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Matrix Elements of Conversion of Gluons in Quark-Gluon Plasma [1]

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Abstract

Non-Abelian induced bremsstrahlung associated with multiple scattering of high-energy parton in a quark gluon plasma (QGP) is investigated in the context of semi-classical gauge-covariant kinetic theory. New collisions terms are derived involving the scattering of virtual gluons by the self-charge of the high-energy parton (Compton bremsstrahlung) as well as by dynamical polarization (nonlinear bremsstrahlung).

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1 Introduction

The estimation of energy loss of a high-energy parton traversing a quark-gluon plasma (QGP) is essential in connection with hadron jets signature of QGP production in ultrarelativistic heavy-ion collisions [1] Parton moving through QGP loses its energy due to interaction with collective modes [2, 3, 4], due to hard elastic collisions [3], and due to induced bremsstrahlung[1]. Perturbative QCD estimates [1] indicate that the dominant contribution to the energy loss of a parton moving through a QGP is likely to be induced radiation. For a better estimation of the energy loss in the framework of kinetic theory it is therefore necessary to derive collision terms taking into account bremsstrahlung of gluons. In the case of QED plasmas this has been done in refs. [5, 6]. In this paper, the non-Abelian radiative kinetic terms are derived.

Bremsstrahlung in the QGP is a very complicated process. One has to take into account the effect of multiple scattering (Landau—Pomeranchuk effect [7] and of polarization of the medium (density effect [8] and nonlinear or transitional bremsstrahlung [5, 6, 9]) on bremsstrahlung in the medium with non-Abelian interactions.

In this paper, which has a formal character, we develop a gauge-covariant semi-classical kinetic approach to the description of QGP which can be a base for derivation of bremsstrahlung collision terms in QGP taking into account effect of multiple scattering and of polarization of the medium on bremsstrahlung.

In our previous works [10, 11] with the help of method based on unification of the gauge-covariant self-consistent kinetic approach [12] and of fluctuation theory [13] the collision terms of the Lenard—Balescu type taking account of the dynamics of color degrees of freedom and effect of dynamical screening of interactions in QGP have been obtained in semi-classical limit and with neglect of spin effects. In this work which continues and develops works [10, 11] we generalize this formalism in order to take into account higher order corrections in powers of g (g is the QCD coupling constant) which describe bremsstrahlung process.

This paper is organized as follows: In Sec.2 we obtain kinetic equations for mean quantities (Wigner function of (anti) quarks, gluons and mean gluon field), equations for quantum fluctuations and study the self-consistency of these equations which in this approach ensure the gauge-covariance.

In Sec.3 by using kinetic equations for fluctuations we obtain matrix elements of scattering of gluons (transversal and longitudinal) by the self-charge of parton (Compton bremsstrahlung) as well as by dynamical polarization (nonlinear bremsstrahlung). We choose units such that $\hbar = c = 1$.

2 Kinetic Equations for Averages and Fluctuations

We will consider covariant quark ($\hat{Q}^{+}(p,x)$) and antiquark ($\hat{Q}^{-}(p,x)$) Wigner operators which are $N \times N$ matrices in color space (for $SU(N)$ color group). $\hat{Q}^{\pm}(p,x) = \theta(\pm p_0)\delta(p^2 - m^2)\hat{W}(p,x)$, where $\hat{W}(p,x)$ is full covariant quark Wigner operator [14]

$$\hat{W}(p,x) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} (\exp(y\cdot \hat{D}^{\dagger}/2)\bar{\psi}(x))(\exp(-y\cdot \hat{D}/2)\psi(x)) \,, \tag{1}$$

where $\hat{D}_\mu = \partial_\mu + \imath g\hat{A}_\mu$, $\hat{A}_\mu = \hat{A}^a_\mu t^a$; $\hat{A}^a_\mu$ is the operator of gluon field and t^a are hermitian generators of $SU(N)$ color group in fundumental representation.

