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CMB Data Analysis in the example of the POLARBEAR experiment and the LiteBIRD
experiment

by

Kolen Cheung

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Physics

and the Designated Emphasis

in

Computational & Data Science & Engineering

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Adrian Lee, Co-chair

Doctor Julian Borrill, Co-chair

Professor Katherine Yelick

Professor Matt Pyle

Fall 2022

CMB Data Analysis in the example of the POLARBEAR experiment and the LiteBIRD
experiment

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by
Kolen Cheung

Abstract

CMB Data Analysis in the example of the POLARBEAR experiment and the LiteBIRD experiment

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Kolen Cheung

Doctor of Philosophy in Physics

and the Designated Emphasis in

Computational & Data Science & Engineering

University of California, Berkeley

Professor Adrian Lee, Co-chair

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In this dissertation, I briefly go through my subject areas—from Astrophysics, Cosmology, to Cosmic Microwave Background (CMB) radiation; and Data Science & High-Performance Computing. I go through the beginning of the Universe briefly in part [I](#) and introduce the Physics of the CMB in part [II](#). Then I move on to how it can be observed experimentally, mathematically, and computationally in part [III](#). In parts [IV](#) to [VI](#), I write about 2 specific CMB experiments—POLARBEAR & LiteBIRD, together with my research involved in them.

In part [V](#), an original research on a measurement of the CMB B -mode at sub-degree scales is presented using the 3rd–5th seasons of POLARBEAR observations. This demonstrates the capability of a single medium aperture telescope with a Continuously Rotating Half-Wave Plate (CRHWP) to target both the low- ℓ and high- ℓ B -mode science at $50 \leq \ell \leq 3000$.

In part [VI](#), the effect of detector crosstalk systematics on the CMB power-spectra is studied in detailed in the example of the LiteBIRD experiment. An analytical, map-domain solution is developed, a computational simulation method is implemented, and the two predictions are compared using the LiteBIRD experiment design specification. The general agreement of the two shows that the analytical model well describes the LiteBIRD experiment, and can be used to calculate the systematic effects without performing time-consuming time-domain simulations with high degree of accuracy. This

can be used to forecast the amount of crosstalk systematics and to develop mitigation strategies. A mitigation strategy is presented, which is a hardware-based mitigation method that minimizes the crosstalk systematic effects in the 2pt power-spectra.

While the author claims that this dissertation is original, unpublished, and is an independent work, only those in parts [V](#) and [VI](#) are original research. Other materials in parts [I](#) to [IV](#) are introductory materials summarizing well-known knowledge in the field. In particular, many plots, graphs and visualizations from these parts are from existing publications which should fall under “fair use” and are properly cited. There are a few exceptions that images are obtained through internal circulations including presentations from others in the collaborations where original source is not known. This is documented in sec. [38.6](#).

To HER

To my grandma Pick Yee Lau,
who chose to care for her small grandson over a surgery for full recovery of her leg.

To my mother Joyce Tsui,
who combats depression alone when her only son pursues a PhD study in a foreign country.

To my wife Ivy Cheung,
who sacrifices her career and follows me till the end of the world.

To my beloved Belle the cat,
who has accompanied me throughout my study, and is our only family in the US.

I miss you, grandma and Belle,
I wish you would be with me in the graduation,
and I look forward to meet you again in the new heaven and new earth.

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¹Takakura et al., “Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

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Part I

The beginning of our Universe

Chapter 1

The Big Bang

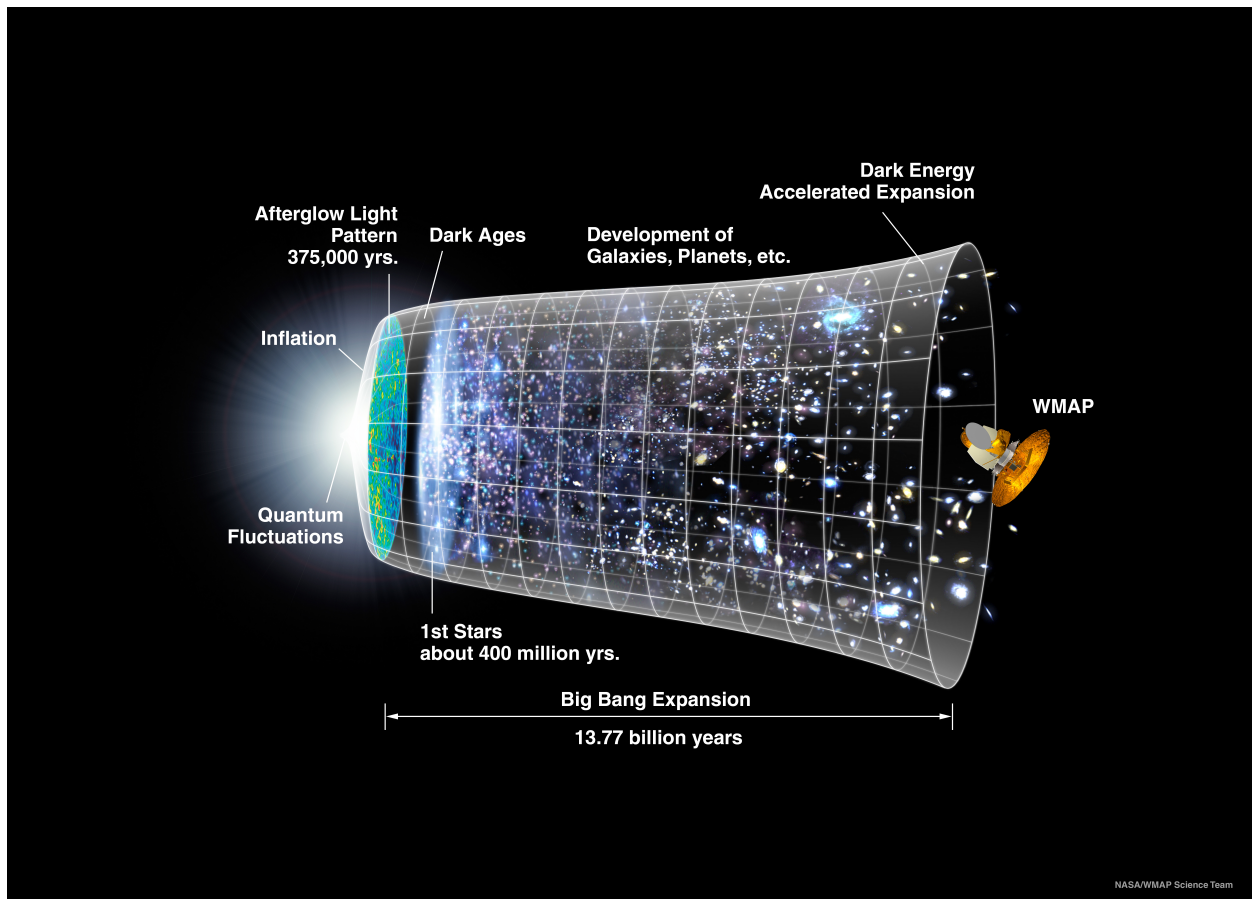


Figure 1.1: Timeline of the Universe¹

¹NASA / WMAP Science Team, *Timeline of the Universe*.

The Universe starts with a Big Bang.

To a good approximation, it started with a uniform density soup of particles and as they are so hot, they are in thermal equilibrium. As the Universe expands they are diluted and cooled down, and eventually they fell out of equilibrium and decoupled with others one by one.

One particular important moment is when the free electron and proton starts to form neutral Hydrogen and it becomes transparent for photon to start free-streaming to us. This is called recombination. The time was around year 375,000. The majority of these photons have not been re-scattered when they reached us. Since they are now peaked at microwave range, this is known as the Cosmic Microwave Background (CMB) Radiation.

However, the Universe was not completely uniform. It started with tiny primordial fluctuations which grew under the influence of gravity. Before any substantial structure was formed, there was the (cosmic) Dark Ages. Then the first stars were born as the fluctuations grew non-linear. And the Universe has grown more complex ever since.

In recent history of the Universe, the expansion is accelerating. This is one of the key signature of our Universe and is caused by Dark Energy.

That briefly summarized the Big Bang theory, in particular, the Λ CDM model that is well-understood via general relativity, thermodynamics, and particle physics, and is confirmed observationally to high precision. Basically, all known branches of fundamental physics lead to this successful standard model of cosmology.

Chapter 2

Fine tuning problems

However, the Big Bang theory left us unsatisfied for multiple fundamental problems.

For example, the best-fit Λ CDM model tells us that the Universe is consistent with zero curvature today whereas there are no fundamental reasons why it should be exactly zero. And because as the Universe grows, curvature also grows, the initial curvature might be incredibly close to zero—this is one example of a class of problems known as fine-tuning problems.

Another fine-tuning problem is the horizon problem. Given our understanding in the Λ CDM model, we can ask what was the size of region that was in causal contact at the time the CMB formed. It turns out it corresponds to about 2° on our sky. Yet, we found that the Universe (and the CMB) is pretty uniform across the whole sky.

Chapter 3

Cosmic inflation

A solution to the fine tuning problems from last chapter is inflationary models. The basic feature of inflation is that the Universe initially goes through a period of rapidly accelerating expansion, expanded by a factor as much as $\sim e^{60}$. The “initial condition” of Λ CDM model is the aftermath of this, which explains why the curvature is so small “initially”. Also, the seemingly causally disconnected region was once in causal contact before the inflation, which explains the horizon problem.

Inflationary models also made some general predictions, and some of these have been observationally verified, such as the scalar spectral index $n_s < 1$.

Although inflation provided explanations to these problems and there are hints showing that inflation happened, it is still unproven, and the details of inflation are largely unknown. Unlike the situation with the Big Bang model, which has a concrete, parameterized Λ CDM model that is derived from known fundamental physics, there are plenty theoretical models that is beyond the Standard Models and fits the general picture of our known Universe without much experimental or observational evidence as which one is the true fundamental theory of physics.

The relevant energy scales for inflationary physics cannot be reached experimentally unless there is fundamental paradigm shift in how we accelerate particles. Therefore, observational probes are our best hope in studying inflationary physics.

In the next part, we are going to look at one such probe, the Cosmic Microwave Background (CMB) Radiation. It is the earliest direct signal from the Universe we can possibly observed. Not only does it currently provide some of the most precise constraints in the known Λ CDM model, but it also offers hope on probing inflationary physics by observing the imprint of primordial gravitational wave on CMB.

Part II

The Cosmic Microwave Background (CMB) radiation

Chapter 4

The CMB spectrum

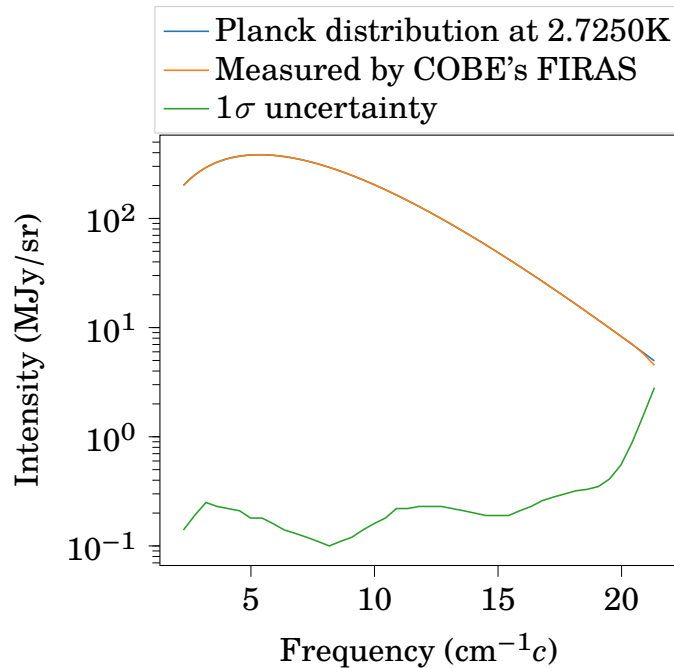


Figure 4.1: CMB blackbody spectrum

As mentioned in part I, when the photons were last scattered during recombination, it was at thermal equilibrium. Fig. 4.1 shows the measurement of the CMB spectrum by COBE's FIRAS instrument¹ which is in close agreement with a blackbody prediction with a temperature of 2.7255(6) K. The last scattering happened at around $z \approx 1100$, corresponding to a temperature of ~ 3000 K.

¹Fixsen et al., "The Cosmic Microwave Background Spectrum from the Full COBE FIRAS Data Set"; NASA, *CMB Monopole Spectrum*.

It also shows that the CMB is incredibly smooth. No matter which part of the sky we are looking at, it is the same temperature within this error-bar.

Chapter 5

The CMB anisotropy

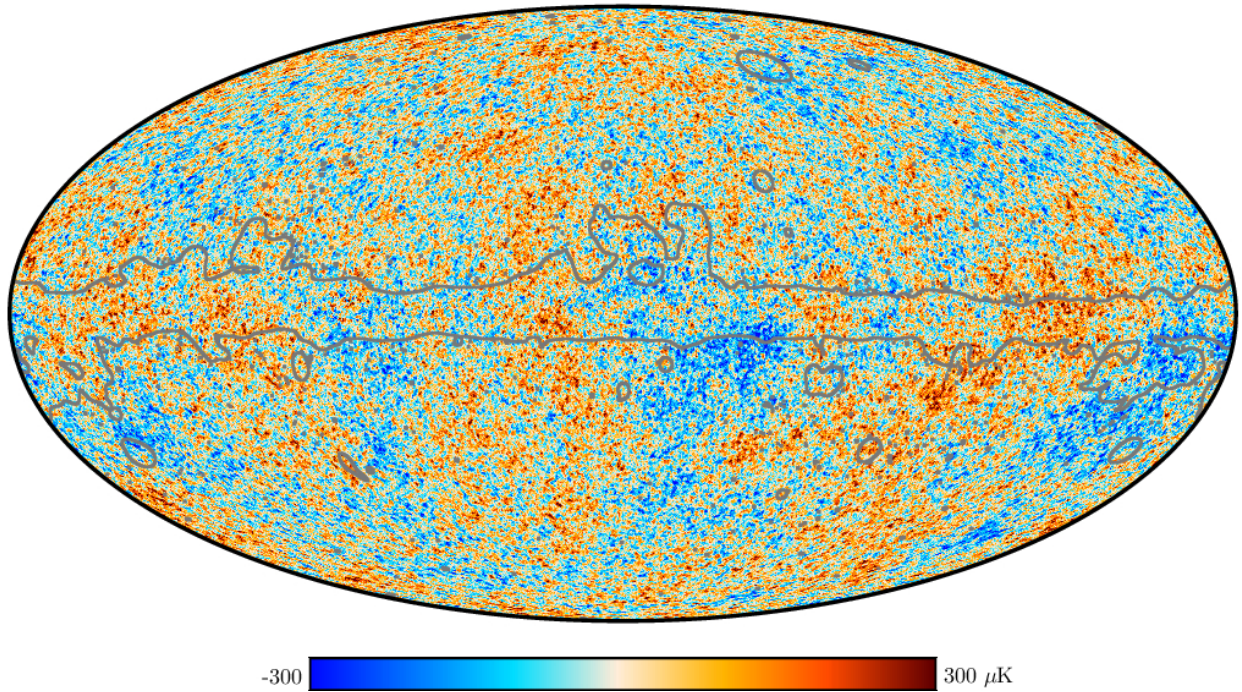


Figure 5.1: Intensity¹

In fact, as seen in the intensity map of CMB in fig. 5.1, the fluctuation of the intensity is about 1 part in 10^5 ,

Moreover, it is known that light waves can be polarized. The primary mechanism to generates polarization is through Thomson scattering of the anisotropic intensity. As Thomson scattering generates only linear polarization, circular polarization is

¹Planck Collaboration, Aghanim, Akrami, Arroja, et al., “*Planck* 2018 Results”.

therefore not expected to be present². Therefore, we can focus our attention to the linear polarization of the CMB.

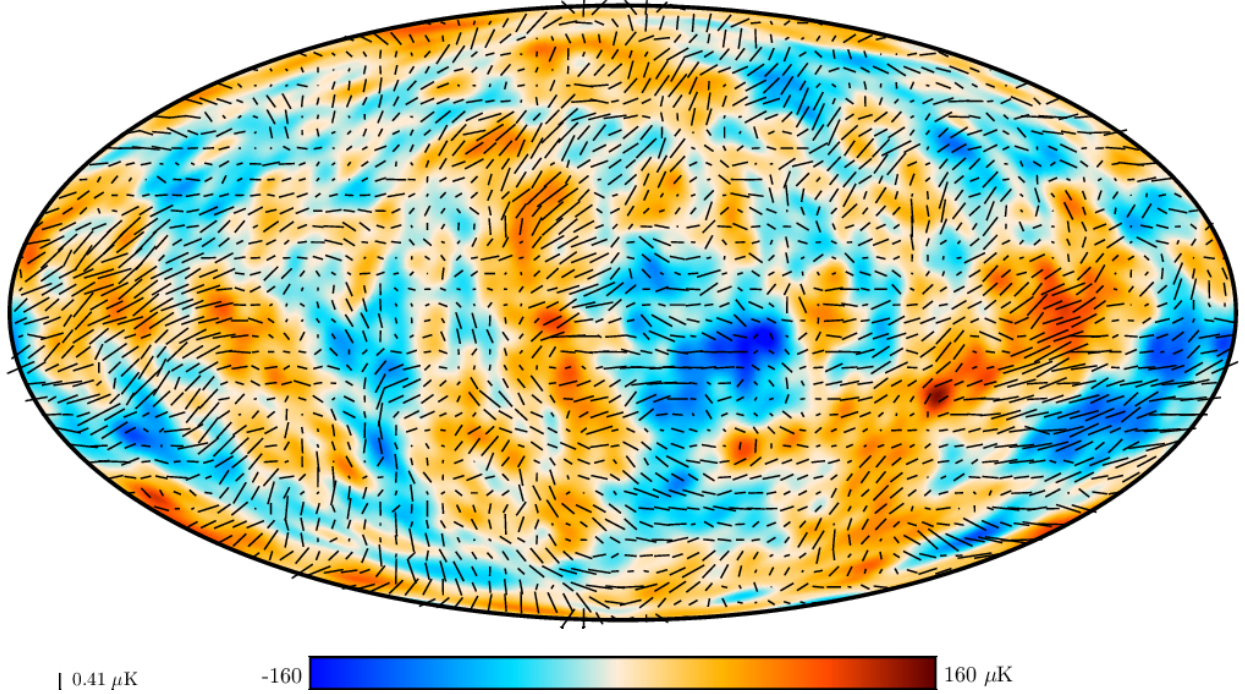


Figure 5.2: Polarization³

Fig. 5.2 shows the smoothed intensity overlaid with the polarization arrows indicating the polarization direction. First, we can see that the arrows and color are quite correlated. And the arrows mainly has divergent patterns, but not so much of a curl. In fact, it is a good idea to decompose the 2 degree of freedom from the linear polarization into this divergence and curl pattern, known as the E-mode and B-mode of the CMB, as this is independent of the choice of coordinate system. In fig. 5.3, we can visualize B-mode using a template constructed by Planck.

Putting all together, T , E & B uniquely characterizes the CMB.

²This assumption has been experimentally constrained, such as in Nagy et al., “[A New Limit on CMB Circular Polarization from SPIDER](#)”. There exists known mechanisms to generate non-zero but tiny contributions of circular polarization in the CMB, such as Zarei et al., “[Generation of Circular Polarization of the CMB](#)”; Mohammadi, “[Evidence for Cosmic Neutrino Background from CMB Circular Polarization](#)”; King and Lubin, “[Circular Polarization of the CMB](#)”; Sadegh, Mohammadi, and Motie, “[Generation of Circular Polarization in CMB Radiation via Nonlinear Photon-Photon Interaction](#)”; Montero-Camacho and Hirata, “[Exploring Circular Polarization in the CMB Due to Conventional Sources of Cosmic Birefringence](#)”; Inomata and Kamionkowski, “[Circular Polarization of the Cosmic Microwave Background from Vector and Tensor Perturbations](#)”.

³Planck Collaboration, Aghanim, Akrami, Arroja, et al., “[Planck 2018 Results](#)”.

⁴Planck Collaboration, Ade, et al., “[Planck Intermediate Results](#)”.

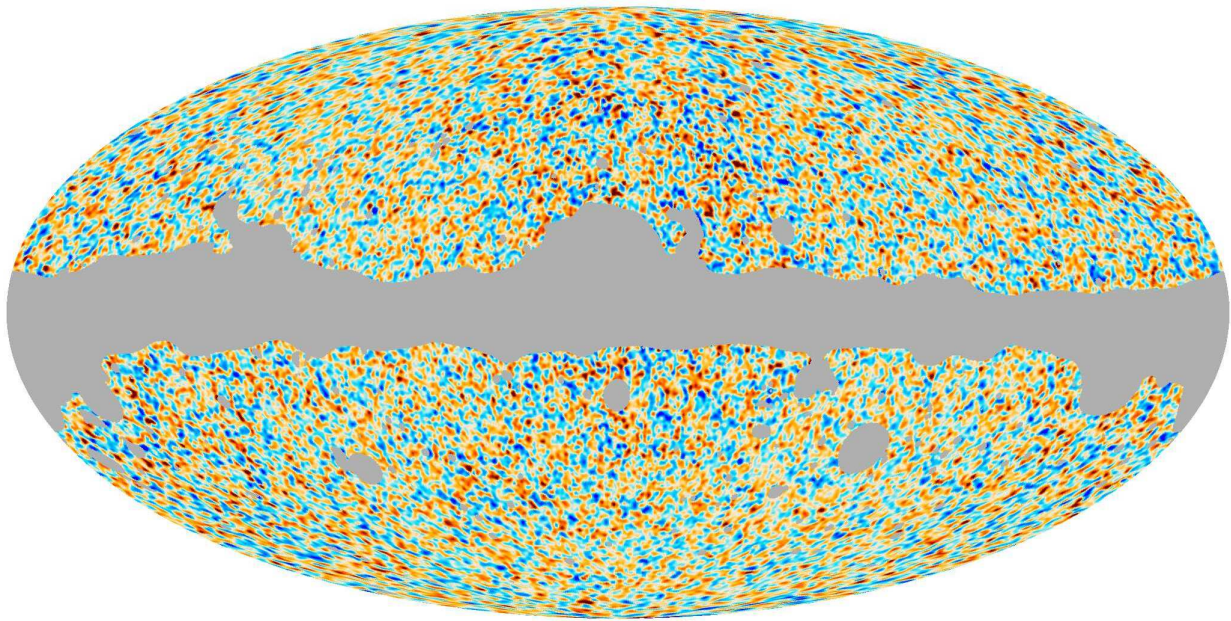


Figure 5.3: B-mode template⁴

Chapter 6

The CMB power-spectra

As these maps are on a 2-sphere, we can perform spherical harmonics transformation on these, and calculate the 2-pt correlation function between them.

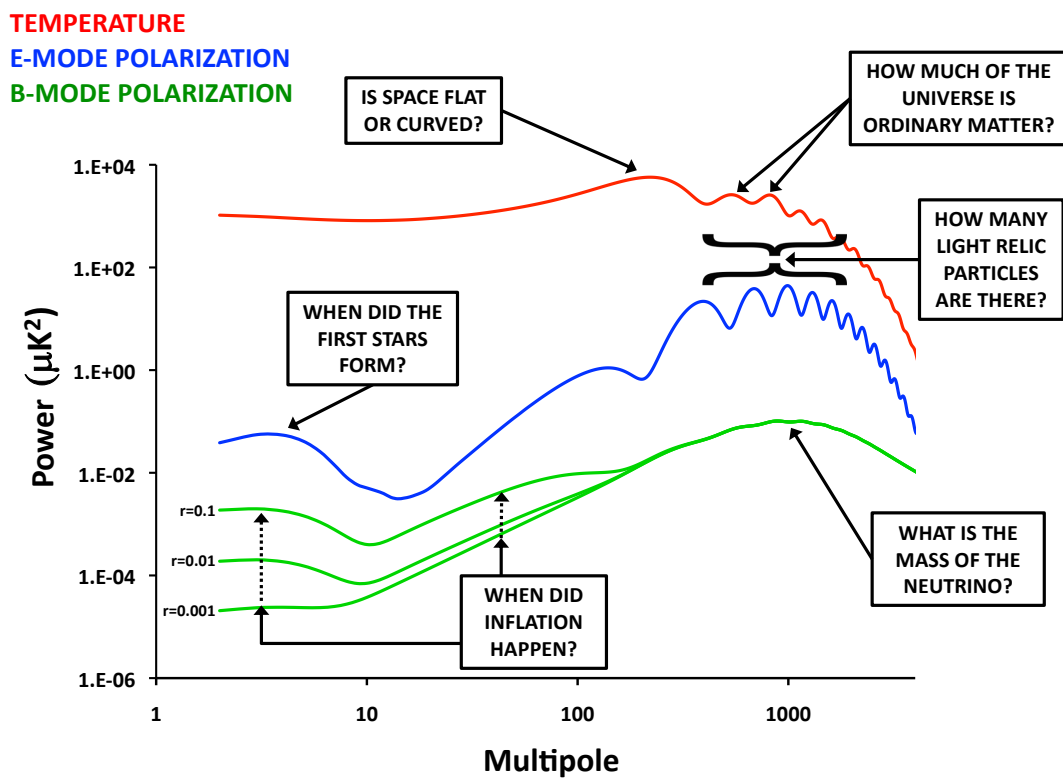
We can form TT , EE , BB , and TE , EB , TB combinations out of them. And by underlying parity symmetry, we expect EB and TB to be exactly zero¹. Hence, we are left with TT , TE , EE & BB .

fig. 6.1 shows the 2-pt correlations a.k.a. power-spectra we theoretically expect. This only depends on the initial parameters unlike the maps.

In this plot we start to see how CMB can be used to extract information about our Universe. Here the horizontal axis is the multipole, roughly corresponds to inverse angular scale. For example, on the top left the low- ℓ TT has a Sachs-Wolfe Plateau, corresponding to the horizon scale. On the right, TT , EE has these oscillatory peaks, a signature of the Baryon Acoustic Oscillations (BAO) between gravity and pressure of all matters. Hence it is sensitive to spatial curvature and matter compositions including matter, dark matter, and dark energy. On the bottom, we see that BB is the weakest, as it can only be contributed by gravitational effects. For example, on the right we have the high- ℓ BB dominated by the gravitational lensing of EE , and is sensitive to the mass of neutrino.

¹Similar to the absence of circular polarization in the CMB, and in fact most if not all null-hypothesis in physics, it can and should be experimentally tested, and discovery can leads to new physics. This phenomenon of non-zero TB , EB is known as cosmic birefringence, and upper limit has been placed by experiments such as Planck Collaboration, Aghanim, et al., “[Planck Intermediate Results](#)”. Potential new physics that can produce cosmic birefringence includes parity violation, Lorentz violation, or axion-photon mixing (Particle Data Group et al., “[Review of Particle Physics](#)”).

²Borrill, [CMB Power-Spectra](#).

Figure 6.1: CMB Power-Spectra²

Chapter 7

The primordial B-mode

Everything that have been mentioned so far are measured to various degree of accuracy. But, on the lower-left, there is something that have yet to be measured. This is the extreme low- ℓ BB , large scale region where no other known form of physics can generate a significant signal of, except for primordial gravitational waves.

This is a tensor perturbation expected from inflation. In other words, while the inflation may have many subtle consequence in different part of the CMB, this is the region where we expect the highest signal-to-noise ratio to detect the signature of inflation. It gives a rare chance for us to glimpse at a direct evidence of inflation. Moreover, it may offer us the capability to start falsifying different inflationary models.

Part III

CMB Observations

Chapter 8

Foreground of the CMB

Now that we know what knowledge we might gain from the CMB, we turn our attention to how we might observe and analyze it.

8.1 Intensity

We already peeked ahead at fig. 5.1, this time we want to focus on some peculiar grey line on the maps. These represent masking regions within which we actually did not observe the CMB.

The reason is simply that when we look for the Cosmic Microwave Background, there are foreground contaminations from many sources. For example, there is a big horizontal region, corresponding to our own galaxy.

Here in fig. 8.1 we can see maps of different components. We are not going into the details here, but it suffices to say that around 20% of the sky is too bright corresponding to the masked out region in fig. 5.1. And also different components may be stronger at different frequencies.

It is more obvious in power-spectra domain, here in fig. 8.2 we can see the foreground components has different frequency dependence from the CMB, and importantly for the Intensity, at some frequencies the CMB is brighter than all foregrounds combined.

8.2 Polarization

Here in fig. 8.3 & fig. 8.4, we see the situation is quite different for polarizations. First, there are only 2 sources of significant foreground polarization—synchrotron and dust.

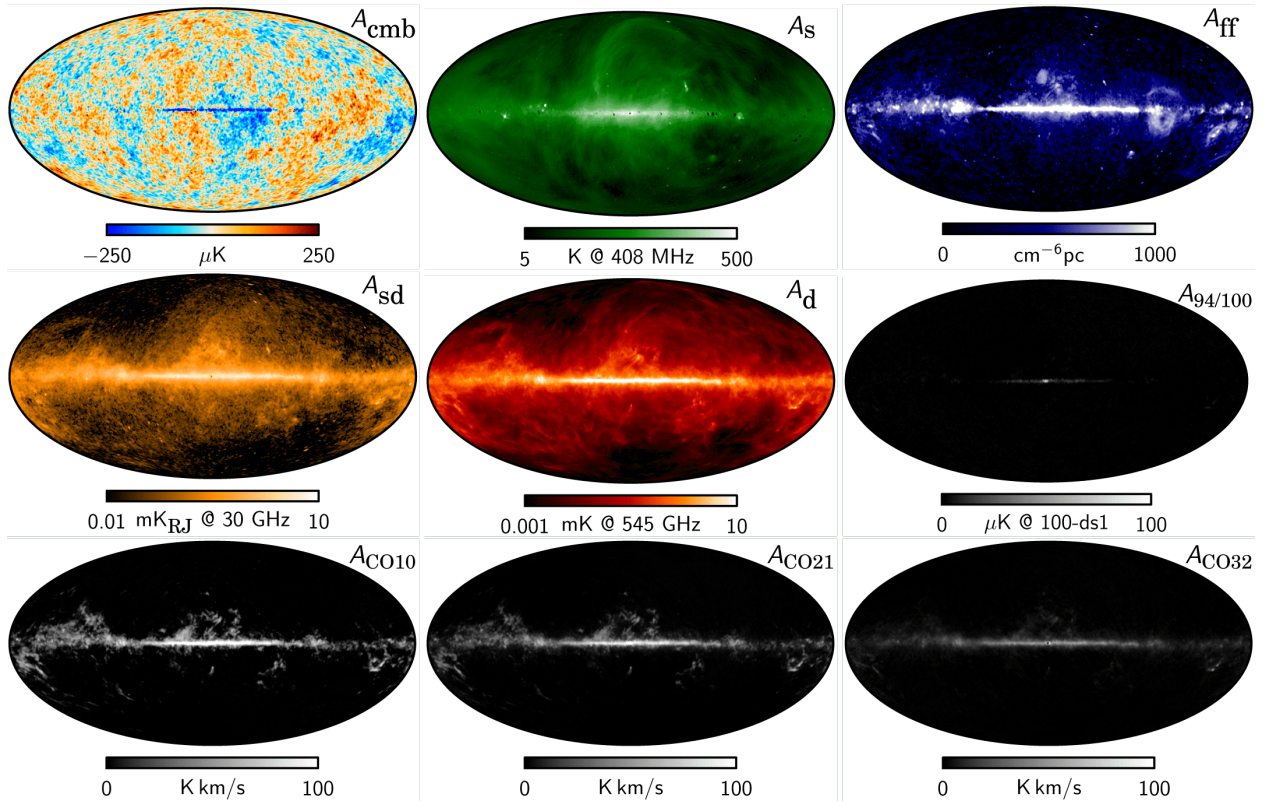
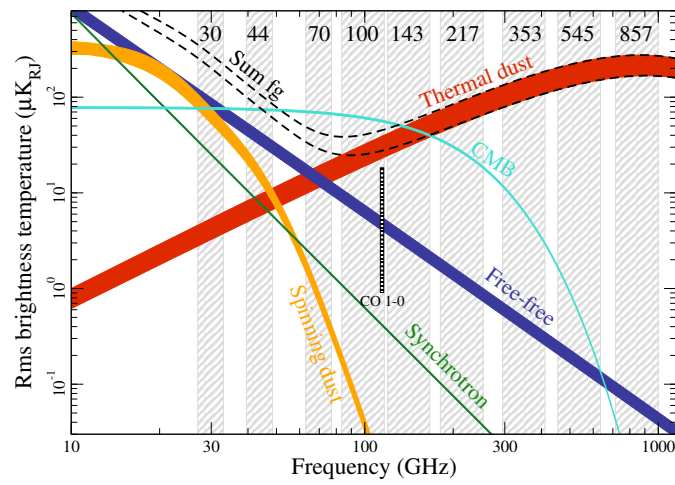
¹Planck Collaboration et al., “Planck 2015 Results”.

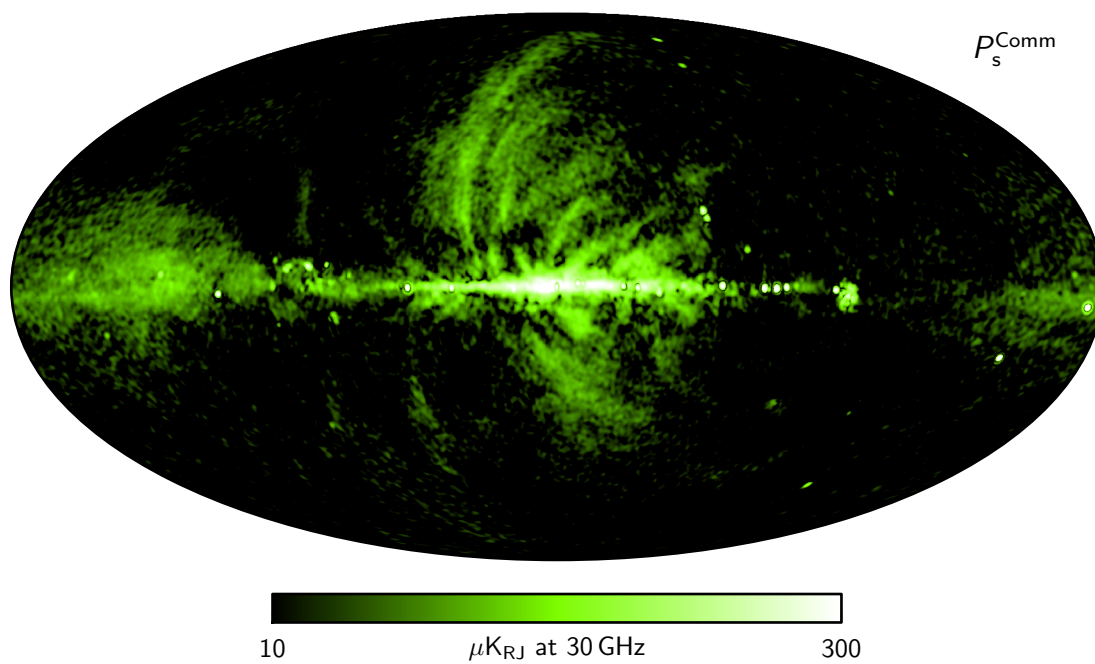
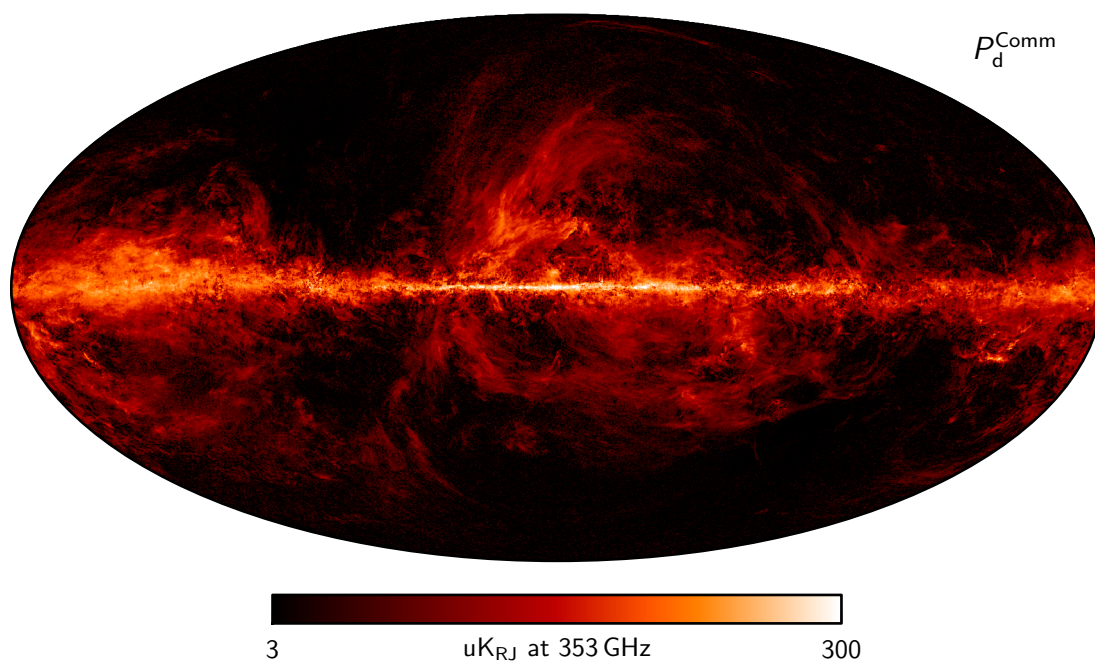
²Planck Collaboration et al., “Planck 2015 Results”.

