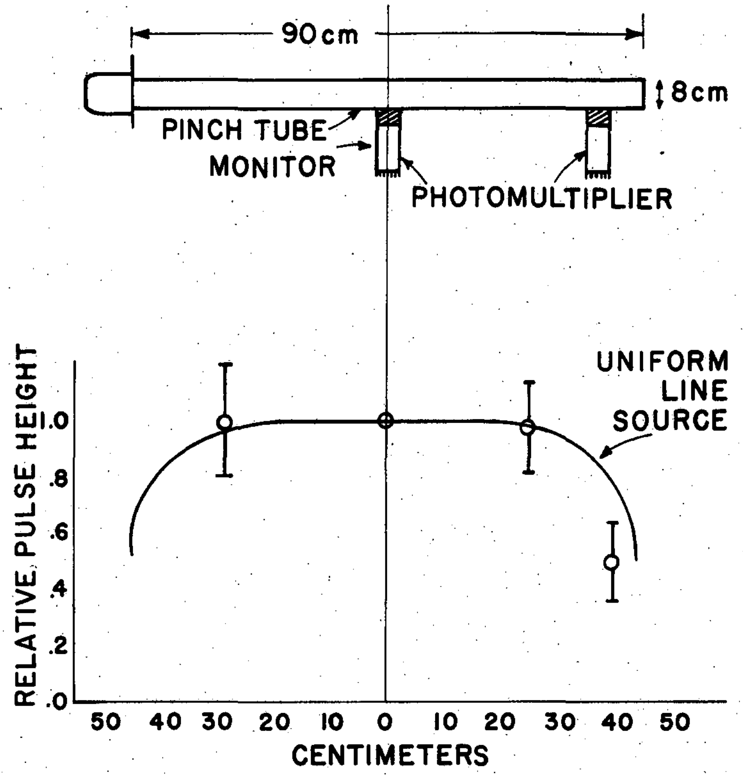

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Fig. 9. The axial variation in neutron yield compared to the calculated yield from a uniform line source. The results are the same with either polarity of applied voltage.

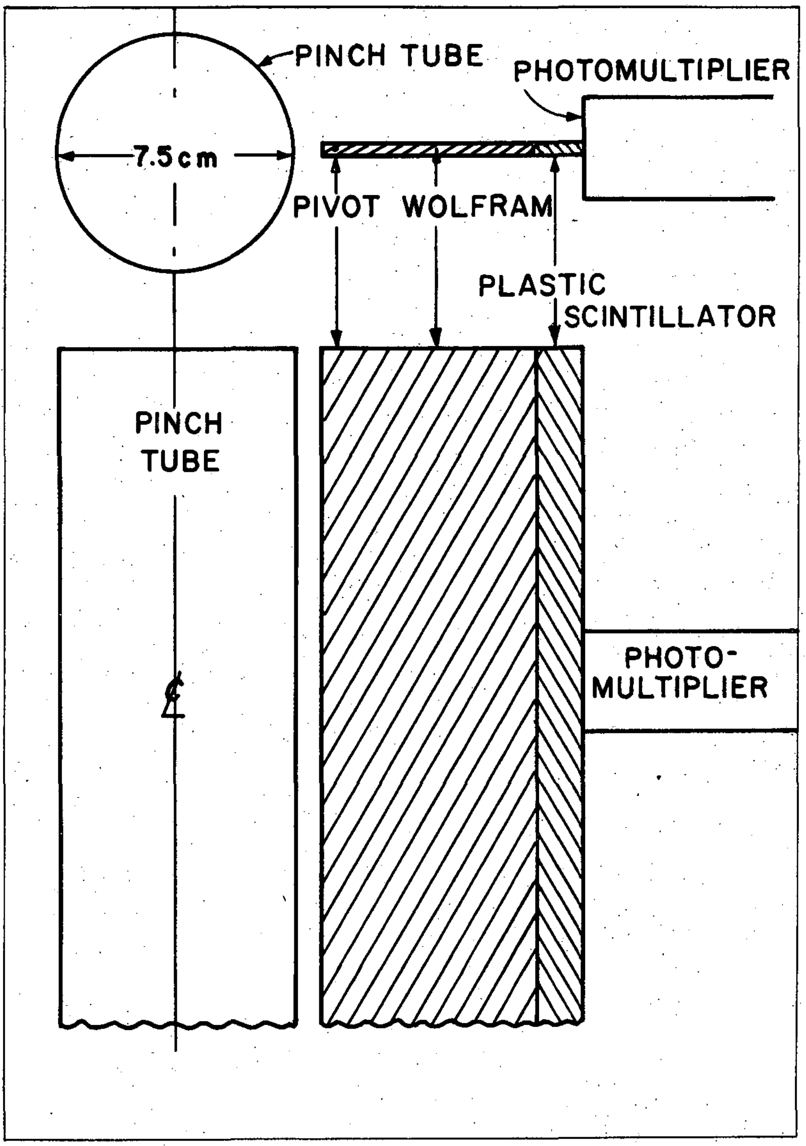

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Fig. 10. Apparatus for radial neutron distribution measurement.