Covariant gluon Wigner operator $\hat{G}(p,x)$ is $(N^2-1)\times(N^2-1)$ matrix in color space. $\hat{G}(p,x) = \hat{G}^\mu_\mu(p,x)$, where

$$\hat{G}^{ab}_{\mu\nu}(p,x) = \int \frac{d^4y}{(2\pi)^4} e^{-\imath p\cdot y} (\exp(y\cdot\hat{\tilde{D}}/2)^{aa'}\delta A^{a'}_\mu(x))(\exp(-y\cdot\hat{\tilde{D}}/2)^{bb'}\delta A^{b'}_\nu(x)) , \tag{2}$$

where $\hat{\tilde{D}} = \partial_\mu + \imath g\hat{\tilde{A}}_\mu$, $\hat{\tilde{A}}_\mu = \hat{A}^a_\mu T^a$; T^a are hermitian generators of $SU(N)$ color group in adjoint representation. The fluctuation of gluon field $\delta A^a_\mu = \hat{A}^a_\mu - A^a_\mu$, where $A^a_\mu = \langle\hat{A}^a_\mu\rangle$ ($\langle...\rangle = Tr(\hat{\rho}...)$ and $\hat{\rho}$ is the density matrix of the system).

We will neglect spin effects and use averaged over spin indices Wigner operators. Wigner functions are defined as quantum—statistical averages of the corresponding operators: $Q^\pm(p,x) = \langle\hat{Q}^\pm(p,x)\rangle$, $G(p,x) = \langle\hat{G}(p,x)\rangle$. The quantum fluctuations about mean values of corresponding operator quantities are defined by the relations $\delta Q^\pm = \hat{Q}^\pm - Q^\pm$, $\delta G = \hat{G} - G$.

We denote the trace over color indices in the fundumental representation by Sp and do in adjoint representation by Tr. So $Sp(t^at^b) = \frac{1}{2}\delta^{ab}$, $Tr(T^aT^b) = N\delta^{ab}$ $((T^a)^{bc} = -\imath f^{abc})$. Covariant derivatives act on color matrices as follows $D_\mu = \partial_\mu + \imath g[A_\mu, ...]$. The gluon field strength tensor is given by $F^a_{\mu\nu}t^a = F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \imath g[A_\mu, A_\nu]$, $\tilde{F}_{\mu\nu} = F^a_{\mu\nu}T^a$.

It has been shown in [10] that in the semiclassical limit and with neglect spin effects operator quantities obey dynamical equations which follows from the Dirac and Yang—Mills equations

$$p^\mu\hat{D}_\mu\hat{Q}^\pm(p,x) + gp^\mu\partial^\nu_p\frac{1}{2}\{\hat{F}_{\nu\mu}, \hat{Q}^\pm(p,x)\} = 0 , \tag{3}$$

$$\begin{aligned}&(p^\mu\hat{\tilde{D}}_\mu\hat{G}(p,x) + gp^\mu\partial^\nu_p\frac{1}{2}\{\hat{\tilde{F}}_{\nu\mu}, \hat{G}(p,x)\})^{ab} = \\ &-\frac{\imath}{2}\delta^4(p)((\hat{\tilde{D}}^2\delta A_\nu(x))^a\delta A^{b\nu}(x) - \delta A^{a\nu}(x)(\hat{\tilde{D}}^2\delta A_\nu(x))^b) ,\end{aligned} \tag{4}$$

$$D_\mu F^{\mu\nu} + D_\mu\delta F^{\mu\nu} + \imath g[\delta A_\mu, F^{\mu\nu}] = \hat{\jmath}^\nu + \hat{\jmath}^\nu_{corr} , \tag{5}$$

$$\hat{\jmath}^{\nu a} = g\int dpp^\nu(Sp(t^a(\hat{Q}^+ + \hat{Q}^-)) + Tr(T^a\hat{G})) , \tag{6}$$

$$\hat{\jmath}^\nu_{corr} = -\imath g(D_\mu[\delta A^\mu, \delta A^\nu] + [\delta A_\mu, D^\mu\delta A^\nu]) . \tag{7}$$

Here $\delta F^{\mu\nu} = D_\mu\delta A_\nu - D_\nu\delta A_\mu$. Note that

$$\begin{aligned}\hat{\jmath}^\nu_g &= gt^a\int dpp^\nu Tr(T^a\hat{G}) = \imath g[\delta A_\mu, \hat{D}^\nu\delta A^\mu] \\ &= \imath g[\delta A_\mu, D^\nu\delta A^\mu] - (\imath g)^2[\delta A_\mu, [\delta A^\mu, \delta A^\nu]] .\end{aligned} \tag{8}$$

As it follows from eqs. (2,8)

$$\hat{D}_\nu \hat{j}^\nu_g = \imath g[\delta A_\mu, \hat{D}^2 \delta A^\mu] \,. \tag{9}$$

Eq. (9) is the first type of self consistency conditions which play an important role in this formalism since they ensure the gauge-covariance.