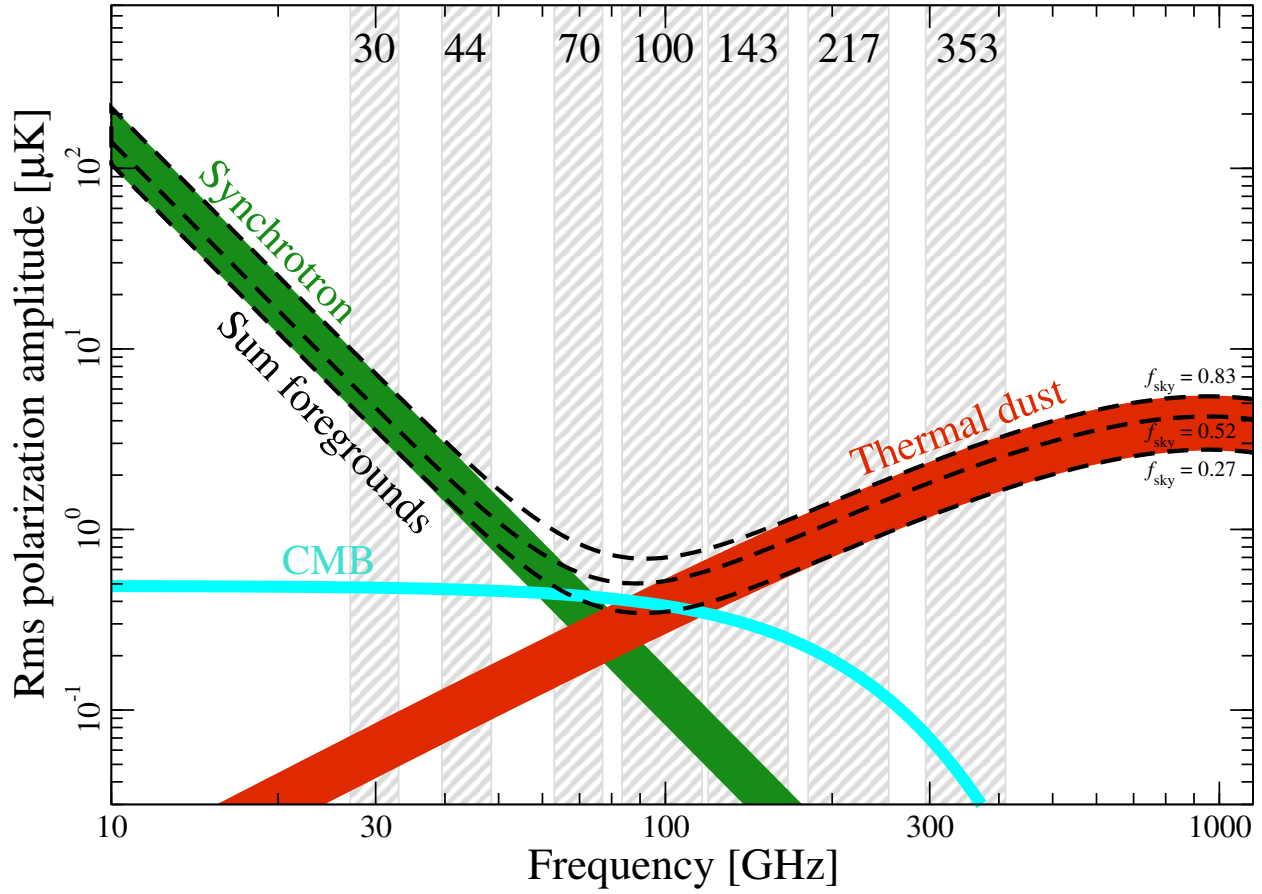
³Planck Collaboration, Akrami, Ashdown, et al., “*Planck* 2018 Results”.

⁴Planck Collaboration, Akrami, Ashdown, et al., “*Planck* 2018 Results”.

⁵Planck Collaboration, Akrami, Ashdown, et al., “*Planck* 2018 Results”.


 Figure 8.1: Foreground Intensity maps¹

 Figure 8.2: Foreground Intensity spectra²

Figure 8.3: Foreground Synchrotron Polarization³Figure 8.4: Foreground Dust Polarization⁴

Figure 8.5: Foreground Polarization spectra⁵

Second, as it becomes obvious in power-spectra domain in fig. 8.5, nowhere is the CMB dominated over foreground contaminants. Therefore, in measuring the polarization of the CMB, it is particularly important to measure in multiple frequencies.

Chapter 9

Noise in CMB observations

Now let us switch gear to talk about the noise properties. This has important implications on how the CMB is observed, how CMB experiments are designed, and even how CMB data is analyzed.

9.1 Atmospheric noise

When we talk about observation frequencies, ground based CMB experiments has extra constraints here.

Take a look at fig. 9.1, we see that our choice of observation frequencies is limited by the atmospheric windows.

Not only that, from the precipitable water vapor (PWV) level across the world shown in fig. 9.2, we see that only some regions in the world favors CMB observations. It is because higher PWV, corresponding to red color in the figure, results in more absorption and therefore is less ideal for CMB experiments.

In fact, this is so important that all current ground based CMB experiments are in the low PWV regions shown as dark blue here².

¹[Barron et al., “Optimization Study for the Experimental Configuration of CMB-S4”.](#)

²This includes 4 regions—Chile, Antarctica, Greenland, and Tibet. The South Pole in Antarctica and the Atacama Desert in Chile are well-established sites for CMB observations for reasons outlined in Barron et al., [“Optimization Study for the Experimental Configuration of CMB-S4”](#). 3 more sites—Ali in Tibet, Dome A in Antarctica, and Summit Camp in Greenland—are explored in Kuo, [“Assessments of Ali, Dome A, and Summit Camp for Mm-Wave Observations Using MERRA-2 Reanalysis”](#). It concludes that, except for Greenland where high level of liquid water clouds is detected, all remaining 4 sites are promising for CMB observations, where Ali in Tibet is the only one on the list which is “in the Northern Hemisphere that will complement well-established Southern sites”. Lastly, it should be noted that existing infrastructure in the South Pole and the Atacama Desert makes it favorable for future deployments.

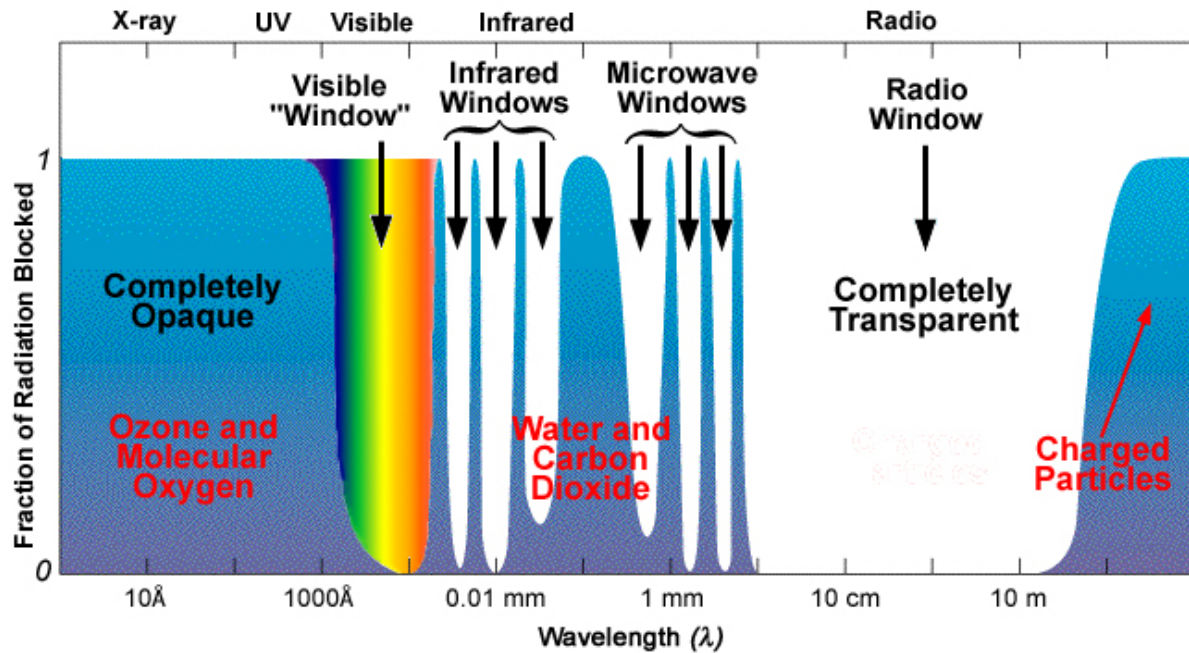
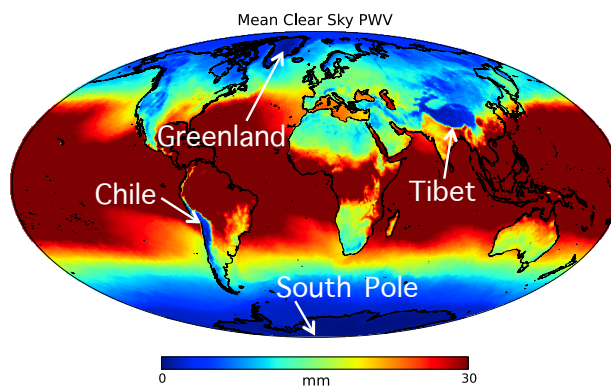


Figure 9.1: Atmospherical windows

Figure 9.2: PWV throughout the World¹

9.2 $1/f$ -noise

Having mentioned the external factors involved in CMB experiments, we will briefly go through the actual instrument CMB experimentalists can design.

There are many interesting physics involved in the design of these instruments. For example, it involves optical design of the telescope. And in detectors, there is Transition Edge Sensor (TES) bolometer which utilize superconductivity to achieve photon-noise limited sensitivity. In detector readout, Superconducting Quantum Interference Devices (SQUIDs) is used to couple TESes together to achieve multiplexing, which enables us to pack a lot of detectors in a focal plane to beat down the noise.

But few of the characteristics of an instrument are as ubiquitous and has as profound implications as the following one we are going to mention: $1/f$ noise.

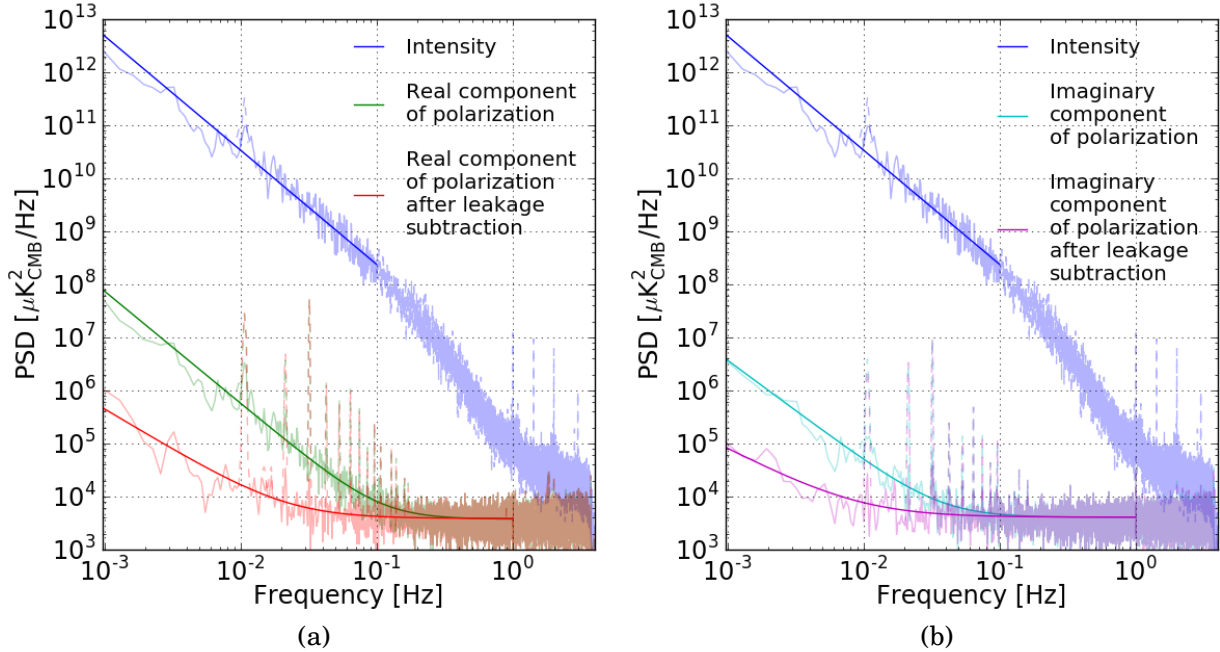


Figure 9.3: “PSDs of coadded timestreams for all focal plane detectors for the real part (left panel) and for the imaginary part (right panel). The blue line shows the intensity fluctuation and the green (cyan) and red (magenta) lines show the real (imaginary) part of the polarization signal before and after leakage subtraction, respectively. The spikes at the harmonics of 0.01 Hz are the scan synchronous signals.”³

From fig. 9.3, we can see an example spectrum of a detector TOD from POLARBEAR.

³Takakura et al., “Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

$$N \left[1 + \left(\frac{f_{\text{knee}}}{f} \right)^\alpha \right] \quad (9.1)$$

Eq. (9.1) describes the empirical form of noise like this. The first term is a white noise floor, and the second term is a straight line with slope α on the log-log scale. This is the infamous $1/f$ noise.

There are multiple sources that can lead to this empirical form, including atmosphere, electronics, temperature drifts, etc.

In this specific example, the atmosphere is the dominant contribution. But this is a fairly generic feature that appears in all our CMB measurements, and this characteristic affects how we design our experiments.

9.3 Scanning Strategy

The main implication here is that we cannot fixate on a point in space and measure the CMB, because in this static limit, where the frequencies approaches 0, the noise level virtually diverges leading to a large uncertainty.

Instead, the rules of the game here is to modulate the signal, meaning that we make it change. i.e. we change the signal such that it is no longer at zero frequency and away from the singularity.

That is why all CMB experiments have to design a scanning strategy to scan across the sky. This process is also called a differential measurement as effectively we are measuring the difference in the signal, the anisotropy of the CMB.

Because the CMB is not a scalar, having multiple components, cross-linking is also important. Cross-linking basically means that you revisit the same pixel with a different signal composition so that you can eliminate the degeneracy and solve for different signal components.

We are not going into the details of designing scanning strategy here, but we now focus on an interesting twist on scanning strategy below.

9.4 Modulation of the CMB signal via HWP

In measuring the CMB, $1/f$ noise is *lesser* of a problem in measuring the Intensity, simply because the signal is big at low- ℓ , corresponding to low-frequency in the time-domain, which means we can afford larger noise-level there. But this is not the case for CMB polarization.

CMB polarization can be measured by pair-differencing—it takes 2 detectors at 90° and take the difference between them. But any non-ideality between the 2 is going to create Intensity to Polarization (I-to-P) leakage. For example, the 2 detectors might not be exactly at 90° , or might have different beam shape or band pass.

These make CMB polarization measurements more challenging.

One idea to solve these problems is to modulate the signal again, but to much higher frequencies utilizing polarization itself.

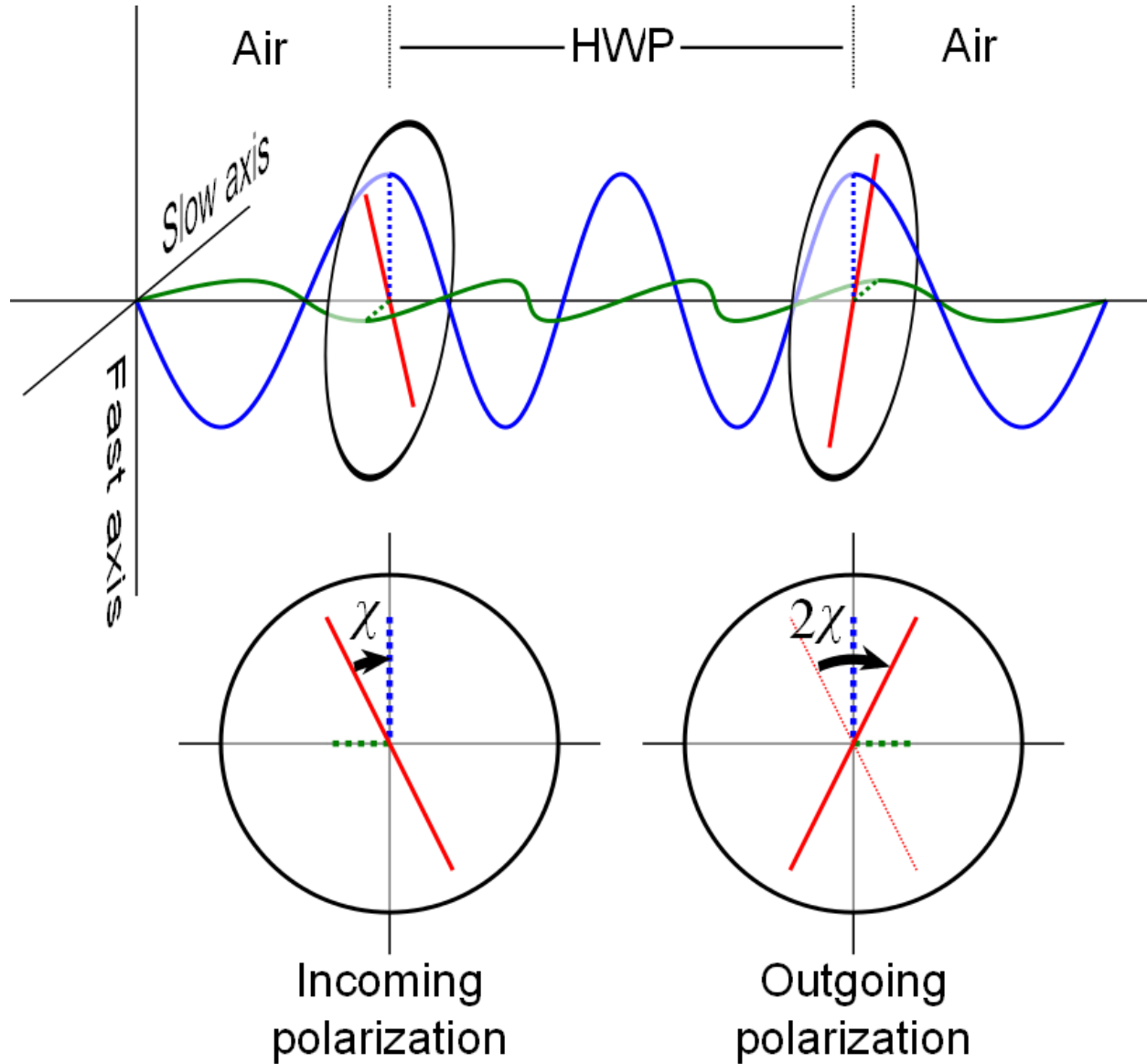


Figure 9.4: HWP

Fig. 9.4 is a Half-Wave Plate (HWP). What it does is that when light transmitted through a HWP, the polarization angle will be reflected. We can then put the HWP in Continuous Rotation (CRHWP), and it will modulates any polarized components on the sky-side of the HWP by 4 times the frequency we rotate it. Note that this means

the unpolarized components including atmospheric noise is left unmodulated. This separates the $1/f$ infrared divergence region from the polarized signal at $4f$.

In fig. 9.3, we see the effect of HWP demodulation, one of the techniques to reconstruct the original polarized components from the modulated signals which is detailed in chapter 22. We can see from the middle curve that $1/f$ noise is hugely suppressed.

This kills 2 birds with one stone—it modulates the signals away from the $1/f$ infrared divergence, and also enables a single detector to see both polarization modes, hence eliminating the need of pair-differencing and the associated systematics.

But there is still residual I-to-P leakage due to HWP systematics. It can include detector non-linearity, off-axis mirror, etc. It can be further suppressed in analysis using leakage subtraction as seen in the lowest curve.

In addition to these systematics, mitigation of $1/f$ noise using HWP modulation is also limited by the presence of polarized noise components. HWP modulation can modulate such noise too, so that the signal is no longer separated from the infrared divergence. Takakura et al.⁴ provides one such example, where ice crystals in tropospheric ice clouds can create such polarized $1/f$ noise components.

⁴“Measurements of Tropospheric Ice Clouds with a Ground-Based CMB Polarization Experiment, POLARBEAR”.

Chapter 10

CMB data

With the help of experimentalists, let us say that we have observed the CMB and have our data. What would it look like and how we should process it?

$$d_t = P_{tp} m_p + n_t \quad (10.1)$$

Eq. (10.1) represents scanning the sky and observing a signal, where

m_p sky signal where p is our pixelization scheme.

P_{tp} pointing matrix which represents where we are pointing at the sky, and also which signal components we are picking up if it is sensitive to polarization.

n_t noise in time-domain.

d_t the observed timestream of data (TOD).

Now our task is to invert this process and obtain the map.

Chapter 11

CMB mapmaking

11.1 The “naïve physicist”

The “naïve physicist” looks at this and says, “it is simple to reconstruct the CMB maps. Wherever the telescope is pointing to, we can simply average down the signal to the corresponding pixel.”

Except that it would not work, take a look at fig. 11.1:

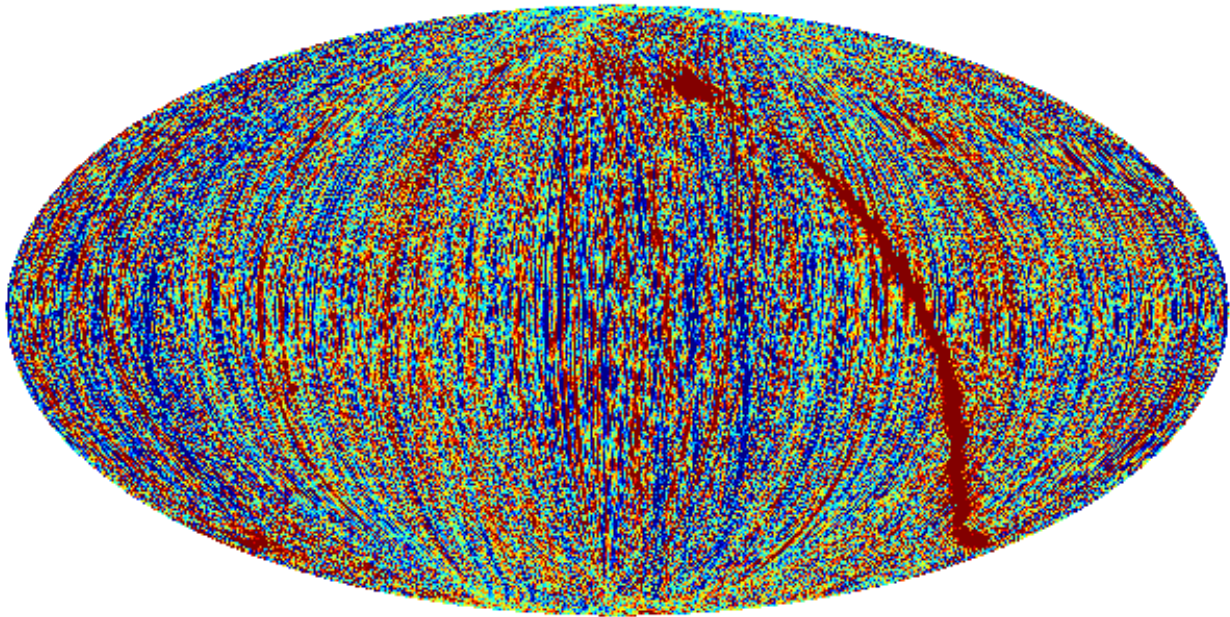


Figure 11.1: Binned map¹

¹Keihänen et al., “Making Cosmic Microwave Background Temperature and Polarization Maps with MADAM”.

We can see visible vertical stripes here. These stripes will appear in the scanning directions because of the infamous $1/f$ noise. The high amplitude, long time-scale noise is manifest in the next scanning line which now has a vastly different amplitude.

No wonder this is literally called naïve mapmaker.

11.2 The “naïve mathematician”

The “naïve mathematician” looks at eq. (10.1) and says, “wait a minute, I recognize this, this is just a standard linear regression problem, and we can write down the solution immediately by Generalized Least Squares (GLS).”

Indeed, by GLS, we know the solution to eq. (10.1) is²

$$\begin{aligned} N_{tt'} &= \langle n_t n_{t'} \rangle \\ W_{pp'} &= P_{pt}^T N_{tt'}^{-1} P_{t'p'} \\ \hat{m}_p &= W_{pp'}^{-1} P_{p't}^T N_{tt'}^{-1} d_{t'} \\ \text{Cov}(\hat{m}) &= W^{-1} \end{aligned} \tag{11.1}$$

$$\tag{11.2}$$

Moreover, it can be shown that this solution is unbiased, and lossless in the sense that the Fisher information matrix w.r.t. the input parameters is preserved.

This is called maximal likelihood mapmaker, which is one example of optimal mapmakers that is lossless.

Indeed, this has been the mapmaker used in CMB experiments such as COBE (1990s) in the beginning. But as pointed out by Borrill³, this has been computationally prohibitive for about 2 decades, because of the $O(N_p^3)$ behavior of the algorithm involved in eq. 11.1 & 11.2.⁴

Today, minority of CMB experiments still use this to solve for the map, but not the covariance. Hence their mapmaking is unbiased, but not lossless.

²How may $N_{tt'}$ be estimated? In the framework of GLS, it is part of the model. I.e. it is assumed to be a known quantity. In practice, one needs to build a model to describe the noise properties and then estimate the parameters of the noise model using the data. Note that in CMB observations, the signal is so faint that the TOD is dominated by the noise. In the special case that the noise model is uncorrelated white noise, one can simply estimate the noise variance using FFT on the TOD utilizing this fact. Incidentally, this technique is used in POLARBEAR. See Keihänen et al., “[Making Cosmic Microwave Background Temperature and Polarization Maps with MADAM](#)” for another assumption and technique to model and estimate the noise covariance.

³“[The Challenge of Data Analysis for Future CMB Observations](#)”.

⁴Actual time complexity is $O(N_p^3)$ with a larger prefactor involving multiple Cholesky decompositions in later steps. See Borrill for details.

11.3 Naïve meeting naïve becomes not so naïve

One day the naïve physicist met the naïve mathematician, and they realized when the noise is uncorrelated white noise, then the naïve mapmaker is the optimal mapmaker.

But we have seen that the noise are far from white. We could, however, cheat and make it white.

The trick is to high-pass-filter our data to remove excessive low frequency noise. Then, as we have seen earlier, CRHWP modulation already makes the noise quite white to begin with.

But now we have a new problem, since we filtered our data, the map is now biased. And in practice, we typically have thrown away on the order of dozens percent of information which is the cost to pay for this naïveness.

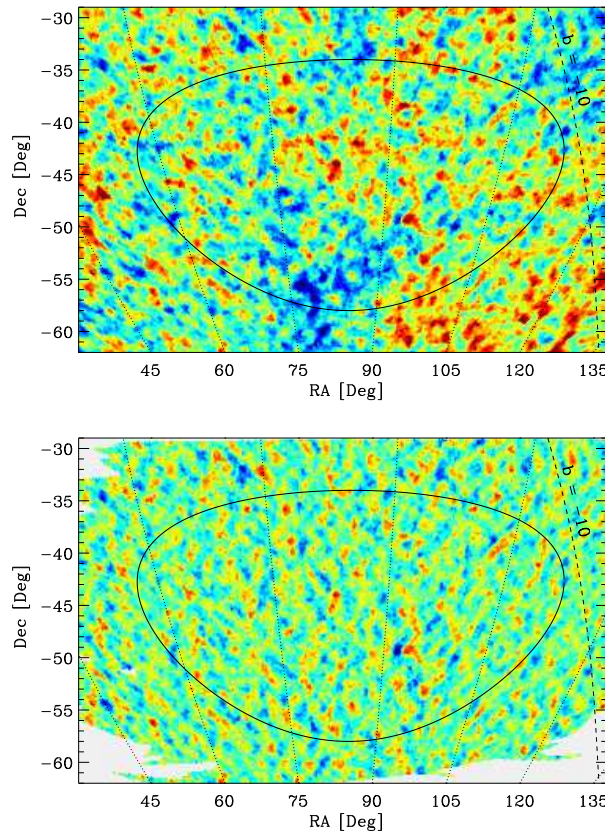


Figure 11.2: Biased mapmaking⁵

Fig. 11.2 shows us an example high-pass filtering, we can see that the biased map below has much less large scale structure than the original input above. While the

⁵Hivon et al., “MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

map is biased, its effect can be corrected in the power-spectrum domain via a method called MASTER devised in the same paper.

The tradeoff for the lossy mapmaking is now we have superior computational requirement. The naïve mapmaker, as we are simply binning the data, is a streaming algorithm, it has $O(N_t)$ time complexity, and is embarrassingly parallel.

For example, this is the mapmaker of choice in the POLARBEAR experiment.

11.4 Unbiased mapmaking

But unbiased mapmaking is desirable if it is achievable. At the very least it is a cost tradeoff between data and computational time. To illustrate, it might be cheaper to build more detectors for ground-based experiments, but for satellite-based experiments, we definitely want to squeeze the last bit of information from the data. Not to mention unbiased maps can provide useful information within and beyond the CMB communities.

There are many unbiased mapmakers with different iterative solutions, examples are one used in a subset of POLARBEAR analysis⁶, and the Madam mapmaker⁷ used in Planck analysis and will be used in many future CMB experiments including LiteBIRD.

They are often solving the mapmaking equation iteratively to achieve a lower time complexity, therefore they are unbiased and often asymptotically optimal.

From the example of Madam mapmaker in fig. 11.3 & fig. 11.4, we can see that the two are very similar, meaning that the destriped map successfully recover the noiseless map, comparing to the visible stripes in fig. 11.1.

⁶Poletti et al., “Making Maps of Cosmic Microwave Background Polarization for B-Mode Studies”.

⁷Keihänen et al., “Making Cosmic Microwave Background Temperature and Polarization Maps with MADAM”.

⁸Keihänen et al., “Making Cosmic Microwave Background Temperature and Polarization Maps with MADAM”.

⁹Keihänen et al.

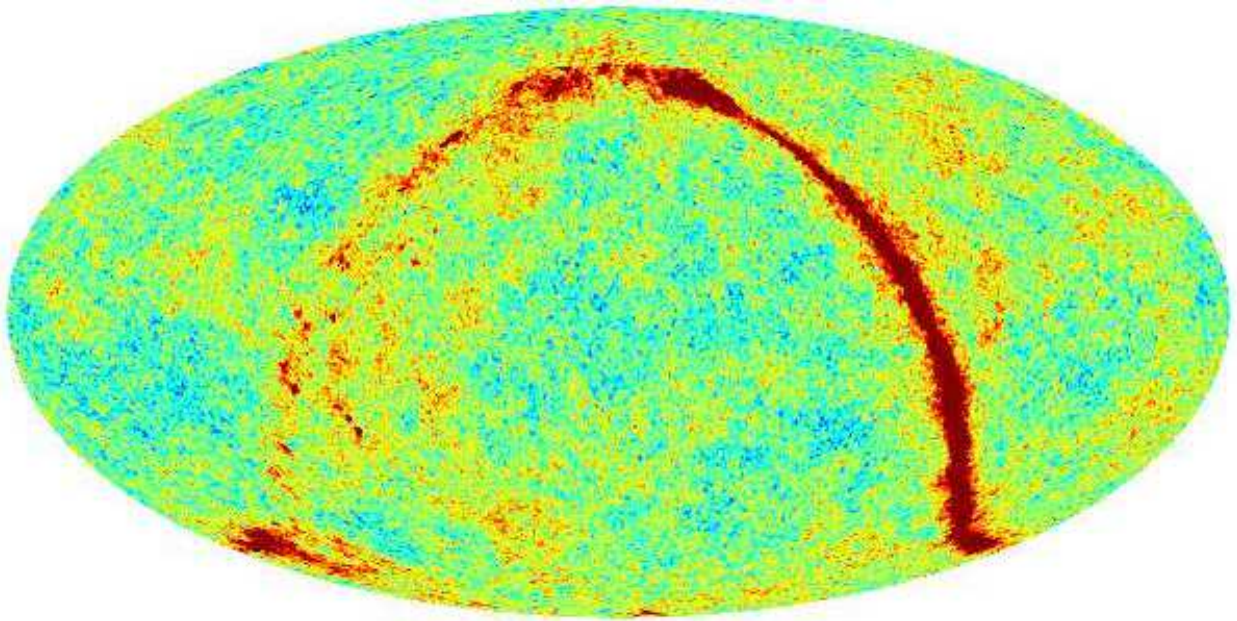


Figure 11.3: Noiseless map⁸

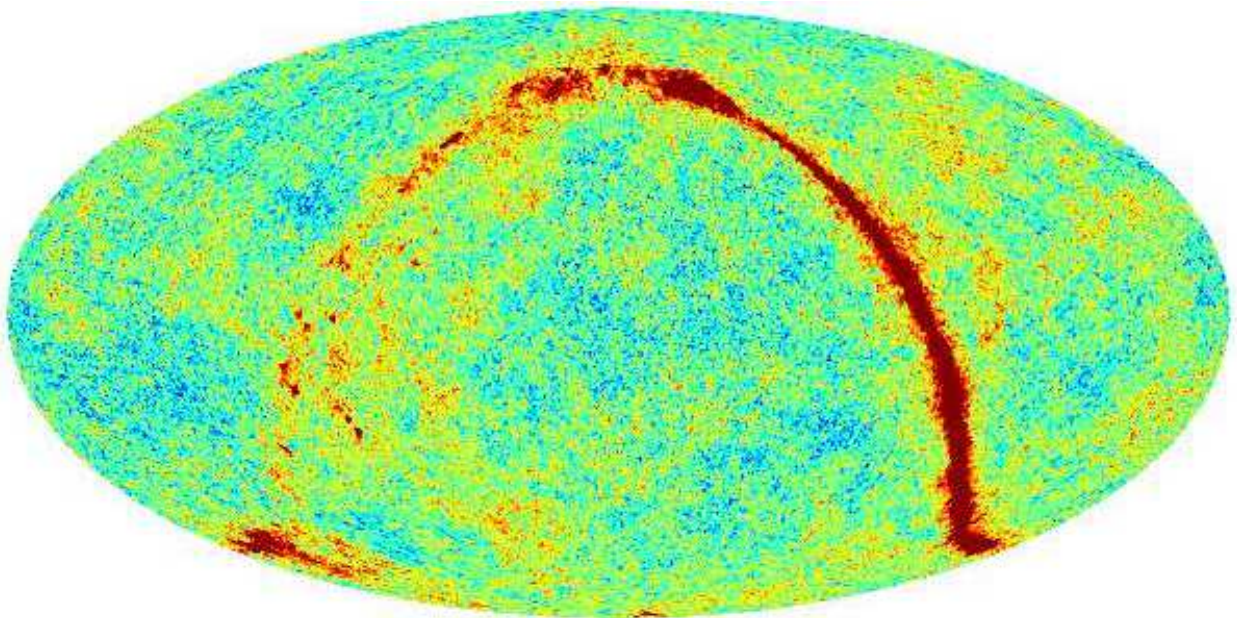


Figure 11.4: Destriped map⁹

Chapter 12

Systematics characterization and mitigation strategies

12.1 Timestream processing

Now that we know how we generally obtain a map, and we understand its statistical properties, we turn our attention on systematics.