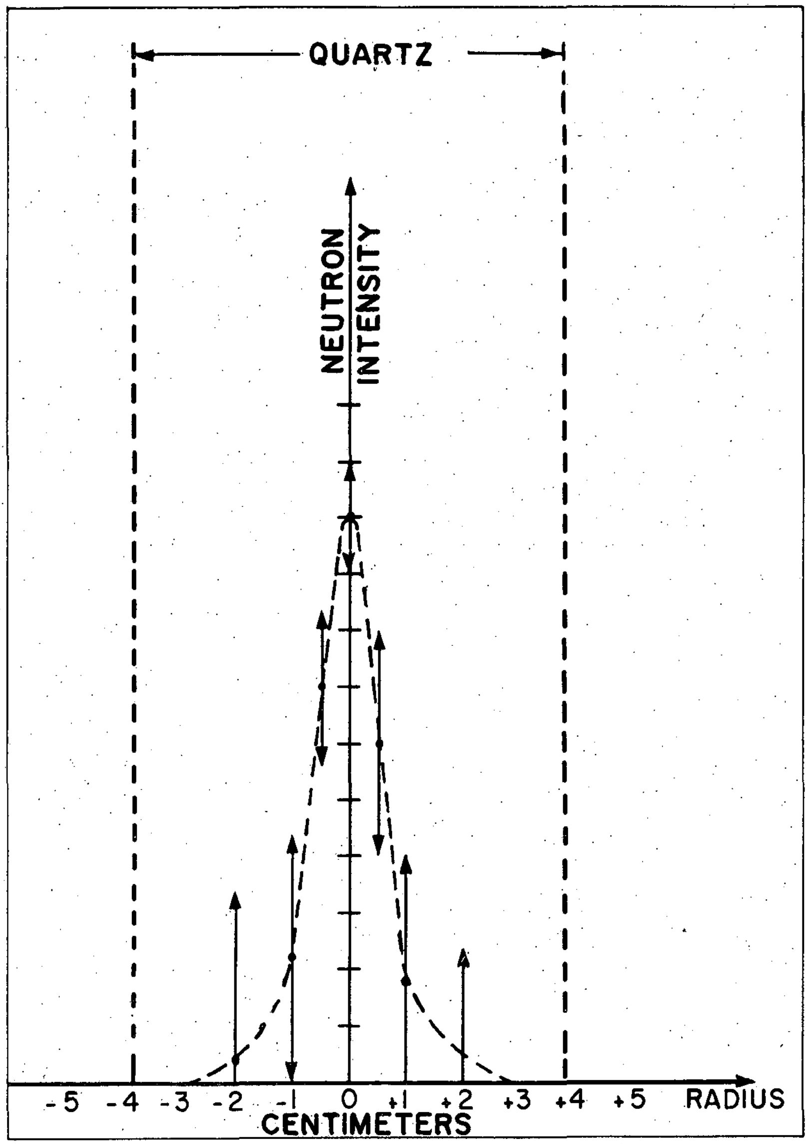

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Fig. 11. Radial neutron distribution.

unexpectedly high yields forced a continued search for some other production mechansim. The voltage across the pinch tube was a few tens of kilovolts at the time of neutron emission; if deuterons were accelerated to ten or more kev in a particular direction, then it should be possible to find this out by careful measurements of the neutron energies (Fig. 12). The histograms of the forward direction ($\pm 10^{o}$) recoil protons produced in Ilford C2 nuclear emulsions by neutrons emitted at 0^{o} and 90^{o} (laboratory system) from a deuterium-loaded metal target bombarded by 45 kev deuterons are shown in Fig. 13.

Ilford C2 nuclear emulsions were exposed at the sides and at 10 cm from each end of the pinch tube. An additional set of emulsions was exposed at the ends with the voltage applied to the pinch tube reversed in polarity. The tube used for these measurements contained a 46 cm long, 7.5 cm inner diameter quartz insulator and was filled with 100 μ deuterium. The condenser bank was charged to 35 kilovolts, but the measured voltage across the pinch tube at the time of neutron emission was 10-20 kilovolts.

The recoil proton histograms from neutrons emitted from the pinch tube at 0^{o} and 180^{o} to the direction of the applied electric field are shown in Fig. 14. Equivalent neutron and bombarding deuteron energies are also shown. One sees that the spectra may be interpreted as being produced by deuterons of 50 to 100 kev which are moving parallel to the axis of the pinch tube and bombarding other deuterons at rest. In some of the cases the bombarding energies must have been at least 200 kev. The thermonuclear contribution, if any, must have been quite small.

G. High Energy Electron Search

It seemed possible that under certain conditions high energy electrons might be produced by the same mechanism that accelerated the deuterons. Newly poured Ilford G5 electron-sensitive emulsions were exposed at the ends of the discharge tube and scanned for electrons of 100 kev or more, this being the minimum energy for penetration of the end foil. Some electrons of many Mev were detected in an apparently statistically significant fashion. However, it is possible that the effect is spurious, and in any event the electrons accelerated to such energies are very few at best.

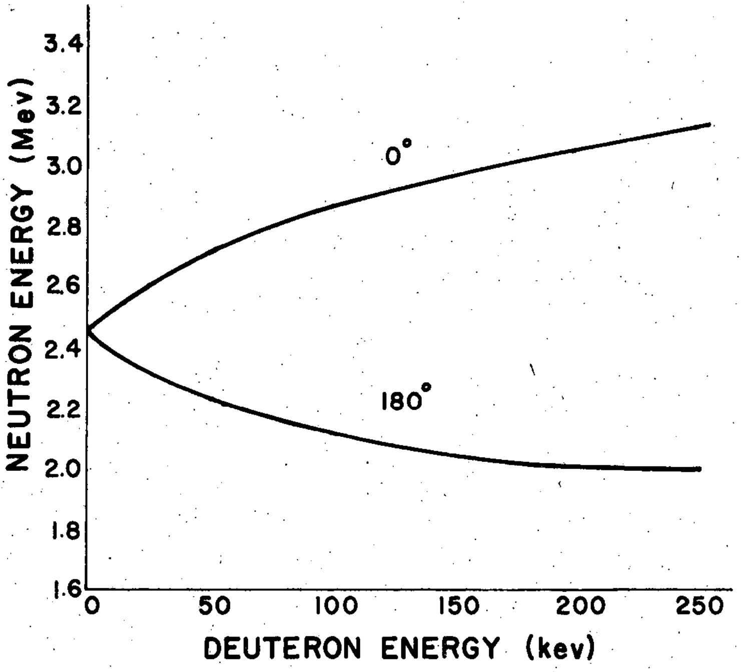

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Fig. 12. Neutron energies from D-D reactions as a function of incident deuteron energy.