As the result of averaging of dynamical equations we get (see [10]) the system of kinetic equations for mean quantities (Wigner functions and mean gluon field)

$$p^\mu D_\mu Q^\pm(p,x) + g p^\mu \partial^\nu_p \frac{1}{2}\{F_{\nu\mu}, Q^\pm(p,x)\} = C_{Q^\pm} \,, \tag{10}$$

$$p^\mu \tilde{D}_\mu G(p,x) + g p^\mu \partial^\nu_p \frac{1}{2}\{\tilde{F}_{\nu\mu}, G(p,x)\} = C_G \,, \tag{11}$$

$$D_\mu F^{\mu\nu} = j^\nu \,, \tag{12}$$

$$j^{\nu a} = g \int dp p^\nu (Sp(t^a(Q^+ + Q^-)) + Tr(T^a G)) \,, \tag{13}$$

Here we set

$$j^\nu_{corr} = -\imath g \langle D_\mu [\delta A^\mu, \delta A^\nu] + [\delta A_\mu, D^\mu \delta A^\nu] \rangle = 0 \tag{14}$$

in accordance with neglect spin effects. Otherwise we have to use gluon Wigner function $G_{\mu\nu}$ instead of $G = G^\mu_\mu$ in order to obtain self-consistent system of equations. This case is much more complicated since equation for $G_{\mu\nu}$ is gauge dependent and also it is nesessary to consider Wigner functions for ghost fields (see [12]). The essential feature of spinless approximation is that we deal with gauge-covariant equation (11) for gluon Wigner function. In accordance with neglect spin effects we restrict the full equation for fluctuation of gluon field

$$\hat{D}^2 \delta A^\nu = \delta j^\nu + (\imath g)^2 [\delta A_\mu, [\delta A^\mu, \delta A^\nu]] + D_\mu D^\nu \delta A^\mu - \imath g [\delta A_\mu, F^{\mu\nu}] \,, \tag{15}$$

where covariant background gauge [12] is assumed, to equation

$$\hat{D}^2 \delta A^\nu = \delta j^\nu \,, \tag{16}$$

when we substitute it in equation for gluon Wigner function (operator).

Eq. (15) can be obtained from equation for fluctuation of gluon field

$$D_\mu \delta F^{\mu\nu} + \imath g [\delta A_\mu, F^{\mu\nu}] = \delta j^\nu + \hat{j}^\nu_{corr} \,, \tag{17}$$

which, in a turn, is obtained under subtraction eq. (12) from eq. (5).

That way we obtain the expressions for collision terms in the r.h.s. of eqs. (10,11)

$$\begin{aligned} C_{Q^\pm} = \; & -\imath g p^\mu \langle [\delta A_\mu, \delta Q^\pm] \rangle - g p^\mu \partial^\nu_p \tfrac{1}{2} \langle \{\delta F_{\nu\mu}, \delta Q^\pm\} \rangle \\ & -\imath g^2 p^\mu \partial^\nu_p \tfrac{1}{2} \langle \{[\delta A_\nu, \delta A_\mu], (Q^\pm + \delta Q^\pm)\} \rangle \,, \end{aligned} \tag{18}$$

$$C_G^{ab} = -\imath g p^\mu \langle [\delta\tilde{A}_\mu, \delta G] \rangle^{ab} - g p^\mu \partial_p^\nu \frac{1}{2} \langle \{\delta\tilde{F}_{\nu\mu}, \delta G\} \rangle^{ab}$$
$$-\imath g^2 p^\mu \partial_p^\nu \frac{1}{2} \langle \{[\delta\tilde{A}_\nu, \delta\tilde{A}_\mu], (G + \delta G)\} \rangle^{ab} - \frac{\imath}{2} \delta^4(p) \langle \delta j_\nu^a \delta A^{b\nu} - \delta A_\nu^a \delta j^{b\nu} \rangle . \qquad (19)$$

Here $\delta\tilde{A}_\mu = \delta A_\mu^a T^a$, $\delta\tilde{F}_{\nu\mu} = \delta F_{\nu\mu}^a T^a$. In derivation of eq. (19) we have used eq. (16).

The forth term in the r.h.s. of eq. (19) ensures the self-consistency of the system of eqs. (10–13). The conservation of mean color current (13) follows from eq. (12):

$$D_\mu j^\mu = 0 . \qquad (20)$$

Substituting eq. (13) into eq. (20) and using kinetic equations (10,11) with collision terms given by eqs. (18,19), one convinces of the validity of eq. (20).