In fact, before we pass the TOD to the mapmaker, we need to do some cleanup first. For example detectors has glitches from sources such as cosmic rays as in fig. [12.1](#), which are not part of our sky signal and thus should be removed.

There is also scan-synchronous signal, such as ground pickup. As the telescope is scanning the sky, the signal from the surroundings of the telescope can contaminate the sky signal it is observing. One mitigation strategy is to construct a ground template and subtract that from the TOD.

Other examples that we have already mentioned are I-to-P leakage subtraction, high pass filtering, foreground masking when building templates.

12.2 Map-domain processing

Having the TOD made into maps, not only did we perform data compression, but also it boosts the signal-to-noise ratio. Hence, we can perform further systematic mitigation.

It can be as simple as masking out the foreground such as point sources and our galaxy as in fig. [5.1](#). And if we have multi-frequency observations, we can extract foreground components as in fig. [8.1](#), fig. [8.3](#), and fig. [8.4](#).

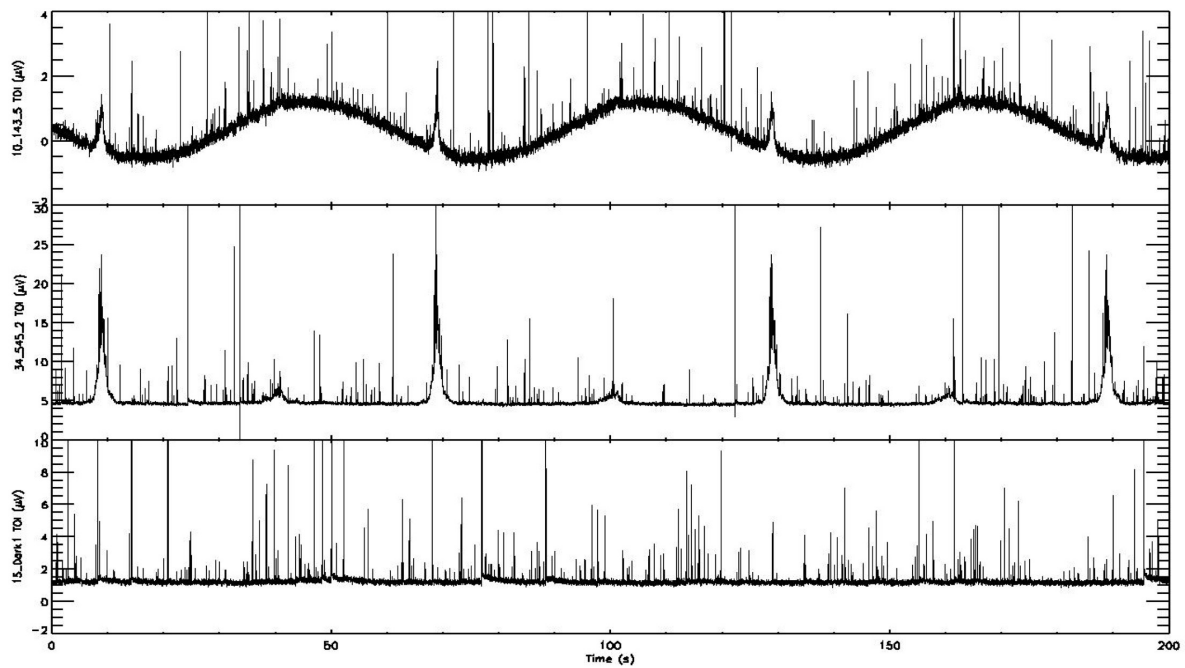


Figure 12.1: Cosmic ray seen by Planck

Chapter 13

CMB power-spectra

We have previously mentioned that we can obtain the power-spectra by performing spherical harmonics transformations and form TT , TE , EE , BB spectra. But we have not mentioned why.

One reason is that maps are not unique. Our Universe is a random realization of fundamental physics and its parameters, and we as observers in the Universe is a random point-of-view. Changing either of these will change the map but not the power-spectra, as it is probing on the expectation values of these random fields.

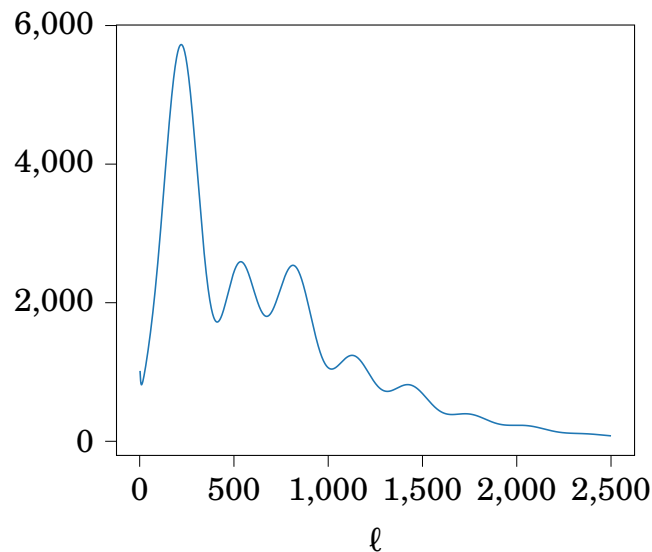


Figure 13.1: TT spectrum

Here is a simple example that from the same power-spectrum in fig. 13.1, the one we measured here in the Universe is fig. 13.2, but fig. 13.3 is another random realization that yields the same spectrum!

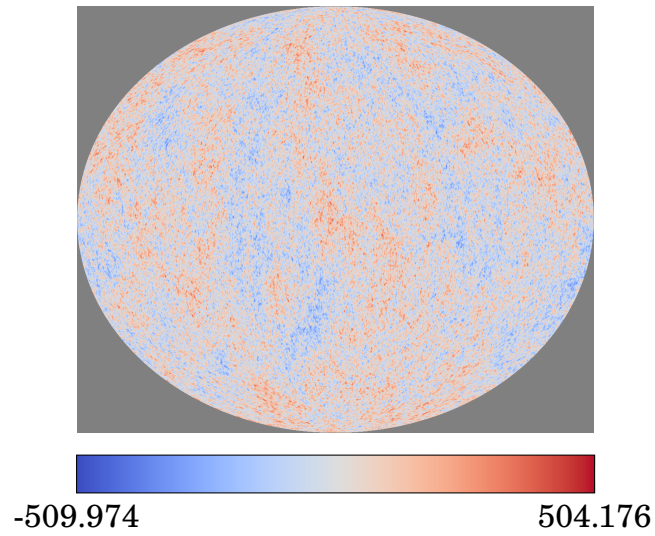


Figure 13.2: Intensity map (real)

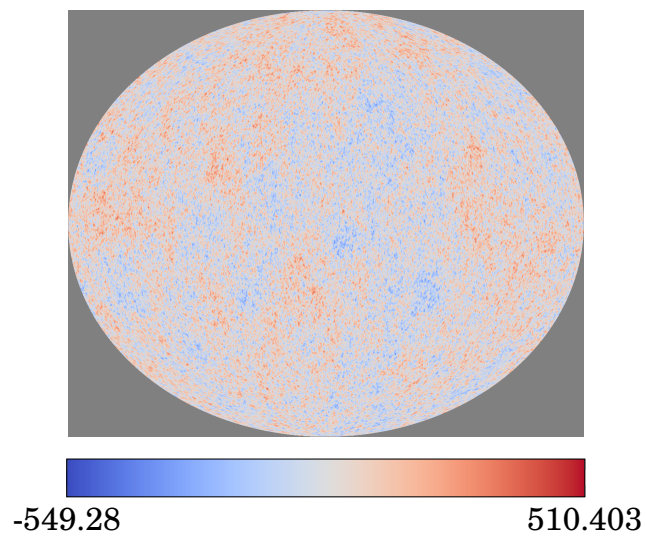


Figure 13.3: Intensity map (random)

Furthermore, theoretically we expect the primordial random fields to be nearly Gaussian, which is also not violated observationally. This means that the power-spectra as 2nd order statistics is sufficient statistics. i.e. this is a lossless data compression, which further helps us to boost the signal-to-noise ratio.

This means that we can perform further systematic mitigation, such as unresolved point sources, gravitational lensing of the CMB by the foreground, or we can construct templates for systematic effects that are not mitigated in earlier steps.

13.1 Beyond 2-pt

Although we said that 2-pt is sufficient statistics for the primordial CMB, the CMB we observed has non-linear, hence non-Gaussian distortion, for example by the gravitational lensing from the foreground. While this distorts all power-spectra, this affects B-mode the most as we have seen earlier.

In fact, the lensing potential can be constructed to describe the gravitational lensing effects, which can then be used together with the E -mode to produced a lensing B -mode template, which is what we have seen in fig. 5.3.

Chapter 14

Simulations of CMB experiments

Now that we have gone through all the major components needed for CMB data analysis, we can now look at the big picture of a general analysis pipeline in Fig. [14.1](#).

The only missing piece so far is the ability to simulate everything we said with an instrument model that captures the important characteristics such as scanning strategy, noise, systematic effects, etc.

There are many reasons we would need this. But the fundamental reason in our formulation here that we need simulation is that our mapmakers do not produce the covariance information due to computational constraints.

Therefore, we need to perform Monte Carlo (MC) simulations. We inject random noise with physical characteristics and use Monte Carlo method to estimate the statistical error in our final power-spectra. The typical number would be $O(100 - 1000)$, which means that computationally we are trading off a higher time complexity to a lower one with a larger pre-factor.

Once we have this facility in place, we can use this to perform pipeline validation, often early in the process of designing an experiment, to demonstrate the correctness of the pipeline, and the capability of handling the observed dataset.

Also, we can use this to simulate systematic effects by injecting them into the data and measure the outcome such as any biases and its associated error, and then devise systematic mitigation strategies and quantify their effectiveness.

Being able to simulate the statistical and systematic error, we can use this to perform forecasts which can inform experimental design.

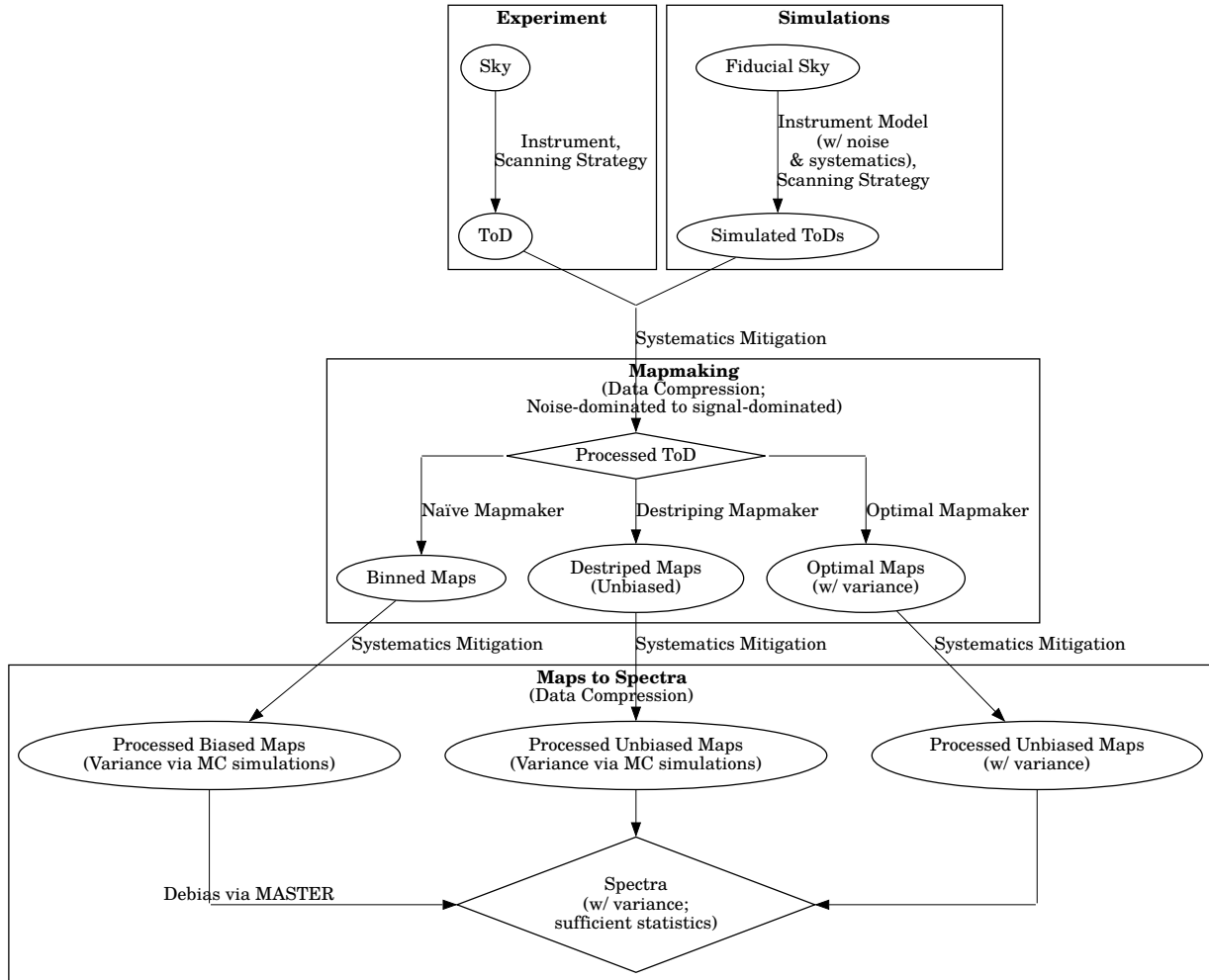


Figure 14.1: Analysis pipelines

Chapter 15

Posterior estimation from the data and its likelihood

Now that is where the fun begins. From here, we can start to infer about our universe.

In Fig. 15.1, the pipeline enables us to obtain the CMB power-spectra with covariance information.

Our theoretical understanding of the Universe enables us to build parametric Physical models, which includes cosmological parameters such as those in the Λ CDM model including matter compositions & spatial curvature, new physics, or parameterized systematics such as foreground components & their spectral indices.

Gathering all the pieces, we can perform Markov chain Monte Carlo (MCMC) method to “invert” from observed power-spectra to parameters, It works like this:

1. construct a likelihood function using the measured power-spectra assuming Gaussianity,
2. write down the prior information from what we already know in physics, such as the Universe is flat from other measurements,
3. makes random parameter choices and use our model to predict the output power-spectra,
4. accepts or rejects the choice based on the likelihood and prior,
5. obtains a Markov chain of parameters,

which gives us an approximate maximum a posteriori (MAP) probability estimation.

This leads to refinements of our understanding of the Universe or even discovery of new physics depending on the significance of its parameters.



Chapter 16

Current measurements

Now let us take a look at the current measurements of the CMB.

In fig. 16.1, we can see the spectra. We can see most of these are measured very accurately over a wide range of scale. In particular, the TT spectrum at $\ell \lesssim 1600$ is measured approaching the cosmic variance limit meaning that observationally we cannot further improve the error-bar beyond a factor of a few fundamentally.

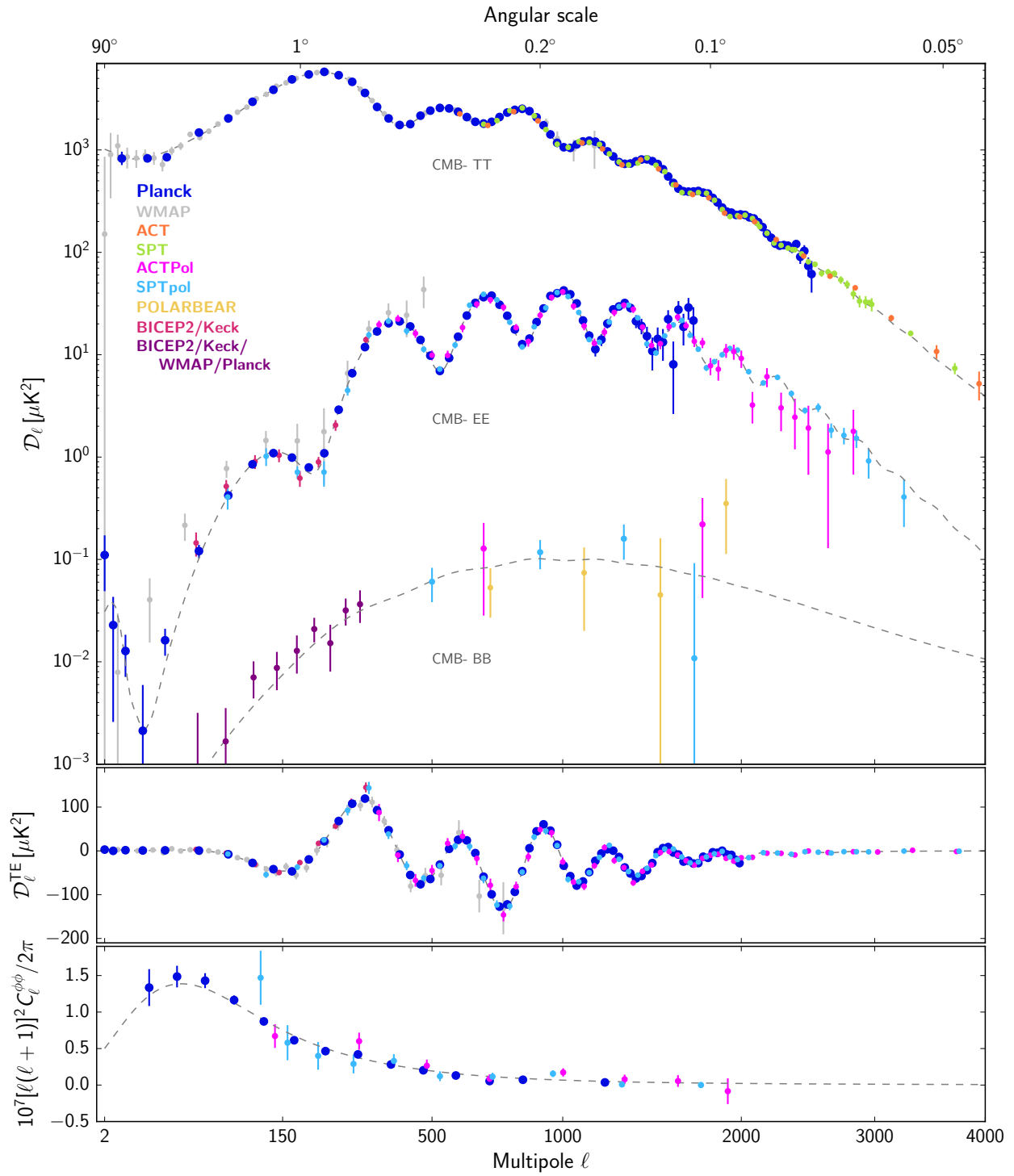
As we explore each of the curves, we can see boundaries that can be pushed in future CMB experiments. For example, the tails and the lensing constraints can be improved, resulted in better understanding of Neutrino physics such as the effective number of species, sum of Neutrino masses, and its mass hierarchy.

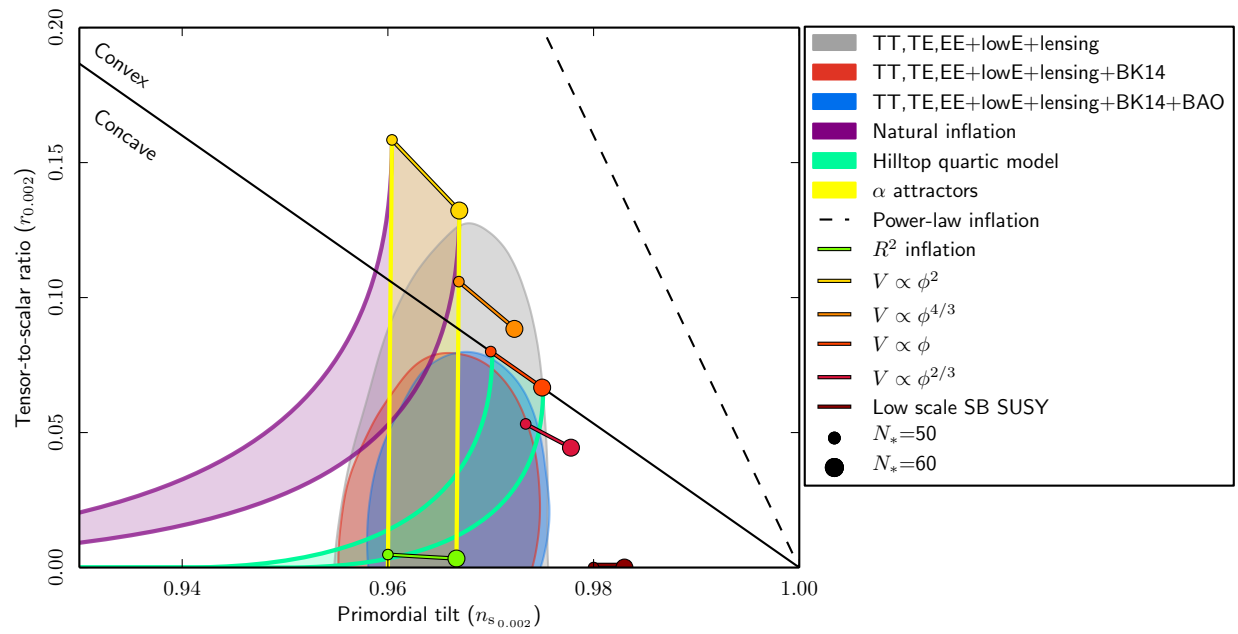
Furthermore, we can see that the B-mode polarization, where evidence is first reported by POLARBEAR, is the least constrained. Not shown on the graph, the primordial B-mode at $\ell \lesssim 100$ is not measured yet. This is the science goal of many upcoming CMB experiments including LiteBIRD. This primordial B-mode is often parameterized by the tensor-to-scalar-ratio r . Let us see what can be achieved by measuring this.

In fig. 16.2, the 2 axes are r, n_s . As we mentioned in chapter 3, we know that $n_s < 1$ is a sign of inflation. But it alone cannot break the degeneracy in inflationary models as seen in this plot. This r offers us a chance to glimpse into the details or even to falsify inflationary models.

¹Planck Collaboration, Aghanim, Akrami, Arroja, et al., “Planck 2018 Results”.

²Planck Collaboration, Akrami, Arroja, et al., “Planck 2018 Results”.

Figure 16.1: CMB Power-spectra summary¹

Figure 16.2: ²

Chapter 17

Next stage of CMB experiments

In order to achieve these goals, there are a lot of upcoming challenges.

Regardless of the science goal of a particular CMB experiment, a common enemy is noise. The remaining battlegrounds as we have seen are weaker signals. And as we are probing weaker and weaker signals, we need to achieve lower noise levels.

As our detectors are already photon-noise limited, it means that we need to increase the number of detectors. This means inevitably the CMB data volumes will grow, and the growth is exponential as we can see in Fig. 17.1.

In general, all the datasets are growing exponentially across different sectors. On the other hand, HPC growth is slowing down as we are approaching the end of Moore's law, basically because we are hitting a Physical limit there.

In fact, if we look at the ground-based observations, the CMB data growth will soon surpass the HPC growth.

This means we need to stay on the cutting edge in hardware and software to deliver the computational requirements of CMB analysis.

Moreover, systematics does not average down. As we are probing weaker signals, systematic effects that was once negligible might matter now. This imposes challenges to both experimental design and analysis software design to minimize, quantify, and mitigate these systematic effects.

With the big picture in our mind, let us focus on my research in 2 specific CMB experiments—the POLARBEAR experiment and the LiteBIRD experiment. In part IV, we will briefly introduce the POLARBEAR experiment, a ground based CMB experiment in the past decade. In part V, we will focus on my research topic on the large-patch high- ℓ B -mode measurement. Lastly, in part VI, we will study the crosstalk systematic effects in a future satellite-based CMB experiment LiteBIRD.

¹Borrill, *Exponential CMB Data Growth*.

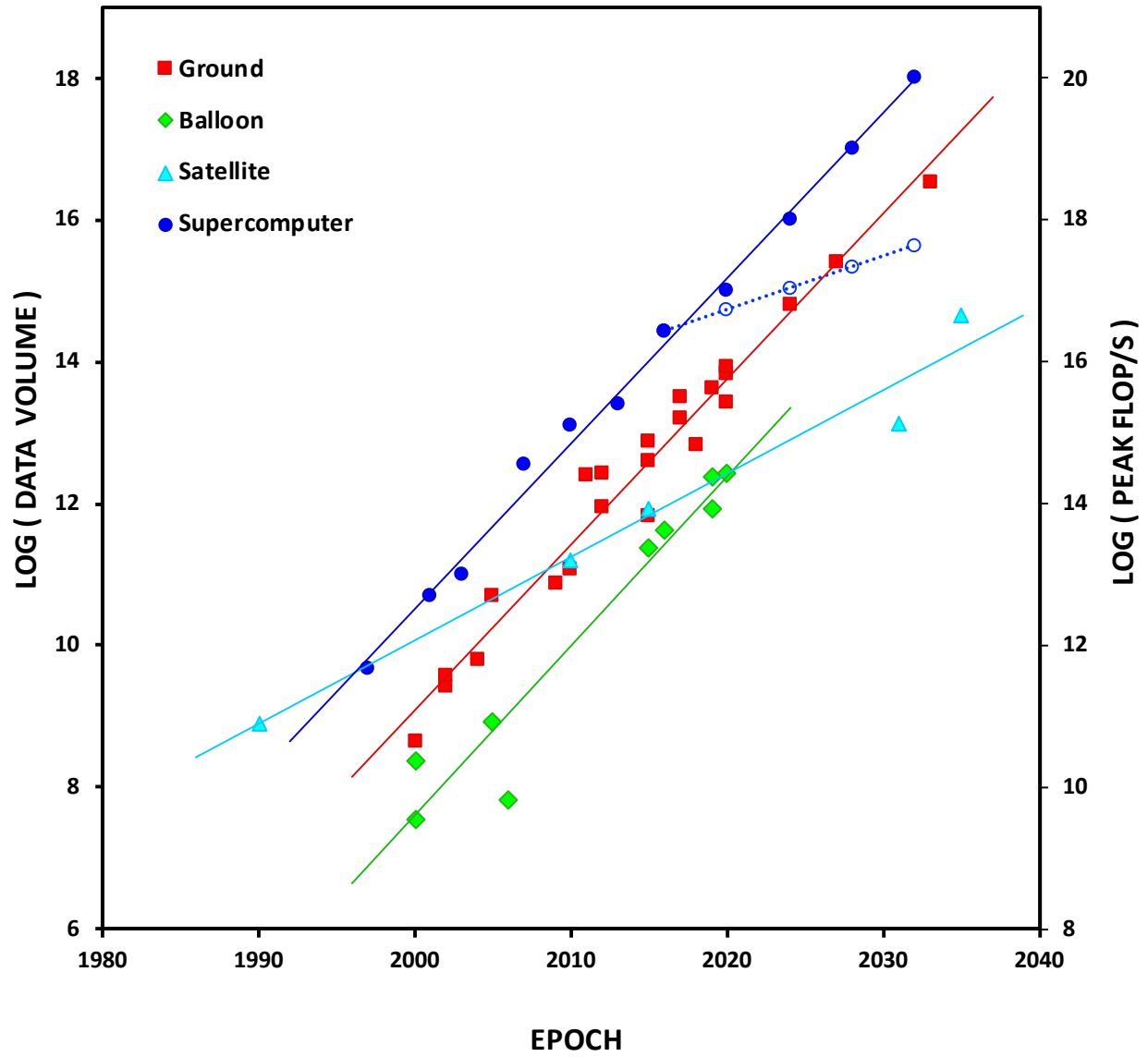


Figure 17.1: Exponential CMB data growth¹. The dotted line represents the projected growth in the post-Moore's law era.

Part IV

The POLARBEAR experiment

Chapter 18

The story behind the name POLARBEAR

To introduce the POLARBEAR experiment, let us start with its name, which has a story too good to be skipped.

For a long while I do not know its meaning. From some casual conversation, I thought it was because POLARBEAR started at the University of California, Berkeley (UCB) and Cal Bears¹ is its identity, and we measured the polarization in the CMB, so obviously POLARBEAR it is.

Until when I was preparing for this and try searching for its actual etymology. A quick search on “acronym POLARBEAR cosmology” will lands you to [Dumb Or Overly Forced Astronomical Acronyms Site \(DOOFAAS\)](#), a site created by Dr. Glen R. Petitpas from Harvard dedicated to dumb astronomical acronyms. POLARBEAR stands for **POLAR**ization of the **B**ackground millimEter bAckground **R**adiation.

Except that it is wrong. Probably the misunderstanding was originated from a 2003 article² that somehow think it is true without any citations on how they knew that.

It turns out it stands for **POLAR**ization of the **B**ackground **R**adiation, which is defined rarely in some of the POLARBEAR papers.³

However, that is not the end of the story. How should it be typeset? I have seen the following forms used in various POLARBEAR publications:

1. PolarBeaR in the early days to emphasize its construction;
2. POLARBEAR, POLARBEAR or sometimes POLARBEAR which uses SMALL CAPS as a stylistic typographical choice; and
3. Polarbear very occasionally.

¹The athletic teams representing the UCB is the California Golden Bears, often called Cal Bears in short. See <https://www.calbears.com/>.

²Delabrouille et al., “Polarization Experiments”.

³Ade et al., “POLARBEAR Constraints on Cosmic Birefringence and Primordial Magnetic Fields”.

Polarbear, and its stylistic derivative POLARBEAR, is clearly wrong as it represents a capitalization of a non-existing word “polarbear”. PolarBeaR does not share widespread use. POLARBEAR and POLARBEAR are both correct, while the former is used more often.

Hence, POLARBEAR is used consistently as the name of the experiment throughout.

Chapter 19

The POLARBEAR telescope



Figure 19.1: POLARBEAR Huan Tran Telescope at the James Ax Observatory located at Cerro Toco in the Atacama Desert in Northern Chile¹

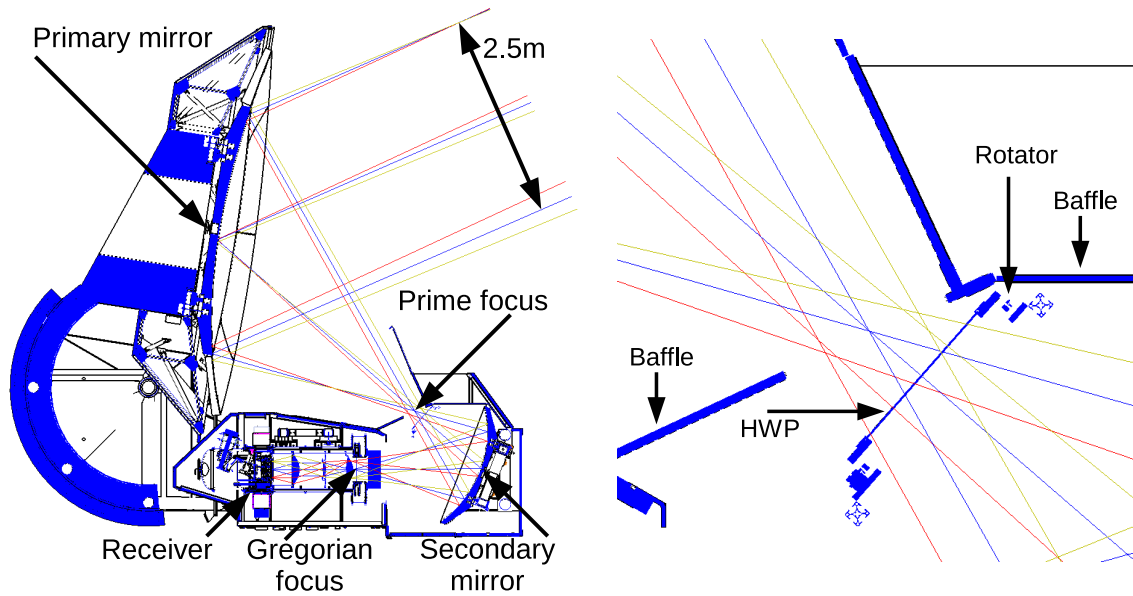


Figure 19.2: Cross section of POLARBEAR (left) and detail of the area around the prime focus (right)².

POLARBEAR is a CMB experiment installed on the 2.5 m aperture Huan Tran Telescope as shown in fig. 19.1, located at the James Ax Observatory in the Atacama Desert in Chile³ (see fig. 9.2 and sec. 9.1), with a $3.5'$ beam. As shown in fig. 19.2, it has an off-axis Gregorian design with the advantage of a clear aperture⁴. It packs 1274 transition edge sensor (TES) bolometers observing at 150 GHz.

Detectors are cooled to 250 mK, whereas the lenses is at 4 K. It achieves $\text{NET}_{\text{array}} = 23 \mu\text{K}\sqrt{\text{s}}$.

POLARBEAR has 5 seasons of observations. In the 1st–2nd seasons (2012–2014), it observed 3 different patches on the sky with a total of 25°^2 sky coverage. It achieved a sensitivity of $8.3 \mu\text{K}'$ in one of the patches named RA23. A B-mode spectrum was constrained at $500 \leq \ell \leq 2100$, where the 1st season of data provided the first direct evidence of polarization lensing.⁵

After the 2nd season, a Continuously Rotating Half-Wave Plate (CRHWP) was installed to modulate the polarization signal so that the $1/f$ noise was reduced to target

¹POLARBEAR collaboration, *POLARBEAR Huan Tran Telescope at the James Ax Observatory Located at Cerro Toco in the Atacama Desert in Northern Chile*.

²Takakura et al., “Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

³The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B-Mode Angular Power Spectrum with POLARBEAR”.

⁴Kermish et al., “The POLARBEAR Experiment”.

⁵Ade et al., “Measurement of the Cosmic Microwave Background Polarization Lensing Power Spectrum with the POLARBEAR Experiment”.

large scale, low- ℓ signal. Optimally, it should be placed as far skyward as possible to avoid I-to-P leakage caused by the asymmetry in the optical system. But the limitation of HWP technology had put its maximum size to about 0.5 m in diameter. Due to this constraint, the optimal point to place it is between the primary and secondary mirrors⁶ as shown in fig. 19.2.

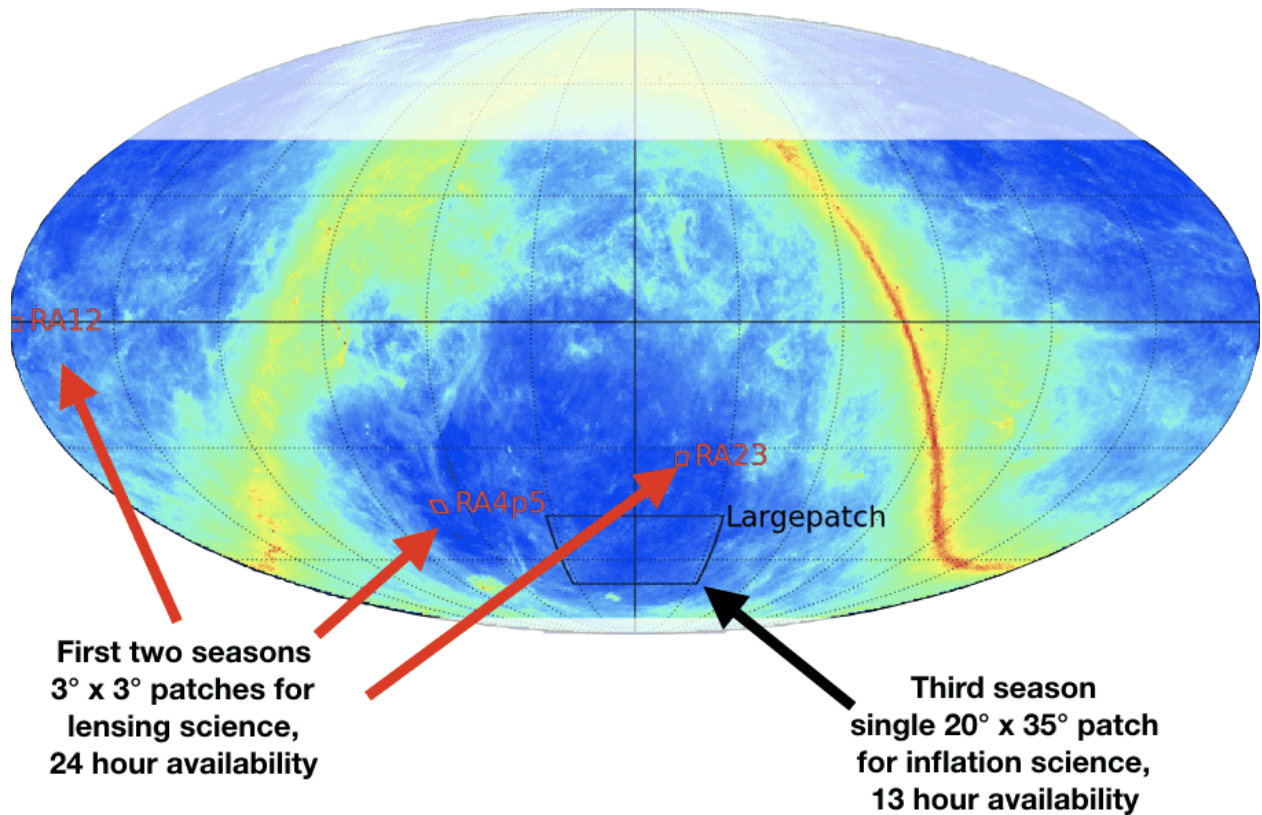


Figure 19.3: POLARBEAR sky coverage

In the remaining 3rd–5th seasons (2014–2017), it observed a sky area of 670°^2 , as shown in fig. 19.3. The presence of the CRHWP and the larger sky area enables us to target low- ℓ science. The resultant observed ℓ -range is $50 \leq \ell \leq 3000$. We called this informally the “large-patch”. This unique design of a medium aperture telescope with a CRHWP enables us to target both low- ℓ for primordial B-mode and high- ℓ for lensing B-mode. It achieves a sensitivity of $32 \mu\text{K}'$.