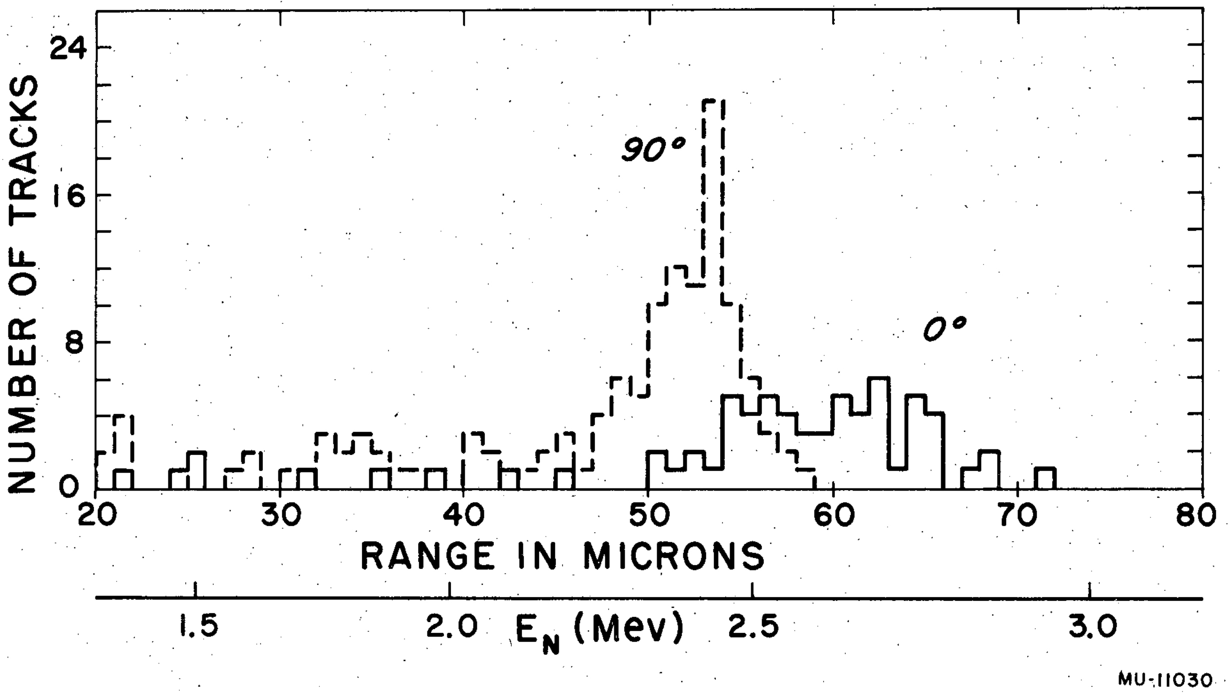

Fig. 13. D(D, n)He^3 neutron energies and recoil proton ranges in Ilford C2 emulsion at 0° and 90° to the 45 kilovolt Cockcroft-Walton beam.

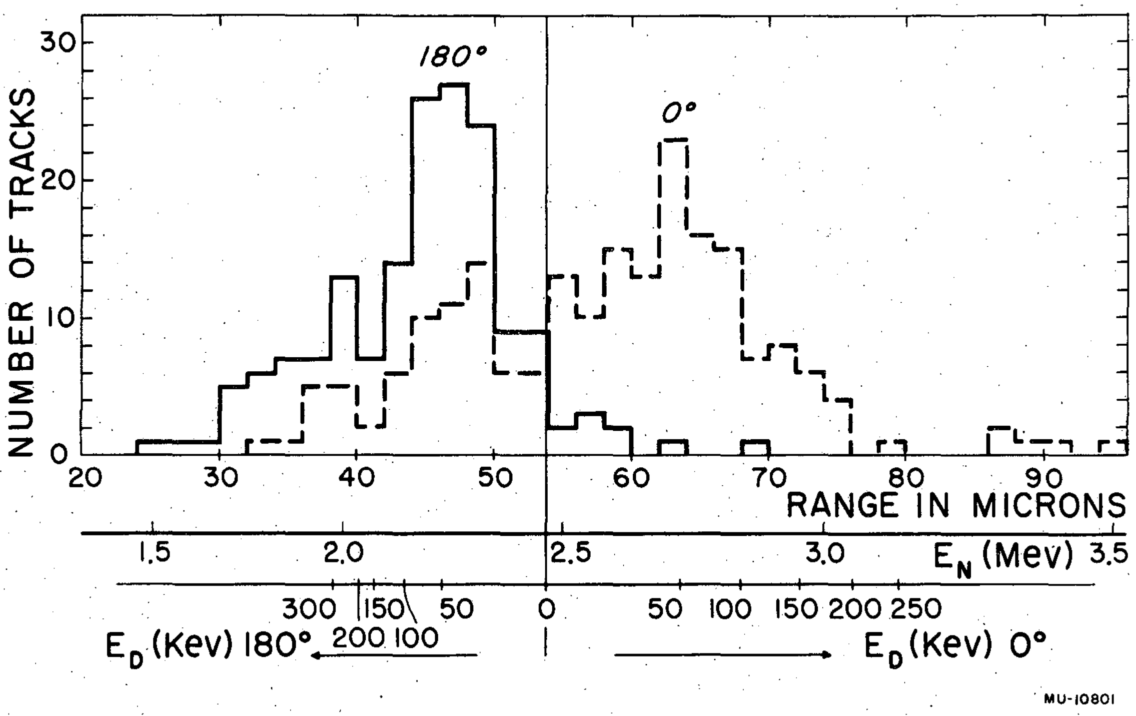

Fig. 14. Recoil proton histograms from $D(D, n)He^3$ neutrons emitted at 0^o and 180^o to the applied electric field.

V. INTERPRETATION

The results of the experiment in which the neutrons from the dynamic pinch were observed are inconsistent with a thermonuclear origin. The proposed explanation of the pinch neutron phenomena that will be discussed here is dependent upon the formation of the m = 0, or sausage-type pinch instability mode. The rapid dynamic growth of this instability mode results in a rapid change of inductance of the pinch. The axial electric field across each instability resulting from this change in inductance accelerates a small group of deuterons to approximately 50 kev per instability. The collisions of these deuterons with the remainder of the deuterons of the pinch give rise to the observed neutron production.

The emphasis upon deciding for or against a possible thermonuclear origin is based upon the recognized principle that, in general, useful power cannot be obtained from a fusion reaction unless the ions are in approximately thermal equilibrium with one another. In other words, an accelerated deuterium or tritium 'beam' incident upon a target cannot produce useful thermonuclear power because the irreversible energy loss of slowing-down in the target is always greater than the reaction energy produced.[8] Even though these pinch experiments produce a relatively large number of neutrons, unless these neutrons were from a thermonuclear origin there would be no hope of extending the yield to a power-producing value. It is therefore worth while to consider the evidence that argues against a thermonuclear origin.