Note that we hold the third terms in expressions for collision terms (18,19) which have been omitted in [10]. They are essential for derivation of bremsstrahlung collision terms [15].

The kinetic equations for fluctuations are obtained under subtraction the kinetic equations for mean quantities (10–14) from the dynamical ones (3–7) and can be written as follows

$$p^\mu \partial_\mu \delta Q^\pm + \imath g p^\mu [\delta A_\mu, Q^\pm] + g p^\mu \partial_p^\nu \frac{1}{2} \{\delta F_{\nu\mu}, Q^\pm\} =$$
$$-\imath g p^\mu ([\delta A_\mu, \delta Q^\pm] - \langle [\delta A_\mu, \delta Q^\pm] \rangle)$$
$$-g p^\mu \partial_p^\nu \frac{1}{2} (\{\delta F_{\nu\mu}, \delta Q^\pm\} - \langle \{\delta F_{\nu\mu}, \delta Q^\pm\} \rangle)$$
$$-\imath g^2 p^\mu \partial_p^\nu \frac{1}{2} (\{[\delta A_\nu, \delta A_\mu], Q^\pm\} - \{\langle [\delta A_\nu, \delta A_\mu] \rangle, Q^\pm\}) , \qquad (21)$$

$$p^\mu \partial_\mu \delta G + \imath g p^\mu [\delta\tilde{A}_\mu, G] + g p^\mu \partial_p^\nu \frac{1}{2} \{\delta\tilde{F}_{\nu\mu}, G\} =$$
$$-\imath g p^\mu ([\delta\tilde{A}_\mu, \delta G] - \langle [\delta\tilde{A}_\mu, \delta G] \rangle)$$
$$-g p^\mu \partial_p^\nu \frac{1}{2} (\{\delta\tilde{F}_{\nu\mu}, \delta G\} - \langle \{\delta\tilde{F}_{\nu\mu}, \delta G\} \rangle)$$
$$-\imath g^2 p^\mu \partial_p^\nu \frac{1}{2} (\{[\delta\tilde{A}_\nu, \delta\tilde{A}_\mu], G\} - \{\langle [\delta\tilde{A}_\nu, \delta\tilde{A}_\mu] \rangle, G\}) , \qquad (22)$$

$$\partial_\mu \delta F^{\mu\nu} = \delta j^\nu + \hat{j}^\nu_{corr} , \qquad (23)$$

$$\delta j^{\nu a} = g \int dp p^\nu (Sp(t^a(\delta Q^+ + \delta Q^-)) + Tr(T^a \delta G)) . \qquad (24)$$

In eqs.(21–23) we have omitted the terms of higher orders and also terms containing mean gluon field (here and below $\delta F_{\mu\nu} = \partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu$). We assume that mean gluon field is weak and doesn't act on the collision process.

We now study the self-consistency of the system (21–24). The $\hat{j}^\nu_{corr}$ current obey the equation:

$$D_\nu \hat{j}^\nu_{corr} = \imath g [(\hat{D}^2 \delta A_\nu - \delta j_\nu), \delta A^\nu] . \qquad (25)$$

¿From eq. (17) we get

$$D_\nu(\delta j^\nu + \hat{j}^\nu_{corr}) = -\imath g[\delta A_\nu, j^\nu] \; . \tag{26}$$

As it follows from dynamical equations (see eq. (9))

$$\hat{D}_\nu \hat{j}^\nu = \imath g[\delta A_\mu, \hat{D}^2 \delta A^\mu] \; . \tag{27}$$

Taking into account eq. (12), rewrite eq. (27) as follows

$$D_\nu \delta j^\nu = -\imath g[\delta A_\nu, j^\nu] - \imath g[(\hat{D}^2 \delta A_\nu - \delta j_\nu), \delta A^\nu] \; . \tag{28}$$

Eq. (25) follows from eqs. (26,28).

