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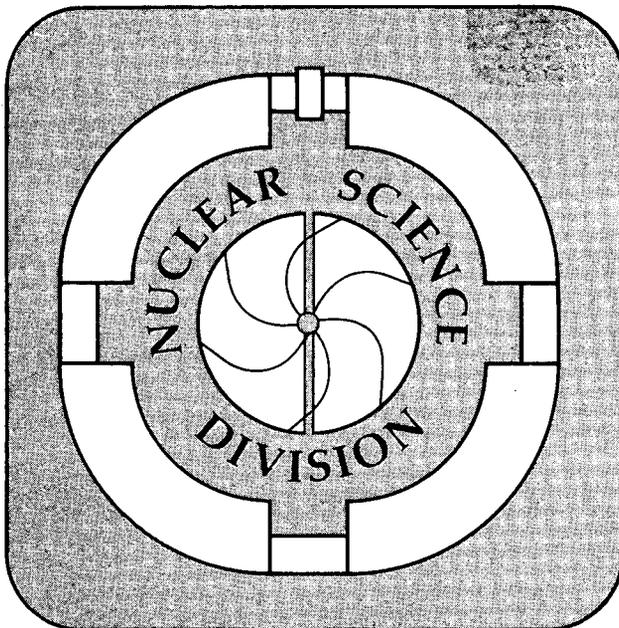
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## Pion Probes of Heavy Ion Collision Dynamics

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## PION PROBES OF HEAVY ION COLLISION DYNAMICS

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## ABSTRACT

Pion interferometry data (2-pion correlation) are examined for information on size and lifetime of the pion-emitting matter. The temperatures inferred from pion, proton and kaon spectra are considered. An explanation consistent with the above size and temperature data is proposed. New theoretical Monte Carlo results on spectator effects on heavy-ion pion spectra are presented.

1. Introduction

Determining the dynamics of nucleus-nucleus collisions in the BEVALAC projectile velocity range of 0.4-0.95 times the speed of light is a formidable task. One would like ideally to have as a function of impact parameter and time the local density, composition, temperature, and collective flow vectors throughout the system. From such information we could try to catalog the behaviour of nuclear matter under exotic conditions of temperature and pressure not otherwise found in the universe since the big bang in the beginning.

There have been scarcely a dozen years that controlled accelerator experiments with relativistic heavy ions could take over from cosmic ray beginnings. Only for the past two years have ions heavier than mass 56 been available. Renewed activity in cosmic rays, as exemplified by the JACEE collaboration<sup>1)</sup>, gives us glimpses of the heavy ion energy regime beyond at multi-TeV per nucleon.

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\* A.N.U. Visiting Fellow, February-March, 1984.

Though the accelerator work has necessarily centred in Berkeley, the research program has been unusually international - I should say intercontinental or global. The Europeans, especially from West Germany, have carried a major load in the high-geometry, high-multiplicity scintillator array development, culminating in the plastic ball-plastic wall. Scientists from Japan have played the major role in the complementary development of wire-chamber magnetic spectroscopy, culminating in the large arrays in the large superconducting HISS spectrometer.

We are just now entering a new regime in heavy ion studies, where symmetric systems large enough for complete stopping can be studied. Plastic ball studies show that mass 40 on mass 40 in the GeV/nucleon range does not show complete stopping but that  $^{91}\text{Nb}$  on  $^{91}\text{Nb}$  does<sup>2)</sup>. Furthermore, the heavier system shows sidesplash not predicted by cascade collision models. Gold on gold data on the plastic ball had been taken but not yet analysed when I left Berkeley. Pion data from the plastic ball and from the Japanese HISS spectrometer runs were not yet available.

In this paper I will present some new analyses from our JANUS spectrometer group and some new theoretical interpretations, but the main data base as of a year ago was well summarised by Nagamiya last year at Lake Balaton<sup>4)</sup> and at Florence<sup>5)</sup>.

## 2. Size of Collision Region

I will not repeat the equations of boson interferometry as given in several references<sup>3-6)</sup>. The idealized Hanbury-Brown-Twiss effect<sup>7)</sup> for non-interacting identical bosons emitted from an extended, chaotic source shows a peaked correlation function at twice the uncorrelated value for momenta  $\vec{q}_1$  and  $\vec{q}_2$  identical. The width in  $q$  (MeV/c) of the momentum correlation function gives a measure of r.m.s. source size  $R$  in that it equals  $h/R$ . Likewise, the width in pion energy difference  $q_0$  (MeV) of the correlation function gives a measure of lifetime  $\tau$  of the source in that the energy width equals  $h/\tau$ .

Fig. 1 shows data of our group from Zajc et al.<sup>3)</sup> These data have been Gamow-corrected for the Coulomb repulsion between the like-charged

pions. Our data were taken around  $45^\circ$  (lab. frame) for near-symmetric systems and for beam kinetic energy of 1.8A GeV.

The U.C. Riverside group has analysed  $2\pi^-$  correlations from streamer-chamber photographs at slightly lower beam energies of 1.5A and 1.2A GeV<sup>6)</sup>. They take  $\pi^-$  tracks over the large angular and momentum range of the streamer chamber acceptance.