⁶Takakura et al., “Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

Chapter 20

The POLARBEAR null-test framework

Kendrick Smith first proposed a null-test framework for systematics detection in CMB observations¹ in the QUIET experiment in 2011². This concept of a null-test framework is used in POLARBEAR extensively.

It is based on null hypothesis testing. If we can construct a null estimator that should be identically zero in the absence of certain systematic effect, then a “discovery” of this estimator being non-zero implies the existence of a systematic effect that is not well controlled. Mathematically,

$$\hat{C}_b^{\text{null}} = \hat{C}_b^{AA} + \hat{C}_b^{BB} - 2\hat{C}_b^{AB},$$

where A, B represents the two halves of a null-split³, i.e. A and B together contains the entirety of the observed dataset. All the $\hat{C}$ here are de-biased signal power-spectrum, defined by the procedure in sec. 27.3. By construction, we have $\langle \hat{C}_b^{\text{null}} \rangle = 0$ in the absence of systematics⁴, hence a null estimator.

Fig. 28.1 shows us both the null-split involved and their correlations in our large-patch analysis.

Each null-split is chosen to be a physical variable that can be used to split the data into two halves. It is particularly useful as a diagnostic device to reveal unknown systematic effects.

For example, QU_PIXEL splits the data into two halves based on the polarization angles of the detectors fabricated on the wafer. This can help us to check against some unknown fabrication systematics.

¹Chinone, “Polarbear Null-Test Framework”.

²QUIET Collaboration et al., “FIRST SEASON QUIET OBSERVATIONS”.

³Here, each null-split refers to a choice of splitting the data into two halves— A & B .

⁴i.e. the signal does not contain a systematic component that is different between A & B .

In addition, POLARBEAR is a blinded experiment to avoid observer bias and confirmation bias. Only after passing the whole null-test suite and other unblinding criteria such as Kolmogorov–Smirnov test, we are allowed to look at our science product—the measured power-spectra.

20.1 Null split definitions

The following are the null-split used in the large-patch null-test suite, first appeared in The POLARBEAR Collaboration et al.⁵

FIRST_SECOND “First half versus second half”

the dataset is split into two equal-weight halves chronologically to probe for time dependent miscalibration or changes in the instrument.

RS_MIDDLE, RM_SET, SM_RISE “middle versus rising and setting”, “setting versus rising and middle”, “Rising versus middle and setting”

the three different CES types are split in all possible combinations to detect elevation-dependent miscalibration or residual ground synchronous signal.

LR_SUBSCAN “Left-going versus right-going subscans”

the dataset is split in half according to the direction of motion of the telescope to test for microphonic or magnetic pickup in the data.

GAIN_BY_CES “High gain versus low gain observations”

the dataset is split into observations with above and below average mean detector gain coefficients to search for problems with the gain calibration.

PWV “High PWV versus low PWV”

the dataset is split by PWV as measured by the nearby APEX radiometer to check for loading or weather dependent effects.

LEAK_BY_BOLO “Mean temperature to polarization leakage by channel”

split the dataset into detectors that see small and large temperature leakage coefficients to test the subtraction and search for residual contamination.

LEAK_BY_CES “Mean temperature to polarization leakage by observation”

split the dataset into observations that see small and large temperature leakage coefficients to test the subtraction and search for residual contamination.

AMP_2F_BY_BOLO, AMP_4F_BY_BOLO “2f amplitude by channel”, “4f amplitude by channel”

split the data by HWP signal amplitude to check for problems removing the HWP structure or systematic contamination coupling into the data through these terms.

AMP_2F_BY_CES, AMP_4F_BY_CES “2f amplitude by observation”, “4f amplitude by observation”

⁵[“A Measurement of the Degree-Scale CMB B-Mode Angular Power Spectrum with POLARBEAR”](#).

split the data by HWP signal amplitude to check for problems removing the HWP structure or systematic contamination coupling into the data through these terms.

QU_PIXEL “Q versus U pixels”

each detector wafer is fabricated with two sets of polarization angles. We split the data into the two pixel types to check for problems in the device fabrication.

SUN_HORIZON, MOON_HORIZON “Sun above or below the horizon”, “Moon above or below the horizon”

we split observations based on whether or not the sun or moon is up to check for residual sidelobe contamination.

SUN_DIST, MOON_DIST “sun boresight distance”, “moon boresight distance”

we split observations based on the sun or moon boresight distance to check for residual sidelobe contamination.

THS_BHS, LHS_RHS “Top half versus bottom half”, “left half versus right half”

we split detectors by the boresight axis of the telescope to check for optical distortion and problems due to far sidelobes.

TOP_BOTTOM “Top versus bottom bolometers”

with a continuous HWP each bolometer TOD independently measures Q and U. We explicitly separate detector pairs to check for temperature aliasing or device mismatch.

Chapter 21

The POLARBEAR analysis pipelines

Because of this null-tests setup and the unblinding criteria, it means that we need to be able to run the pipeline for all these null-tests combinations and to rerun it repeatedly until we can pass the null-tests. We need to figure out a way to fail fast, speculate often on how to mitigate the systematics, and repeat.

21.1 Naïve Mapmaking

That is why the naïve mapmaker¹ is the mapmaker of choice. Since it is just binning the data, per TOD we can perform binning all at once with different null-split. This makes the computational cost of having more null-splits almost free. (In fact, in one paper with the small-patch dataset², both the naïve mapmaker and an unbiased mapmaker are used and they demonstrated they can only manage the full null-tests suite on the naïve mapmaker pipeline because of computational constraints, and the results from both pipelines are consistent.)

Per Constant Elevation Scan (CES), which is about an hour length of TOD observation³, mapmaking can be performed independently.

Mathematically, we can make it abstract and write

$$m^i = \mathcal{M}\{d^i\}, i = 1, \dots, n_{\text{CES}}, \quad (21.1)$$

where d represents the TOD, m represents the map, i represents the i -th CES, and $n_{\text{CES}} = 3391$ represents the total number of CES. $\mathcal{M}$ represent the mapmaking procedure that turns TOD into maps. While this procedure involves a lot of oper-

¹see chapter 11.

²The POLARBEAR Collaboration et al., “A Measurement of the Cosmic Microwave Background B -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data”.

³Constant Elevation Scan refers to the fact that those hour-long observations are scanning the sky at constant elevation.

ations including filtering and TOD processing, the key feature here is that it is a mathematically well-defined operator.

Note that here many indices are suppressed, explicitly, we have $m_p^{i,X,n}$, where X represents the different I, Q, U components, n represents different null-splits, such as 1A for the first half of the 1st null-split, 3B for the second half of the 3rd null-split, etc. p represents the pixel index of the map, which is inherently a 2-dimensional object. As we take the flat-sky approximation here, we can understand p as enumerating over $(1, 1), (1, 2), \dots, (2, 1), (2, 2), \dots$

Finally, note that m here is a random variable. Mathematically,

$$m_p^{i,X,n} \sim \mathcal{N}(\mu_p^{i,X,n}, 1/w_p^{i,X,n}).$$

Uncorrelated Gaussian white noise in the TOD is assumed in order for this to be strictly true.⁴ Here μ is what people usually called the map, where w is statistically called precision—the inverse of the variance, and is usually called the weights among physicists.

21.2 Coadd

After that, another pipeline called coadd simply adds individual CES maps together, weighted by inverse-variance.

Mathematically, first we should mention the variables that we propagate in the mapmaking and coadd pipelines to be M, w , where $M \equiv w\mu$.

And then we write down the inverse-variance weighting scheme,

$$w^I = \sum_{i \in I} w^i \tag{21.2}$$

$$\begin{aligned} \mu^I &= \frac{\sum_{i \in I} w^i \mu^i}{\sum_{i \in I} w^i} \\ \Rightarrow M^I &= \sum_{i \in I} M^i \end{aligned} \tag{21.3}$$

Here the I index represents a map-bundle—a collection of CES maps that together forms a complete map of the patch we observed. Each map bundle amount to about 10 days worth of observation, And we have 38 map-bundles in total.

This explains the verb “coadd” and why we keep track of the combination of M, w , where $\mu = \frac{M}{w}$ is the quantity of interest. It also explains why naïve mapmaking hugely

⁴In the limit that this assumption is exact, naïve mapmaker is the optimal mapmaker (See chapter 11.) This implies that we need filtering to turn $1/f$ -noise into approximately white noise, and deviation from this assumption results in less-than-optimal but usable maps, where usability subjects to other criteria including null-test passing and de-biasing.

simplifies the pipelines computationally. To put it in programming terms, the naïve mapmaker can be implemented using the MapReduce paradigm where the mapmaking-pipeline *maps* each TOD per CES into a map, and the coadd-pipeline *reduces* these maps into map bundles.

Note again that many indices are suppressed and can be expanded to $m_p^{I,X,n}$. Note that per map bundle & null-split, this is embarrassingly parallel.

21.3 Cross-pseudo-power-spectra

The cross-pseudo-power-spectra, often called pseudo-spectra for brevity, between the null-split map bundles will be computed by the pseudo-spectra pipeline.⁵

Schematically,

$$\tilde{C}_b^{X'Y',n} = \mathcal{P}\{m_p^{I,X,n}, m_p^{J,Y,n}\},$$

where $\mathcal{P}$ is a mathematically well-defined operator⁶ that turns map-bundles into cross-pseudo-power-spectra. Note that $X' = T, E, B$ while $X = I, Q, U$ where $T \equiv I$. For the purpose here, we would note only that $\mathcal{P}$ involves Fourier transform, a procedure to “rotate” Q, U components into E, B , as well as a procedure to obtain a pure B-mode estimator that suppress E-to-B leakage⁷. Computationally, we note that per null-split, the spectra can be computed independently.

21.4 Monte Carlo (MC) simulations

And the whole process will be repeated, once with the real data, and n_{sim} times each with a random simulation⁸.

Since all these are embarrassingly parallel, it should be any HPC (High-Performance Computing) programmer’s dream.

And because of the simplicity of the pipeline, it is a policy in POLARBEAR to implement all these in pure Python.

21.5 Splitting the pipeline into two

All these worked fabulously in small-patch. But when processing the large-patch dataset, it failed spectacularly. Simply put, the software have not been expecting nor

⁵This is called pseudo because it is so far biased and will be corrected for in later steps.

⁶See Ade et al., “A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”, eq. 16.

⁷Smith, “Pseudo- C_ℓ Estimators Which Do Not Mix E and B Modes”.

⁸See chapter 14.

testing against the larger map size⁹.

In the early days of large-patch analysis, it was decided to make maps at much larger pixel at $10'$ which is good enough for $\ell \lesssim 600$. This decision split the pipeline into two, the low- ℓ pipeline for $50 \leq \ell \leq 600$ targeting primordial B-mode, and the high- ℓ pipeline for $600 \leq \ell \leq 3000$, targeting lensing B-mode, E-mode, and the lensing potential.

In addition to “solving” the computational problem in the interim, splitting such a large ℓ -range into two enables each pipeline to have their own optimization and mitigation strategies, such as filtering, data-selection, etc.

⁹Large-patch is about 2 order of magnitude larger than small-patch. When maps are made at the same resolution of $2'$, small-patch would be hundreds of pixel wide, and large-patch would be thousands of pixel wide.

Chapter 22

CRHWP demodulation

The simplicity and efficiency in POLARBEAR’s mapmaking strategy was discussed earlier, but a detail was glossed over about how different I, Q, U map components can be decomposed during mapmaking.

One commonly used scheme is called pair-differencing¹. But as the large-patch is observed after the installation of CRHWP, a huge simplification can be used instead.

We already know that CRHWP modulates the signal to higher frequency such that the polarization signal is away from the $1/f$ noise.

fig. 22.1 illustrates a frequency-domain based demodulation scheme. The signal is well separated in the frequency domain nicely provided that we choose the scanning speed and rotational speed carefully². This allows us to obtain the intensity signal I by low pass filtering, and the polarization signal first by high pass filtering, then by a translation in the frequency domain because the HWP is rotating at constant speed. And if we do this in phasor then we can obtain Q & U as the real and imaginary part respectively.

And all these are obtained at the time-domain. So it enables a workflow that the TOD is pre-processed with lower level processing including demodulation, and the life cycle of null-test iterations normally starts from this processed TOD. The mapmaking step is then very simple, as the components are already separated.

This concludes the brief overview of the POLARBEAR analysis pipelines, for more details, see for example Ade et al.³, Takakura et al.⁴, The POLARBEAR Collaboration et al.⁵

¹See Ade et al., “A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”, eq. 14, 15.

²Takakura et al., “Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

³“A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”.

⁴“Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

⁵“A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

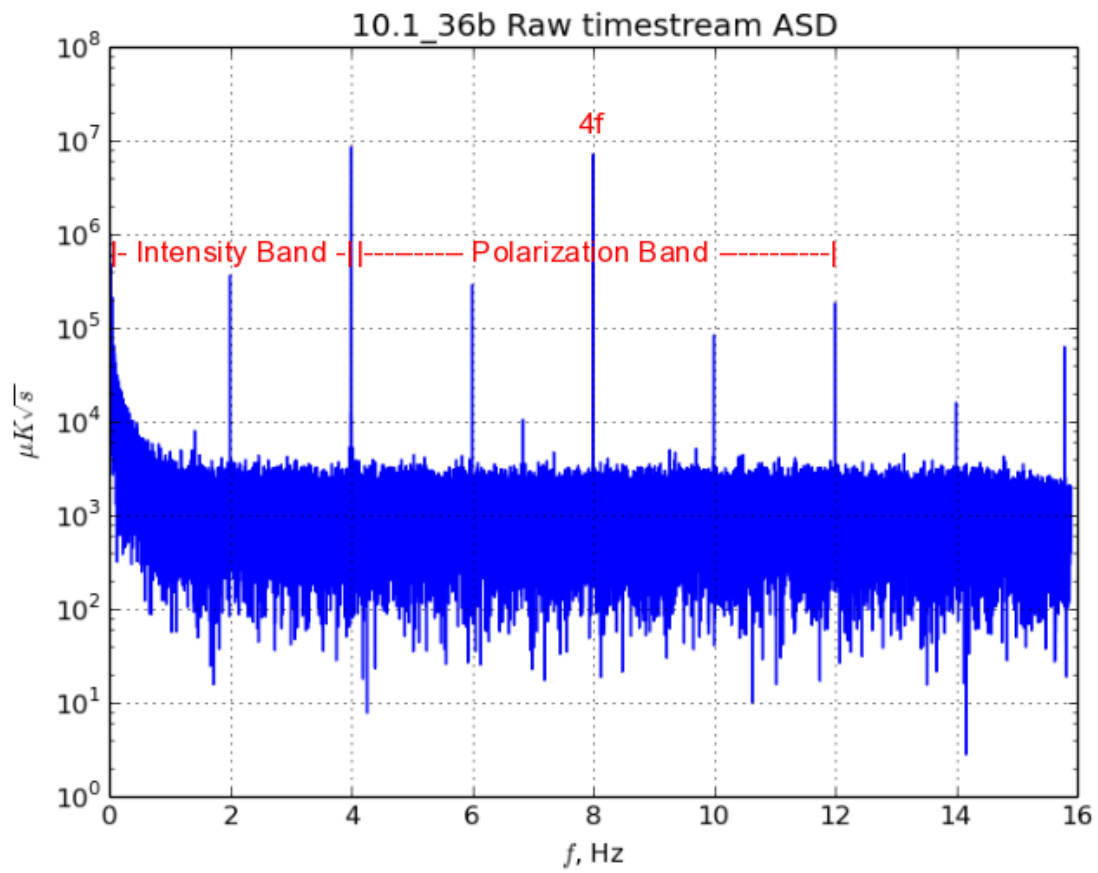


Figure 22.1: Frequency-domain based demodulation

Part V

A Measurement of the CMB *B*-mode Angular Power Spectrum at Sub-degree Scales from 670 Square Degrees of POLARBEAR Data

Chapter 23

Sensitivity projection

As we have mentioned in chapter 19, in the 3rd–5th seasons, POLARBEAR observed a sky area of 670° ². This dissertation focused on the high- ℓ B -mode science product from this observation. Let us start with a back of the envelop calculation to see what result we should be expecting.

Since we have a $3.5'$ beam. We can estimate the maximum ℓ that we can resolve by using the rule of thumb

$$\Delta\theta \sim \frac{180^\circ}{\ell}. \quad (23.1)$$

This is because for the spherical harmonics used in decomposing the CMB¹,

$$Y_\ell^m(\theta, \varphi) = \sqrt{\frac{(2\ell+1)}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta) e^{im\varphi}.$$

Since $|m| \leq \ell$, the smallest periodicity of the magnitude in the φ -direction is exactly $\frac{180^\circ}{\ell}$.

So, from a resolution of a $3.5'$ beam, we expect

$$\ell_{\max} = \frac{180^\circ}{3.5'} \sim 3000.$$

For the minimum ℓ we can measure, that is subjected to a different criteria. First, our large-patch is roughly 45° wide, so using a similar estimate seems to suggest we can measure down to $\ell \sim 4$. But there are other factors come into play. The most important one is the ℓ_{knee} , a characteristic ℓ scale determined by the presence of low frequency noise. It turns out that $\ell_{\text{knee}} = 90$, which is the topic of the low- ℓ paper.²

As mentioned in sec. 21.5, the pipelines are split into two at $\ell = 600$. This work focuses on the high- ℓ range of 600–3000.

¹See sec. 38.6 for details.

²The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

In this ℓ -range, we are observing the lensing B -mode. This lensing B -mode is predicted by, as the name suggested, the lensing effect of the E -mode power-spectrum that becomes a B -mode pattern. It has first been observed by POLARBEAR in its first two seasons of data in the range of $500 \leq \ell \leq 2100$.³ A natural question to ask is how would the sensitivity of this project compares to that.

To do this, we need to know how the sensitivity should scales with sky-area and map-depth.

23.1 Knox Formula

According to Hivon et al.⁴, eq. 16-17,

$$\Delta C_\ell \approx \sqrt{\frac{2}{\nu_\ell}} \left(C_\ell + \frac{N_\ell}{B_\ell^2} \right) \quad (23.2)$$

$$\nu_b \approx (2\ell + 1) \Delta \ell f_{\text{sky, eff.}}$$

$$f_{\text{sky, eff.}} \equiv f_{\text{sky}} \frac{w_2^2}{w_4}$$

where it is understood that the l -bins are half open intervals: $b = [l - \frac{\Delta l}{2}, l + \frac{\Delta l}{2})$.

Here, C_ℓ is the power-spectrum, ΔC_ℓ is its error-bar, ν is the d.o.f., B_ℓ is the beam, N_ℓ is the noise spectrum, f_{sky} is the fraction of the sky observed. $f_{\text{sky, eff.}}$ is the effective f_{sky} adjusted by the 2nd and 4th moment (w_2 and w_4 respectively) of the mask used in calculating the power-spectra from the maps.

Note that this formula is exact given ℓ if the underlying random variables (including the C_ℓ term and the noise term) are all Gaussian and independent. $\nu = 2\ell + 1$ per ℓ , and the factor of f_{sky} adjusted for the fraction of number of modes lost in not measuring the full sky. Hence the validity of this formula is based on how well these assumptions are in practice.

Understanding what it means, we can estimate ν better⁵,

$$\nu_b \equiv \sum_{l=\frac{\Delta l}{2}}^{l+\frac{\Delta l}{2}-1} (2\ell + 1) f_{\text{sky, eff.}} = 2\ell \Delta \ell f_{\text{sky, eff.}}$$

³The POLARBEAR Collaboration et al., “A Measurement of the Cosmic Microwave Background B -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data”.

⁴“MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

⁵Hamilton et al., “Sensitivity of a Bolometric Interferometer to the Cosmic Microwave Background Power Spectrum”, eq. 20.

i.e. the one Hivon et al.⁶ used over-estimated the error-bar a little bit, becoming relatively negligible when ℓ is large.

Now, let us move on to see how we can project the sensitivity knowing only how the noise and f_{sky} scales. Assuming noise-limited,

$$\Delta C_b \sim N_b \sqrt{\frac{2}{\nu_b}} \propto \frac{n^2}{\sqrt{f_{\text{sky}}}}. \quad (23.3)$$

Here we introduce the map-depth n which is the square-root of the noise spectrum, and also usually put in a different unit. Typically, the noise spectrum has a steradian implicitly in its unit⁷. But map-depth is usually put in the unit of $\mu\text{K}'$ so a conversion from sr to $'^2$ (square-arcminute) is needed.

Method 1: Scaling by the change of sky area only

Consider only the change in sky coverage, if we *decrease* the sky area by a factor of r ,

$$f_{\text{sky}} \rightarrow \frac{f_{\text{sky}}}{r},$$

$$n \rightarrow \frac{n}{\sqrt{r}},$$

$$\Delta C_\ell \rightarrow \frac{\Delta C_\ell}{\sqrt{r}},$$

where the scaling of n is deduced from the scaling of the standard error of the mean, assuming the total amount of observation stays constant.

That is, the signal-to-noise ratio is increased by $\sqrt{r}$.

In the first two seasons, POLARBEAR observed a sky area of $25^\circ^2 \rightarrow r = 25/670$. The signal-to-noise ratio is expected to decrease by 0.19.

Since The POLARBEAR Collaboration et al.⁸ reports a statistical error of 0.26 on A_L , we expect it to be around 1.3 in this measurement.

But we can do slightly better. To account for the difference in the number of season (2 vs. 3), deducing from the standard error of the mean again, the signal-to-noise ratio will decrease by $0.19 \times \sqrt{\frac{3}{2}} = 0.23$, leading to an error of A_L to be around 1.1.

All these should have been known before the beginning of the 3rd season of observation⁹. And since A_L simply is the ratio of what we observed comparing to what we

⁶“MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

⁷i.e. the unit is simply μK^2 with the steradian not written explicitly.

⁸“A Measurement of the Cosmic Microwave Background B -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data”.

⁹While the slight difference in the use of ℓ -range is not accounted for so far, it turns out to be negligible.

expect, its fiducial value is 1, i.e. we should have expected the sensitivity is less than 1σ .

Before we move on, let us improve this calculation slightly by using the actual number of observation hours. The POLARBEAR Collaboration et al.¹⁰ reports 2800 h of observations, while we have 2985 h. i.e. it is much less data volume then we would expect from the increase from 2 to 3 seasons. One of the reason is the eruption of a nearby volcano on October 30th, 2015¹¹, among other mechanical problems. This means the signal-to-noise ratio is decreased by a factor of $0.19 \times \sqrt{\frac{2985}{2800}} = 0.20$ instead, resulting in an error of A_L to be around 1.3.

Method 2: Estimated using the actual map depth in map domain

The map depth n can be estimated in the map domain directly.

Naïvely, this can be estimated by the deepest part of the map. Internally, it is reported to be $4.5 \mu\text{K}'$ ¹² from the first two seasons of data, and $16 \mu\text{K}'$ for the later three seasons.

Using eq. (23.3), we would expects the error-bar to be 2.4 times more, resulted in $\Delta A_L = 0.63$.

But this number is highly deceptive for two reasons. First, the absolute gain was not known at the time. Taking into the account of the absolute gain in the hindsight, the map depth is $18.4 \mu\text{K}'$, resulted in $\Delta A_L = 0.84$.

The more serious problem is that the map depth is highly irregular as seen in fig. 23.1, while the first two seasons have very uniform maps.

So a better metric would be to compare by their weighted average map depth. This is known internally to be $5.5 \mu\text{K}'$ for the first two seasons and $23.8 \mu\text{K}'$ for last three. This amounts to $\Delta A_L = 0.94$.

Method 3: Estimated using the actual map depth in power-spectra domain

In fig. 23.2 and fig. 23.3, we plotted the noise spectrum. $\sqrt{N_\ell}$ is the noise-spectrum, and the $\sqrt{N_b}$ is the noise-spectrum after de-biased using the MASTER algorithm,¹⁴

¹⁰“A Measurement of the Cosmic Microwave Background B -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data”.

¹¹The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

¹²Hereafter for the RA23 patch out of 3 different patches only. See The POLARBEAR Collaboration et al., “A Measurement of the Cosmic Microwave Background B -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data” for its definition.

¹³“A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

¹⁴Hivon et al., “MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

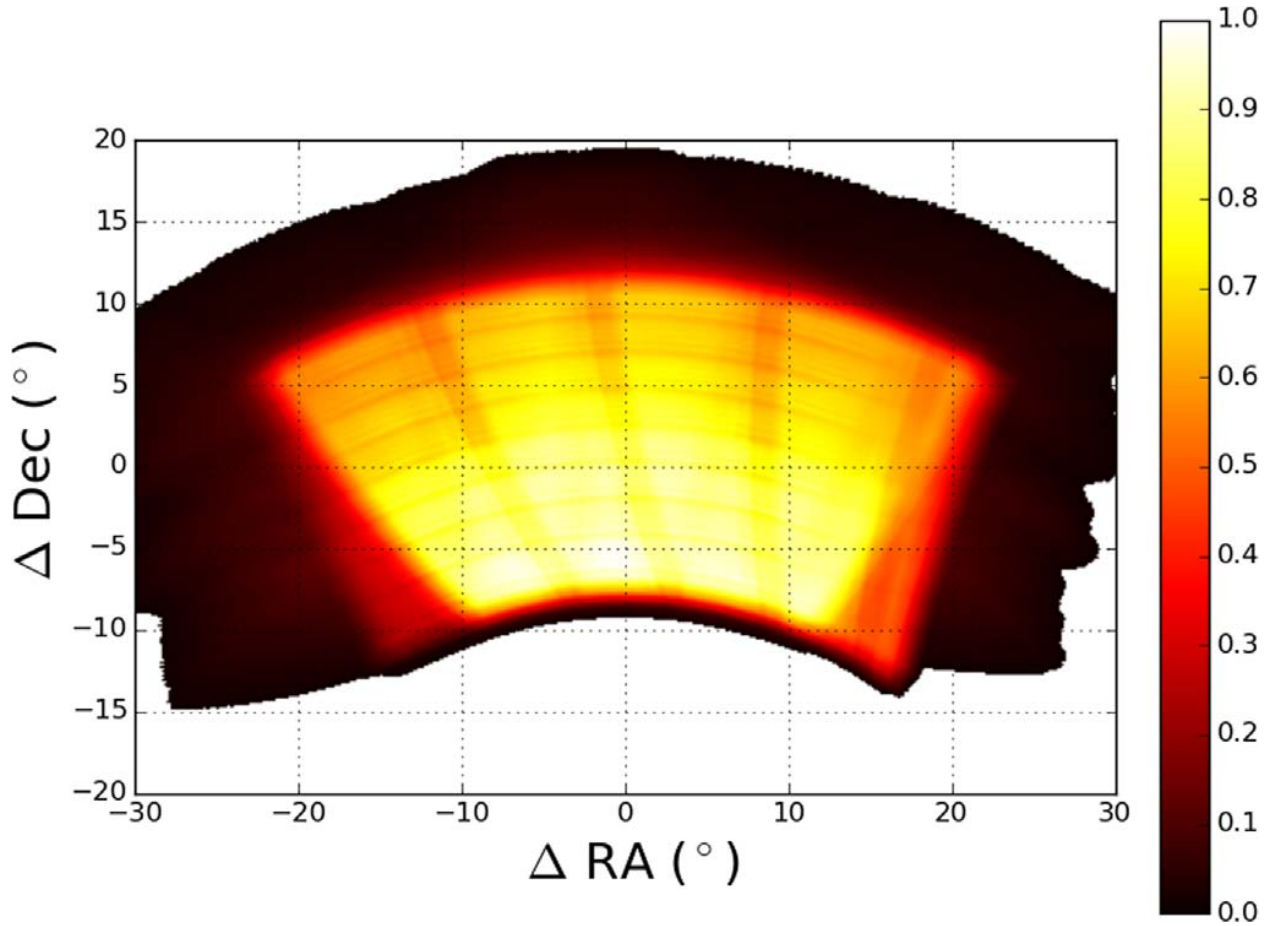


Figure 23.1: Normalized map depth illustrating the scan pattern, The POLARBEAR Collaboration et al.¹³

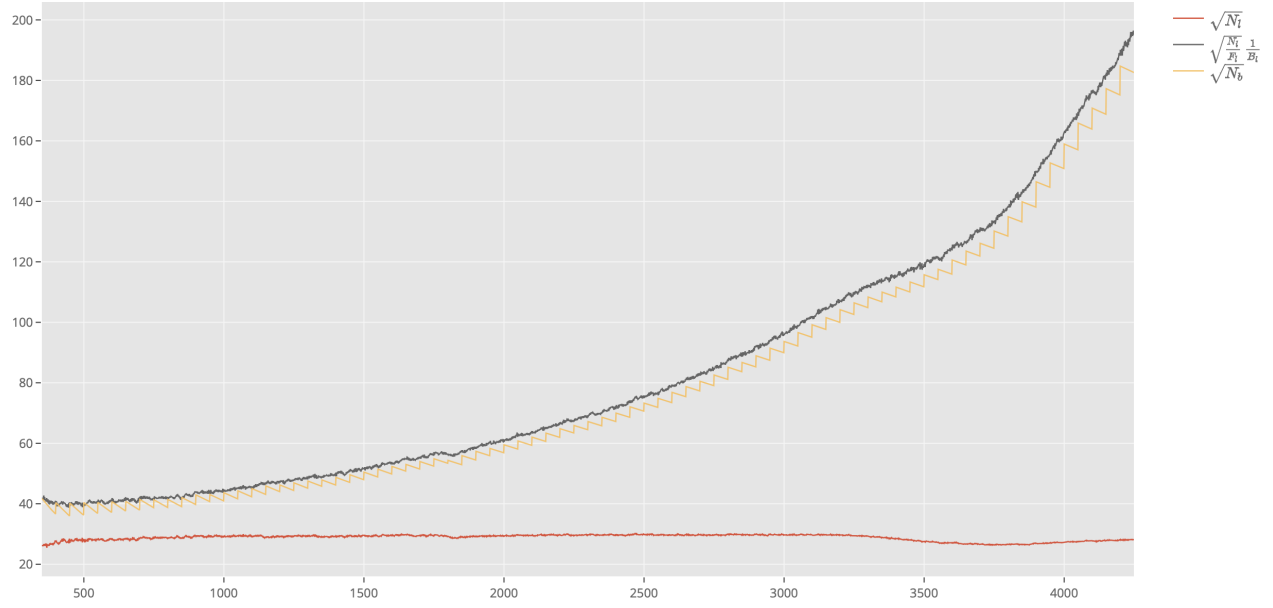
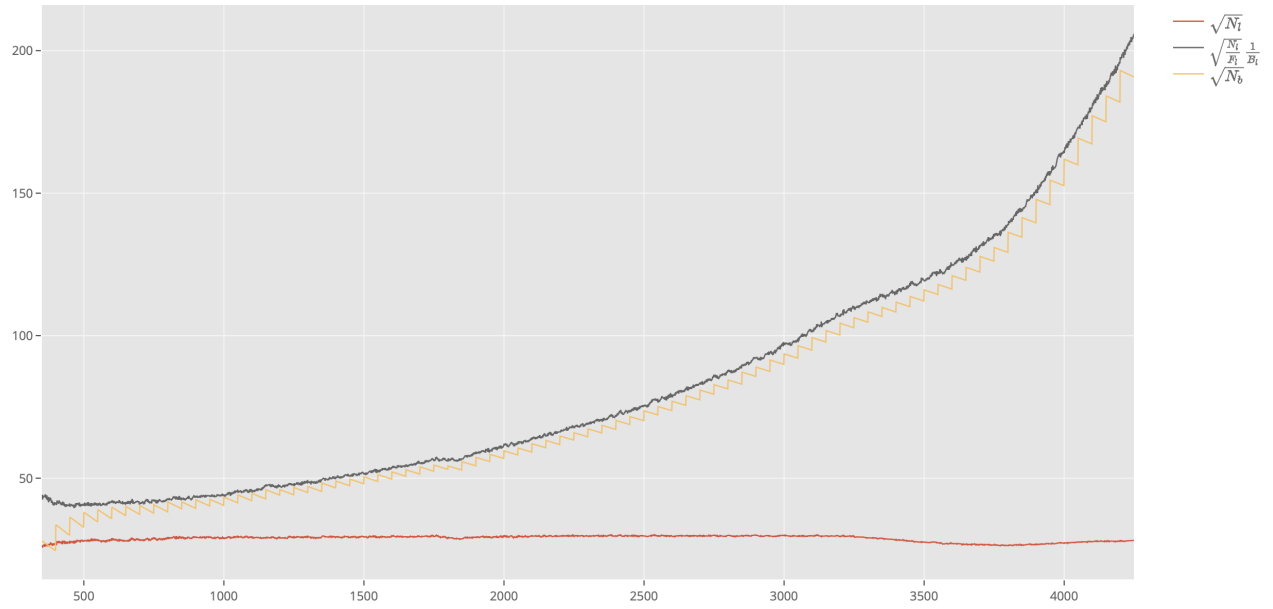
binned at a bin-width of 50. Since our science product is the de-biased power-spectra, $\sqrt{N_b}$ should be the focus here.

Even if we are using the smallest value at $\ell = 525$ which equals to $36.2 \mu K'$, comparing to an internally provided value from the first two seasons at $6.3 \mu K'$, it would project to be $\Delta A_L = 1.7$.

Lesson learnt

To summarize, we should have projected the sensitivity to be $\Delta A_L = 1.1$ before the observation even started, and $\Delta A_L = 1.7$ in the beginning of the analysis.

But, all these was unknown to anyone in the collaboration at the time, including the very author who spent a few years in analysis. Hence, this is perhaps the most

Figure 23.2: *EE* noise spectrum ($\mu\text{K}'$)Figure 23.3: *BB* noise spectrum ($\mu\text{K}'$)

important but simple physics one should understand and calculate. Sometimes we got buried in details and got lost in the big picture.

However, all is not lost. As we shall soon see, this experiment shows that we can measure the polarization of the CMB in a wide range of $50 \leq \ell \leq 3000$ ¹⁵ using a single medium aperture telescope.

¹⁵For $50 \leq \ell \leq 600$, E -mode is only used for calibration.

Chapter 24

Reproducible research

24.1 Introduction

Distilled to the essence, one may define science as a discipline to predict. In this aspect, reproducibility can be viewed as the cornerstone of science—without reproducibility, one cannot reliably predict even the outcome of a repeated experiment, it is not science.

However, it has been coined in recent history in science the term replication crisis or reproducibility crisis.¹

There are two layers of reproducibility. The first is reproducibility in the general sense which focuses on the reproducibility of the experiment in general, and is a standard expectations in scientific publications. The second is restricted to a subset of reproducibility in an experiment, called computationally reproducible research or often simply reproducible research.²

Regarding the general reproducibility, it is central to any CMB experiments. For example, systematic effects are an important area that many CMB physicists deeply worried about. Systematic effects can be viewed as one form of reproducibility requirements—i.e. when constructing an independent experiment which comes with a different set of systematics, they better reproduce the results from each other by excluding any significant systematic effects. In POLARBEAR, the null-test framework is used because it helps to avoid these systematic effects. POLARBEAR is designed to be a “Blind Analysis as a Correction for Confirmatory Bias”³, which is one example of many form of biases that resulted in non-reproducible science. The general agreements between results from CMB experiments indicate that the general reproducibility crisis is not a major problem here.