1.) The first and primary evidence is the energy distribution of the recoil proton tracks in photographic emulsion, exposed at either end and the sides of the linear pinch tube. These showed that in one axial direction the neutrons causing the recoils were of a higher mean energy than would come from a deuteron collision whose combined center-of-mass was at rest relative to the emulsion plates. The interpretation of this center-of-mass velocity is that a deuteron of mean axial kinetic energy of 50 kev strikes a deuteron at rest. The resulting nuclear reaction occurs with a moving center-of-mass so that neutrons emitted in the forward direction have a higher energy than those directed backward.

The observed neutron energy distribution corresponded to deuterons with a mean axial energy of 50 kev with a half-width of ± 50 kev. Some deuterons must have been accelerated up to 200 kev to have created the observed neutron energy. If the voltage across the pinch tube was reversed, or the plates exposed at the opposite end of the pinch tube, the corresponding lower energy neutron group was observed. In other words the neutrons originate from a small class of deuterons that are accelerated axially to a high velocity in the direction that would correspond to the applied electric field.

2.) A weak axial magnetic field of 50 to 100 gauss quenches the neutron production. The dynamics of the pinch[10] would indicate that a compression of 50 might be expected on the first bounce, increasing to 100 on the second bounce. This agrees with density measurements made on a helium pinch by observing the Stark broadening of spectral lines. With a compression of 100, an initial axial field of 100 gauss trapped inside the pinch would be increased to 10,000 gauss which is a trivial pressure compared to the pinch field of approximately 100,000 gauss. The effect of an internal axial field on a thermonuclear pinch would be dependent upon the internal H_z pressure compared to the external H_θ pressure. For these to be comparable we solve the equilibrium equation:

$$H_z = H_\theta$$

$$H_z = R^2 \frac{H_{z_o}}{r^2} \qquad H_{z_o} = 100 \text{ gauss}$$

$$H_\theta = \frac{I}{5r} \qquad I = 150{,}000 \text{ amp}$$

therefore

$$\frac{R^2}{r^2}\, 100 = \frac{150{,}000}{5r} \qquad R = 3.75 \text{ cm.}$$

therefore

$$r = 0.047 \text{ cm.}$$

This is a considerably smaller radius than would be expected due to the limitation of shock phenomena. It implies a compression of 6500 which is two orders of magnitude larger than measured for a helium pinch, and similarly unlikely for a deuterium pinch. In other words, the quenching of the neutron production by 10-fold by such a small amount of axial field suggests that the production phenomena is critically associated with radii less than 0.05 cm.

3.) The neutron yield does not increase with increasing voltage as rapidly as would be expected of a thermonuclear process. The yield curve versus applied voltage actually decreases slightly between 40 and 50 kilovolts. This could be interpreted in terms of poorer initial ionization with resulting impaired pinch dynamics; however, the timing of the dynamic bounces agrees with the scaling from much lower voltages so that the ionization process must be sufficiently complete to trap the major fraction of the gas. The higher voltage gives an observed shorter bounce time and therefore should give a higher temperature. If the neutron origin were thermonuclear, then an increase of temperature (proportional to the applied voltage) should result in 5 times the rate of increase of neutron yield. The fact that an actual decrease in yield was observed between 40 and 50 kev is inconsistent with a thermonuclear origin.

4.) The neutron yield is much greater than would be expected from a simple analysis of the dynamic pinch behavior. (See Appendix I.)

The remaining experimental evidence supports a thermonuclear origin and must also be consistent with any other theory of neutron production.

5.) The quenching effect of impurities in a thermonuclear pinch should be proportional to the specific heat of the impurity relative to the thermal energy of single particles. The radiated bremsstrahlung power in these short times (0.1 μ sec) is trivial - because there is hardly an electron-ion thermalization time available. The specific heat of an impurity of atomic number Z is comprised of the energy needed to strip approximately Z electrons and given them each KT energy. Therefore the specific heat of an impurity relative to a deuteron atom will be approximately $\frac{2ZKT}{2KT} = Z$. Therefore an argon atom should add 20 times the specific heat of a deuterium atom, so that 1% impurity of argon should be a 20% effect on temperature. If the temperature were high enough to give the observed yield thermonuclearly, then a 20% change in temperature would be 100% change in yield - in approximate agreement with an observed factor of 4.

6.) The timing of the neutron burst occurs at either the second or third dynamic bounce of the pinch. (The radius versus time behavior of the pinch can be obtained qualitatively from the inductance behavior of the pinch as a circuit element.)

7.) The initial voltage applied across the tube is 50 kilovolts or less, and at the time of neutron production is measured to be between 10 or 20 kilovolts.

8.) The neutrons are produced uniformly along the axial length of the tube within a statistical magnitude of ± 15% and with an axial resolution of ± 10%. This excludes the possibility of the neutrons being generated by deuteron bombardment of deuterium occluded on one or both electrode surfaces.

9.) The radial distribution of neutron production agrees with a radially centered distribution of full width at half maximum of 1 cm. diameter or less. The quartz tube diameter is 7.5 cm.

10.) The neutrons are produced simultaneously along the axial length of the tube to within 5% of the transit time of a 50-kev deuteron from one end of the tube to the other. This excludes the possibility of deuterons being accelerated by a sheath drop at one electrode, and traveling focused within the pinch, making collisions along the full axial length.

The proposed mechanism for neutron production depends upon the instability breakup of the pinch. The voltage between the electrodes of the pinch as a function of time depends upon the inductance of the pinch as a circuit element. If we assume axial uniformity so that the radius of the pinch is uniform with axial position, then the inductance measurements give the pinch radius as a function of time. The first bounce agrees with the concept that the pinch compresses to 1/7 to 1/8 of the original tube diameter, hence the compression of 50. The second, and sometimes third, bounces are observable, but thereafter the circuit behavior is only interpretable in terms of an instability breaking up the pinch. After breakup (usually after what would appear as the third bounce), the circuit appears as if the current were flowing on the inside of the quartz tube wall. At very low condenser voltages, 20 kilovolts and below, there sometimes appears a very high voltage spike (100,000 to 200,000 kilovolts) at the time of the second or third bounce. The above behavior is in agreement with the idea that after one or two bounces the inherent instabilities have had a chance to grow to sufficient magnitude to perturb seriously the inductive behavior of the pinch. When this rate of change of inductance is high enough, the resulting voltage can reach a value many times the applied voltage and may be large enough to break down (voltage-wise) the inside of the quartz wall - causing a surface current to flow that is large enough to isolate electrically the pinch from the external circuit. This is why the very high cumulative voltage during neutron production need not appear in the external circuit.