A problem hitherto in comparing results has been the ambiguity in the size R and time  $\tau$  determinations. Often a  $\tau$  value was assumed in order to specify R. Now both groups have analysed results on likelihood contour plots in the  $\tau$ -R plane. Fig. 2 gives combined results of Zajc et al. counter experiments<sup>3)</sup>. The streamer chamber results (with trigger bias toward central collision) give somewhat larger sizes and lifetimes for  $\pi^-$  correlations in Ar + KCl. The difference in results may result from the lower momentum cut off of the streamer chamber, as will be discussed later. The differences do not affect the qualitative conclusions here. On Fig. 2 are indicated size and lifetimes expected from two models. The "schematic model" assumes the source is just the geometric overlap in space and time of nuclear density distributions of the two colliding nuclei, and the "Cascade Code" is based on theoretical results of Cugnon and Koonin collaborations<sup>8,9)</sup>.

Measured lifetimes are consistent with cascade codes, but the measured source sizes are much larger. Furthermore, the source sizes indicated by 2-proton interferometry<sup>10)</sup> are at least 50% smaller than those from pion interferometry.

Furthermore, the 1.5A GeV streamer chamber pion work<sup>6)</sup> shows the source to be spherical, not the lens shape of a schematic overlap model at some mean impact parameter. Beavis et al.<sup>6)</sup> also show the source size does not seem to depend on pion multiplicity, hence, on impact parameter. They do find the higher momentum pions ( $P_{c.m.} > 150$  MeV/c) show a source size 50% smaller than that measured by lower momentum pions ( $P_{c.m.} < 150$  MeV/c). They have since Nagamiya's reviews<sup>4,5)</sup> published their 1.2A GeV analyses, showing 20% smaller source size and somewhat longer lifetime.

What are we to make of all these observations? I would suggest that the pions, being of lower mass and consequently higher velocity

than fireball protons, propagate within the spectator matter to give large effective source sizes, independent of impact parameter. We defer to a later section the problem of size dependence on pion momentum. Next we turn to the old puzzle of different particles yielding different effective temperatures.

### 3. Temperature of Collision Region

Here I again call attention to the reviews of Shoji Nagamiya<sup>4,5)</sup>, whose group is responsible for the most of the proton, pion and kaon inclusive spectra near  $90^\circ$  (c.m.). My Fig. 3 is taken directly from their work, and it shows the  $E_0$  slope parameter (effective temperature) for various heavy ion collision systems and particles. A long-standing puzzle has been why pions, protons, and kaons show successively hotter temperatures for the same system. Nagamiya has developed the scenario of expanding, cooling fireball with kaons, protons, and pions decoupling at successive times in order of their decreasing mean free paths in nuclear matter. This scenario in its simplest form has been criticized by saying the nucleons are the thermal "bath", and protons can hardly decouple from themselves at an earlier stage than pions. Furthermore, from the Joule-Thomson experiment of expanding gases into a vacuum we know that expanding gases only cool by doing work on something.

Let us try to devise a modification of the collision scenario consistent with the qualitative observations we have today. Indeed the drop in temperature from kaon probe to proton probe may come from a decoupling of kaons earlier in the collision time. The cooling of the participant matter may come from doing pressure work on the confining spectator matter, and clear evidence for a resulting bounce-off effect is emerging from plastic-ball work<sup>2)</sup>.

The nucleons, however, are too slow-moving to equilibrate temperature between participants and spectators. This principle has been the basis of the much-used Swiatecki abrasion-ablation model. For collision systems of nuclei below mass 60 the participant matter does not fully lose forward momentum<sup>2)</sup>, so the two-fireball model of das Gupta<sup>11)</sup> is a refinement. Now that there is plastic ball evidence<sup>2)</sup> for sidewise

collective flow a minimal model for proton spectra will involve four sources, the two spectators and the two fireballs, all with finite amounts of perpendicular momentum. The fireballs are hot sources giving the main contribution to the  $90^\circ$  (c.m.) spectra and temperature of Fig. 3, while the spectator sources will have temperatures of only a few MeV, dominating the spectra near beam and target velocity.

Finally, we come to the pions, which appear slightly cooler than protons. I have already suggested in the previous section that their large source sizes come because the light, fast moving, interactive pions spread out into the spectator region. In this process pions will undergo inelastic scattering in spectator matter, hence experience cooling analogous to fast neutrons in a cold moderator.

One interesting point is the Riverside finding<sup>6)</sup> that the fast pions indicate a smaller source size than the slower pions. This problem leads us to consider the next subject, calculation of spectator influence on pion spectra. Qualitatively, it would seem consistent that the slower pions had spread more completely into the spectator region and consequently cooled more and showed a larger source size.

#### 4. Pion Interactions with Spectators

Even the humble Coulomb interaction has proved troublesome to deal with for the pion heavy ion system. Various treatments successfully reproduced the  $\pi^-$  cusp at beam velocity and corresponding  $\pi^+$  depression<sup>12)</sup>. This  $\pi^-$  spectral feature has been used to probe the initial velocity dispersion of projectile fragments before nucleon evaporation. There are no new data since the published analyses, so I will not repeat the material. It will be most interesting when the Japanese HISS experimental data of Hashimoto and collaborators are worked up to provide  $\pi^-$  spectra associated with particular projectile fragments of measured momenta.