¹Ioannidis, “Why Most Published Research Findings Are False”; Schooler, “Metascience Could Rescue the ‘Replication Crisis’”.

²Alston and Rick, “A Beginner’s Guide to Conducting Reproducible Research”; Peng, “Reproducible Research in Computational Science”.

³MacCoun and Perlmutter, “Blind Analysis as a Correction for Confirmatory Bias in Physics and in Psychology”.

However, in terms of computationally reproducible research, CMB experiments fail in this regard to various extents, especially in first generations of ground-based CMB experiments including POLARBEAR.

What is (computationally) reproducible research? Peng⁴ said,

There are two major components to a reproducible study: that the raw data from the experiment are available; and that the statistical code and documentation to reproduce the analysis are also available.

Hutson⁵ mentioned in an article focusing on Artificial Intelligence (AI) with the following subtitle: “Unpublished code and sensitivity to training conditions make many claims hard to verify”, while Miyakawa⁶ even claim in the title: “No raw data, no science: another possible source of the reproducibility crisis”.

To summarize, (computationally) reproducible research focuses on what happened after an experiment gathered data. Reproducing an experiment is expensive, making the entry barrier to reproduce very high. Reproducible research requires everything but the building of the experiment itself to be completely reproducible in the sense that the observed data and software written to come to the conclusion of the research are open for anyone to examine and come to an independent conclusion. A prerequisite of this would requires all raw data collected to be released openly, and all essential softwares that is used in a publication are made at the very least source-available if not open-source⁷.

In this sense, none of the CMB experiments are reproducible to the best of the author’s knowledge. For example, all WMAP data are made available in LAMBDA⁸ but not all its associated code and recipe for complete reproducibility⁹. Planck releases many of their data products and software packages, but not all.¹⁰ Simons Observatory releases most of its software in a [public GitHub organization](#),¹¹ no commitment has been made yet for complete (computationally) reproducibility¹². Not to mention POLARBEAR has released their data products sparingly, has made no raw data available to the public, nor most, if not all, of the softwares they used internally.

⁴[“The Reproducibility Crisis in Science”](#).

⁵[“Artificial Intelligence Faces Reproducibility Crisis”](#).

⁶[“No Raw Data, No Science”](#).

⁷Source-available simply means the source is available for audit, open-source has additional requirements concerning the rights of the users receiving the source codes as well.

⁸Bennett et al., [“NINE-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE \(WMAP \) OBSERVATIONS”](#).

⁹Incidentally, because of the availability of full WMAP data, a third party effort has been made to claim the first reproducible CMB map in Bobin, Sureau, and Starck, [“Cosmic Microwave Background Reconstruction from WMAP and Planck PR2 Data”](#).

¹⁰[“Planck Mission Products - Planck Legacy Archive Wiki”](#).

¹¹Koopman et al., [“The Simons Observatory”](#); Li et al., [“The Simons Observatory”](#).

¹²Ade et al., [“The Simons Observatory”](#).

While this is not something the author can change, the author decompose the different extents of reproducibility in the following way, and make every effort to make it as reproducible as humanly possible given the constraints.

First, we already mention that (computationally) reproducible research has components of data and software. Second, we can define the degree of reproducibility in different level of audiences—the general public, and internal collaborators. This can be illustrated in tbl. 24.1.

Table 24.1: Different extents of (computationally) reproducible research

	Collaborators	General public
Data	Is the data accessible to internal collaborators? Is the data format documented?	Is the data available to the public?
Software	Is the softwares available to internal collaborators? Is the recipe to reproduce a product available and documented, including the software environment?	Is the softwares used in research released as open-source softwares?

24.2 Making this research more reproducible

Software

As mentioned above, the raw data used in POLARBEAR is not available to the general public. The softwares, where the bulk of data analysis depends on resides in a software repository named AnalysisBackend, is not made source-available.

While there are nothing the author can do to change this, the author makes an effort to factor the code in a way that most newly written code do not resides in AnalysisBackend but in other repositories such that the author can freely open-source them without subjecting to the agreements of any existing authors in the original repositories.

Because of that, most of the descriptions here focus on reproducibility to internal collaborators.

While POLARBEAR's data is in principle available to any internal collaborators, they resides in a project directory in the [Community File System \(CFS\)](#) at NERSC. Due to the lack of maintainers in that space, any collaborators can write to that space with a default permission that is not readable to anyone but the one who writes the data. Hence, while all files in this space are supposed to be group-readable among all collaborators, some files such as some instrumental metadata are not readable. While this does not impact the results of analysis, it does cause troubles in, for example, staging raw data to scratch space for further analysis, and is a sign of not designing with the goal of reproducibility towards even internal collaborators. While I can volunteer

to manage the project account, since there is no maintainer, there is not much I can do to change the situation.

To make my contribution reproducible to internal collaborators, I setup a GitHub organization for internal collaborators. Using that platform, I provide a repository for all the scripts I submitted to NERSC—this resembles how a lab notebook is used in a laboratory setting, whereas I recorded every scripts I run at NERSC so that everything I did is traceable, a first step towards being reproducible.

Soon I found that the software environment can be part of the puzzle that makes something not reproducible. At first it was provided by another collaborator as [HPC-Ports](#), as it is abandoned and no longer updated, I volunteered to maintain a software environment to run our software stack at NERSC. To make this software environment reproducible, I had to automate the building of the software environments (which is basically what HPCPorts did) in the form of scripts. Eventually, I rewrote it as a Python package called [pmpm](#) which stands for Python manual package manager.

Another pain point in reproducibility is the software we used—AnalysisBackend. It has no maintainer, because of that, every collaborator just create their own branch and merging their contribution is often not done. One consequence of this is that in many occasions in our analysis meetings, after discussing about an analysis problem for a long while, someone would often starts to realize the root of the problem and ask, “wait a minute, which branch are you using?”. While this situation cannot be changed in AnalysisBackend without top-down management, the main collaborators working on large-patch analysis try to converge on a common branch. While each of us still maintains our own branch, we mostly keep the differences between these branches minimal.

Since I opt for writing in separate repositories as much as possible, there I can opt for semantic versioning¹³ with changelogs and documentations.

Regarding reproducibility between collaborators, oftentimes the pipeline scripts are passed with a long chain of command line arguments that is hard to trace the differences because there can be hundreds of options involved. A better way would be to put this configuration arguments in a format that is more tractable. In order to do this while still being able to run arbitrary command line program with arguments from any software including AnalysisBackend, [yaml2cli](#) is written to “organizes cli args by YAML” which can generates bash scripts including SLURM batch scripts from a configuration written in YAML. YAML is chosen because of its readability and features such as alias that facilitate “code reuse” within the configuration file. See sec. [38.6](#) for examples used in this analysis.

¹³[Semantic Versioning](#)

Data

Data generated in my analysis also uses the semantic versioning scheme. See sec. [38.6](#) for the versions of null-test perform in this analysis.

The data container itself should be chosen for performance and stable archival purpose. While many collaborators like to save data in the Python pickle format due to its ease of use, “you should be wary of using pickles for long-term storage where the underlying code is not highly stable”¹⁴. HDF5¹⁵ is a standard data container format designed for these purpose. It has wide adoption in the field of cosmology.

Not only it is important to use a stable container for archival purpose, how data is stored should be documented carefully to ensure it is used and interpreted correctly. In this analysis, the data released internally are documented and the documentation is made available in sec. [38.6](#).

¹⁴[UsingPickle - Python Wiki](#)

¹⁵[The HDF5® Library & File Format - The HDF Group](#)

Chapter 25

Computational challenges

25.1 Orientation

In the beginning, I was told to just run the whole pipeline with a smaller pixel at 2' and see what are the null-test failures. It was null-test v0.1.0¹.

However, I encountered multiple job failures because the nodes generated out-of-memory errors and/or exceeding the requested allocation time. It involved a lot of manual labor including fixing and rerunning whatever corrupted files when the jobs got killed.

Long story short, with 25 realizations², it took me 1 month and the following amount of NERSC hours³:

1. mapmaking: 33690 NERSC hours
2. coadd: 55339 NERSC hours
3. final-coadd: 86 NERSC hours
4. mode-coupling matrices: ~70 NERSC hours
5. pseudo-spectra: ~60,000 NERSC hours

I actually got the last one for free using the big-memory node at NERSC⁴. Had it been charged, it would have been a total of ~150,000 NERSC hours.

¹I adopt [semantic versioning](#) in my null-test runs. See sec. [38.6](#).

²1 for the real data, 8 each for 3 kinds of simulations.

³NERSC hour is a unit of computer time at the National Energy Research Scientific Computing Center (NERSC). It was calibrated to the computational power of 1 CPU core in an hour for an older NERSC system called Hopper. Exactly how much 1 NERSC hour is worth is upon calibration and it has been changed over time. During the time this analysis was done, on a Cori Haswell system where each node holds two Intel Xeon Processor E5-2698 v3 processors with 128GB memory (See [Cori - NERSC Documentation](#)), 80 NERSC hours were charged when using 1 such node for an hour.

⁴There are 2 such big-memory nodes at NERSC with 750 GB. During the time of analysis, it was free to use but very congested.

And if you are laughing right now, you understand that mapmaking is supposed to be the only expensive part. But here, co-adding is more expensive than mapmaking (where it is only doing weighted-sum), and pseudo-spectra pipeline is more expensive than co-add.

To put that time cost into perspective, I ran approximately 31 different null-test versions⁵ that involve at least the post-mapmaking pipelines.

Not only that, near the end of the analysis where we are closer to passing the null-tests, we increased our total number of realizations from 25 to 97. And we used 769 realizations in the final run. Had I been using the original pipeline for this, I would have used ~5.2 million NERSC hrs and 31 months of turnaround time for this alone. So by the time of publishing this dissertation, I would have been nowhere near passing the null-tests, and would have exhausted my NERSC allocation per year a dozen times already.

25.2 Low lying fruit

Essentially embarrassing parallelism can only be 100% effective if you can feed each processor a job and the load is perfectly balanced. It turns out both of these are far from true in our pipelines with our large-patch map size requirements.

For example, in the mapmaking pipeline, recall from eq. (21.1), $m^i = \mathcal{M}\{d^i\}$, $i = 1, \dots, n_{\text{CES}}$, the default configuration is to run an MPI job with as much processes as the number of CESes where $n_{\text{CES}} = 3391$. So each job is waiting for the longest running job to finish.

In the end, I wrapped the AnalysisBackend mapmaking command line interface with a Python function interface⁶ and call it from an MPI script. With naïve random scheduling and using fewer MPI processes, the load balancing issue is solved and the total NERSC hours needed is decreased by around a factor of 5.

Another example is the coadd-pipeline from eq. (21.2) & eq. (21.3), $w_p^{I,X,n} = \sum_{i \in I} w_p^{i,X,n}$, $M_p^{I,X,n} = \sum_{i \in I} M_p^{i,X,n}$. It is very inefficient that requires 40GiB of memory per process and has bad reading pattern⁷. By rewriting it from scratch, it has more balanced read pattern, memory use is reduced to ~135 MiB, and since the problem is IO bound, multithreading is used, resulted in negligible NERSC hours needed comparing to mapmaking. Notice also from the inherent parallelism involved⁸,

⁵around 30–40 depending on how you counts. See sec. 38.6.

⁶It is a peculiarity in the design of AnalysisBackend that mapmaking can only be called from the command line using argparse.

⁷It loads all combinations of X, n, p and $i \in I$ in memory first.

⁸I decided not to dispatch over X due to how the datasets are stored and the fact that there is enough parallelism.

$I = 1, \dots, 38, n = 1A, 1B, \dots, 21A, 21B$, with a total of 1535⁹ embarrassingly parallel tasks.

The last remaining computational problems are the mode-coupling matrices¹⁰ and maps-to-pseudo-spectra pipelines¹¹, which I called them the “post-mapmaking pipeline” in short. While the former does not seem to be expensive, it is a blocking dependency of the next step, making it also a target to shave the turn-around time.

I have a few iterations in accelerating these pipelines. Earlier attempts are more Pythonic and trying to be backward compatible. But these “duct-tape” solutions do not work well as it relies on the big-memory node which is a bottleneck both in terms of scaling out the applications to more number of simulations and its availability¹².

25.3 Accelerating the post-mapmaking pipeline

Post-mapmaking pipeline includes everything after obtaining the maps, including the calculation of mode-coupling matrices and calculating the pseudo-spectra from maps.

Both of these are examples where it cannot be written efficiently using pure Python code that heavily relies on vectorization¹³.

For example, in fig. 25.1, when calculating the pseudo-spectra from maps, the maps are first converted to frequency domain using Fast Fourier Transform (FFT), and since we are interested in ℓ , corresponding to the magnitude of the frequencies in the 2D-FFT, the values needed to be binned down to individual ℓ .

In fact, the original code relies on vectorization to first create a mask per ℓ , and calculate the averages by indexing arrays by this mask. Obviously it is very inefficient, but it also means that in order to get the job done in reasonable amount of time, the averaging is done across an intermediate bin-width as shown in fig. 25.2.

Besides the computational problem, I found that the resultant binned null-spectra from this pipeline has different qualitative behavior depending on the bin-width used, for example, comparing fig. 29.1 & fig. 25.3. I suspect the use of this intermediate bin-width might be where the problem lies in its implementation in AnalysisBackend.

The obvious first step is to proof-read the whole codebase to see where the problem lies and also how I might speed it up. Eventually, I found that it was easier to completely rewrite it.

⁹It does not exactly equals $38 \times 21 \times 2$ because some of the null-split maps would be empty.

¹⁰See sec. 27.4.

¹¹See sec. 21.3.

¹²I often need to wait from days to weeks to get into the queue.

¹³The de facto standard in writing pure Python code in scientific computing is to use Numpy and the ecosystem built around it including Scipy. Numpy defines an array interface with syntactic sugars in Python with operations built around the concept of vectorization, and most of the operations is written as C module, resulted in mostly efficient code. Limitations on its effectiveness originate from the Python data model, the generality of this array interface which is compiled AOT, and how effective vectorized programming can be on a particular algorithm.

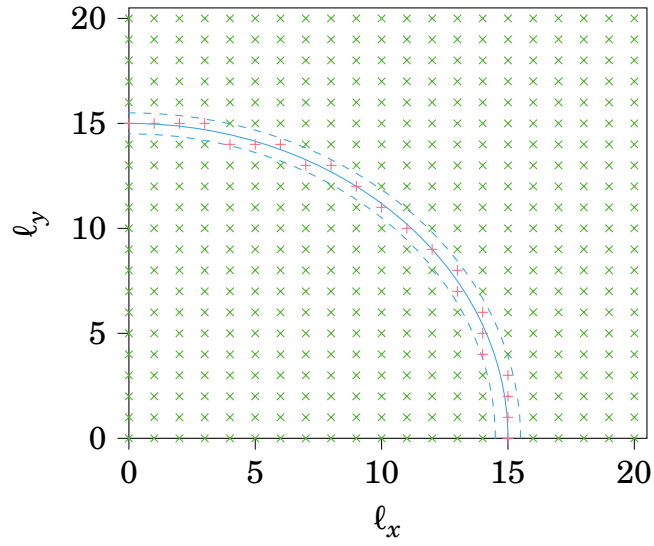


Figure 25.1: Illustration of 2D FFT to 1D power-spectra

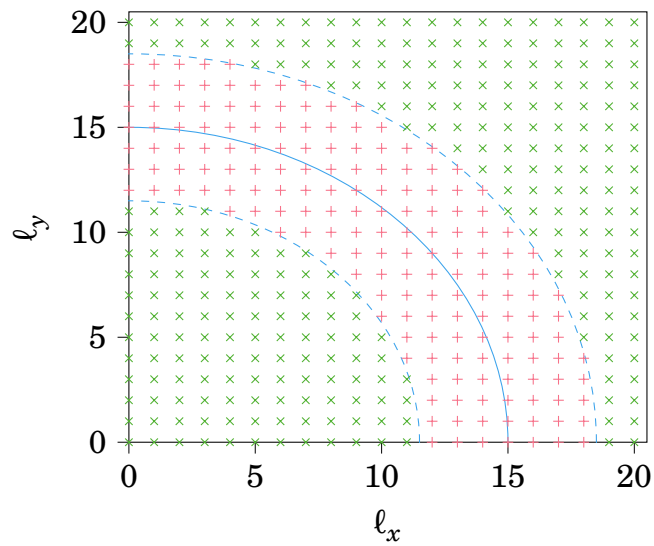


Figure 25.2: Illustration of 2D FFT to 1D power-spectra with intermediate bin-width

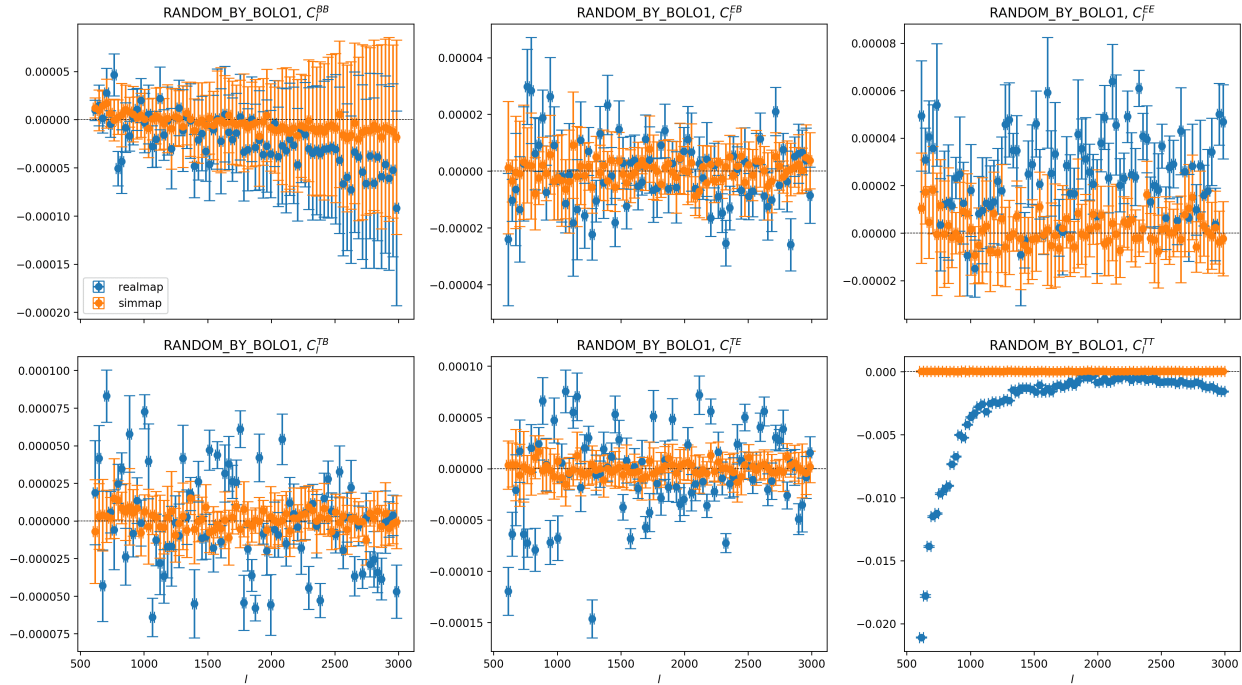


Figure 25.3: Null test v0.6.2 RANDOM_BY_BOLO1 null-spectra at finer bin-width

In order for it to be performant, and as we have a policy of using Python only, I evaluated Cython and Numba, and in the end I chose Numba because it is easier to write Numba code that is guaranteed to release the GIL (`nogil`), is compiled to machine code (`nopython`), and is parallelized (`parallel`). Since Numba uses LLVM to just-in-time (JIT) compile the function, distribution is also effortless which is a bonus¹⁴.

This code is [released as open source software on GitHub named TAIL](#), which stands for This Ain't Interpreted Language.

For example, one function from the pseudo-spectra pipeline is shown in [lst. 25.1](#). Its style is quite C-like, and the `prange` there is similar to `omp parallel` for to explicitly parallelized the loop.

An innovation in this code is the use of the Python language as a meta-programming language of the Numba language, for example as shown in [lst. 25.2](#). The reason is that only a subset of the Python language can be compiled to machine code using Numba. So by writing higher order function using the Python language which returns a Numba-jit-compiled function, we can achieve generality while being performant.

¹⁴In traditional ahead-of-time (AOT) compilation, the pre-compiled binaries should be compiled to optimize per architecture for each system. Since JIT compiler compiles at run-time which sees the CPU, it auto-optimizes for the target architecture without extra work, such as the Cori II KNL system which uses Intel Xeon Phi processors.

25.4 Results

In terms of turnaround time, note that each step of the pipeline are independent and each has a fundamentally different number of parallel jobs to do. There are $n_{\text{sim}} \times n_{\text{CES}}$ mapmaking tasks, but has $n_{\text{sim}} \times n_{\text{map bundle}} \times n_{\text{null-split}}$ coadd tasks, $n_{\text{sim}} \times n_{\text{null-split}}$ pseudo-spectra tasks. A key is to find the right MPI job size such that multiple pipelines can be put together into one single job script with one `srun` after another¹⁵.

By designing this carefully, The full null-tests pipelines can be finished with 2 submissions to the NERSC queue via SLURM workload manager to minimized the queue time and hence the total turnaround time.

In the earlier phase when these are put together and we are still in the phase of solving null-test failures, the turnaround time is shorten from 1 month to typically a day, running exactly the same number of simulations.

As a testimony of the effectiveness of this short turnaround time, shortly after this is put in place, in 2 weeks I finished 6 iterations of null-test suites, leading to eventually the discovery and solution to our null-test failures.

In the final run with 769 realizations, instead of around 5.2 million NERSC hrs and 31 months of turnaround time, it only took me about 440,000 NERSC hrs and a few days to finish first few hundreds of simulations (where I use the job array here to march through the queue more quickly.) This is a reduction of a factor of 12 in compute time, and a factor of a few hundreds in turnaround time.

Now that we have the computational challenge under control, we can turn our attention to what sort of problems we are facing when we are working towards unblinding.

¹⁵It is a limitation of the design in AnalysisBackend, which take advantage of the embarrassingly parallel nature of our algorithms, to have each tasks reading the inputs from disk and dumping the outputs to disk. While this is an inefficient way of handling intermediate data by having IO on disk instead of holding it in memory, it does simplify the way to chain these independent pipelines together by using one `srun` after another when each pipeline has very different topology from each other.

Listing 25.1 Binning 2D spectra to 1D in Numba

```
# https://github.com/ickc/TAIL/blob/master/tail/pseudospectra.py
```

```
@jit(nopython=True, nogil=True, parallel=True)
```

```
def _bin_psd(pixel_size, l_max, psd):
```

```
    N = psd.shape[0]
```

```
    freq = fftfreq(N, pixel_size)
```

```
    n = l_max + 1
```

```
    psd_1d = np.zeros(n)
```

```
    hit = np.zeros(n, dtype=np.int64)
```

```
    pi_2 = np.pi * 2.
```

```
    for i in prange(N):
```

```
        freq_i = freq[i]
```

```
        for j in range(N):
```

```
            freq_j = freq[j]
```

```
            l = int(round(
                pi_2 * np.sqrt(
                    freq_i * freq_i +
                    freq_j * freq_j
                )
            ))
```

```
            idx = l if l < l_max else l_max
```

```
            hit[idx] += 1
```

```
            psd_1d[idx] += psd[i, j]
```

```
    psd_1d = psd_1d[:-1]
```

```
    hit = hit[:-1]
```

```
    ak_to_dl_scale = N * pixel_size
```

```
    ak_to_dl_scale *= 0.5 * ak_to_dl_scale / np.pi
```

```
    for i in range(l_max):
```

```
        hit_ = hit[i]
```

```
        psd_1d[i] = (
            (psd_1d[i] * ak_to_dl_scale / hit_) *
            (i * (i + 1))
        )
```

```
        if hit_ > 0 else np.nan
```

```
    fill_nan(psd_1d, fill_boundary=False)
```

```
    return psd_1d
```

Listing 25.2 Python as Metaprogramming language of Numba

```

# https://github.com/ickc/TAIL/blob/master/tail/modcoupling.py#L128-L203
def _get_M_gen(Mtype, pure='hybrid'):
    if Mtype == 'TTTT':
        _J = _J_t
    else:
        _J_p = _get_J_p(Mtype, pure='hybrid')

    @jit(nopython=True, nogil=True)
    def _J(k1, k2, k3):
        return _J_t(k1, k2, k3) * \
            _J_p(k1, k2, k3)

    @jit(nopython=True, nogil=True)
    def simps(W, k1, k2):
        k3_min = np.abs(k1 - k2)
        k3_max = k1 + k2
        result = 0.
        for i, k3 in enumerate(
            range(k3_min + 1, k3_max)
        ):
            result += (2 - i % 2) * \
                _J(k1, k2, k3) * W[k3] * k3
        return result

    @jit(nopython=True, nogil=True, parallel=True)
    def _get_M(W, l_max, dl):
        bin_width = dl // 2
        n = l_max // dl
        M = np.empty((n, n))
        for i in prange(n):
            k1 = bin_width + dl * i
            for j in range(n):
                k2 = bin_width + dl * j
                M[i, j] = k2 * simps(W, k1, k2)
        return M

    return _get_M

```

Chapter 26

Timestream of Data (TOD)

As we have some general ideas about the pipelines from previous sections, let us go back and revisits some of the details involved. We should note that the low- ℓ B-mode large-patch analysis is published already. All large-patch analyses share similar pipelines. The earlier a task is in the pipeline, the more are shared among them. So here we are not repeating all the details but will refer to The POLARBEAR Collaboration et al.¹ whenever possible.

We will start from the Timestream of Data (TOD). As we mentioned in sec. 9.3, when observing the CMB which is an inherently a 2D object, we instead scan the sky and obtain a TOD first².

To expand upon eq. (10.1) to show all complications that arise in real world observations,

$$m'_p = \mathcal{B}\{m_p\} \quad (26.1)$$

$$d(t) = P_{tp} \left(m_p'^I + \eta \left(m_p'^Q \cos(2\psi_t) + m_p'^U \sin(2\psi_t) \right) \right) + N_t \quad (26.2)$$

$$d'(t) = \mathcal{D}\{d(t)\} \approx (1 + g d(t)) d(t - \tau d(t)) \quad (26.3)$$

Here, m_p represents the physical CMB map on the sky. And eventually $d'(t)$ is the TOD recorded as raw data.

Eq. (26.1) represents the map that the telescope sees. This normally means it is a beam-convolved map, assuming the beam is symmetric for simplicity.

Eq. (26.2) represents how the telescope scans the sky and obtains a TOD. Note that the HWP angle ψ and the pointing P are involved, where both together constitute the scanning strategy which determines the observed signal. η represents the polarization efficiency of the detectors. And N is the noise of the detectors.

¹“A Measurement of the Degree-Scale CMB B-Mode Angular Power Spectrum with POLARBEAR”.

²Also see chapter 22.

Lastly, eq. (26.3) represents a non-linear effects from the detectors which makes the observed TOD a convolved version of the actual TOD. The approximation is characterized in Takakura et al.³, eq. 2.9.

In fact, these describe the TOD observed per detector, so all parameters involved can depend on individual detector.

While a scanning strategy determines P and ψ , the observing pointing P and HWP angle ψ are recorded. For the pointing, as outlined in Ade et al.⁴, a 5-parameter pointing model is used to reconstruct the pointing to reduce error from the actual pointing. In large-patch analysis, we followed a similar procedure to construct the pointing model using point sources.⁵

Eq. (26.1) means that we need to characterize the beam as well. The beam window function⁶ is measured using dedicated observations of Jupiter,⁷ an effective beam is used across all detectors although they differs slightly across wafers. This is good enough as the differences between the beams are much less than the statistical error-bar at this noise level.

Then, we need to characterize the detectors including the polarization efficiency and the effect of detector non-linearity. The methodology of the latter is studied in Takakura et al.⁸ Detector time constants are measured by using a thermal source on the telescope, and polarization efficiency together with their polarization angles are calibrated using Tau A observation, as indicated in The POLARBEAR Collaboration et al.⁹

After all these calibrations, the TOD are deconvolved¹⁰ as an inversion of eq. (26.3), the mapmaker is effectively inverting eq. (26.2), and at a later stage the power-spectra is de-biased by the beam profile to invert the effect of eq. (26.1).

26.1 Time domain filtering

One key difference between the low- ℓ and high- ℓ pipelines is illustrated below.

As the naïve mapmaker is used in our analysis, filtering the TOD is one of the techniques we can count on as we are not performing un-biased mapmaking. One

³“Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

⁴“A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”.

⁵The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

⁶This inherently assumes the beam is symmetric.

⁷The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

⁸“Performance of a Continuously Rotating Half-Wave Plate on the POLARBEAR Telescope”.

⁹“A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

¹⁰This only applies to the real data but not the simulations as we do not simulate the detector non-linearity effects.

of the benefits of splitting the large-patch analysis into low- ℓ and high- ℓ pipelines is that we can use different amounts of filtering to suppress different levels of unwanted systematics. In high- ℓ , we can afford heavier filtering from the low-frequency end, as we do not need the low- ℓ signals.

TOD filtering is an optimization problem. If too little, systematics is too large and null-tests will fail. If too much, we are eating into the signal and hurting the sensitivity.

One such example is the common-mode filter. Common mode means the mode common to all detectors. And it is estimated as the sample mean of those detectors at time t . It can be understood in terms of Principle Component Analysis (PCA) that the common mode approximately corresponds to the principle component among the detectors. The subtraction of the common mode helps reduce the amount of $1/f$ noise.

Note that common mode is constructed among a set of detectors. In low- ℓ , a single common mode is constructed over all detectors on the focal plane. As seen in fig. 26.1, a single wafer corresponds to a smaller angular scale on the sky, comparing to the full focal plane¹². Hence, the high- ℓ pipeline can afford more aggressive filtering by constructing a common mode per wafer.

Similarly, we have polynomial filtering per sub-scan¹³ of the telescope. A n -th order polynomial are fitted in the TOD within each sub-scan and is subtracted from the TOD. This attempts to remove the long time scale fluctuation from the $1/f$ noise, but also inevitably removes some of our large angular scale signal. Again, the high- ℓ pipeline can afford a larger degree polynomial without eating too much into our science range here. In this specific research, a 9-degree polynomial is fitted and subtracted from the TOD for all 3 components I, Q, U .

¹¹Kermish et al., “The POLARBEAR Experiment”.

¹²As a total of 7 wafers are on the focal plane, where smaller region on the focal plane corresponds to smaller field of view on the sky.

¹³A sub-scan corresponds to the duration where the telescope is constantly scanning in one direction. I.e. each turn-around point marks the next sub-scan.

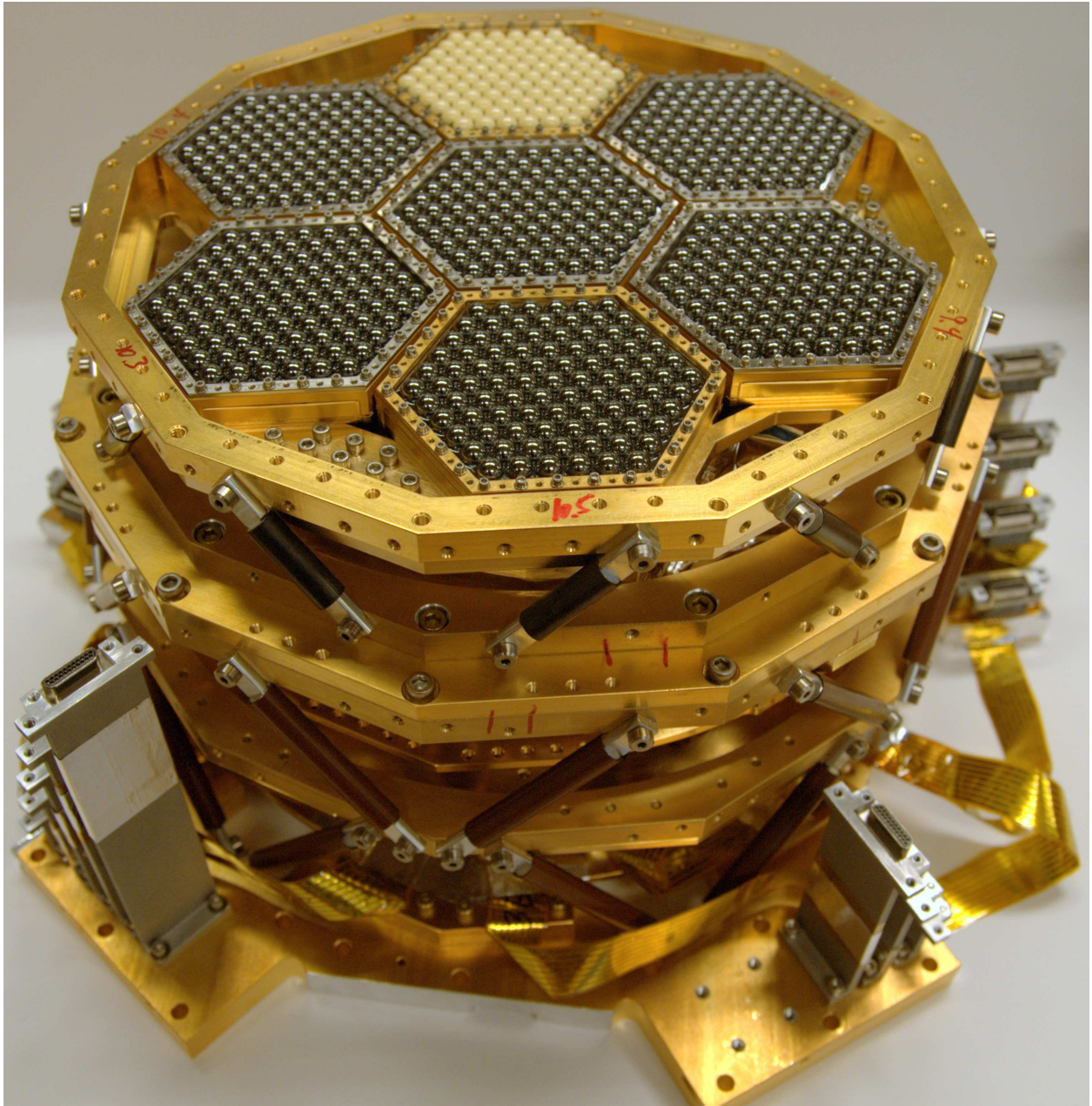


Figure 26.1: A photograph of the fully assembled focal plane, support structure and milliKelvin wiring.¹¹

Chapter 27

Analytical methods

We will go through the main analytical methods involved here. The general framework is the same with previous B-mode papers from POLARBEAR¹ The basic approach will be outlined here, and will provide more details whenever they differs.

27.1 Cross-pseudo-power-spectra calculation

We briefly outlined the cross-pseudo-power-spectra in sec. 21.3. Equation-wise, it is well-documented in previous POLARBEAR publications on B-mode. It is based on Hinshaw et al.²

We mentioned earlier 38 map-bundles were formed. Each of them amounts to about 10-days worth of observations, and each forms a complete map of the observed patch of sky.

This cross-map approach assumes that each map-bundle is an independent observation of the sky. So a cross-pseudo-power-spectra would be noise-free and bias-free. The validity of the assumption is from empirical knowledge that the correlations in the TOD has a timescale of around 20 min, and from empirical studies including the null-test suite. Hence, this estimator provides a signal-only estimation.

The noise pseudo-power-spectra can be estimated by first calculating the pseudo-power-spectra of the full map (that is coadded from all 38 map-bundles), often called the auto-power-spectra, and subtracting the signal-only estimate from previous step.

¹Ade et al., “A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”; The POLARBEAR Collaboration et al., “A Measurement of the Cosmic Microwave Background *B* -Mode Polarization Power Spectrum at Subdegree Scales from Two Years of Polarbear Data”; The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB *B*-Mode Angular Power Spectrum with POLARBEAR”.

²“First-Year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) Observations”.

27.2 Smoothing and tapering of the maps

As the estimate of E -mode and B -mode involves 2nd order derivatives³. Therefore we need to make sure the input maps are at least twice differentiable. Also, at the edge of the map, derivatives can be problematic. Hence, a tapering is usually done to ensure gradual transition to zero. Finally, in the calculation of pseudo-power-spectra, we would want the region with lower level of noise to be weighted more heavily to reduce resultant error-bar. Naturally, as we already have the weight map⁴, it is used for this purpose.