The instability mode that would grow fastest is the sausage type ($m = 0$) where the plasma is 'necked' off in one or more places. This mode should grow fastest because the rate of increase of pressure ($H_\theta^2/8\pi$) is more rapid for a small change in pinch radius r than is the differential pressure for the kink instability ($m \geq 1$) for a small amplitude. The rate of growth of a given wavelength instability is exponential for an amplitude small compared to both the wavelength λ and pinch radius r.

As the amplitude becomes larger, the troughs of magnetic field growing radially inward speed up due to the increased magnetic field pressure of the smaller radius pinch. The resulting 'necking off' of the pinch causes an axial flow of plasma that expands the pinch immediately adjacent to the 'neck'. This non-linear coupling tends to space the instabilities regularly so that the configuration expected at the time of the second or third bounce is a series of $m = 0$ instabilities regularly spaced along the length of the pinch. This is confirmed by time-resolved pictures taken at Los Alamos of the pinch at this time. The spacing corresponds to a wavelength equal to the pinch diameter at the second bounce which is in agreement with the predicted structure of the non-linear growth of the trough.

The shape of the 'neck' or trough as it grows radially inward can be derived on the basis of a simplified hydrodynamical model. Once the initial instability has grown to an amplitude equal to 1/4 the pinch radius, then the magnetic field pressure at the point of minimum radius is approximately twice as great as the plasma pressure - assuming that the original growth was from an equilibrium radius. For an overpressure of 2 or greater, the flow velocities are governed primarily by shock hydrodynamics. In this case we will assume the snow-plow model which is sufficiently accurate for the approximation. The plasma magnetic field interface will then move at a velocity determined by the pressure difference. Since we are primarily interested in the instability shape at small radius, we will assume the magnetic field pressure is always much greater than the plasma pressure.

If an initial instability (Fig. 15a) grows from an equilibrium pinch of radius r, then the velocity of every point on the r, z curve describing the interface shape will be dependent upon the pressure $H^2/8\pi$. For snow-plow hydrodynamics the pressure equals the time rate-of-change of momentum, so that to first approximation

$$P = \frac{H^2}{8\pi} = \rho\, u^2 \qquad \begin{array}{l}\rho = \text{density}\\ u = \text{velocity normal}\\ \quad\text{to the shock.}\end{array} \tag{1}$$

The shock velocity in the two directions r and z depends upon the component of the pressure in these two directions

$$F_r = \frac{H_o^2}{8\pi}\ \frac{1}{r^2}\ \frac{1}{\sqrt{1 + (dz/dr)^2}} \tag{2}$$

$$F_z = \frac{H_o^2}{8\pi}\ \frac{1}{r^2}\ \frac{1}{\sqrt{1 + (dr/dz)^2}} \tag{3}$$

H_o = magnetic field where $r = 1$

$= \frac{I}{5}$

therefore

$$\frac{dr}{dt} = \sqrt{\frac{H_o^2}{8\pi\rho\, r^2}\ \frac{1}{\sqrt{1 + (dz/dr)^2}}} \tag{4}$$

and

$$\frac{dz}{dt} = \sqrt{\frac{H_o^2}{8\pi\,\rho\, r^2}\ \frac{1}{\sqrt{1 + (dr/dz)^2}}}\ . \tag{5}$$

Since neither velocity depends upon time, there is a time independent shape which can be obtained by dividing (4) by (5) giving:

$$\frac{dr}{dz} = \frac{\frac{dz}{dr}\sqrt{1 + (dr/dz)^2}}{\sqrt{1 + (dr/dz)}} \tag{6}$$

or

$$\frac{dr}{dz} = 1.$$

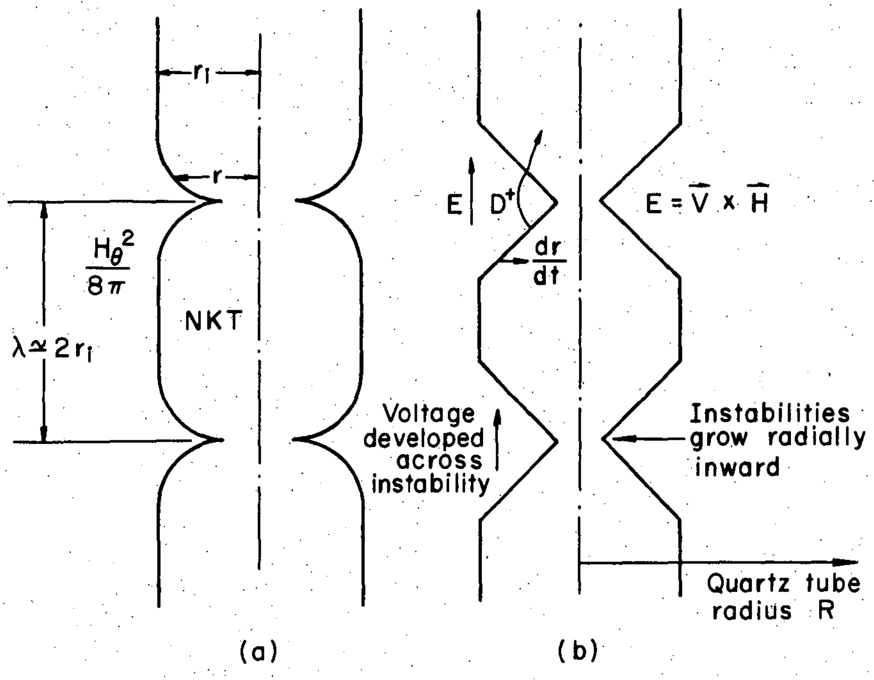

Fig. 13 - Growth of sausage instability with resulting electric field.

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Fig. 15. Growth of sausage instability with resulting electric field.