The theoretical Coulomb treatments that preceded  $\pi^-/\pi^+$  ratio measurements near rest in the centre-of-mass greatly overestimated the ratio<sup>13,14)</sup>. Radi et al.<sup>12)</sup> restudied the problem by Monte Carlo trajectory methods after experiment showed central ratios near 1.5 for

$^{20}\text{Ne} + \text{NaF}$  and  $^{40}\text{Ar} + \text{Ca}$ . The new Monte Carlo studies gave consistency with experiment, and we attribute the lack of agreement of the earlier Monte Carlo work<sup>13)</sup> to their not including pionic absorption in nuclear matter. We should mention that Harris<sup>15)</sup> reports  $\pi^-/\pi^+$  ratios near unity for  $^{40}\text{Ca}$  on Ca at 1.05A GeV, although he has not reported a final analysis of their data.

As with so much of the first-generation BEVALAC work, the impact-parameter averaging inherent in (singles) spectra can smear out and obscure details. We look forward to the Japanese HISS experimental data on  $\pi^-$  and  $\pi^+$  spectra near rest in the centre-of-mass but tagged by specific projectile fragments.

Indeed, the latest Monte Carlo trajectory study of Radi et al.<sup>16)</sup> anticipates such tagged data. To make the problem well defined, with the least dependence on collision models we treat only the grazing collision with minimal overlap. The pions are assumed formed in an isotropic Boltzmann distribution at the instant the two nuclei are tangent. Specifically, we have first treated  $^{207}\text{Pb}$  on  $^{207}\text{Pb}$  at 0.4A GeV laboratory kinetic energy. Given this geometry most pions will enter spectator matter, and we must deal with nuclear as well as Coulomb interactions. (In our previous work<sup>12)</sup> we arbitrarily ruled that total pion absorption abruptly occurs for trajectories passing within 0.8R of a spectator centre.) Pion scattering and absorption in nuclei cannot be handled by the familiar static optical model potentials. The dominant role of the (3,3) resonance ( $\Delta$ -intermediate) for the pion-nucleon P-state makes the interaction strongly velocity-dependent. An approximation widely used for pionic atom analysis is the Kisslinger potential, with a gradient term. Michigan State theorists<sup>17)</sup> have applied potentials of Kisslinger form also to pion scattering data. R. Seki<sup>18)</sup> has also made extensive analyses with potentials of this kind. On the other hand, various pion scattering researchers<sup>19)</sup> have been critical of the Kisslinger potential for scattering and have worked in momentum space with potentials that are explicitly non-local. Mehrem et al.<sup>20)</sup> have chosen to use the Kisslinger potential with parameters deduced by Carr, McManus, and Stricker-Bauer. Since we are dealing mostly with pions of

larger wave length than the diffuseness of the nuclear surface, we have chosen to take a uniform nuclear-density distribution, sharply falling to zero at the nuclear surface. Then we handle the pions incident on the nuclear surface as reflecting or refracting like light at a boundary where the optical index of refraction changes.

We have re-examined and revised the pion mean-free-path calculations of Hecking<sup>21)</sup>. He encountered some problem with the Schrödinger equation in that growing solutions (negative mean free paths) were encountered for the lowest energy pions, and his procedure for handling this problem was somewhat arbitrary. The problem arises because the static potential term is 20-70 MeV repulsive, so lowest energy pions are tunneling. Thus, with an imaginary wave number in nuclear matter the imaginary part of the velocity-dependent Kisslinger term becomes a source, not a sink, and solutions grow. We go back to the original Klein-Gordon equation

$$\left[ i\hbar \frac{\partial}{\partial t} - V_{\text{opt}}^2(\tau) \right]^2 \psi(\vec{\tau}, t) = [-(\hbar c)^2 \Delta^2 + \mu^2] \psi(\vec{\tau}, t)$$

with  $\omega = [p_0 c]^2 + \mu^2]^{1/2}$  and  $\mu = m_\pi c^2 = 140$  MeV. We find four plane-wave solutions and select the physically meaningful damped solution.

Fig. 4 shows (above) the "mean free path" in  $^{207}\text{Pb}$  for  $\pi^0$  plotted against external (vacuum) wave number  $k_0$ . The lower half of the figure gives the real ( $k_1$ ) and imaginary ( $k_2$ ) components of the wave number within the nucleus.

The results for mean free path are not much different from those of Hecking, but we now understand the drops of mean free path at lowest wave numbers. The drop has nothing to do with true absorption but is merely a manifestation of the tunneling in the repulsive static potential, as can be seen by the predominantly imaginary wave number at low  $k_0$  values. The longest mean free path  $\sim 2.7$  fm occurs for  $k_0$  of  $0.4 \text{ fm}^{-1}$ , a kinetic energy of 20 MeV.