Putting all together, the weight map will be smoothed, tapered—the product of which is often called the weight-mask or simply mask—and will be multiplied to the map before performing pseudo-power-spectra estimation in the previous section.

In previous POLARBEAR B -mode analyses, hamming window with a scale of 0.53° is used to smooth the map, and a C^2 -tapering of 0.5° wide is performed at the edge. These weight maps are calculated per map within the map-bundles.

In this analysis, a gaussian blur of $3.301\,333\,811\,031\,487\,4''$ ⁵ is used to smooth the map, and a C^∞ -tapering with a scale of 1° applied at the edge.

27.3 Monte Carlo Apodized Spherical Transform Estimator (MASTER)

MASTER is “a fast and accurate method for estimation of the cosmic microwave background (CMB) anisotropy angular power spectrum”⁶.

This is the central concept that enables the naïve mapmaker that biases (filtering) is allowed but will be de-biased in the power-spectra domain. And the effect of the weight mask from the previous section will also be accounted for. Mathematically,

$$\tilde{C}_\ell = \sum_{\ell'} M_{\ell\ell'} F_{\ell'} B_{\ell'}^2 C_{\ell'}$$

Here, C is the original, un-biased (cross-)power-spectrum. B is the beam profile, F is the filter transfer function, and M is the mode-coupling matrices. $\tilde{C}$ is the pseudo-(cross-)power-spectra that we can measure in sec. 27.1.

The beam is measured in previous steps⁷ and is known at this stage. Mode-coupling matrices account for the fact that multiplying a weight-mask would change the resultant

³See sec. 38.6.

⁴To recall, the weight map is precision—the inverse of the variance at each pixel on the map.

⁵The number is chosen so that this gaussian window is minimally different from the hamming window above.

⁶Hivon et al., “MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

⁷See chapter 26.

power-spectra and is the topic later in sec. 27.4 and can be calculated analytically given the weight-mask.

Had we known the filter transfer function F , we could have used that to invert the formula and would have obtained an unbiased estimate of C .

The key idea in using this equation is that we can perform Monte Carlo (MC) simulations to estimate F . Specifically, we can run MC simulations that replicating the actual experiment including any filtering effects, resulting in

$$\tilde{C}_\ell^{\text{MC}} = \sum_{\ell'} M_{\ell\ell'} F_{\ell'} B_{\ell'}^2 C_{\ell'}^{\text{MC}} \quad (27.1)$$

Here, C^{MC} is a known input, $\tilde{C}^{\text{MC}}$ can be calculated as outlined in sec. 27.1. Hence we can solve this equation and obtain an estimate of F .

In fact, since there are $E \rightarrow B$ leakage and vice versa, we can perform different MC simulations with one set of maps having only T, E signals, and another set having only T, B . This allows us to estimate $F^{EE \rightarrow BB}, F^{BB \rightarrow EE}$ in addition to $F^{TT \rightarrow TT}, F^{EE \rightarrow EE}, F^{BB \rightarrow BB}$.

Now, the devil is in the details. How do we invert eq. (27.1)? Note that the mode-coupling matrices has very high condition number unless we have close to full-sky coverage⁸. Inverting it numerically is technically possible but the resultant F would have huge numerical error making it not usable.

Hivon et al.⁹ has mentioned an iterative approach to estimate F . This approach inherently depends on the number of iteration parameter n_{iter} . Note that when $n_{\text{iter}} \rightarrow \infty$ it should recover the F with high numerical error, as the iterative approach is solving the inverse matrix problem iteratively. n_{iter} has to be chosen carefully. Hivon et al.¹⁰ chose $n_{\text{iter}} = 1$ for BOOMERANG, and Ade et al.¹¹ chose $n_{\text{iter}} = 10$ for POLARBEAR's small-patch experiment.

Below we will describe another method that has no such arbitrary parameter for the numerical solution.

Modified MASTER

The key idea is to move the term F to the left,

$$\tilde{C}'_\ell = \sum_{\ell'} F'_\ell M_{\ell\ell'} B_{\ell'}^2 C_{\ell'}. \quad (27.2)$$

The prescription here is clearly wrong. The filter transfer function captures the effect of the filtering performed on the TOD. The mode-coupling effect happened at

⁸This can be understood by observing in the limit of full-sky coverage, $M \rightarrow I$ the identity matrix.

⁹"MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum".

¹⁰"MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum".

¹¹"A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR".

a later stage when calculating the pseudo-(cross-)power-spectra. So only the order prescribed in MASTER is correct.

Analytically, if M is diagonal or F is a constant, the modified MASTER and the original MASTER are the same. We know that M is close to diagonal (in the sense that main diagonal is dominant in magnitude) and F is slowly varying across ℓ .

Empirically, it should be mentioned that the estimate of F is never exact here, because it is estimated using MC simulations, and because it is numerically unstable to construct F exactly.

So phenomenologically, as long as the error made is small, eq. (27.2) should be ok. Note that if we define

$$\tilde{C}_\ell'' = \sum_{\ell'} M_{\ell\ell'} F_{\ell'}' B_{\ell'}^2 C_{\ell'} \quad (27.3)$$

Then

$$\tilde{C}_\ell' - \tilde{C}_\ell'' = \sum_{\ell'} (F_\ell' M_{\ell\ell'} - M_{\ell\ell'} F_{\ell'}') B_{\ell'}^2 C_{\ell'}$$

So the difference is roughly speaking a commutator between M, F .

To find out the error made when using the modified MASTER, we can use MC simulations eq. (27.1). Note that in this equation, both $\tilde{C}$ and C are known. We can then use eq. (27.2) to estimate F' , plug that in eq. (27.3), and calculate the quantity

$$\frac{\tilde{C}_\ell'' - \tilde{C}_\ell}{\tilde{C}_\ell}.$$

This represents the relative error made in terms of the resultant pseudo-power-spectra had we knew the true F but used the modified MASTER to estimate F' instead.

Estimating this relative error in the pseudo-power-spectra cases of TT, EE, BB , which is shown in fig. 27.1, fig. 27.2, fig. 27.3 using 128 MC simulations, we see that the relative error is small (RMS at $\sim 0.5\%$ across $600 \leq \ell \leq 3000$) and unbiased. As the bin-width used in later step is 100, the error would be much smaller.

Not only that, empirically, I find that this relative error is roughly inversely proportional to F' . Since $0 < F \leq 1$, empirically as long as F' is not close to zero¹², the approximation performed by the modified MASTER should be small.

Note that the relative error mentioned here is comparing between modified MASTER and the original MASTER, in my pipeline I consistently always use the modified MASTER throughout the pipelines so the actual error made should be even smaller.

To conclude, we find that the modified MASTER is parameter-free requiring no tuning, the error made is negligible, and is good enough phenomenologically in this particular analysis.

¹²This is a region that the de-biased power-spectra would have close to infinite error which is not usable anyway.

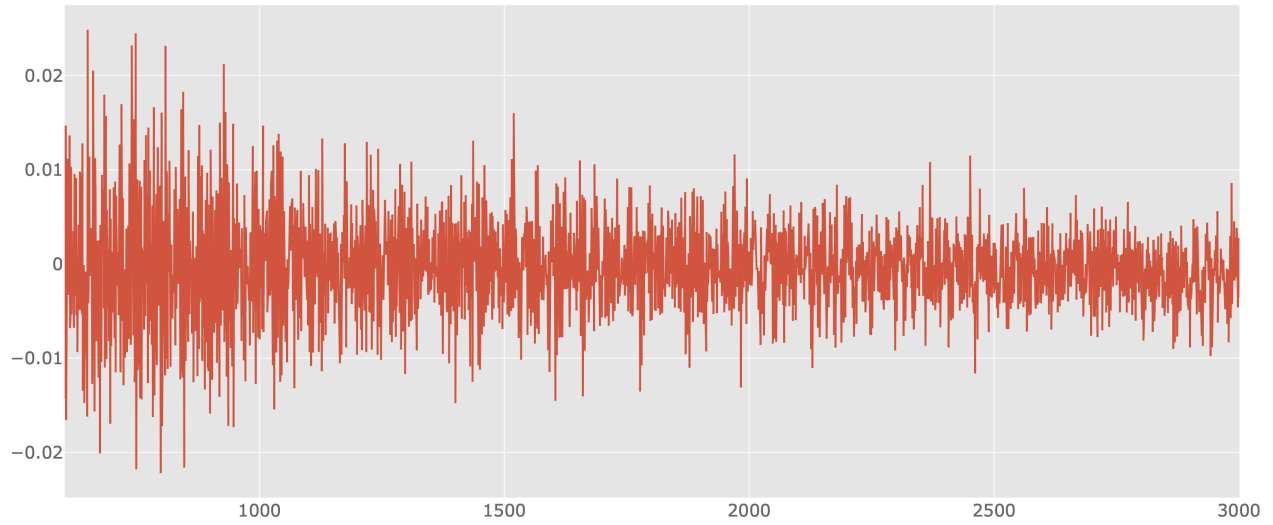


Figure 27.1: Relative error made in TT pseudo-power-spectra using modified MASTER

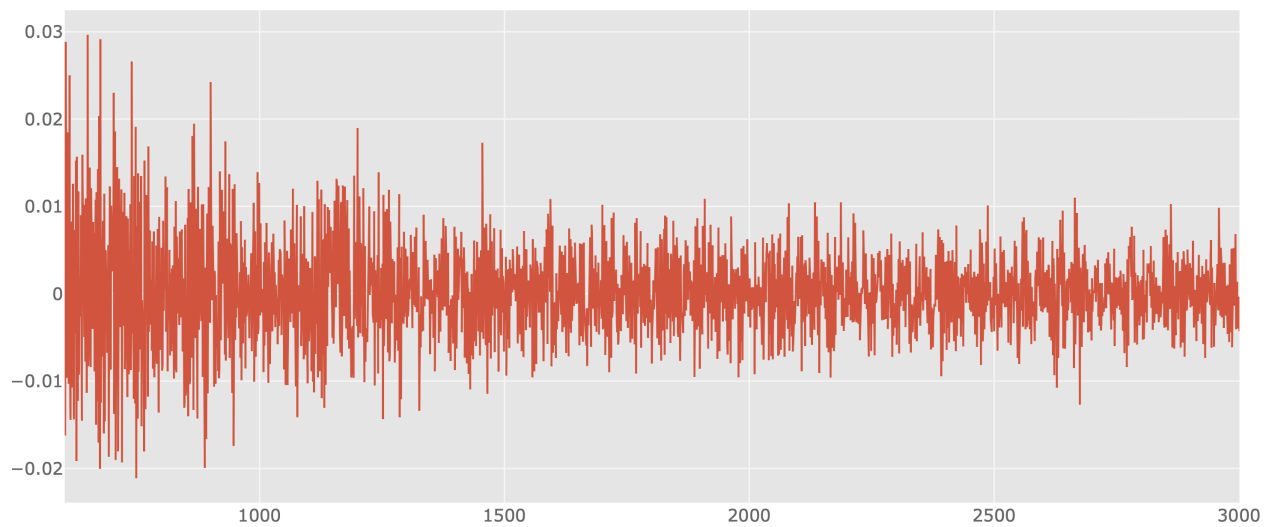


Figure 27.2: Relative error made in EE pseudo-power-spectra using modified MASTER

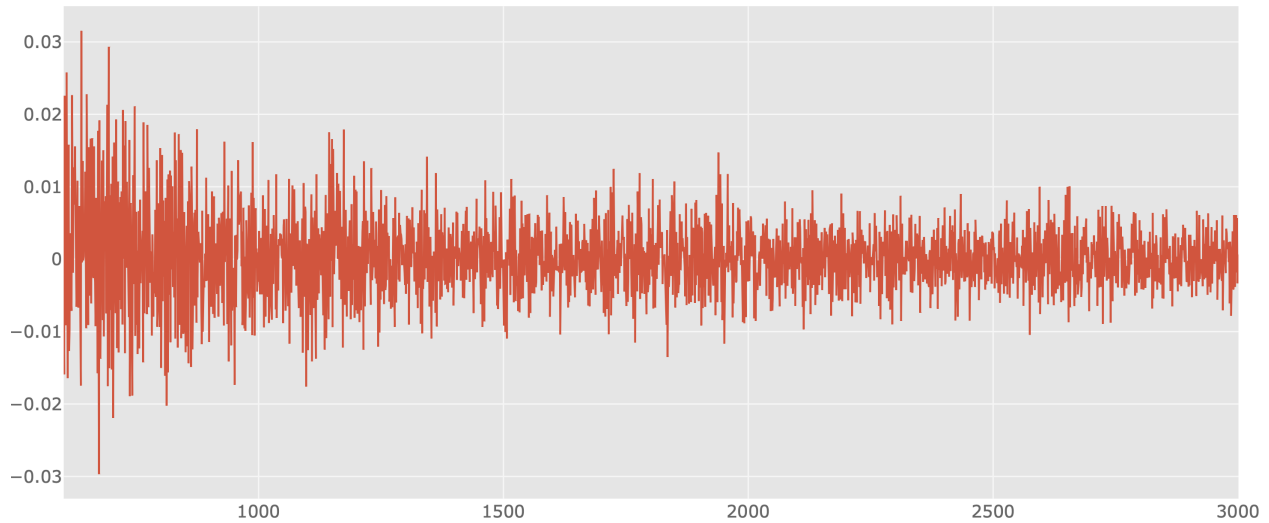


Figure 27.3: Relative error made in BB pseudo-power-spectra using modified MASTER

Finally, it should be noted that this modified MASTER may be used elsewhere. It may have been mentioned in casual conversations with other CMB scientists. The justification here may be new in the sense that it is shown here explicitly why we can do this.

27.4 Mode-coupling calculations

We will briefly go through how the mode-coupling matrices are estimated.

First, the mode-coupling matrix is often written as M^{XYXY} , or $M^{XY \rightarrow XY}$, representing the mode-coupling matrix from the XY power-spectra to the XY pseudo-power-spectra.

XY can be any cross-spectra TT, EE, BB, TE, TB, EB .

For the TT case, Hivon et al.¹³ has derived it in the appendix. For the rest which includes polarizations, Miller¹⁴ documented them in an internally circulated document, and implemented it in AnalysisBackend. In these equations, the B-mode is assumed to be constructed by the pure B-mode estimator.¹⁵

Later, I found some problems in AnalysisBackend regarding the mode-coupling matrix calculations.

First, there is a bug that in the case of EB , the pure B-mode estimator is not used. Second, there are various numerical problem in the calculation that the matrices deviate from actual values when it is far away from the main diagonal.

¹³“MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

¹⁴“Mode Mixing Calculation on the Flat Sky”.

¹⁵Smith, “Pseudo- C_ℓ Estimators Which Do Not Mix E and B Modes”.

The mode-coupling matrix code is completely rewritten to fix this, in addition to making it running much faster as explained in chapter 25. [This module is released in TAIL.](#)

There is another subtlety related to the calculation of mode-coupling matrices. In the previous pipelines of POLARBEAR *B*-mode analysis, each map-bundle uses their own weight-mask, which implies that the mode-coupling matrix calculated here are not correct (as the analytical formula assumes all share the same weight-mask.) Phenomenologically, using MC simulations, previous pipelines can recover unbiased power-spectra empirically. Alternatively, we can use a single weight-mask (constructed using the fully coadded map.) The disadvantage of this approach is that the weight-mask applied is less than optimal, and therefore potentially reducing sensitivity in the end. Both approaches have been compared in this study, and no major differences between them are found. In the end, the latter approach is taken as it is better safe (less than optimal) than sorry (wrong mode-coupling matrices applied.)

27.5 Zero padding of the map for the calculation of the power-spectra

In the calculation of the pseudo-(cross-)power-spectra in sec. 27.1, because an FFT is needed to first transform the map into frequency domain, zero-padding is required as FFT assumes periodicity.

However, how much zero-padding is needed? If we simply estimate from eq. (23.1), as the lowest ℓ here is 600, it corresponds to around $18'$ on both end. This amounts to only 9 pixels wide in $2'$ resolution.

However, as Kuo et al.¹⁶ said,

For a strictly stationary process this is not a problem since the correlation usually vanishes after a sufficiently long time lag. For piecewise stationary processes, a straightforward application of FFT produces false correlation for lag $> n/2$, where n is the number of elements. This can be prevented by zero padding.

So a $2x$ -zero-padding is recommended when calculating the pseudo-power-spectra.

The effects of zero-padding between $1.3x$ and $2x$ are compared in this study and found negligible differences from simulations. However, as a caution, we still follow the standard practice of having $2x$ zero-padding in the final analysis.

¹⁶[“High-Resolution Observations of the Cosmic Microwave Background Power Spectrum with ACBAR”.](#)

27.6 Noise

Noise estimate is used in 2 places, one in the coadd pipeline where the noise is used to determine the weight, and another when noise is injected in the simulations to mimic the real observations where the statistical fluctuations in the resultant power-spectra are used to estimate the statistical error-bar of the power-spectra.

There are 2 noise pipelines. In the first one, we assume an uncorrelated white noise model, and estimate them in the TOD. This is always the noise model used to construct the weight-maps. Optionally, this white noise model is also used in the simulation to simulate noisy observations.

Another noise pipeline is called sign-flipped noise. Assuming the maps containing signal and noise terms only, if we randomly assign a sign in the coadd process to the maps, then the coadded map should on average contains no signal and only has noise.

These 2 noise pipelines are compared, and they are found to be in general agreements in N^{EE}, N^{BB} , as the assumption of uncorrelated white noise is very good there because of the CRHWP modulation away from the $1/f$ noise¹⁷. This is not true for the temperature maps and its noise power-spectra N^{TT} simply because of the $1/f$ noise.

As explained before, less than optimal construction of the weight-maps only results in less than optimal (bigger error-bar), but usable resultant maps.

To obtain realistic error-bar in simulations including the intensity map, we opt for the sign-flip noise. With this realistic estimate of statistical errors, we shall soon see we can pass the null-test suite even if we include the temperature-related power-spectra (including TT, TE, TB) for the first time in POLARBEAR. These power-spectra are used for absolute gain calibration later.

¹⁷Ade et al., “A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR” further compared between correlated and uncorrelated white noise models and found them to be consistent.

Chapter 28

Statistical methods

28.1 Absolute Gain Calibration

Keating, Shimon, and Yadav¹ shows that we can use the CMB itself to perform absolute gain and angle calibration. Absolute gain refers to an overall factor multiplying the power-spectra, absolute angle refers to an overall angle rotating the linear polarization of the CMB. In a later section, we will show how they are estimated and corrected for in this analysis.

Cosmic variance limit

Let us first start by investigating a toy model where there is no noise. This is often coined as the cosmic variance limit, as it is an intrinsic uncertainty from the universe even if we can observe perfectly.

We have already seen the Knox formula eq. (23.2), in the limit of no noise,

$$\Delta C_\ell \approx \sqrt{\frac{2}{\nu_\ell}} C_\ell.$$

Defining absolute gain α ² as

$$C_b^{\text{obs.}} = \alpha C_b^{\text{Th.}}$$

$$\alpha_b = \frac{C_b^{\text{obs.}}}{C_b^{\text{Th.}}}$$

¹“SELF-CALIBRATION OF COSMIC MICROWAVE BACKGROUND POLARIZATION EXPERIMENTS”.

²Note that if we define absolute gain β in the TOD and hence the map, $\alpha \equiv \beta^2$.

$$\Delta \alpha_b \approx \sqrt{\frac{2}{\nu_b}}$$

Now, we assume a naïve OLS procedure assuming they are uncorrelated³ random variables as a function of b , the corresponding standard error will then be

$$\begin{aligned} \Delta &= \left(\sum_b \frac{1}{(\Delta \alpha_b)^2} \right)^{-\frac{1}{2}} \\ &= \left(\sum_b \frac{\nu_b}{2} \right)^{-\frac{1}{2}} \\ &= \left(\sum_b \frac{\sum_{l \in b} (2l+1) f_{\text{sky, eff.}}}{2} \right)^{-\frac{1}{2}} \\ &= \left(\sum_{l=l_{\min}}^{l_{\max}} (2l+1) \frac{f_{\text{sky, eff.}}}{2} \right)^{-\frac{1}{2}} \\ &= \left(\frac{(l_{\max} + 1 - l_{\min})(l_{\max} + 1 + l_{\min})}{2} f_{\text{sky, eff.}} \right)^{-\frac{1}{2}} \end{aligned}$$

Note that this is independent to the actual binning b used.

Using our actual data, $f_{\text{sky, eff.}} \approx 0.0159$.

With $l_{\min} = 600, l_{\max} + 1 = 3000, \Delta \approx 0.00381$.

Method 1

Using C_l^{TT} , the error of the absolute gain calibration α is Δ .

Method 2

Using C_l^{EE} , the α estimated above is the product of the actual absolute gain times the polarization efficiency squared. i.e.

$$\begin{aligned} \hat{\alpha} &= \frac{\alpha^E}{\eta^2} \\ \left(\frac{\Delta \alpha}{\alpha} \right)^2 &= \left(\frac{\Delta}{\alpha^E} \right)^2 + \left(2 \frac{\Delta \eta}{\eta} \right)^2 \end{aligned}$$

³It will not be exactly uncorrelated in our case since we have far from a full sky map.

Assuming $\alpha \sim 1$, $\frac{\Delta}{\alpha E} \sim \Delta \approx 0.00381$.

From internally circulated data, $\frac{\Delta\eta}{\eta} \approx 0.0151$.

Method 3

Inverse variance weighted between method 1 & 2.

Since $2\frac{\Delta\eta}{\eta} \gg \frac{\Delta}{\alpha}$, the error bar obtained here is virtually identical to method 1.

Conclusion

In the cosmic variance limit, using the temperature data to estimate the absolute gain is much more accurate as the uncertainty in the polarization efficiency dominates the uncertainty in the case of polarization⁴.

This toy model illustrates why we would want to include the temperature power-spectrum for absolute gain calibration. With it, another bonus is that we can cross-check if the polarization efficiency is correct in the sense that the “absolute polarization efficiency” should be consistent with 1. Later we will see how this compares with the absolute gain inferred using the actual data.

28.2 POLARBEAR null-test framework in equations

In chapter 20, we already have an overview of the POLARBEAR null-test framework.

We see null-spectra are defined there. During the early phase of running null-tests, we can intuitively see what null-tests are failing where there are huge outliers. But we can do better to decide this objectively.

It involves constructing null-estimators from null-power-spectra. Exactly how this statistics is constructed is somewhat arbitrary. Ade et al.⁵ described the 5 different statistics used in the small-patch analysis over 3 different sky region resulting in a total of 15 statistics. The definitions of the 5 statistics are somewhat vague, and the way the 15 statistics is combined is not specified.

There is also an internally circulated document Chinone⁶ documenting this null-test framework in more details.

Below we will mathematically define these quantities in this analysis.

⁴Note that this does not reduce statistical uncertainty in determining the absolute scale of the CMB polarizations *per se*. This is because there are other contributions to this absolute scale, such as the “absolute polarization efficiency”. If it can be shown that other kinds of systematics in determining this absolute scale are well controlled, then the benefits of getting a well-categorized absolute gain can result in meaningful reduction of statistical error in the CMB polarization measurements.

⁵“A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”.

⁶“Polarbear Null-Test Framework”.

Definition 1. Null-power-spectra.

Define random variable $\chi(X, n, b) \equiv \frac{\hat{C}_b^{Xn, \text{null}}}{\hat{\sigma}_b^{Xn}}$ as a function of

X : power-spectrum. This includes TT, TE, TB, EE, EB, BB .

n : null-split, such as AMP_4F_BY_BOL0, ...

b : an ℓ -bin, such as $600 \leq \ell < 700$.

Note that by construction, χ is a standard score, assuming the theoretical expected value of 0, and estimated using the sample standard deviation via MC simulations. Asymptotically in n_{sim} and ν , $\chi \xrightarrow{d} N(0, 1)$.

Definition 2. Null-statistics.

Per spectrum, we collapse the null-power-spectra into the following 5 different ways,

$$Y_1(X) = |\langle \chi(X, n, b) \rangle_{n,b}|$$

$$Y_2(X) = \max_{n,b} \chi^2(X, n, b)$$

$$Y_3(X) = \max_n \sum_b \chi^2(X, n, b)$$

$$Y_4(X) = \max_b \sum_n \chi^2(X, n, b)$$

$$Y_5(X) = \sum_{n,b} \chi^2(X, n, b)$$

Consider sample space Ω .

$$Y_i : \Omega \rightarrow \mathbb{R}$$

$$Y_i : s \mapsto y_i$$

$$G_{Y_i}(y_i) \equiv P(Y_i > y_i)$$

Since $G \equiv 1 - F$, $G_{Y_i} \rightarrow U(0, 1)$

Note that F is the cumulative distribution function (CDF) in statistics, while G is one minus that, often called probability-to-exceed (PTE).

In this analysis, the 5 statistics $G_{Y_i}(y_i)$ is considered, by estimating distribution of Y_i empirically through Monte Carlo simulation, and y_i from measured data. Additionally, 3 spectra are considered in unblinding criteria: BB, EB, EE . i.e. there are a total of 15 p-values.

Theorem 1. *Direct computation of null-statistics assuming $\chi(X, n, b) \xrightarrow[i.i.d.]{d} N(0, 1)$.*

$$\begin{aligned}
G_{Y_1}(y_1) &= \text{erfc}\left(\sqrt{\frac{n_{split}n_b}{2}}|y_1|\right) \\
G_{Y_2}(y_2) &= \frac{n_{split}n_b}{2^{\frac{1}{2}}\Gamma(\frac{1}{2})} \int_{y_2}^{\infty} \left(P\left(\frac{1}{2}, \frac{u}{2}\right)\right)^{n_{split}n_b-1} e^{-\frac{u}{2}} u^{\frac{1}{2}-1} du \\
G_{Y_3}(y_3) &= \frac{n_{split}}{2^{\frac{n_b}{2}}\Gamma(\frac{n_b}{2})} \int_{y_3}^{\infty} \left(P\left(\frac{n_b}{2}, \frac{u}{2}\right)\right)^{n_{split}-1} e^{-\frac{u}{2}} u^{\frac{n_b}{2}-1} du \\
G_{Y_4}(y_4) &= \frac{n_b}{2^{\frac{n_{split}}{2}}\Gamma(\frac{n_{split}}{2})} \int_{y_4}^{\infty} \left(P\left(\frac{n_{split}}{2}, \frac{u}{2}\right)\right)^{n_b-1} e^{-\frac{u}{2}} u^{\frac{n_{split}}{2}-1} du \\
G_{Y_5}(y_5) &= Q\left(\frac{n_{split}n_b}{2}, \frac{y_5}{2}\right)
\end{aligned}$$

P and Q are the regularized lower incomplete gamma function and the regularized upper incomplete gamma function respectively[See @technology_us_nist_2010, eq. 8.2.4.]. n_{split} is the total number of null-splits, n_b is the total number of b-bins.

It is trivial to prove these formula when we realizing max is a special case of ordered statistics where general formula is known. Proof is left as an exercise to the readers.

With these formula, we do not need to estimate the PTE via MC simulations. Hence, it is much faster to evaluate and can act as an independent cross-check by comparing both approaches. We expect them to be similar, but note that the later approach makes a qualitative assumption. Hence, the PTEs from MC simulations are the one used in the unblinding criteria.

Note that all special functions are available in Scipy⁷, and the integral can be evaluated numerically. A Python implementation of these functions is available as `p_1`, ..., `p_5` in the `stats` module in `TAIL`.

Table 28.1: PTE for the null-statistics defined in definition 2 estimated using MC simulations.

power-spectra	G_{Y_1}	G_{Y_2}	G_{Y_3}	G_{Y_4}	G_{Y_5}
BB	82.8 %	40.2 %	52.1 %	46.9 %	14.5 %
EB	79.7 %	31.2 %	35.9 %	55.9 %	7.6 %

⁷“SciPy is a collection of mathematical algorithms and convenience functions built on the NumPy extension of Python.”

power-spectra	G_{Y_1}	G_{Y_2}	G_{Y_3}	G_{Y_4}	G_{Y_5}
EE	36.5 %	97.7 %	90.4 %	99.4 %	83.4 %
TB	8.6 %	59.6 %	98.8 %	96.1 %	89.1 %
TE	17.0 %	84.2 %	59.2 %	86.1 %	85.9 %
TT	17.2 %	1.6 %	9.0 %	3.5 %	6.6 %

Table 28.2: PTE for the null-statistics defined in definition 2 estimated using theorem 1 assuming $\chi(X, n, b) \xrightarrow[i.i.d.]{d} N(0, 1)$.

power-spectra	G_{Y_1}	G_{Y_2}	G_{Y_3}	G_{Y_4}	G_{Y_5}
BB	79.5 %	43.9 %	53.1 %	40.4 %	13.1 %
EB	76.2 %	32.8 %	36.7 %	53.1 %	5.8 %
EE	26.9 %	97.4 %	90.2 %	99.0 %	85.2 %
TB	4.8 %	61.6 %	98.2 %	96.3 %	90.6 %
TE	11.2 %	86.1 %	58.8 %	83.8 %	85.6 %
TT	2.3 %	0.0 %	0.0 %	0.1 %	1.2 %

As an example, here we show the comparison between the two estimates in tbl. 28.1 and tbl. 28.2 using null-test v1.0.0.

The general agreements between the two is a cross-check that they are behaving as we expected. The disagreements from the null-statistics generated using the TT power-spectrum is an evidence that the assumption is not valid there. Fig. 28.4 further confirms the i.i.d. assumption is invalid. As explained in sec. 28.2, tbl. 28.1 is used in the unblinding criteria.

Kolmogorov–Smirnov test

From real map, we obtained $\chi^{\text{real}}(X, n, b)$. From n simulated maps, we obtained $\chi^{\text{sim},i}(X, n, b)$. A Kolmogorov–Smirnov test between $\chi^{\text{real}}(X, n, b)$ and $\chi^{\text{sim},i}(X, n, b)$ is performed. This is often to ensure the simulation is statistically similar to the real observation such that the passing of the null-test would not be too optimistic⁸.

Correlations between null-splits

Fig. 28.1 tells us the empirical correlations between null-splits using their corresponding null-spectra. Fig. 28.2 shows the square of the correlation between the boolean

⁸In other words, the PTE is a $U[0, 1]$ random variable.

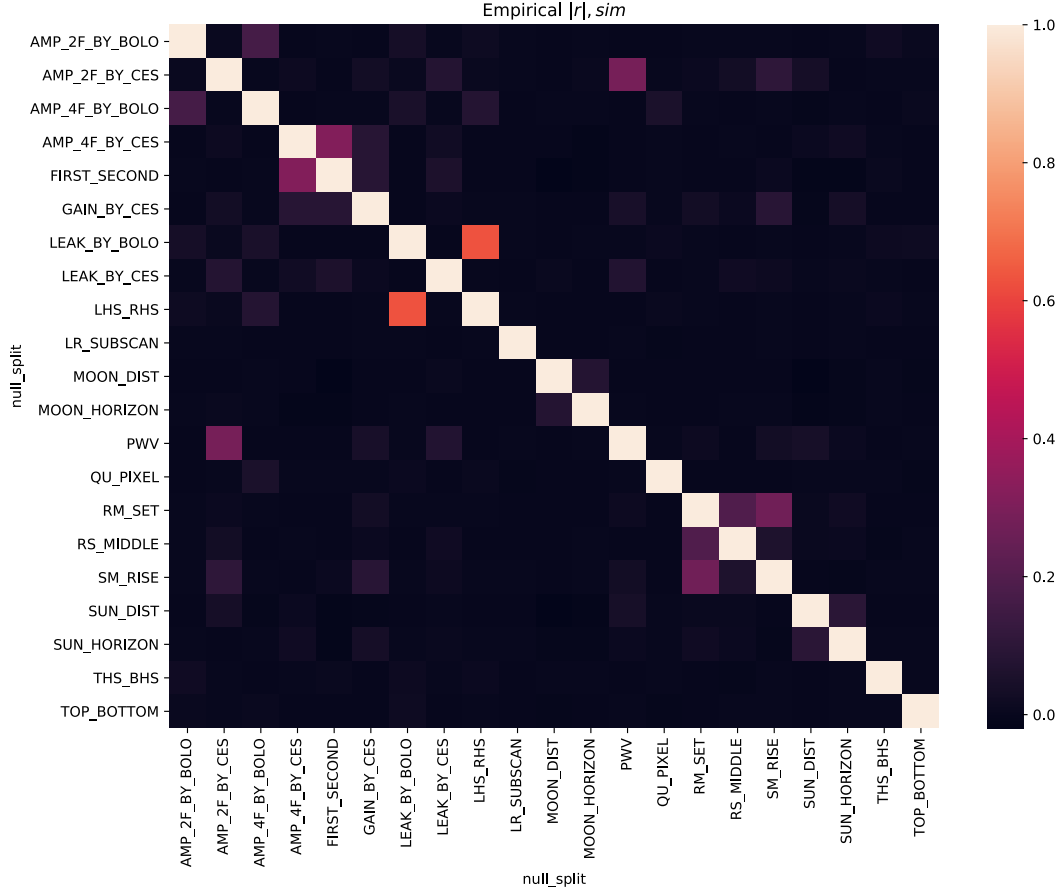


Figure 28.1: Empirical correlation between null-splits estimated using v1.0.0 simulation

flags of the null-split definition⁹. Under ideal condition that the TOD has identical noise properties, we can predict that $r_{\text{empirical}} \approx r_{\text{boolean}}^2$. Fig. 28.3 shows that this is also empirically true.

As you can see, since we are splitting the same dataset differently, they are correlated.

Similarly, from fig. 28.4 we see that the bin-bin correlations are negligible in all but the TT power-spectrum. Note that the HWP $2f$ frequency corresponds to $\ell = 1800$.

And if 2 null-splits are highly correlated, they are somewhat redundant with each other. Since we are performing a kind of sum over them in null-statistics, a natural

⁹Consider the boolean flags as a time-series of boolean values where each indicates if a sub-scan is chosen to be in that null-split or not.

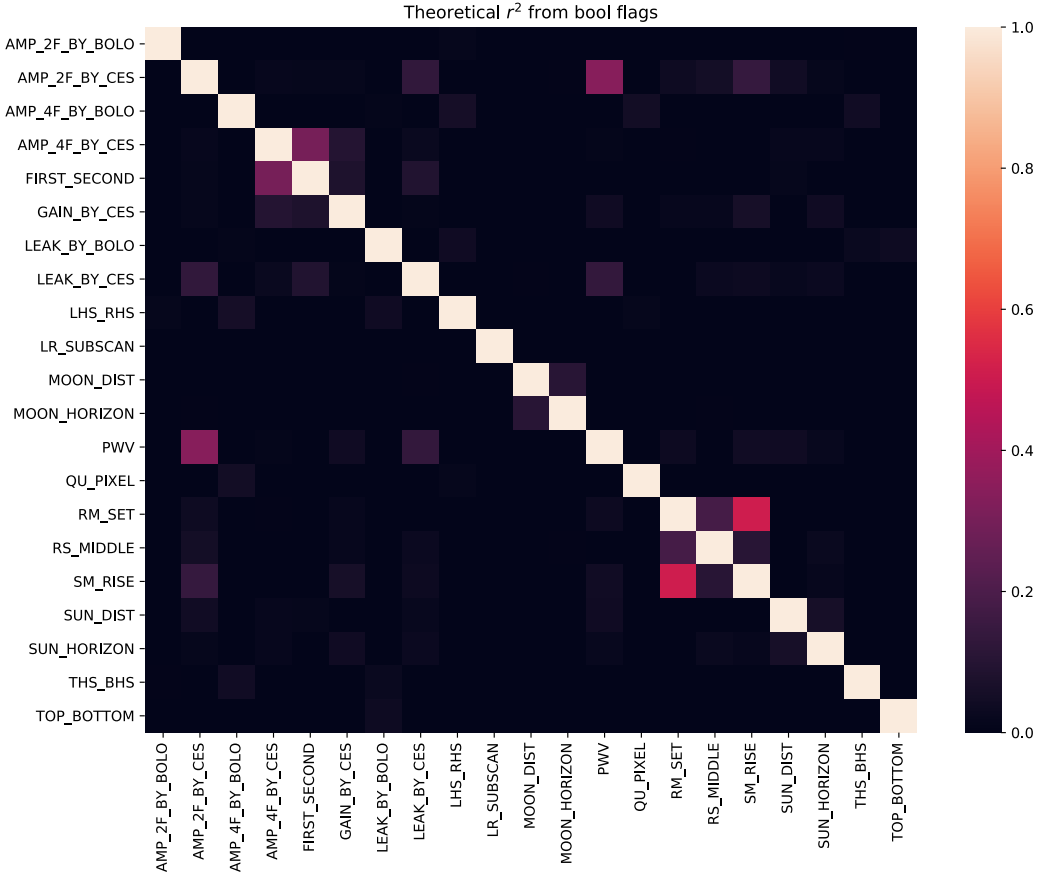


Figure 28.2: Predicted correlation between null-splits estimated using v1.0.0 simulation

question to ask is if given 2 of them has non-negligible correlations, should we choose only one of them? If we only choose one in the passing criteria, how can we be sure that is the correct choice, that is, the other one will equally pass the test?