This gives the 45° triangular shape of Fig. 15b, which is of course only an approximation, but it should be fairly valid in the region of the apex or radius minimum.

A voltage will be developed between each instability lobe because of the rate of change of inductance.

$$V = \frac{d}{dt}(LI) \qquad L = \text{inductance} \quad I = \text{current} \tag{8}$$

$$= I \frac{dL}{dt} + L \frac{dI}{dt} \quad . \tag{9}$$

The total voltage across the pinch is either the applied voltage, which is small at second bounce time, or is less than this because the inside quartz wall has shorted on itself, in which case

$$I \frac{dL}{dt} = L \frac{dI}{dt} \tag{10}$$

or

$$\frac{L_1}{L_2} = \frac{I_1}{I_2} \quad . \tag{11}$$

Since even a large number of regularly spaced instabilities do not change the total inductance of the pinch by very much (~30%), the current I during the instability growth tends to remain constant. Therefore the voltage across a given instability is

$$V = I \frac{dL}{dt} \tag{12}$$

$$L = 2 \times 10^{-9} \times 2 \int_{r_f}^{r_i - r_f} \ell n \frac{R}{r} \, dz \text{ henries/cm.} \tag{13}$$

R = outside conductor radius
r = pinch radius
r_f = minimum instability radius
z = axial position
r_i = initial pinch radius,

but

$$s = r .$$

Therefore

$$L = 4 \times 10^{-9} \int_{r_f}^{r_i - r_f} \ln \frac{R}{r}\, dr \tag{14}$$

$$= 4 \times 10^{-9} \left[\frac{r_i - r_f}{R} \left(\ln \frac{R}{r_i - r_f} - 1 \right) - \frac{r_f}{R} \left(\ln \frac{R}{r_f} - 1 \right) \right] .$$

The time derivative of the inductance during instability growth for $r_f << r_i$ becomes

$$\frac{dL}{dt} = 4 \times 10^{-9} \frac{\dot{r}_f}{R} \left(\ln \frac{R}{r_i - r_f} + \ln \frac{R}{r_f} - 4 \right) . \tag{15}$$

The radial velocity at the 'neck' of the instability, $\dot{r}_f$, can be calculated on the basis of equation (4)

$$\dot{r}_f = 2 \frac{dr}{dt}$$

$$= 2 \sqrt{\frac{H_o^2}{8\pi \rho\, r_f^2 \sqrt{2}}} \tag{16}$$

ρ = density in pinch (170 microns D_2 compressed 100-fold at second bounce time) = 3×10^{-6} g/cm^3.

$H_o = \frac{I}{5}$, $I = 150{,}000$ amp.

Therefore

$$\dot{r}_f = \frac{2I}{25\, r_f\, \rho^{1/2}\, 2^{1/4}} = \frac{6 \times 10^6}{r_f} \text{ cm/sec} \tag{17}$$

From (12) and (15) the voltage per lobe becomes:

$$V = I \frac{dL}{dt} = 150{,}000 \times 4 \times 10^{-9} \frac{6 \times 10^6}{R\, r_f} \left[\ln \frac{R}{r_i - r_f} + \ln \frac{R}{r_f} - 4 \right] \tag{18}$$

r_i = .375 cm

R = 3.75 cm.

Therefore

$$V = 9.6 \times 10^2 \frac{1}{r_f} \left[\ln \frac{3.75}{r_f} - 1.7 \right].$$

In order to reach 50 kev per gap,

$$r_f = .05 \text{ cm}.$$

This is just the radius at which the calculated axial magnetic field pressure of 100 gauss should stabilize further instability growth and quench the neutrons.

The ions can cross the magnetic field between lobes by being accelerated by the electric field, because their Larmor radius is greater than the lobe spacing. The sides of the lobe are 2r apart; the magnetic field is

$$\frac{30{,}000}{r};$$

the Larmor radius

$$r_D = \frac{200\sqrt{E}}{H} \quad \text{or} \quad r_D = \frac{200\sqrt{E}\, r}{30{,}000}.$$

For an ion to cross the gap in a single cycloid orbit

$$1.4\, r_D = 2r$$

therefore

$$E \geq 36 \text{ kev}.$$

The range of an accelerated deuteron in the plasma of the lobes can be calculated from the slowing-down-time τ. The time necessary for the deuteron to lose 1/e of its energy is

$$\tau = 1.1 \times 10^{12} \frac{T^{3/2}}{n_e}$$

T is temperature in kev = 0.1

n_e is electron density = 1.7×10^{18}

therefore

$$\tau = 1.1 \times 10^{12} \frac{0.03}{1.7 \times 10^{18}} = 2 \times 10^{-8} \text{ sec}.$$

The corresponding range for a 50-kev deuteron is 4 cm. which is too long by a factor of 4. A decrease of 2 in mean electron temperature and an increase of 2 in density could easily account for the difference. However, the long-range would indicate that a small class of particles could be accelerated from one instability to the next giving much higher energies and thereby explaining the few very high energy deuterons observed. The same process would apply for electrons if it were not for the difficulty of their crossing the high magnetic fields.

A comparison can be made between the proposed instability acceleration mechanism and the experimental observations.

1.) A small class of high energy axially accelerated deuterons should give rise to a forward peaked neutron distribution. The wide observed width of the distribution can be caused by the bending of the deuteron trajectories in the strong magnetic fields of the instability.

2.) A weak axial magnetic field of 100 gauss should stabilize the $m = 0$ mode of instability, for radii less than 0.05 cm. This is just the critical radius to which the instability must grow to create the accelerating voltages.

3.) The strong impurity quenching cannot be properly explained. It might be associated with the ion injection mechanism into the accelerating electric field. A boundary layer must exist between the magnetic field and plasma during the instability growth. The characteristics of such a layer are that the ions penetrate into the magnetic field further than the electrons. The resulting charge separation electric field turns the ions around in a distance small compared to the ion Larmor radius. An ion of smaller $(Z_{\text{effective}})(e/M)$ will penetrate further into the vacuum region; would feel more of the accelerating electric field, and might be preferentially accelerated in the instability breakup process. Therefore, a small number of impurity ions that have retained some orbit electrons might be preferentially accelerated and result in no neutrons. If this explanation is true, then a small addition of tritium would be expected to give a disproportionately higher neutron yield than the ratio of cross-sections would indicate. This experiment has been tried at Los Alamos with negative results. It is also suggested that a very small impurity addition will cool the electron temperature by inelastic excitation of bound atomic levels. If the electron temperature is lowered, the boundary layer will become thicker and the dynamics at small radii will be impaired.