The mean free path falls to about 0.6 fm in the vicinity of the (3,3) resonance at the highest  $k_0$  values shown. Of course, there is an isospin dependence to the pion potential, so results will differ for  $\pi^+$ ,  $\pi^-$ , or different N/Z ratios. However, the  $\pi^0$  behaviour is represen-

tative. An appreciation for the isospin dependence can be gained from perusal of Fig. 5, where the real and imaginary parts of the optical potential in  $^{207}\text{Pb}$  are shown for various external wave numbers  $k_0$ . Here all three pion isobars are shown.

In Fig. 6 we show the  $\pi^0$  reflectivity from the  $^{207}\text{Pb}$  surface vs. wave number  $k$  for a range of angles of incidence from normal ( $0^\circ$ ) to grazing ( $90^\circ$ ).

The reflectivity reaches a very low value for normal incidence and  $k_0 \sim 0.5$ . At this wave number the magnitude of the optical potential is a minimum (see Fig. 5), and there is consequently almost no refractive-index mismatch at the surface.

The transmitted rays across the nuclear surface are assumed to bend according to Snell's law

$$n = \frac{k_1 + ik_2}{k_0} = \frac{\sin\theta_0}{\sin\theta}$$

Complex values of angles result, but these are tractable in the semi-classical trajectory framework.

With the above methods for pion behaviour in spectator matter and at nuclear boundaries, Radi et al.<sup>16)</sup> carried out the full-fledged Monte Carlo trajectory program for grazing collision of Pb on Pb at 0.4A GeV. Initial Gaussian momentum distributions corresponding to pion temperature of about 60 MeV were used. The results of about 200,000 trials are summarized in Fig. 7, 8, and 9. The Lorentz-invariant cross sections are given for various angles (c.m.) as a function of pion momentum (c.m.) for, respectively,  $\pi^0$ ,  $\pi^+$ , and  $\pi^-$ . The  $\pi^0$  experiences no electromagnetic interaction but does scatter, refract, and absorb in the grazing nuclei. The main feature seems to be a  $\pi^0$  peaking near rest in the c.m. frame. This is reasonable, since the grazing geometry subjects most pions with finite initial momentum to nuclear absorption.

For the  $\pi^+$  in Fig. 8 we see the persistence of the peak for pions at rest in c.m. Now we see a tremendous suppression of cross section at  $0^\circ$  and small angles at velocities near the projectile velocity. In this type of classical calculation  $\pi^+$  will be strictly excluded from having kinetic energies in the projectile (or target) frame less than

the Coulomb repulsive energy on a unit charge at the  $^{207}\text{Pb}$  nuclear surface ( $E_{\text{min}} \sim \frac{Ze^2}{R} \sim 14 \text{ MeV}$ ). Finally in Fig.9 for the  $\pi^-$  we see the same peak for pions at rest in the c.m., but we also see a small peak at  $0^\circ$  near beam velocity ( $\sim 65 \text{ MeV}/c$ ).

### 5. Concluding Discussion

The next year or two should bring much new pion data from heavy ion collisions that could make drawing firm conclusions here quite premature. Nevertheless, it seems clear that pions indicate larger and cooler sources than do protons. I believe these differences arise from the higher propagation velocities of the less massive pions, which can scatter and cool in spectator matter. There are hints of pion source coherence (pion condensation?) in the lowest energy (1.2A GeV) studies<sup>6)</sup>. Lack of space precludes review of the new record low energy  $\pi^0$  production at the Superconducting MSU cyclotron with heavy ions<sup>22)</sup>. Cross sections are orders-of-magnitude higher than non-cooperative mechanisms provide<sup>23)</sup>. There is much fascination in the unfolding of the relativistic heavy ion adventure, and the pion, conveniently provided in three charge states, continues to be a vital probe.

### 6. Acknowledgements

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## FIGURE LEGENDS

Fig.1. Two-pion correlation functions  $(d^6\sigma/dp_1^3 dp_2^3)/(d^3\sigma/dp_1^3)(d^3\sigma/dp_2^3)$  after correction for Coulomb repulsion between the two pions. The left-hand plots have as abscissa the pion momentum difference  $q(= |\vec{p}_1 - \vec{p}_2|)$ , while the right-hand plots are against pion energy difference  $q_0(= |E_1 - E_2|)$ . Data for three systems, all at 1.8A GeV beam kinetic energy, are shown.

Fig.2. Likelihood contour plot for combined data set of the three systems of Fig.1. The dot gives the most likely pion source size and lifetime, while the contours surround regions at the 68% and 95% confidence levels. The triangle and arrows indicate the location of source size and lifetime from two models described in the text.

Fig.3. Values of the "firecloud temperature" parameter  $E_0$  as a function of beam energy in the nucleon-nucleon center-of-mass frame. Results for several heavy-ion systems and for different probe particles are plotted. Dashed lines through the points are merely to guide the eye. This figure is taken from a talk of Nagamiya.

Fig.4. The upper half shows the effective attenuation length (fm) of a  $\pi^0$  in  $^{207}\text{Pb}$  as a function of incident wave number  $k_0$  just outside the nuclear surface. The lower half correspondingly shows the real ( $k_1$ ) and imaginary ( $k_2$ ) components of the  $\pi^0$  wave vector inside the nucleus.