Due to our restriction of neglect spin effects (14,16) we get from eq.(25)

$$D_\nu \hat{j}^\nu_{corr} = 0 \tag{29}$$

or in the case of neglect of influence of mean gluon field, as in the case of system (21–24),

$$\partial_\nu \hat{j}^\nu_{corr} = 0 \; . \tag{30}$$

Considering Fourier transform of $\hat{j}^\nu_{corr}$

$$\hat{j}^\nu_{corr}(k) = -g \int dk_1 (2k^\mu - k_1^\mu)[\delta A_\mu(k_1), \delta A_\nu(k - k_1)] \; , \tag{31}$$

we obtain that in accordance with our assumption of neglect spin effects, we have to set

$$\int dk_1 k_1^\mu [\delta A_\mu(k_1), \delta A_\nu(k - k_1)] = 0 \; , \tag{32}$$

$$\hat{j}^\nu_{corr}(k) = -2g \int dk_1 k^\mu [\delta A_\mu(k_1), \delta A_\nu(k - k_1)] \; , \tag{33}$$

where $\hat{j}^\nu_{corr}$ defined by eq. (33) obeys the condition

$$k_\nu \hat{j}^\nu_{corr}(k) = 0 \tag{34}$$

in agreement with eq. (30).

That way the kinetic equations for fluctuations (21–24) with neglect mean gluon field, where $\hat{j}^\nu_{corr}$ is defined by eq. (33) satisfy the self-consistency condition

$$\partial_\nu (\delta j^\nu + \hat{j}^\nu_{corr}) = 0 \; . \tag{35}$$

3 Matrix Elements of Conversion of Gluons in QGP

We now consider equations for fluctuations which are nesessary for derivation bremsstrahlung collision terms in QGP. Collision terms for QGP without taking account of radiation have been derived earlier (see [10, 11]).

We determine induced fluctuations $\delta Q^{\pm}_{ind}, \delta G_{ind}$ as solutions of equations

$$p^{\mu}\partial_{\mu}\delta Q^{\pm}_{ind} + \imath g p^{\mu}[\delta A_{\mu}, Q^{\pm}] + g p^{\mu}\partial^{\nu}_{p}\frac{1}{2}\{\delta F_{\nu\mu}, Q^{\pm}\} = 0 , \qquad (36)$$

$$p^{\mu}\partial_{\mu}\delta G_{ind} + \imath g p^{\mu}[\delta \tilde{A}_{\mu}, G] + g p^{\mu}\partial^{\nu}_{p}\frac{1}{2}\{\delta \tilde{F}_{\nu\mu}, G\} = 0 . \qquad (37)$$

By using Fourier transform, we get

$$\delta Q^{\pm}_{ind}(p,k) = \frac{g p^{\mu}}{(p\cdot k)}[\delta A_{\mu}(k), Q^{\pm}(p,x)] + \frac{g}{2(p\cdot k)}\hat{b}^{\alpha}(p,k)\{\delta A_{\alpha}(k), Q^{\pm}(p,x)\} , \qquad (38)$$

$$\delta G_{ind}(p,k) = \frac{g p^{\mu}}{(p\cdot k)}[\delta \tilde{A}_{\mu}(k), G(p,x)] + \frac{g}{2(p\cdot k)}\hat{b}^{\alpha}(p,k)\{\delta \tilde{A}_{\alpha}(k), G(p,x)\} , \qquad (39)$$

where

$$\hat{b}^{\alpha}(p,k) = (p\cdot k)\partial^{\alpha}_{p} - p^{\alpha}(k\cdot\partial_{p}) , \qquad (40)$$

and poles in eqs. (38,39) are by-passed according to Landau rule [16]

$$\frac{1}{(p\cdot k)} = \lim_{\epsilon\to 0}\frac{1}{((p\cdot k) + \imath\epsilon p_0)} = V.p.\frac{1}{(p\cdot k)} - \imath\varepsilon(p_0)\pi\delta(p\cdot k) , \qquad (41)$$

where $\varepsilon(p_0) = \theta(p_0) - \theta(-p_0)$.

Now we split following [13]

$$\delta Q^{\pm} = \delta Q^{\pm}_{ind} + \widetilde{\delta Q^{\pm}} , \qquad (42)$$

$$\delta G = \delta G_{ind} + \widetilde{\delta G} . \qquad (43)$$

Substituting eqs. (42,43) into the linearized Yang—Mills equation (23), we get

$$-(k^2\bar{g}^{\nu\mu} - \Pi^{\nu\mu})\delta A_{\mu}(k) = \widetilde{\delta j}^{\nu}(k) + \hat{j}^{\nu}_{corr}(k) , \qquad (44)$$

where

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} , \qquad (45)$$

and singlet polarization tensor $\Pi_{\mu\nu}$ is given by [11]

$$\Pi_{\mu\nu}(k) = -g^2\int dp\frac{1}{(p\cdot k)}p_{\mu}k^{\alpha}p_{[\alpha}\partial^{p}_{\nu]}\mathcal{N}(p) = g^2\int dp X_{\mu\nu}\mathcal{N}(p) , \qquad (46)$$

where

$$\mathcal{N}(p) = \frac{1}{2}(Q^{+}_{eq} + Q^{-}_{eq}) + NG_{eq} \ , \tag{47}$$

$$X_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu k_\nu + p_\nu k_\mu}{(p \cdot k)} + \frac{k^2 p_\mu p_\nu}{(p \cdot k)^2} \ . \tag{48}$$