4.) The neutron yield as a function of applied voltage might be expected to level out once the instability breakup voltages had reached 50 to 100 kev. Certainly rather wide fluctuations in yield might be expected for a statistical process of a small number of instabilities.

5.) The neutron yield from a thermonuclear process is estimated in Appendix I. The expected yield from the instability breakup process can be estimated with a lower limit from a space-charge limited current. This discussion follows this list of comparisons.

6.) The timing of the neutrons agrees with the time in the pinch history where one would expect instabilities to have grown to large amplitude. The radius versus time behavior can be used to calculate the expected breakup voltages.

7.) For weak pinches (small currents) a sudden rise in voltage can be observed at the pinch dynamic time at which one would expect neutrons. However, at higher pinch currents the shorting effect of the insulator (quartz) wall must be invoked to explain the lack of observed external high voltages. The direction of acceleration is in agreement with the change of inductance of the known current.

8.), 9.) Uniformity along the axis and a radially centered source are in agreement with the theory within the resolution of the experiments.

10.) The theory would predict that the instabilities should grow simultaneously within a time small compared to the time of the third bounce. The third bounce time is of the order of 5 to 10 x 10^{-8} sec so that the observed simultaneity of ± 5 x 10^{-8} sec is reasonable.

The accelerated current per lobe should be greater than the space-charge limited value because there should exist both an ion and electron density greater than the space-charge neutralizing value. An ion current of 1000 amps/cm^2 at 50 kev requires an electron density of only 3 x 10^{13} for neutralization so that a strong electric field will accelerate a very much greater current than the space-charge limited value. Indeed if the electron density outside the pinch is 0.2% of the value inside ($\sim 1.7 \times 10^{18}$) it is possible that the whole current of the pinch can be carried space-charge free by the ions. This might occur if the instability actually interrupted the electron current by the mechanism of increasing the resistance of the 'necked' region.

The space-charge limited current, in plane geometry, on the assumption that the magnetic field is strong enough to prevent electrons from being accelerated, is

$$i = \frac{4}{9} K_2 \frac{2e}{M_n} \frac{V^{3/2}}{d^2} \text{ amp/cm}^2.$$

Since we are interested in only an approximate lower limit, we will neglect the logarithmic voltage gradient over the slot. Therefore

$$i = \frac{0.56}{r^2} \text{ amp/cm}^2 \qquad \begin{array}{l} V = \text{voltage} = 50 \text{ kev} \\ d = \text{spacing} = r \end{array}$$

This current must be integrated over the surface of the slot

$$i_t = \int_{r_o}^{r} \frac{0.56}{r^2} 2\pi r \, dr = 3.5 \, \ell n \frac{r_o}{r} = 7.6 \text{ amp per lobe.}$$

The lobes are equally spaced along the length of the tube 70 cm long at $\lambda = 2r_o = 0.7$ cm. Therefore 100 lobes will give a peak current of 760 amp.

The length of time that the instability configuration should exist is roughly the time for another instability to grow to an amplitude equal to the pinch radius r_o. During this time of approximately 0.1 μ sec, the original slot will be growing axially giving roughly the same rate of change of inductance as during the radial growth. This time of .1 μ sec is roughly 1/2 the observed neutron pulse length and 1/2 the voltage spike width observed for weak pinches.

The number of accelerated deuterons becomes

$$N_D = 760 \times 6 \times 10^{18} \times 10^{-7} = 4.6 \times 10^{14}.$$

The number of deuterons that the accelerated ions see in slowing down in a relaxation length is $4 \times 1.7 \times 10^{18} = 6.8 \times 10^{18}$.

The reaction cross-section $\sigma(D\text{-}D)_{50\ kev} = 3.7 \times 10^{-26}\ cm^2$ of which one-half the reactions lead to neutrons, so that the expected neutron production becomes

$$n = N_{Dac} \times N_D \times \frac{\sigma}{2} = 4.6 \times 10^{14} \times 6.8 \times 10^{18} \times \frac{3.7}{2} \times 10^{-26}$$

$$= 6 \times 10^7 \text{ neutrons.}$$

This is the approximate yield observed, although some yields as high as 10^8 neutrons have been observed. If the mean deuteron energy is assumed lower than 50 kev, the reaction cross-section and space-charge limited current are so much lower that space-charge neutralization must be assumed in order to give the observed yield.

Two additional theories of the neutron production have been proposed. The first was briefly referred to by Academician I. V. Kurchatov at the Harwell Conference[6] in which a Fermi type acceleration was proposed in which a small class of ions were accelerated by reflection from moving random magnetic fields. If the field motion is random, then the ion motion must be random and no shift in the center-of-mass motion should be observed.

The theory proposed by Jim Tuck of Los Alamos invokes the m = 0 mode of instability and accelerates just those deuterons that break through the sheath at the 'neck' of the instability. The ions are accelerated to essentially the 'neck' velocity, and then they are bent into the axial direction by the magnetic field. By the snow-plow hydrodynamic model, this requires an instability minimum radius of .01 cm which is considerably smaller than considered here but feasible.

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APPENDIX I.

Subject: The calculation of the length of time to pinch to the first bounce and an estimate of the energy and resultant temperature of the plasma.

The original treatment of the radial motion of the pinch under the assumption of a perfectly conducting boundary was made by Marshal Rosenbluth.[10] The equations of motion were integrated on the basis of a constant applied voltage and snow-plow shock hydrodynamics.

The experimental conditions are more frequently met by the assumption of a sinusoidal current that is primarily determined by the relatively large inductance in series with the pinch. The snow-plow hydrodynamics is probably a very good assumption because the neutral fraction inside the initial current boundary layer may be quite large. This neutral fraction will be pinched and ionized by the current layer. (For n_o greater than 5×10^{15} the current layer is thick compared to neutral-ion scattering, and thick compared to the neutral ionization time multiplied by the layer velocity.) The effective γ (ratio of specific heats) of this neutral fraction is close to unity; i.e., the specific heat of ionization is large so that the compression in an exact hydrodynamic model

$$(\eta = \frac{\gamma + 1}{\gamma - 1})$$

is large (approximately 6 to 10). This is just the limit for which the snow-plow model is valid.