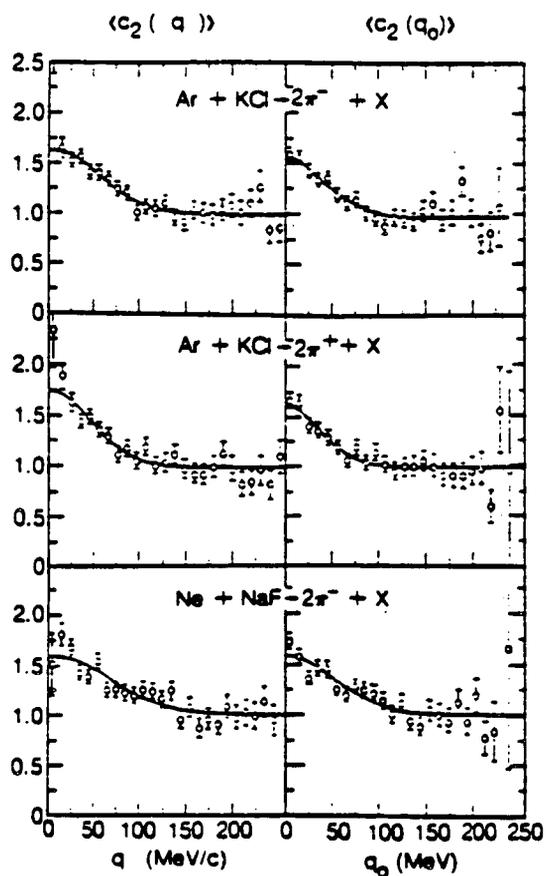
Fig.5. These plots show the real (upper) and imaginary (lower) components of the effective optical potential for  $^{207}\text{Pb}$  vs. wave number  $k_0$  just outside the nuclear surface. All three charge states of the pion are shown to give a feeling for the isospin dependence of the potential.

Fig.6. This family of curves gives the reflectivity of a sharp nuclear surface ( $^{207}\text{Pb}$ ) for  $\pi^0$  at various angles of incidence and wave numbers.

Fig.7. Monte Carlo trajectory results for  $\pi^0$  spectrum expected from a grazing collision of two  $^{207}\text{Pb}$  nuclei. The center-of-mass frame is used.

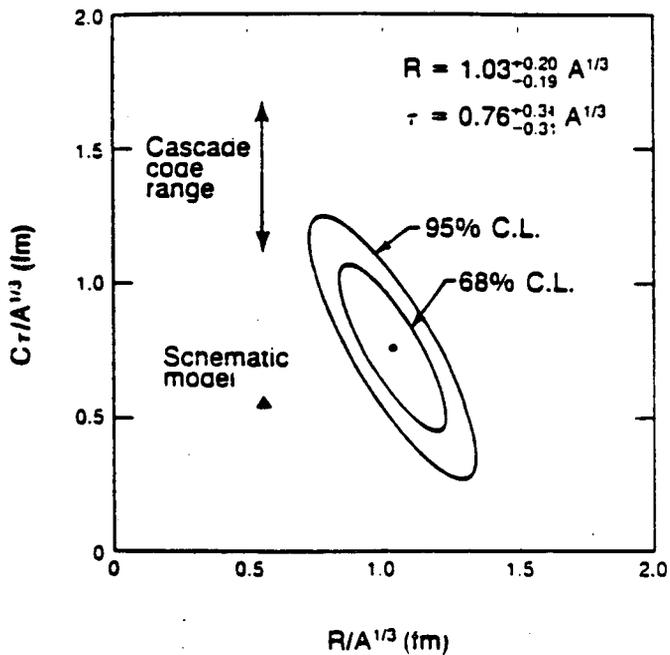
Fig.8. Same as Fig.7. except for  $\pi^+$ .

Fig.9. Same as Fig.8 except for  $\pi^-$ .