Here $Q^{\pm}_{eq} = \frac{1}{N} Sp Q^{\pm}$, $G_{eq} = \frac{1}{N^2-1} Tr G$. We assume that QGP is in colorless and thermal equilibrium state and $Q^{\pm} = Q^{\pm}_{eq} \cdot \mathbf{1}$, $G = G_{eq} \cdot \mathbf{1}$, where $Q^{\pm}_{eq}$, G_{eq} are Fermi— Dirac and Bose—Einstein distribution functions correspondingly.

Using the spectral decomposition of polarization tensor [17]

$$\Pi_{\mu\nu} = \Pi_L Q_{\mu\nu} + \Pi_T P_{\mu\nu} \ , \tag{49}$$

where

$$Q_{\mu\nu} = \frac{\bar{u}_\mu \bar{u}_\nu}{\bar{u}^2} \ , \quad P_{\mu\nu} = \bar{g}_{\mu\nu} - Q_{\mu\nu} \ , \tag{50}$$

$$\bar{u}_\mu = \bar{g}_{\mu\nu} u^\nu \ , \tag{51}$$

$$\Pi_L = \Pi_{\mu\nu} Q^{\mu\nu} \ , \quad \Pi_T = \frac{1}{2} \Pi_{\mu\nu} P^{\mu\nu} \ , \tag{52}$$

(u_μ is the four velocity of QGP) we can invert equation (44)

$$\delta A_\mu(k) = -D_{\mu\nu}(k)(\widetilde{\delta j}^{\nu}(k) + \hat{\jmath}^{\nu}_{corr}(k)) \ , \tag{53}$$

where [17]

$$D_{\mu\nu}(k) = \frac{Q_{\mu\nu}}{k^2 - \Pi_L} + \frac{P_{\mu\nu}}{k^2 - \Pi_T} + \alpha \frac{k_\mu k_\nu}{k^4} \ . \tag{54}$$

The currents in eqs. (44,53) are given by

$$\widetilde{\delta j}^{a\nu} = g \int dp p^\nu (Sp(t^a(\widetilde{\delta Q}^{+} + \widetilde{\delta Q}^{-})) + Tr(T^a \widetilde{\delta G})) \ , \tag{55}$$

$$\hat{\jmath}^{a\nu}_{corr} = -\imath g f^{abc} \int dk_1 k^\mu \delta A^b_\mu(k_1) \delta A^{c\nu}(k - k_1) \ , \tag{56}$$

where fluctuations $\widetilde{\delta Q}^{\pm}$, $\widetilde{\delta G}$ are determined by eqs. (42,43).

Substituting decompositions (42,43) into eqs. (21,22), making Fourier transform and taking account of eqs. (38,39), we get

$$\begin{aligned} \widetilde{\delta Q}^{\pm}(p,k) \ &= \ \delta Q^{\pm}_0(p,k) + \int dk_1 \frac{g p^\mu}{(p \cdot k)} ([\delta A_\mu(k_1), \widetilde{\delta Q}^{\pm}(p, k - k_1)] - \langle \ldots \rangle) \\ &+ \ \int dk_1 \frac{g}{2(p \cdot k)} \hat{b}^\alpha(p, k_1)(\{\delta A_\alpha(k_1), \widetilde{\delta Q}^{\pm}(p, k - k_1)\} - \langle \ldots \rangle) \\ &+ \ \int dk_1 s^{bc}_{Q^\pm \beta\gamma}(p, k, k_1)(\delta A^{b\beta}(k_1) \delta A^{c\gamma}(k - k_1) - \langle \ldots \rangle) \ , \end{aligned} \tag{57}$$

where $\delta Q_0^{\pm}(p,k)$ obey free equations $(p\cdot k)\delta Q_0^{\pm}(p,k)=0$ and

$$s^{bc}_{Q^{\pm}\beta\gamma}(p,k,k_1) = g^2[t^b,t^c]\frac{p^\beta}{(p\cdot k)}(\frac{1}{(p\cdot(k-k_1))}\hat{b}^\gamma(p,k-k_1)+\partial_p^\gamma)Q^{\pm}_{eq}(p) + \frac{g^2}{2}\{t^b,t^c\}\frac{1}{(p\cdot k)}\hat{b}^\beta(p,k_1)\frac{1}{(p\cdot(k-k_1))}\hat{b}^\gamma(p,k-k_1)Q^{\pm}_{eq}(p)\,. \quad (58)$$