Null-test selection and partial correlation

First, independence implies no correlations, but not vice versa.

As defined in definition 2, our null-statistics are complicated functions of the null random variable χ . It is a misconception among some physicists that somehow the Pearson correlation coefficient is more physical, and if we find it to be small between 2 null-splits, their resultant 5 null-statistics will also be fairly independent.

Moreover, even if we find 2 null-splits to be highly correlated, are they still that much correlated after the correlation with other null-splits is taken into accounts?

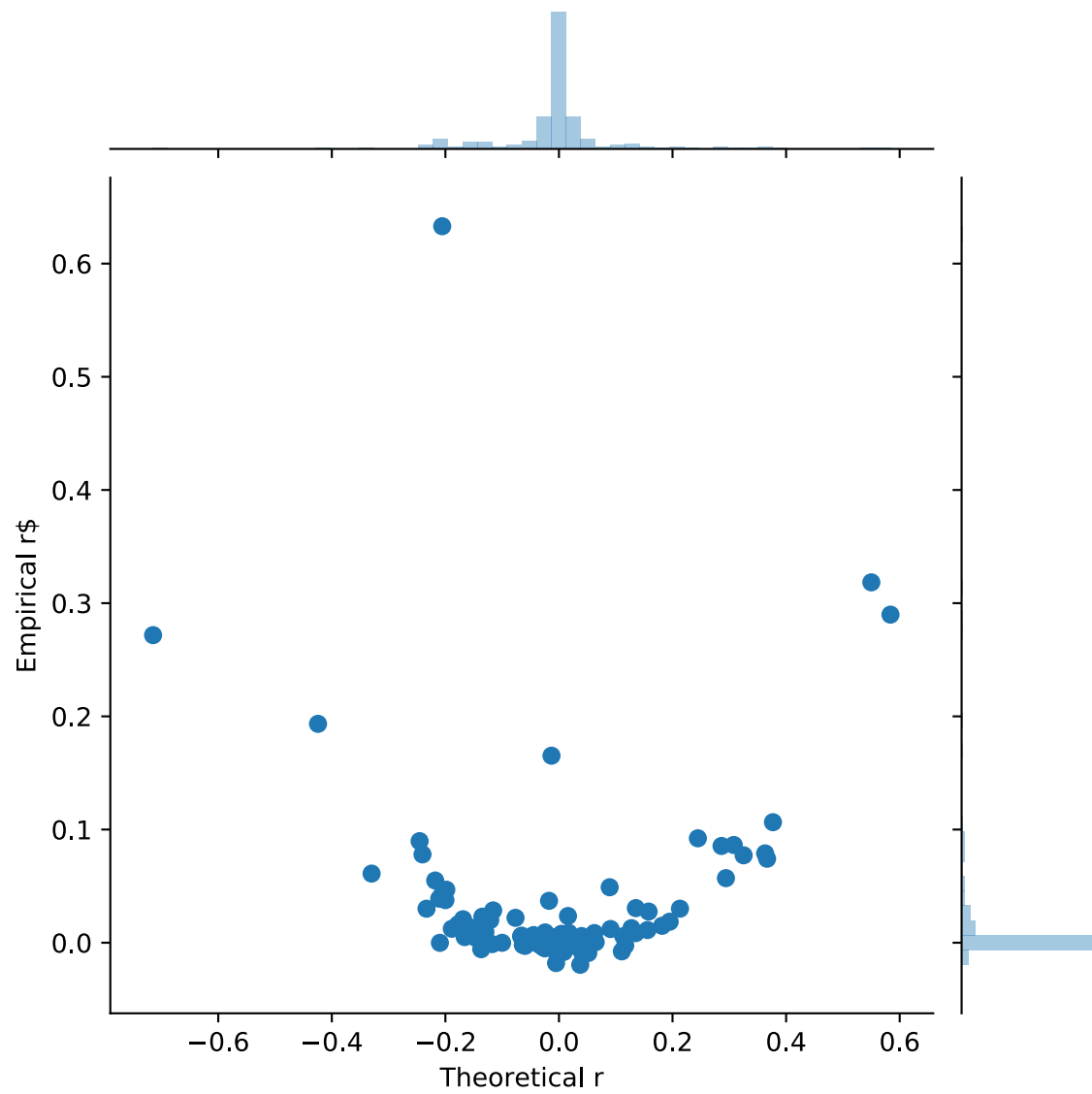


Figure 28.3: Comparison between empirical correlation and predicted correlation

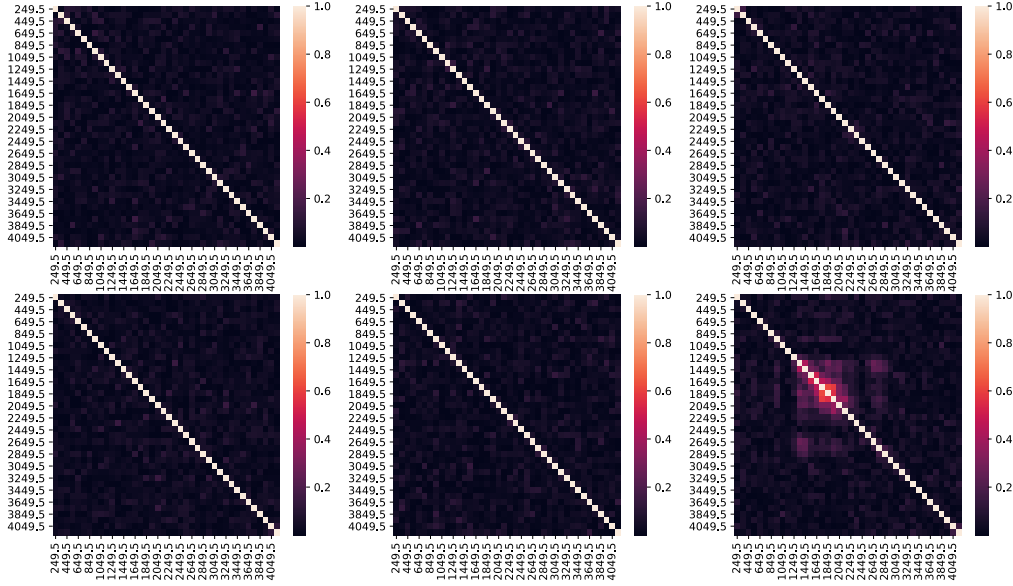


Figure 28.4: Empirical correlation between b -bins estimated using v1.0.0 simulation

A statistical way to quantify this is the partial correlation matrix, which can be written as

$$\rho_{\chi_i, \chi_j | \chi_{k \neq i, j}, \dots},$$

which is defined as the correlations between the residuals $r_{\chi_i | \chi_{k \neq i, j}, \dots}$ and $r_{\chi_j | \chi_{k \neq i, j}, \dots}$.

fig. 28.5; fig. 28.6 shows the difference between the correlation matrix. The takeaway message here is that 2 null-splits can be apparently high-correlated, but much of the correlations can be explained by the rest of the null-splits. Hence, if one of them are excluded in the final null-test suite, it actually helps less than expected in removing dependencies.

In the end, the author suggests not to exclude null-splits in passing criteria based on some apparent correlations between them. Often, a null-split is defined by some physical observables with clearly defined meaning that should increase our confidence in the lack of systematics if the null-test suite passes. However, some can be excluded if there are strong reasons behind them. Here for example, we see that in earlier null-test v0.2.0 shown in fig. 28.5, there were more null-splits including THIRD_SEPARATE, FOURTH_SEPARATE, FIFTH_SEPARATE, which is defined as the 3rd season of data vs. the rest, so on and so forth. Since FIRST_SECOND splits the data in the two halves chronologically,

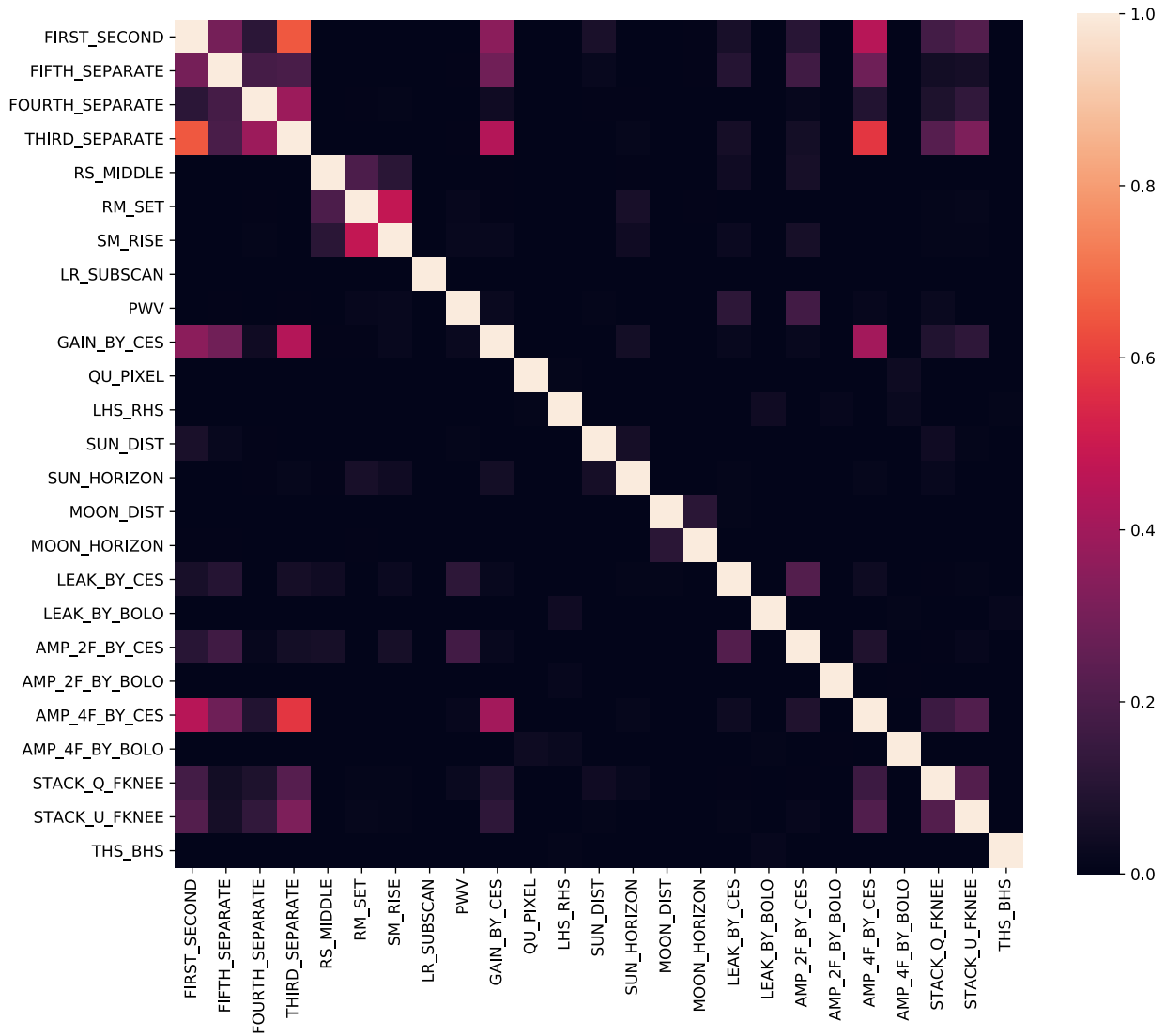


Figure 28.5: Square of correlation between null-splits flags estimated using v0.2.0 simulation

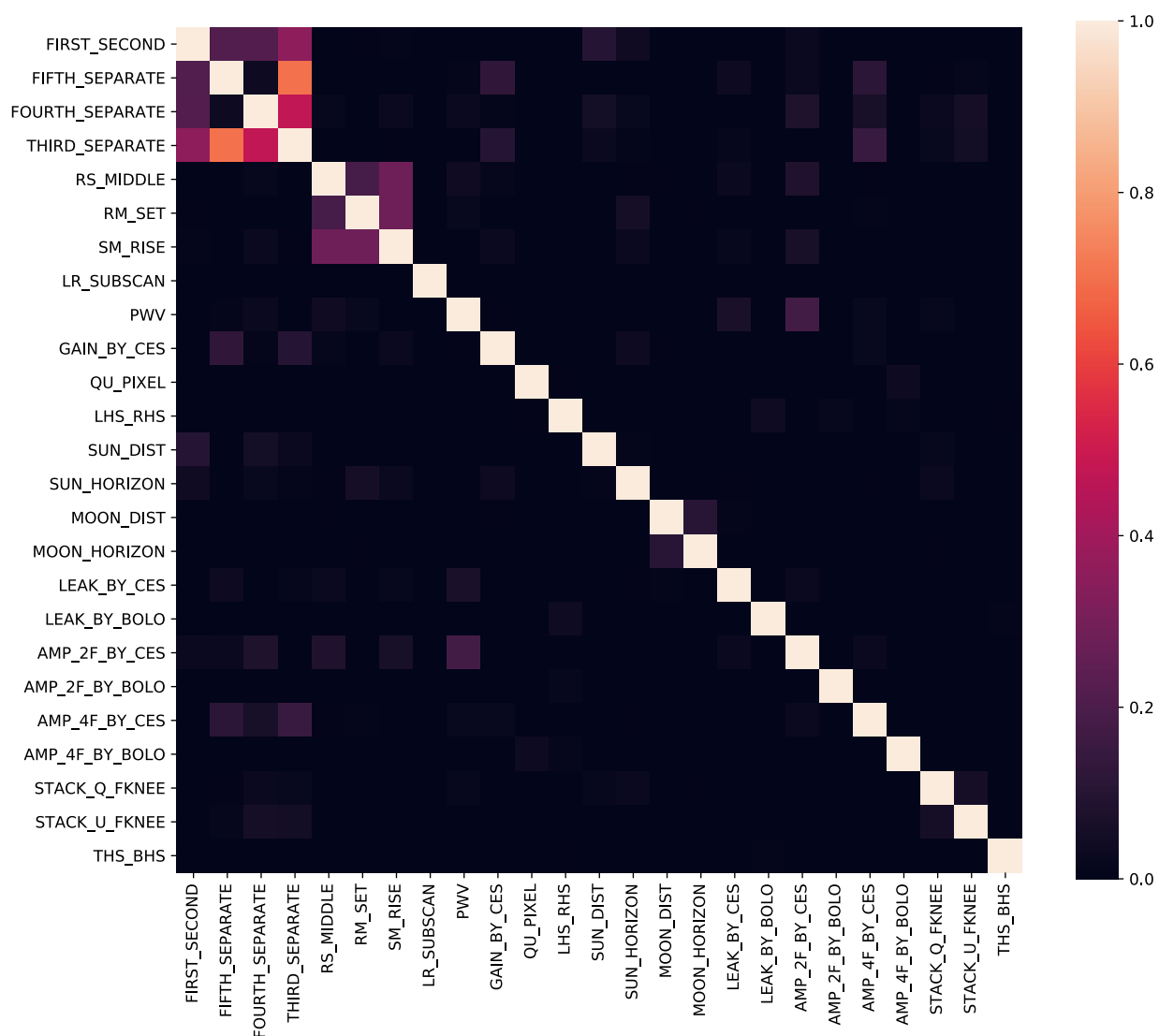


Figure 28.6: Square of partial correlation between null-splits flags estimated using v0.2.0 simulation. Pseudo-inverse is used for this estimation because of numerical stability.

these 4 null-splits are quite redundant with each other. Since splitting the data in a half in data volume maximize the sensitivity of the null-spectra, only FIRST_SECOND is kept in the final analysis.

Combined statistics—Family-wise Error Rate (FWER)

Essentially, we do not want to have false alarm of non-existent systematics. It means that in terms of null-hypothesis testing, we want to control the type I errors. And since we have more than one such statistics, we want to control the Family-Wise Error Rate (FWER).

There are many ways to control the FWER to be of α -level, where we chose this to be 5% as our passing criteria.

Given 15 numbers, one way of controlling the Family-wise error rate (FWER) is the most basic Bonferroni correction—assign $\frac{\alpha}{15}$ to each test. If none is below that level, it passes. But this can let us pass the null-test too easily. Statistically, this is called less *powerful*.

Combined statistics used in previous POLARBEAR *B*-mode publications

While the previous POLARBEAR papers have not written down how the statistics are combined, here is its definition.

Definition 3. Combined statistics used in previous POLARBEAR *B*-mode publications.

$$\mathcal{G}(s) \equiv \min_i (G_{Y_i}(Y_i(s))), s \in \Omega$$

$$\mathcal{F}(s) \equiv P(\mathcal{G} \leq \mathcal{G}(s))$$

$$\mathcal{F} \rightarrow U(0, 1)$$

$\mathcal{F}$ is the combined statistics which is proved to be distributed as $U(0, 1)$, consistent with what one would expect from an FWER.

Informally, it is defined as $\mathcal{G}$ which is the minimum of all p -values and ask how often this minimum (as a random variable) is smaller than the observed minimum.

Note that these functions are construct over the sample space directly to avoid confusion. They take 1 single realization (the real observation) and map it to a number (indirectly through χ, Y_i, G_i .)

Computationally, distribution of $\mathcal{G}$ is estimated empirically through Monte Carlo simulation and $\mathcal{G}(s)$ from measured data. Note that to obtain G empirically, one would need to compare a value from other values obtained using MC simulations—this value can be the real data when we are computing the statistics, or from an MC simulation when we are constructing the sample distribution of G .

Hence, in order to construct the empirical distribution of $\mathcal{G}$, the $n_{\text{sim}} = 512$ simulations are needed to be used recursively, say by comparing values from a simulation to the rest of 511 simulations, etc.

Note that then the empirical distribution of G is discretized, from $G = 0$ when the MC chosen is the maximum among all MC, to $G = \frac{511}{512}$ when the MC chosen is the minimum. I.e. it takes $\frac{0,1,\dots,511}{512}$, exactly 512 different discrete values.

Then when we construct $\mathcal{G}$, which is the minimum among $n_{\text{family}} = 15$ number of G -statistics, can only be at most $n_{\text{sim}} = 512$ different values. And in practice the number of different values would be much less by the mere fact that we are performing a minimum statistics here¹⁰.

This is the subtle mathematical reason why this is inferior, as we are losing resolution (in the sense of having fewer different discrete values) when comparing the statistics obtained from real data $\mathcal{G}(s)$ to this empirical distribution. In other words, a small change in $\mathcal{G}(s)$ can results in a large change in $\mathcal{F}(s)$ depending on the multiplicity in the empirical distribution. This makes the estimated PTE unreliable, or alternatively requires much more number of simulations to counter this effect.

Combined statistics using Holm–Bonferroni method

A better method is Holm–Bonferroni method. Mathematically, it is uniformly more powerful than the Bonferroni correction¹¹, and does not require any assumption on the statistical independence between the statistics that is often imposed by other more powerful methods.

Note that this already solve our problem about the correlations between null-splits in sec. 28.2. In short, we do not have to pre-select null-splits based on arbitrary criteria, we will choose all of them and let the mathematics takes care of the rest.

Comparing to Bonferroni method, instead of uniformly assigning $\frac{\alpha}{15}$ to each test, it assigns $\frac{\alpha}{15}$ to the smallest p-value, $\frac{2\alpha}{15}$ to the 2nd smallest, $\frac{k\alpha}{15}$ to the k-th smallest, α to the last (largest) p-value.

It can be proven, without any assumption on the correlation between the tests, that Holm–Bonferroni method guarantees $\text{FWER} \leq \alpha$.

Also, a critical α value, α_c , can be defined such that if the chosen $\alpha \leq \alpha_c$, then it passes. This single number α_c will be our combined statistics.

Moreover, Holm–Bonferroni method not only allows us to distill many statistics into one α_c , this procedure also tells us, given a critical α value, which of the n_{family} statistics are failing.

¹⁰Minimum statistics is a special case of ordered statistics, where the CDF can be computed analytically given the original CDF if they are independent. In this case the original CDF is from a uniform discrete distribution of $\frac{0,1,\dots,511}{512}$. It can be shown that it is skewed towards the left and more skewed as n_{family} increases.

¹¹Loosely speaking, it means it does not pass as easily.

For example, with the Y_3 statistics in ??, before we take the final maximum, it has the form $\sum_b \chi^2(X, n, b)$ depending on the null-split only. By applying the Holm–Bonferroni method on this $n_n = 21$ number of statistics, it can tell you exactly which null-splits are failing.

For example, since the beginning in null-test v0.1.0, these null-splits are failing by inspection,

- QU_PIXEL
- LEAK_BY_BOLO
- AMP_2F_BY_BOLO
- AMP_4F_BY_BOLO

Running my statistics pipeline with the Holm–Bonferroni method, these are exactly our “discoveries”.

This eliminates an error-prone procedure of manually looking into each null-spectra, as sometimes we may be seeing things that statistically is just a fluke, and it happens from time to time.

Finally, it should be mentioned that the culture in POLARBEAR prefers simple statistics that is simple to understand. In response to that, there is nothing simple in the previous section. In fact it is very complicated and abstract to define that quantity mathematically. Computationally, we also showed with non-simple explanations why subtly it is not well behaved. Instead, Holm–Bonferroni method is easy to describe as above, and only takes a few lines to prove that it controls the FWER. It is first constructed in 1979¹², which is quite old. Put it in other words, we are not doing cutting edge research in statistics if we adopt the Holm–Bonferroni method here, we are simply catching up in putting well-known good tool to use. Cosmology is a science based on statistics¹³, we should not be afraid to adopt better tools offered by the vast field of statistics.

28.3 Likelihood estimation

Simultaneously calibration and fitting lensing amplitude

Note that Einstein summation notation is implied in this section unless otherwise stated.

We can represent a rotated, scaled version of the true power-spectra by

¹²Holm, “A Simple Sequentially Rejective Multiple Test Procedure”.

¹³As we argued elsewhere we only have one observable universe that we cannot experiment on but to observe statistical properties from it.

$$\begin{aligned}
C_{b,kl}^{\text{scaled}} &= a_k a_l C_{b,kl}^{\text{CMB}}, \text{ no summation} \\
C_{b,ij}^{\text{observed}} &= R_i^k R_j^l C_{b,kl}^{\text{scaled}} \\
&= \sum_{kl} R_i^k R_j^l a_k a_l C_{b,kl}^{\text{primordial}} \\
&= T_{ij}^{kl} C_{b,kl}^{\text{primordial}} \\
T_{ij}^{kl} &\equiv R_i^k R_j^l a_k a_l, \text{ no summation}
\end{aligned}$$

Here, $i, j, k, l = T, E, B$, and

$$R_i^k(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{pmatrix}.$$

After expansion this agrees with Keating, Shimon, and Yadav¹⁴, eq. 5 if we choose $a_i = 1$. Written in this form it is trivial to perform the inverse transformation to transform the measured data instead.

Also, note that $a_T = g$ is the absolute gain calibration, $a_E = g\eta$ where η is the absolute polarization efficiency calibration, $a_B = a_E$.

We are going to apply T^{-1} to the measured spectra below. I.e.

$$\hat{C}_{b,ij}(a_T, a_E, \theta) \equiv (T^{-1})_{ij}^{kl} C_{b,kl}^{\text{observed}}$$

In the end, the $C_{b,BB}$ from the theory will have an additional A_{lens} term to fit for lensing amplitude. i.e.

$$C_{b,ij} = \begin{pmatrix} C_{b,TT}^{\text{th}} & C_{b,TE}^{\text{th}} & 0 \\ C_{b,TE}^{\text{th}} & C_{b,EE}^{\text{th}} & 0 \\ 0 & 0 & A_{\text{lens}} C_{b,BB}^{\text{th}} \end{pmatrix}$$

Likelihood procedure

1. The effective d.o.f. ν used below is calculated according to sec. 28.4.
2. Transform the measured (signal) spectra according to previous section to obtain $\hat{C}_{b,ij}(a_T, a_E, \theta)$.

¹⁴“SELF-CALIBRATION OF COSMIC MICROWAVE BACKGROUND POLARIZATION EXPERIMENTS”.

3. Noise spectra scaled with a_i as we are using sign-flip noise which is from the measured data.
4. BB leakage template is constructed using the TE noiseless simulation, i.e. Pixel Window Function (PWF) should not be applied here and hence it has a slightly different Band-Pass Window Function (BPWF).

Following Hamimeche and Lewis¹⁵, we define the likelihood $\mathcal{L}$ as

$$\begin{aligned}
 -2 \ln \mathcal{L}(\mathbf{C}_b | \hat{\mathbf{C}}_b) &= \sum_b \nu_b \{ \text{tr}[\hat{\mathbf{C}}_b \mathbf{C}_b^{-1}] - \ln |\hat{\mathbf{C}}_b \mathbf{C}_b^{-1}| - n \}, \quad n = 3 \\
 \mathbf{C}_b &= C_b^{XY, \text{total}}, \quad \hat{\mathbf{C}}_b = \hat{C}_b^{XY, \text{total}}, \quad X, Y = T, E, B \\
 \hat{C}_b^{XY} &= C_b^{XY, \text{total, measured}} + \hat{N}_b^{XX} \delta^{XY} \\
 C_b^{XY} &= C_b^{XY, \text{total, theory}} + \hat{N}_b^{XX} \delta^{XY} + \hat{C}_b^{BB, \text{leakage-template}} \delta^{XB} \delta^{YB} + \\
 &\quad \hat{C}_b^{BB, \text{foreground-reference}} \delta^{XB} \delta^{YB}
 \end{aligned}$$

Notes on Absolute Calibrations

The procedures outlined above will be used to fit the final A_{lens} once. i.e. the absolute angle and gain will not be used to rotate and scale the maps. The resulting science product, the BB spectrum, will be calculated using the MLE a_i, θ on $(T^{-1})_{ij}^{kl}(a_i, \theta) C_{b,kl}^{\text{measured}}$.

This is a full-Bayesian approach to perform simultaneous self-calibration and parameter estimation.

In previous POLARBEAR B -mode analyses, the maps are rotated and scaled directly. This means that gains and angle has to be predetermined, say by the MLE. As this does not take the error of the gains and angle into account, it requires manual error-propagation of them into the spectra. Furthermore, the ratio between the EE subtracted in measured BB , mainly by the rotation mechanism, is fixed. By having a full likelihood allowing all these to vary (gain, angle, A_{lens}), the likelihood of the A_{lens} is also sensitive to this unknown amount of EE rotated in the BB spectrum. This is better than just scaling the error-bar of the spectra, because EE is not simply a multiple of BB .

This procedure can cause a misunderstanding that this likelihood is mixing the calibration factors into the cosmological factor A_{lens} . It is not mixing them, because

$$\hat{C}_{b,ij}(a_T, a_E, \theta) \equiv (T^{-1})_{ij}^{kl} C_{b,kl}^{\text{observed}}$$

¹⁵“Likelihood Analysis of CMB Temperature and Polarization Power Spectra”.

$$C_{b,ij} = \begin{pmatrix} C_{b,TT}^{\text{th}} & C_{b,TE}^{\text{th}} & 0 \\ C_{b,TE}^{\text{th}} & C_{b,EE}^{\text{th}} & 0 \\ 0 & 0 & A_{\text{lens}} C_{b,BB}^{\text{th}} \end{pmatrix}$$

and

$\ln \mathcal{L}(\mathbf{C}_b | \hat{\mathbf{C}}_b)$ depends on $\hat{\mathbf{C}}_b \mathbf{C}_b^{-1}$ only. Since the $C_{b,ij}$ is block-diagonal, the inverse is easy to calculate in the BB corner. Expanding that term alone, one has

$$\hat{\mathbf{C}}_b \mathbf{C}_b^{-1} \sim \frac{(\dots)_{EE} C_{EE}^{\text{measured}} + (\dots)_{BB} C_{BB}^{\text{measured}}}{A_{\text{lens}} C_{BB}^{\text{th}}} + (\dots)_{\text{others}}$$

So it only couples the theory BB into the measured BB and EE , through the gains and angle in $(\dots)_{BB \text{ or } EE}$, which is unavoidable regardless of approach. If we choose a predetermined gains and angle, the $(\dots)_{BB \text{ or } EE}$ term is fixed. If we do not fix it, then it is equivalent to having $(\dots)_{BB \text{ or } EE}$ set by a prior distribution, where this prior distribution inevitably is still introduced by the $(\dots)_{\text{others}}$ terms.

So rather than mixing the calibration factors into the cosmological factor, this procedure automatically propagate the error-bar in angle and gains into the final estimation of A_{lens} , and doing this better than simple rescaling the error-bars (since EE is not proportional to BB .)

The only caveat is about the pure B-mode estimator, as performing rotation in the map can change this. But given the angle is $\sim 0.4^\circ$, this should not be a problem.

28.4 Analytical error-bar: Effective d.o.f.

From sec. 23.1, we already see the term ν as the effective degree of freedom (d.o.f.) in a b -bin across a range of ℓ .

To understand the Knox formula, first note that

$$C_\ell \equiv \frac{1}{(2\ell + 1)} \sum_{m \leq \ell} \langle |a_{\ell m}|^2 \rangle,$$

where we expect $a_{\ell m}$ are independent Gaussian random variables that shares the same expectation value C_ℓ for all m for each ℓ . This is why we expect C_ℓ to be a χ_ν^2 random variable.

Then define the following random variable such that

$$\begin{aligned}
\mathcal{C}_\ell &\sim C_\ell \frac{\chi_{\nu_\ell}^2}{\nu_\ell} \\
\mathbb{E}[\mathcal{C}_\ell] &= C_\ell \\
\text{Var}(C_\ell) &= \frac{2}{\nu} C_\ell^2 \\
\nu_\ell &\equiv (2\ell + 1) f_{\text{sky}}
\end{aligned} \tag{28.1}$$

Here, for simplicity, C represents the total power-spectrum which includes both the signal and noise term. This proves the Knox formula.

Then the key idea from Hivon et al.¹⁶, mentioned in sec. 23.1, is that in the case of cut-sky, effectively we lose some amount of degree of freedom ν , scaled accordingly w.r.t. the fraction f_{sky} .

Finally, we should notice that the combined effect of the MASTER equation, the binning and interpolation operators together has the effect of¹⁷,

$$\hat{C}_b = \sum_{\ell} w_{b\ell} C_\ell,$$

where $w_{b\ell}$ is often called the Band-Pass Window Function (BPWF) as shown in fig. 28.7.

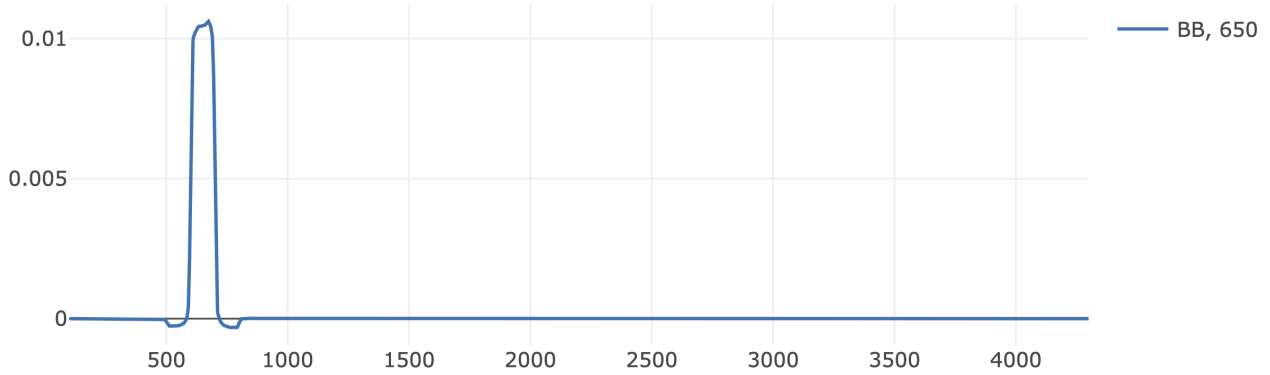


Figure 28.7: The 600-700 bin of band pass window function of B -mode power-spectrum

You can see some of the power is negative. i.e. rather than distribute like a χ_ν^2 distribution, it is more like

¹⁶“MASTER of the Cosmic Microwave Background Anisotropy Power Spectrum”.

¹⁷Ade et al., “A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND-B-MODE POLARIZATION POWER SPECTRUM AT SUB-DEGREE SCALES WITH POLARBEAR”.

$$\begin{aligned}\mathcal{C}_{b,\text{eff.}} &\sim a \left(\chi_{\nu_b^+}^2 - \chi_{\nu_b^-}^2 \right) \\ \mathbb{E}[\mathcal{C}_{b,\text{eff.}}] &= C_b \Rightarrow a \equiv \frac{C_b}{\nu_b^+ - \nu_b^-} \\ \text{Var}(\mathcal{C}_{b,\text{eff.}}) &= a^2 (2\nu_b^+ + 2\nu_b^-) = 2C_b^2 \frac{\nu_b^+ + \nu_b^-}{(\nu_b^+ - \nu_b^-)^2}\end{aligned}$$

comparing this to eq. (28.1), we can define an effective d.o.f.

$$\nu_b^{\text{eff}} = \frac{(\nu^+ - \nu^-)^2}{\nu^+ + \nu^-}.$$

The new formula improves the agreement between MC error-bar and analytical error-bar, from $\sim 10\%$ to $\sim 3\%$ agreement for BB, EB, EE, TB, TE power-spectra, and from $\sim 20\%$ to $\sim 13\%$ agreement for TT power-spectrum¹⁸.

This improvement is crucial, as general agreement between the analytical error-bar and the MC error-bar is a good cross-check, and the likelihood used in sec. 28.3 contains ν which implicitly uses the analytical error-bar rather than the MC error-bar.

Error propagation & miscellaneous likelihood parameters

It is already mentioned that the likelihood is going to propagate the uncertainties of calibration parameters. Here we mention other source of errors.

As mentioned before, the filter transfer functions are estimated via 2 sets of 128 MC simulations, one without the B -mode signal, another without the E -mode signal. This results in a relative error in F_ℓ peaked at 1%-level in the low- ℓ end. The final bin-width used is 100, making relative error of F_b to be at least an order of magnitude smaller.

Empirically, the agreement between the 2 sets of simulations above and the theory with BPWF is peaked at 10^{-6} -level in the low- ℓ end.

Because of these, and the fact that the statistical error from noise is much larger, errors from filter transfer function is negligible and will be ignored here.

Error from beam will be fitted similarly as in previous POLARBEAR B -mode publications, i.e. a ℓ dependent gain. The error-bar will be introduced in the likelihood equation automatically by having the following gain term,

$$a_i = a_{0,i} e^{-\frac{\ell(\ell+1)\sigma^2}{2}},$$

where σ is a free parameter in the likelihood.

¹⁸ TT is not as i.i.d. as the others empirically, so the analytical model fails more here. See sec. 28.2.

Chapter 29

Running the Null-test suite

29.1 Null-spectra

Let us take a look at an example null-spectra in fig. 29.1.