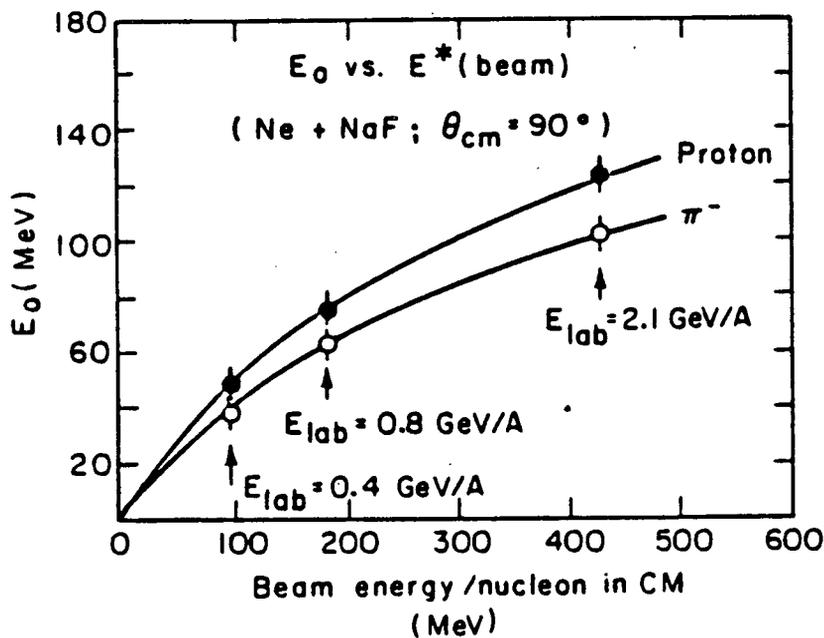
LBL 843-948

Fig. 1



XBL 2212-7487

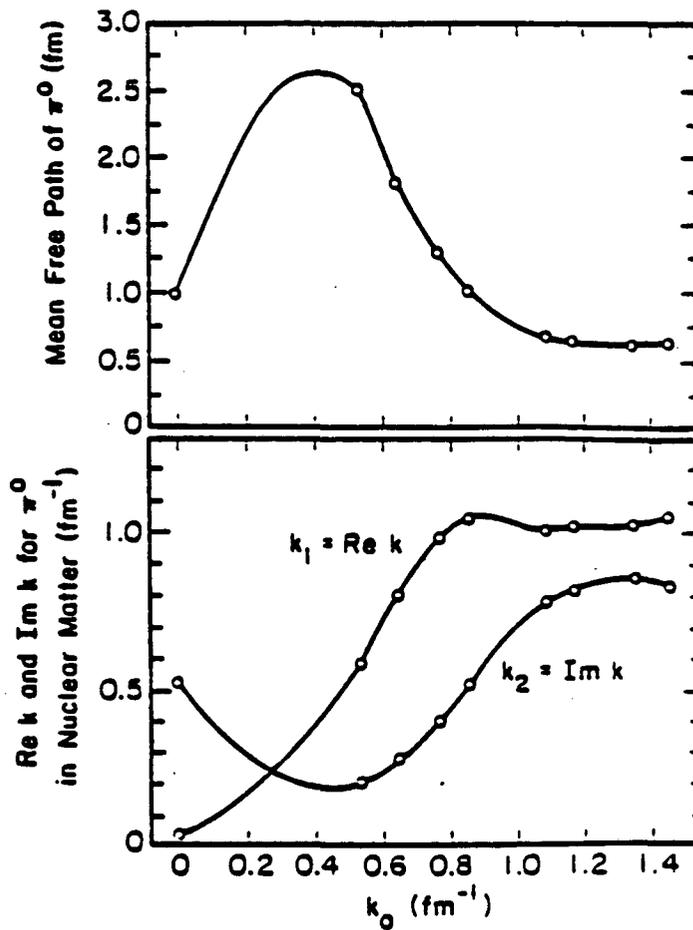
Fig. 2



XBL 788-14958

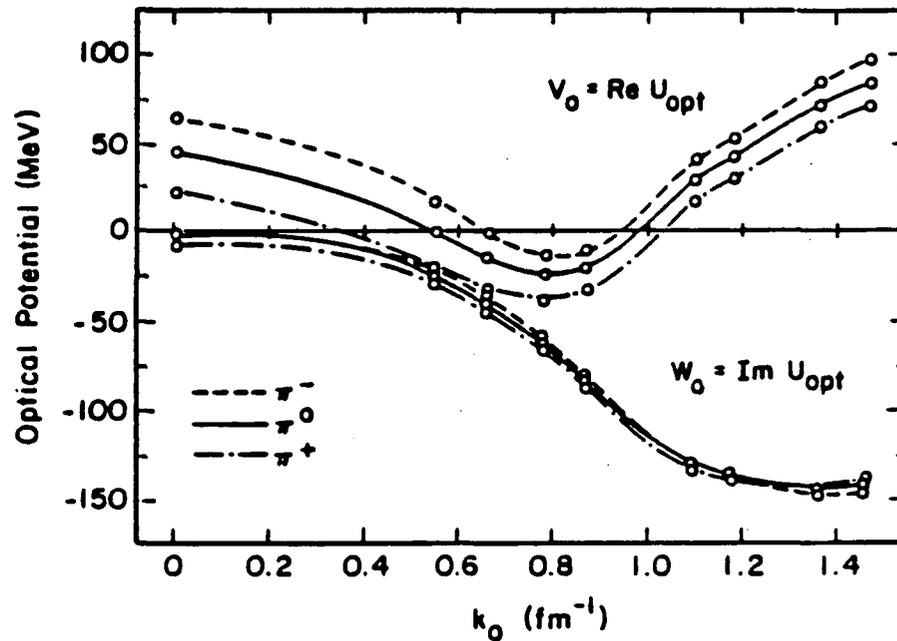
Fig. 3

Fig. 4