Analogously we get for $\widetilde{\delta G}$

$$\widetilde{\delta G}^{de}(p,k) = \delta G_0^{de}(p,k)+\int dk_1\frac{gp^\mu}{(p\cdot k)}([\delta\tilde{A}_\mu(k_1),\widetilde{\delta G}(p,k-k_1)]-\langle\ldots\rangle)^{de} + \int dk_1\frac{g}{2(p\cdot k)}\hat{b}^\alpha(p,k_1)(\{\delta\tilde{A}_\alpha(k_1),\widetilde{\delta G}(p,k-k_1)\}-\langle\ldots\rangle)^{de} + \int dk_1\frac{\delta^4(p)}{2(p\cdot k)}((\widetilde{\delta j}^d_\nu(k_1)\delta A^{e\nu}(k-k_1)-\widetilde{\delta j}^e_\nu(k_1)\delta A^{d\nu}(k-k_1))-\langle\ldots\rangle) + \int dk_1(s^{bc}_{G\beta\gamma})^{de}(p,k,k_1)(\delta A^{b\beta}(k_1)\delta A^{c\gamma}(k-k_1)-\langle\ldots\rangle)\,, \quad (59)$$

where

$$(s^{bc}_{G\beta\gamma})^{de}(p,k,k_1) = g^2[T^b,T^c]^{de}\frac{p^\beta}{(p\cdot k)}(\frac{1}{(p\cdot(k-k_1))}\hat{b}^\gamma(p,k-k_1)+\partial_p^\gamma)G_{eq}(p) + \frac{g^2}{2}\{T^b,T^c\}^{de}\frac{1}{(p\cdot k)}\hat{b}^\beta(p,k_1)\frac{1}{(p\cdot(k-k_1))}\hat{b}^\gamma(p,k-k_1)G_{eq}(p) + \frac{g^2}{2}(\delta^{db}\delta^{ec}-\delta^{eb}\delta^{dc})\frac{\delta^4(p)}{(p\cdot k)}\int dp_1\frac{p_1^\gamma}{(p_1\cdot k_1)}\hat{b}^\beta(p_1,k_1)\mathcal{N}(p_1)\,. \quad (60)$$

Here $\delta G_0(p,k)$ obeys free equation $(p\cdot k)\delta G_0(p,k)=0$ and $\mathcal{N}(p_1)$ is given by eq. (47).

Substituting eqs. (57–60) into eqs. (55,56), we get

$$\widetilde{\delta j}^{a\alpha}(k)+\hat{j}^{a\alpha}(k)=\delta j_0^{a\alpha}(k)+ \int dp_1dk_1R^{\alpha\beta}(p_1,k)(\delta A^b_\beta(k_1)\widetilde{\delta\mathcal{N}}^{[ab]}(p_1,k-k_1)-\langle\ldots\rangle)+ \int dp_1dk_1J^{\beta\alpha}(p_1,k,k_1)(\delta A^b_\beta(k_1)\widetilde{\delta\mathcal{N}}^{\{ab\}}(p_1,k-k_1)-\langle\ldots\rangle)+ \int dk_1\frac{1}{2}(S^{abc}_{\alpha\beta\gamma}(k,k_1,k-k_1)+S^{acb}_{\alpha\gamma\beta}(k,k-k_1,k_1))(\delta A^{b\beta}(k_1)\delta A^{c\gamma}(k-k_1)-\langle\ldots\rangle), \quad (61)$$

where δj_0 is defined by the eq. (55) with the substitution $\delta Q_0^{\pm}$, δG_0 instead of $\widetilde{\delta Q}^{\pm}$, $\widetilde{\delta G}$ and

$$\widetilde{\delta\mathcal{N}}^{ab}=Sp(t^at^b(\widetilde{\delta Q}^+ + \widetilde{\delta Q}^-))+Tr(T^aT^b\widetilde{\delta G})\,, \quad (62)$$