The equation of motion of a snow-plow shock is based upon the principle that the time rate of change of momentum equals the force:

$$-\frac{d}{dt}\left(M\frac{dr}{dt}\right) = P = \frac{B^2}{8\pi} = \frac{I^2}{8\pi\, 25r^2}$$

The mass per centimeter of circumference in the snow-plow shock is the accumulated mass from the outside radius R to the pinch radius r divided by the pinch circumference $2\pi r$.

$$M = \frac{\rho}{2\pi r}\int_R^r 2\pi r\, dr$$

ρ = initial density in g/cm^3
r = pinch radius in cm.

$$= \frac{\rho}{2r}(R^2 - r^2)$$

If the external inductance is large compared to the pinch inductance, then $I = I_o \sin \omega t$. From known values of the condenser capacity, voltage, and time to current maximum, I_o can be calculated.

$$I_o = \omega C V$$

ω = 2π frequency of current oscillation
C = capacity of condenser
V = voltage.

In general the first contraction of the pinch occurs within a time short compared to the 1/4 cycle time of the condenser circuit, so that within this time the current $I_o \sin \omega t$ can be replaced by $I_o \omega t$. Therefore

$$\frac{d}{dt}\left[\frac{\rho}{2r}(R^2 - r^2)\frac{dr}{dt}\right] = -\frac{I_o^{\,2}(\omega t)^2}{200\,\pi\,r^2}$$

If we let

$$\tau = \sqrt[4]{\frac{I_o^{\,2}\,\omega^2}{100\,\pi\,R^4}} \qquad\qquad y = \frac{r}{R}\,,$$

then the equation becomes

$$\frac{d}{d\tau}\left[\frac{1-y^2}{y^2}\;\frac{dy}{d\tau}\right] = -\frac{\tau^2}{y^2}$$

An integration of this equation for the boundary condition $y = 1$, when $\tau = 0$, gives the intercept value $\tau = 1.39$ when $y = .1$, i.e. the probable fractional radius at first bounce time.

The actual time to first bounce therefore becomes:

$$t = 1.39\,R\,\sqrt[4]{\frac{100\,\pi\,\rho}{I_o^{\,2}\,\omega^2}} \quad \text{seconds.}$$

This indicates that the pinch time scales as $\rho^{1/4}$ provided the pinch occurs in a time short compared to the quarter cycle time of the circuit. It should be noted, therefore, that the time to pinch scales as $\rho^{1/4}$ only for a limited fraction of the current cycle. For the experimental values

$\omega = .516 \times 10^6$
$V = 50{,}000$ volts
$C = 12 \times 10^{-6}$ farads
$R = 3.8$ cm
$\rho = 3 \times 10^{-8}$ g/cm^2 at 170 microns pressure D
$t = .72 \times 10^{-6}$ sec.

The observed value is .68 μ sec, which is in excellent agreement considering the simplifying assumptions used.

The radial velocity at the time of first bounce is

$$\frac{dr}{dt} = -1.56 \sqrt[4]{\frac{I_o^2 \omega^2}{100 \pi\rho}} \quad \text{cm/sec.}$$

The kinetic energy per particle is given directly by

$$KT = \frac{M_i}{2} \left(\frac{dr}{dt}\right)^2 \qquad M_i = \text{ion mass.}$$

This energy calculation is not as accurate as the timing information because the maximum transfer from magnetic energy to particle energy occurs at small radius when the current is large — and the variable inductive behavior of the pinch becomes larger than the constant series inductance.

Using the experimental values again,

$$\frac{dr}{dt} = -10^7 \text{ cm/sec}$$

or

$$KT = 100 \text{ ev.}$$

This is an upper limit to the hydromagnetic energy because we have assumed that the current was sinusoidal for the whole pinch period. Actually the back voltage of the changing inductance causes a dip in the current at about the time of each bounce so that the pressure term for part of the pinch implosion is less than the full sine value. This error should be less than 25% because the current reduction is small. The second bounce should increase the temperature somewhat because energy is still being added to the pinch hydrodynamically. The increase should be roughly proportional to the current increase which is 20% during first to second bounce time.

A final temperature of 100 volts is consistent with the assumed compression of 100 at second bounce time.

The particle pressure is

$$P = NKT = 100 \times 1.7 \times 10^{16} \times 100 \times 1.6 \times 10^{-12}$$

$$= 2.7 \times 10^8 \text{ dynes/cm}^2 \qquad \eta = \text{compression} = R^2/r^2.$$

This must be balanced by a magnetic pressure

$$\frac{H^2}{8\pi} = \frac{1}{8\pi}\left(\frac{I}{5r}\right)^2 = 2.7 \times 10^8 \qquad r = 1/10\ R = 0.375 \text{ cm.}$$

Therefore

$$I = 150{,}000 \text{ amp.}$$

This is in agreement with the observed current at the time of the second pinch bounce.

An expected upper limit to the thermonuclear yield can be calculated assuming 100-fold compression, a temperature of 100 ev and a thermal distribution. The thermal relaxation time of the ions at 100 ev and 8.5×10^{17} ion density is approximately 2×10^{-9} sec which is short enough so that within the bounce time of 4×10^{-9} sec ions will be thermalized.

The neutron rate is 1/2 the reaction rate; therefore

$$n = 1/2 \times 1/4 \, N^2 \, \overline{\sigma v} \ \times \text{Vol neutrons per sec}$$

$$N = 8.5 \times 10^{17} \text{ ions/cc}$$

$$\overline{\sigma v} = 4 \times 10^{-31}$$

$$\text{Vol} = \text{compressed Vol} = 20 \text{ cc.}$$

Therefore $n = 7.2 \times 10^{5}$ neutrons/sec.

If the time during which the pinch stays assembled is 4×10^{-8} sec, then the neutron yield is .03 neutrons per pinch. In order to obtain the observed yield thermonuclearly, the reaction rate must be 10^{9} greater, requiring a temperature slightly greater than 1 kev.

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