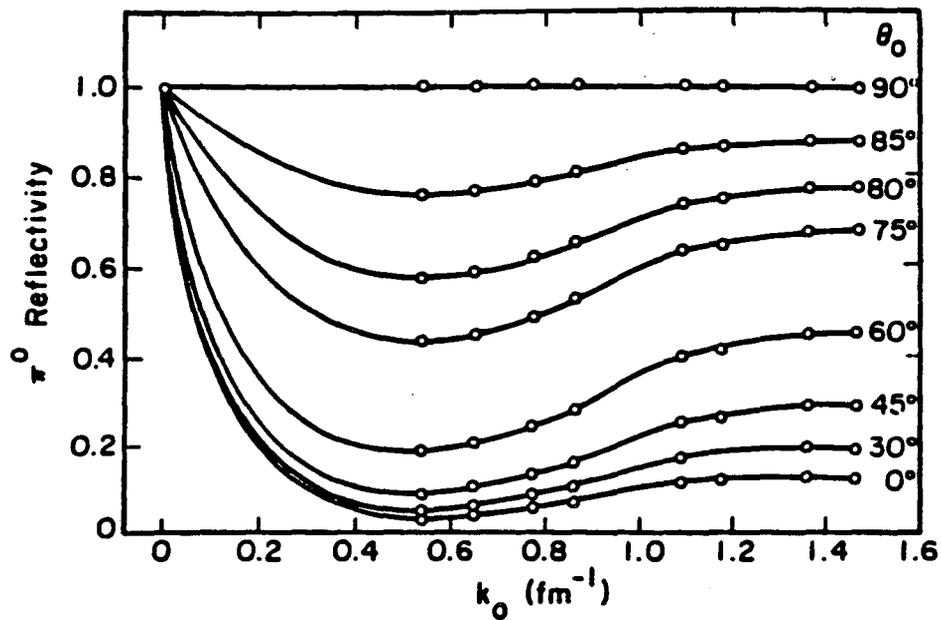
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Fig. 5



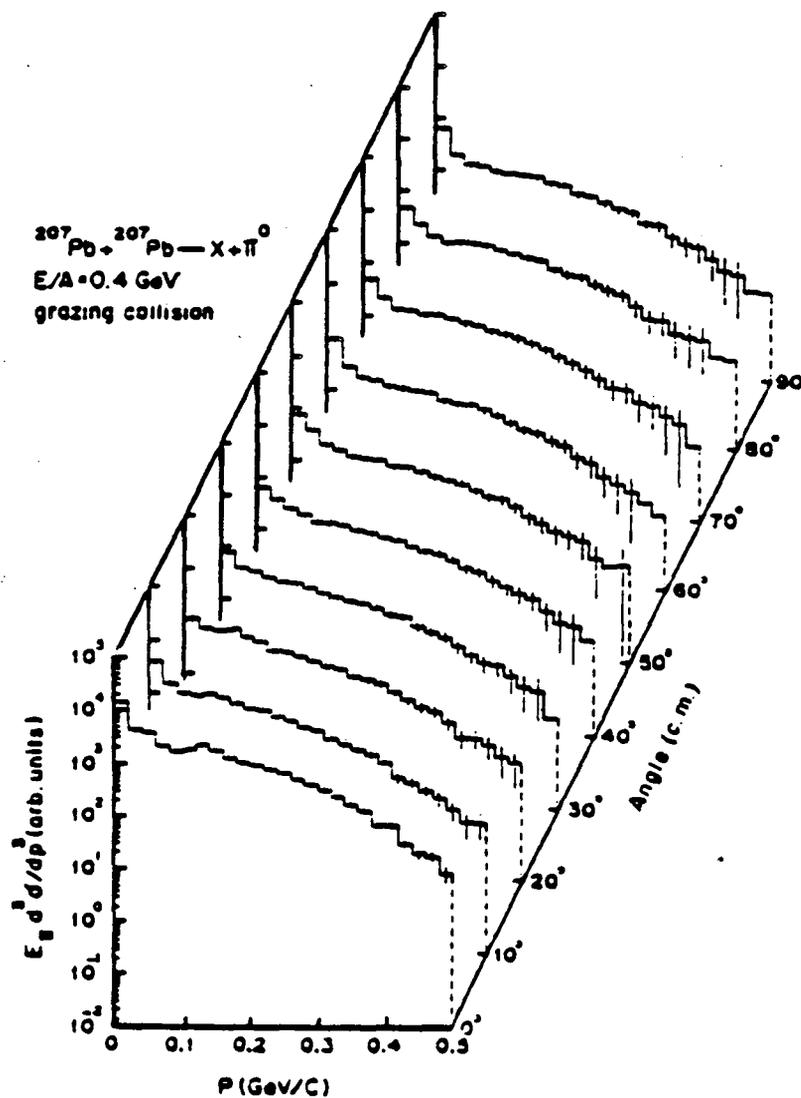
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Fig. 6