$$R^{\alpha\beta}(p_1,k)=g^2\frac{p_1^\beta}{(p_1\cdot k)}(g_{\alpha\sigma}-\frac{k_\alpha k_\sigma}{k^2})p_1^\sigma\,, \quad (63)$$

$$J^{\beta\alpha}(p_1,k,k_1)=-g^2\hat{b}^\beta(p_1,k_1)\frac{p_1^\alpha}{(p_1\cdot k)}\,, \quad (64)$$

$$S^{abc}_{\alpha\beta\gamma}(k,k_1,k-k_1) = -\imath g f^{abc} k_\beta g_{\alpha\gamma} +$$
$$\imath g^3 f^{abc} \int dp_1 \frac{p_1^\beta}{(p_1\cdot k)}((g_{\alpha\sigma} - \frac{k_\alpha k_\sigma}{k^2})\frac{p_1^\sigma}{(p_1\cdot(k-k_1))}\hat{b}^\gamma(p_1,k-k_1) + p_1^\alpha \partial^\gamma_{p_1})\mathcal{N}(p_1)' +$$
$$\frac{g^3}{2} d^{abc} \int dp_1 \frac{p_1^\alpha}{(p_1\cdot k)}\hat{b}^\beta(p_1,k_1)\frac{1}{(p_1\cdot(k-k_1))}\hat{b}^\gamma(p_1,k-k_1) n(p_1)\ , \tag{65}$$

Here d^{abc} is symmetric constant of $SU(N)$ group and $n(p_1) = \frac{1}{2}(Q^+_{eq}(p_1) + Q^-_{eq}(p_1))$.

In derivation of eqs. (61–65) we have used identity

$$\int d^4p \frac{p^\nu}{(p\cdot k) + \imath\epsilon p_0}\delta^4(p) = \frac{k^\nu}{k^2 + \imath\epsilon k_0}\ . \tag{66}$$

The terms in which eq. (66) appeares ensure the conservation of color current

$$k_\nu(\widetilde{\delta j}^{a\nu}(k) + \hat{j}^{a\nu}_{corr}(k)) = 0\ . \tag{67}$$

One can convince in the validity of eq. (67) by using explicit expressions (61–65).

Eqs. (53,61–65) describe the conversion of gluons in the quark-gluon plasma in the semiclassical limit and with neglect of spin effects. The scattering of gluons by self-charge of parton (Compton scattering) are given by the second and the third terms in the r.h.s. of eq. (61), where amplitudes R and J are given by the eqs. (63,64) correspondingly. The terms $\widetilde{\delta Q}^\pm$ in eq. (62) correspond to parton—(anti) quark while $\widetilde{\delta G}$ does parton— gluon. The third term in the r.h.s. of eq. (61) is non-Abelian generalization of QED Compton amplitude [5, 6] while the second one has pure non-Abelian origin and is absent in QED case.

The scattering of gluons by dynamic polarization (nonlinear bremsstrahlung) is given by the forth term in the r.h.s. of eq. (61), where effective three—gluon vertex is given by eq. (65). We note that the third term in eq. (65) has a QED analog which describes the transitional bremsstrahlung [5, 6]. The first and the second ones have pure non-Abelian character. Note that the first term in eq. (65) appeares due to nonlinearity of the Yang—Mills equation.

The physical sense of matrix elements and their use for derivation bremsstrahlung collision terms in QGP will be consider in [15].

4 Conclusion

In this paper we have developed a semiclassical gauge-covariant approach to the description of a QGP which can be a base for derivation of bremsstrahlung collision terms in QGP. The neglect of spin effects with the use of semiclassical approximation allow us to avoid the ambiguity connected with gauge dependence.

The investigation of bremsstrahlung process is of interest from the point of view of estimation of energy loss of high-energy parton in QGP in connection with hadron jets signature of QGP production in ultrarelativistic nucleus—nucleus collisions.

The matrix elements of scattering of gluons by the self-charge of parton (Compton bremsstrahlung) as well as by dynamic polarization (nonlinear bremsstrahlung) obtained in this paper contain the terms which have pure non-Abelian origin and are absent in the case of QED plasma. They can essentially change the picture of bremsstrahlung in a medium which has been developed in the case of electromagnetic interaction.

The derivation of bremsstrahlung collision terms for the quark-gluon plasma and the study of effect of multiple scattering (Landau— Pomeranchuk efffect) and of polarization of the medium (density effect) on bremsstrahlung in QGP will be given in the forthcoming publication [15].

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