XBL 8310-12032A

Fig. 7



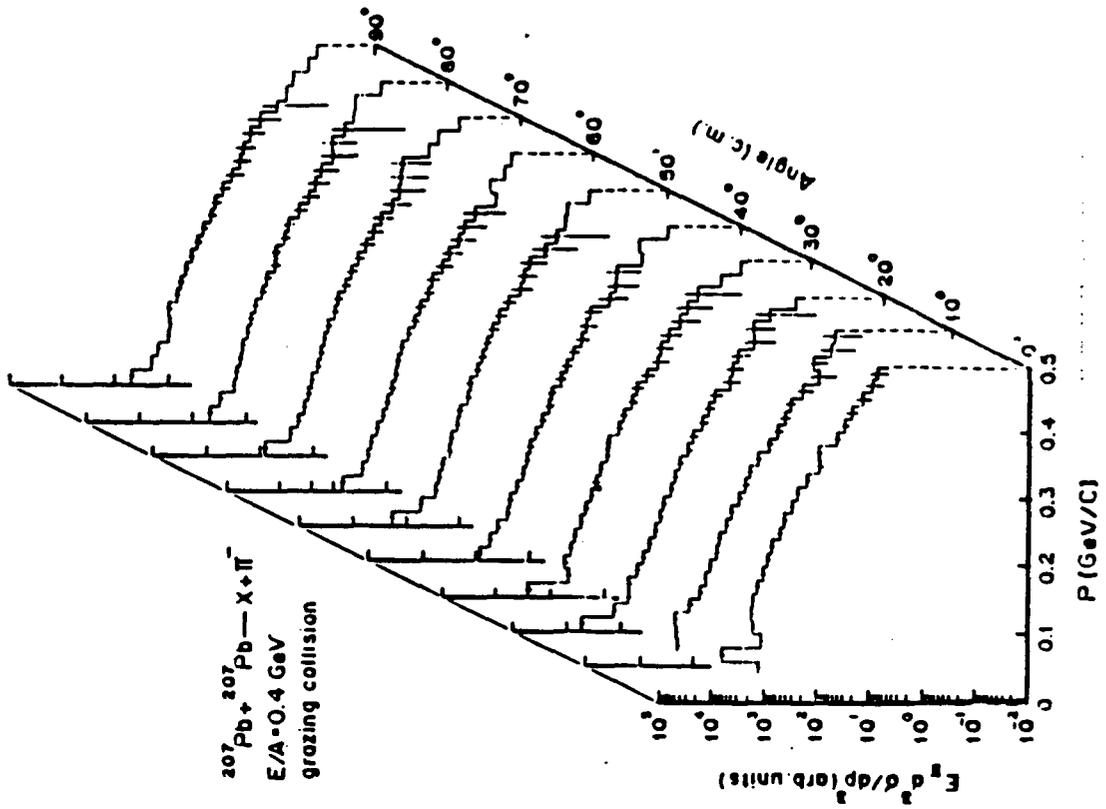


Fig. 9

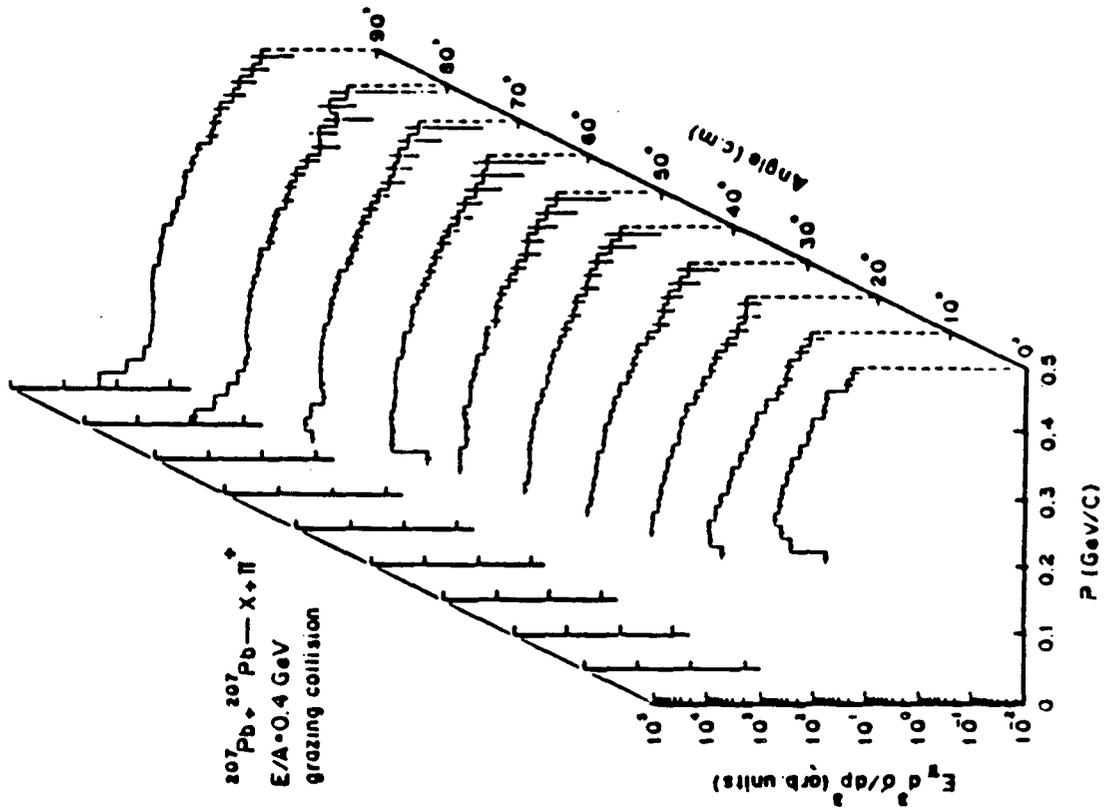


Fig. 8



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