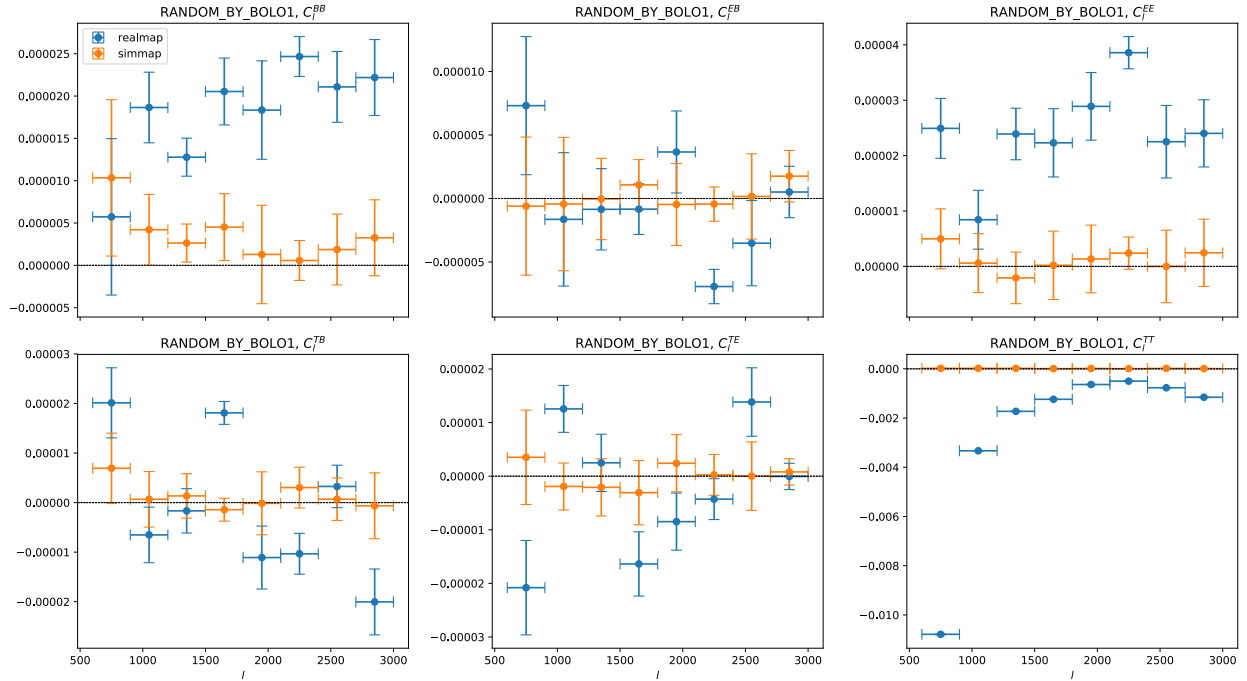


Figure 29.1: Null test v0.6.2 RANDOM_BY_BOL01 null-spectra

1. The error-bars are estimated using $n_{\text{sim}} = 8$ simulations that includes fiducial signals and an uncorrelated white noise model with magnitude estimated from the TOD.

2. The values of the simulations are averaged from all simulations to show you that this null-estimator is unbiased as we expect it to be consistent with 0 statistically.
3. The values from the real data is the same null-estimator constructed using the observed TOD. Hence, this is where we might make “discovery” of unknown systematic effects.

(Normally, the vertical axis is in D_ℓ scale, but here it is $C_\ell B^2 \ell$ for a reason that is going to be clear later.)

Just by looking at it, we can see that something is wrong with the BB, EE spectra. In fact, something is very wrong, as this null-split, `RANDOM_BY_BOLO1`, is randomly splitting the data into two halves according to which detectors it belongs to. And we do not normally expect a random-split to be failing.

Other null-test failures includes

- `QU_PIXEL`
- `LEAK_BY_BOLO`
- `AMP_2F_BY_BOLO`
- `AMP_4F_BY_BOLO`

which are our discoveries in sec. 28.2.

29.2 Null-test failures

We named these failures as “local-by-bolo” splits, as they are all null-split by detectors that are separated locally on the focal plane.

For a long while, these null-test failures are not understood, and there are a long list of speculations as to why they are failing. And I checked each of these theories and falsified them:

- I-to-P leakage?
- Gain mismatch?
- $1/f$ noise aliasing into $4f$ frequency band contaminating the polarization signal?
- Requiring pairing of top and bottom detectors?

The actual problem and solution are discussed below.

The first hint is from fig. 29.1 when it was first plotted in $C_\ell B^2 \ell$ scale. Looking like white noise here is a huge hint. So we now know that this is a noise problem.

Further digging into the noise spectra, when the ratio between the real noise spectra and the simulated noise spectra is plotted in fig. 29.2, we have found the same set of null-splits that stand out from the others¹.

¹Do not worry about the overall normalization as it would be fixed later. Also, the two random null-splits are added when searching for hints on why this problem exists.

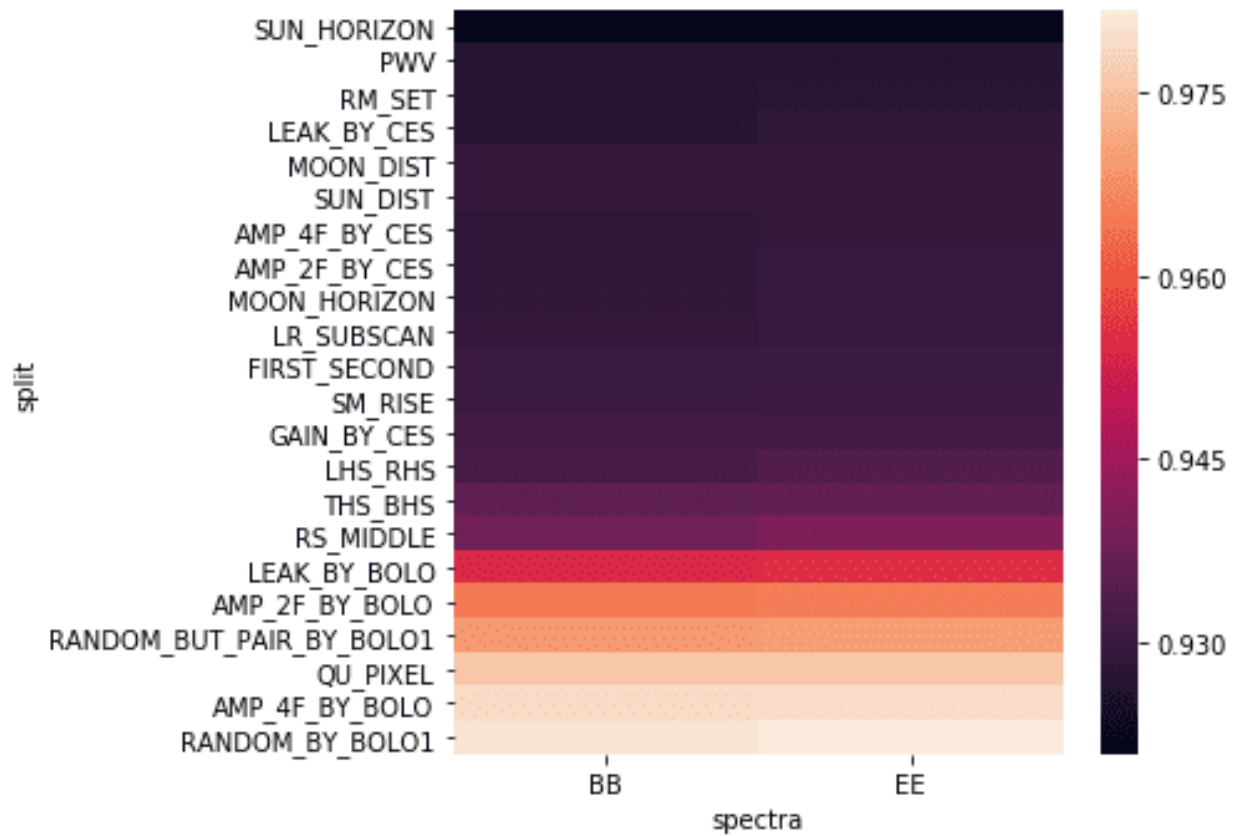


Figure 29.2: $\frac{N_{l,\text{real}}}{N_{l,\text{sim.}}}$ as a function of null-split

Digging further, as pointed out by Goeckner-Wald² in an internally circulated document, there is a bug in AnalysisBackend that added a term that should not exist in the calculation of the null-spectra, which normally would not be a problem as the expected value of this term is zero if the noise between the two splits are uncorrelated. But the very bug also gave us a hint of the origin of the problem—correlated detector noise. (Since different null-splits are splitting the same data in different halves, we know that the total noise level must be a constant and therefore this detector noise correlation is anti-correlating with some of the nearby detectors.)

The detector-detector correlations are calculated and a bad example CES is shown in fig. 29.3. These correlations are used to calculate a Kernel Density Estimation (K.D.E.) and there is a second peak. This gives us a data selection criteria, which is a classification problem, using the feature on the K.D.E. as the classification rule.

Our rule is then to find this 2nd peak per CES and cut away the data from the

²“Expanding Out the QUIET Cross Null Spectrum”.

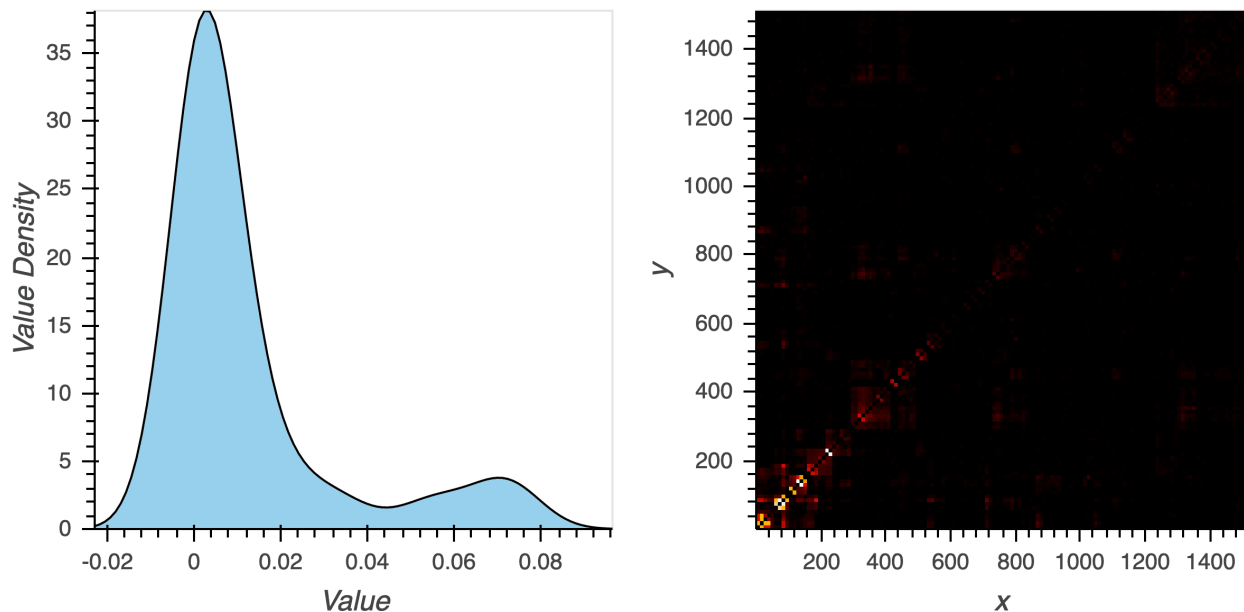


Figure 29.3: $|r_{ij}|^2$, where r_{ij} is the correlation between detectors TOD estimated using $Q + iU$ with an example CES named 20141211_173206. R.H.S. shows a heatmap with the detectors reordered to reveal the clustering. On the L.H.S., $\tilde{R}_i \equiv \max_j |r_{ij}|^2$ is first computed, and the Kernel Density Estimation (KDE) of $\tilde{R}_i$ is calculated and shown here.

valley between them. This cuts away an additional amount of data volume at around 5%.

With all these fixes in-place, the null-spectra look like fig. 29.4.

We can see it is a huge improvement over fig. 29.1 for all of our science products EE, EB, BB power-spectra³.

29.3 When can we unblind?

As we are marching through the null-test iterations, the situation is improving.

The actual passing criteria are complicated, which includes other criteria such as Kolmogorov–Smirnov test mentioned in sec. 28.2.

But even focusing on null-test suite alone, when are we going to end this endless pursue of hidden systematics and be confident that we are good enough to unblind? In other words, what is the stopping criteria?

³Note that the white noise model instead of the sign-flip noise is used, so the TT null-power-spectra is still not good. This is fixed in the final analysis. See sec. 27.6.

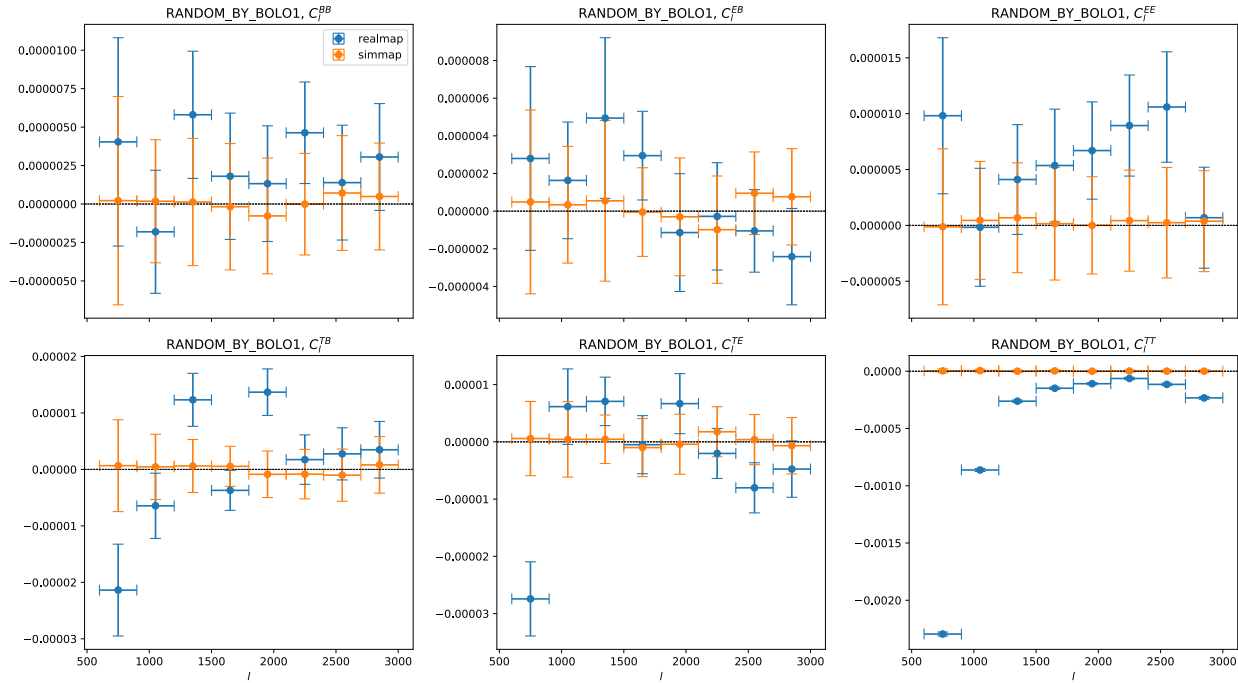


Figure 29.4: Null test v0.10.0 RANDOM_BY_BOLO1 null-spectra

The first boring answer is to increase the number of simulations because they are not failing as dramatically and hence we need more accurate statistics to discern between fluke and discovery. But how much is enough?

This is a statistical problem. In short, we want to quantify how confident we are in passing the null-test before running the final run with hundreds of simulations.

The answer is confidence interval. But how do we estimate it? In Chinone⁴, an erroneous argument is made following Clopper and Pearson⁵, which uses an approximation that fails at the two extremes of the binomial, and is applied at one of the extreme $p = 0$.⁶

But we do not even have to fix that formula, instead, we can apply the bootstrap method on the MC simulations from existing preliminary null-test result using a smaller number of MC simulations.

For example, in null-test v0.13.4, I have $n_{\text{sim}} = 48$ simulations. Using the bootstrap method, I can resample the n_{sim} simulations and estimate the confidence interval $P(\alpha_c \geq 0.05)$.

⁴“Polarbear Null-Test Framework”.

⁵“THE USE OF CONFIDENCE OR FIDUCIAL LIMITS ILLUSTRATED IN THE CASE OF THE BINOMIAL”.

⁶See more from Agresti and Coull, “Approximate Is Better Than ‘Exact’ for Interval Estimation of Binomial Proportions”; Thulin, “The Cost of Using Exact Confidence Intervals for a Binomial Proportion”.

Applying this procedure, the weakest statistics in v0.13.4 is from *EB*, where I have $\alpha = 6.2\%$ and $P(\alpha_c \geq 0.05) = 81.4\%$.

Since we require all *EE*, *EB*, *BB* to be passing, this is good enough and a sign that we are good to go.

Chapter 30

The limitation of the null-test framework

30.1 Hypothesis testing, discovery, and default action

As mentioned before, the POLARBEAR null-test framework is based on hypothesis testing in statistics where our null-hypothesis H_0 is the absence of systematics. Hence, our discovery in this hypothesis testing is the detection of systematics, which contaminates our data.

But here we are using hypothesis testing in a very different context comparing to how it usually is used in science. Namely, typically the null-hypothesis is the lack of new discovery leading to new science. To put it in other words, as Oleszak¹ put it as “the null hypothesis reflects our default action”. In the case of making new scientific discoveries, when no discovery (null-hypothesis not rejected) is found, the default action would be not to proceed any further including publications².

Here, as we made no discovery (null-test suite makes no discovery on systematic bias) we proceed and go to unblind the data resulting in a publication. We are taking a much more risky default action—unblind and publish.

On one hand, certainly the null-test suite is not the only procedure to guard against biases. But on the other hand, biases can arise even after all these and passing any null-test criteria.

For example, we found that the absolute angle calibration is too large after unblinding the data, which is due to not having deconvolvedchapter 26 the TOD by the detector time constant, which turns out to be an internal (lack of) communication error in applying a single command line argument!

¹“The Hypothesis Tester’s Guide”.

²See Banerjee et al., “Hypothesis Testing, Type I and Type II Errors” for more about the basics of good hypothesis testing.

Despite the presence of this systematics, the null-test suite passes anyway because it represents a lag in the measured detector signal, which has an effect that is degenerate from an additional rotation (resulting in a large apparent absolute angle in calibration.) This effect would not cause a null-test failure (as rotating null still results in null.)

Besides being an example of a limitation in the null-test framework, this perhaps reiterate the point in chapter 24 that even reproducing a result within a collaboration is crucial to avoid an error like this. It is a systemic issue that cultivates a problem as important as this to be overlooked.

30.2 The action of unblinding

Another limitation of the null-test framework is implicit in its default action—if null-test is passed, then we unblind the data.

But what is blindness? It is a somewhat arbitrary entity with no clear definitions. For example, it is argued that looking at the signal maps is ok, because the map itself is not the science product in our case. And it is abstract enough that from looking at the map, it is hard to draw conclusions. Nonetheless, this makes defining blindness rigorously impossible.

Another more subtle case would be to look at the null-spectra. Since they are expected to be zero, there is no harm done by looking at them, right? But the nature of systematics is that we may not know every source of systematics beforehand, and some, which can be as simple as a gain mismatch, can result in the signal power-spectra leaking into the null-spectra. One can be accidentally looking at a rescaled signal when looking at null-spectra.

There is an even more philosophical example, that when one runs the pipelines from AnalysisBackend, by default it will print the numerical power-spectra on screen. Those are usually scrolling so fast that a normal human's eyes cannot catch up, but nonetheless this is an arbitrary blindness criteria which relies on the human's incapability (in reading numbers fast enough).

Not being able to write down a formal definition of blindness has consequence. In an early study of absolute gain calibration, I accidentally used the real data to project the level of absolute gain expected. In the hindsight, it clearly violates blindness criteria, but it is easy to lump all these together and think that “since the gain is not our science product, it would be fine.”

30.3 After unblinding

Another deficiency of the null-test framework is that it implicitly assumes an unblinding procedure comes from what can be changed after unblinding.

If one changes an analysis technique for example, it can be a source of observer bias. So ideally one should not change anything after unblinding. But in our framework, a practical problem is that we do not strongly specify the analysis techniques involved after unblinding.

We already went through great length in chapter 28 discussing the procedure involved, but that is not a strong specification in the sense that there are devils in the details that can be changed.

To give one example, recall that we use sign-flip noise in the final analysis. It means that noise estimated from the real data are injected into maps simulated using fiducial maps, which are generated using the 2018 Planck cosmology³. Hence, the noise components and signal components have a different Pixel Window Function (PWF). Admittedly, the implication or even existence of this has not been well thought out before unblinding. It also means after unblinding, new codes are written to take this effect into account.

A solution is offered by reproducible research⁴, that we should have required the whole software stack and pipelines to be reproducible before unblinding. The unblinding stage should run exactly the same pipelines over the newly generated datasets.

30.4 Miscellaneous problems

Lastly, after the final analysis, I found 2 programming bugs such that the counter term of the pure B -mode estimator is incorrect.

This increases the amount of E -to- B leakage. Phenomenologically, $\sim 20\%$ of B -mode power comes from leakage peaked at the lower- ℓ end. While the bug makes this larger, it is much smaller than the total statistical error in this analysis.

One solution to problem like this is to have maintainers for research softwares, and to require code-review before merging, which is a standard software engineering practice elsewhere.

³Planck Collaboration, Aghanim, Akrami, Ashdown, et al., “Planck 2018 Results”.

⁴See chapter 24.

Chapter 31

Our measurement

In null-test v0.13.4¹, 24 T, E noiseless simulations and 24 T, B noiseless simulations are used to estimate the filter transfer functions. 48 T, E, B with sign-flip noise simulations are used as MC simulations for the estimation of statistical properties including the error-bar. Null-test suite is passing preliminarily. Following the reasoning outlined in sec. 29.3, it is scaled up to a set of 128, 128, 512 simulations respectively in null-test v1.0.0.

The deconvolution problem mentioned in chapter 30 is fixed in v1.0.1. There is another minor problem concerning how the ground-template is constructed and is fixed in v1.0.2. This is the final analysis. Below are its null-statistics and the measurements.

The ℓ -range for BB, EB, EE is 600–3000, using a bin-width of 100. The ℓ -range for TT, TE, TB is 600–1600 instead as the T -filter transfer function has numerical stability issues above $\ell \simeq 1600$.

31.1 Passing Null Test suite

$\alpha = 5\%$ is required in the null-test suite passing requirements.

For Kolmogorov–Smirnov test, when it is performed using null random variable $\chi(X, n, b)$ with $X = BB, EB, EE$, overall p-value is 13.0% with a confidence level of $P(p > \alpha) = 99.9\%$, estimated using 1,000 bootstrap.

Table 31.1: PTE for the null-statistics defined in definition 2 estimated using MC simulations.

power-spectra	G_{Y_1}	G_{Y_2}	G_{Y_3}	G_{Y_4}	G_{Y_5}
BB	72.07%	81.25%	75.39%	85.74%	14.45%
EB	80.27%	37.30%	63.28%	27.93%	6.05%

¹See sec. 38.6.

power-spectra	G_{Y_1}	G_{Y_2}	G_{Y_3}	G_{Y_4}	G_{Y_5}
EE	37.30%	83.20%	80.66%	98.44%	65.82%
TB	8.01%	41.02%	99.22%	81.45%	80.66%
TE	11.33%	81.45%	31.45%	88.67%	48.83%
TT	29.10%	84.77%	40.63%	94.53%	91.60%

Tbl. 31.1, shows the null-statistics as defined in definition 2 estimated using MC simulations. Using the 15 null-statistics with $X = BB, EB, EE$, overall p -value is at $\alpha_c = 90.8\%$ with a confidence level of $P(p > \alpha) = 100\%$, estimated using 10,000 bootstrap. This is the unblinding requirement.

For reference, if all 30 null-statistics with $X = BB, EB, EE, TE, TB, TT$ are to be included, overall p -value is at $\alpha_c = 99.2\%$ with a confidence level of $P(p > \alpha) = 100\%$, estimated using 10,000 bootstrap.

31.2 Likelihood & posterior

Method detailed in sec. 28.3 is followed to perform likelihood estimation.

Foreground model

Prior of dust foreground is included following the same model used in Sayre et al.²

$$D_\ell = A_{d,150 \text{ GHz}} \left(\frac{\ell}{80} \right)^{-0.58}.$$

A different gaussian prior is used however, as a slightly different numerical value is found when it is derived from the source of the input in the model from Ade et al.^{3 4},

$$\begin{aligned} \beta &= 1.59(11) \\ T &= 19.6 \text{ K} \\ A_{d,353 \text{ GHz}} &= 4.6(11) \\ \Rightarrow A_{d,150 \text{ GHz}} &= 0.009\,554\,078\,346\,271\,361(2907862359357405). \end{aligned}$$

$$\text{Lastly, } D_b \equiv \sum_\ell w_{b\ell}^{BB} D_\ell.$$

²“Measurements of B-Mode Polarization of the Cosmic Microwave Background from 500 Square Degrees of SPTpol Data”.

³“Constraints on Primordial Gravitational Waves Using Planck, WMAP, and New BICEP2/Keck Observations Through the 2015 Season”.

⁴Instead of $A_{\ell=80;150 \text{ GHz}}^{\text{dust}} = 0.0094(21) \mu\text{K}^2$ from Sayre et al., “Measurements of B-Mode Polarization of the Cosmic Microwave Background from 500 Square Degrees of SPTpol Data”.

Calibration / nuance parameters

Using the likelihood procedure defined in sec. 28.3, an affine-invariant ensemble sampler for Markov chain Monte Carlo (MCMC)⁵ is used to estimate the posterior distribution of the input parameters.

The posterior distribution of $A_{\ell=80;150\text{ GHz}}^{\text{dust}}$ is similar to the prior.

Regarding calibration parameters, the absolute gain of the intensity map is $a_T = 1.091(12)$, and the absolute gain of the polarization maps is $a_E = 1.107(13)$. The agreement of both means the polarization efficiency calibration is unbiased.

For the absolute gain calibration, $\theta = -0.32(19)^\circ$. For the σ^2 parameter in the beam model, $\sigma^2 = 0.48(19)'^2$.

These are consistent with the result from the large-patch low- ℓ pipeline⁶.

Cosmological parameter

The cosmological parameter here is A_{lens} which is the ratio between the measured lensing B -mode and the theoretically expected amount of lensing. Hence, its fiducial value is 1. There is a couple of ways to interpret this result.

It can be started with the MCMC chain, and one can estimates that

$$P(A_{\text{lens}} > 0) = 99.6\%.$$

Wilks' theorem can also be used⁷ to convert the likelihood ratio to a probability of $p = 99.2\%$. The agreement between the two is reassuring, and the result from MCMC is used in the end as it makes no assumption on the behavior of the likelihood.

Lastly, $A_{\text{lens}} = 3.90(151)$, where 3.90 is the MLE and 1.51 is the standard deviation. The 90% CI is $1.46 < A_{\text{lens}} < 6.42$, while the 90% CI upper bound is $A_{\text{lens}} < 5.85$.

⁵Foreman-Mackey et al., “Emcee”.

⁶The POLARBEAR Collaboration et al., “A Measurement of the Degree-Scale CMB B -Mode Angular Power Spectrum with POLARBEAR”.

⁷Since the assumption made in Wilk's theorem is not exactly true here, this is an approximation.

Part VI

The effect of detector crosstalk systematics on the CMB power-spectra in the example of the LiteBIRD experiment

Chapter 32

The LiteBIRD satellite

LiteBIRD stands for Lite (Light) satellite for the studies of B-mode polarization and inflation from cosmic background Radiation Detection.

32.1 Introduction

LiteBIRD is a satellite-based CMB experiment expected to launch in the late 2020s. It will be the successor of Planck and will become the 4th satellite dedicated to CMB observation. Like its predecessors since the 2nd generation WMAP, it will orbit around L2 point with a quasi-stable Lissajous orbit as shown in fig. 32.1, far away from the contamination by the Earth and the Moon.

32.2 Science target

The primary science target of LiteBIRD is the primordial B -mode and hence it is a probe of inflationary physics.

In fig. 32.2 & fig. 32.3, which can be compared to fig. 16.1 & fig. 16.2, They show the projected sensitivity of LiteBIRD in terms of the B-mode spectrum, parameterized by the tensor-to-scalar ratio r .

The current constraint is $r < 0.044$. LiteBIRD aimed to constrain $\delta r < 1 \times 10^{-3}$. This can either make a discovery or rule out well-motivated inflationary models⁴.

This makes it one of the most exciting CMB experiment in the coming decade. And to date, hundreds of Scientists around the globe is working on the project to achieve this goal.

¹Hazumi et al., “LiteBIRD Satellite”.

²Hazumi et al., “LiteBIRD Satellite”.

³Hazumi et al., “LiteBIRD Satellite”.

⁴Hazumi et al., “LiteBIRD Satellite”.

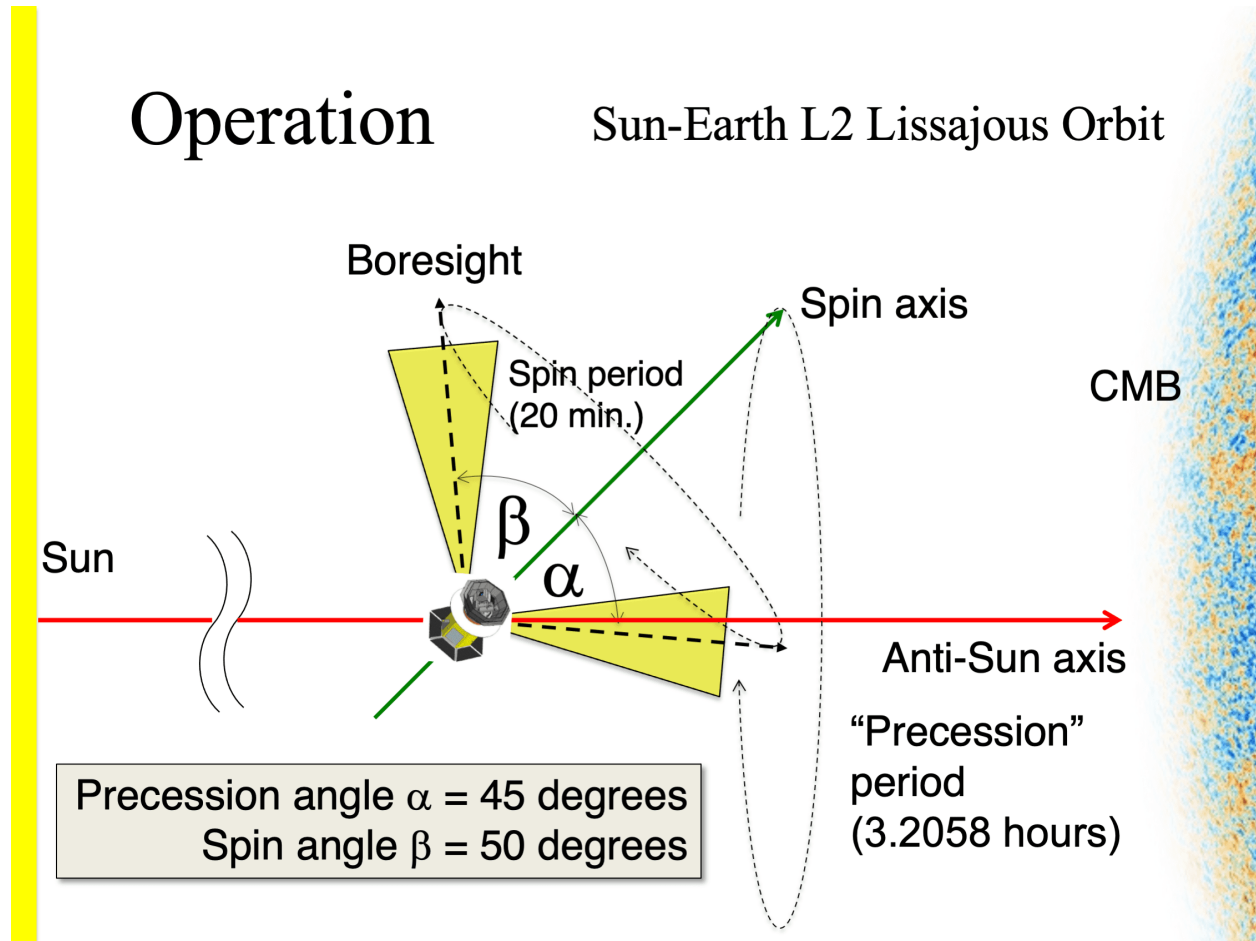


Figure 32.1: Scan strategy of LiteBIRD in a Lissajous orbit around L2.¹

32.3 Multiplexed readout system

LiteBIRD is designed to be a 3 years mission, with the current design holding 4508 detectors, reading out data from all those detectors is challenging. These TES bolometers are daisy-chained together using digital frequency-domain multiplexing (fMUX) through superconducting quantum interference device (SQUID) to achieve simultaneous readout.

For example, in POLARBEAR, 8 bolometers are chained through SQUID. In LiteBIRD, the current baseline design is to chain around 68 bolometers, with possibly more in the order of 100s as the design improves.

This is also a general trend in CMB experiments. As seen in chapter 17, number of detectors are growing exponentially. For reasons laid out in Montgomery et al.⁵, one of

⁵[Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System](#)

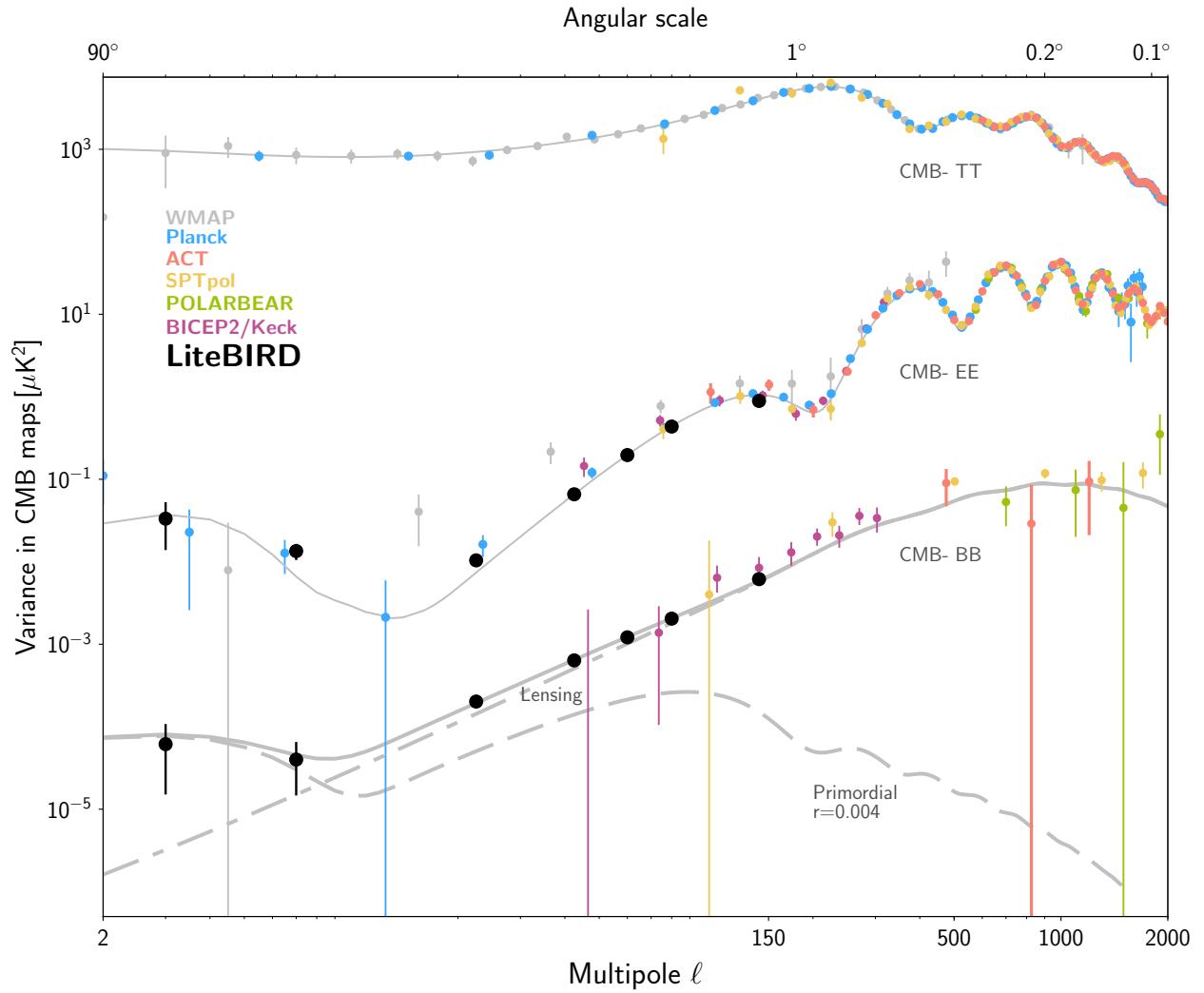


Figure 32.2: Summary of present measurements of CMB power spectra and expected polarization sensitivity of LiteBIRD.²

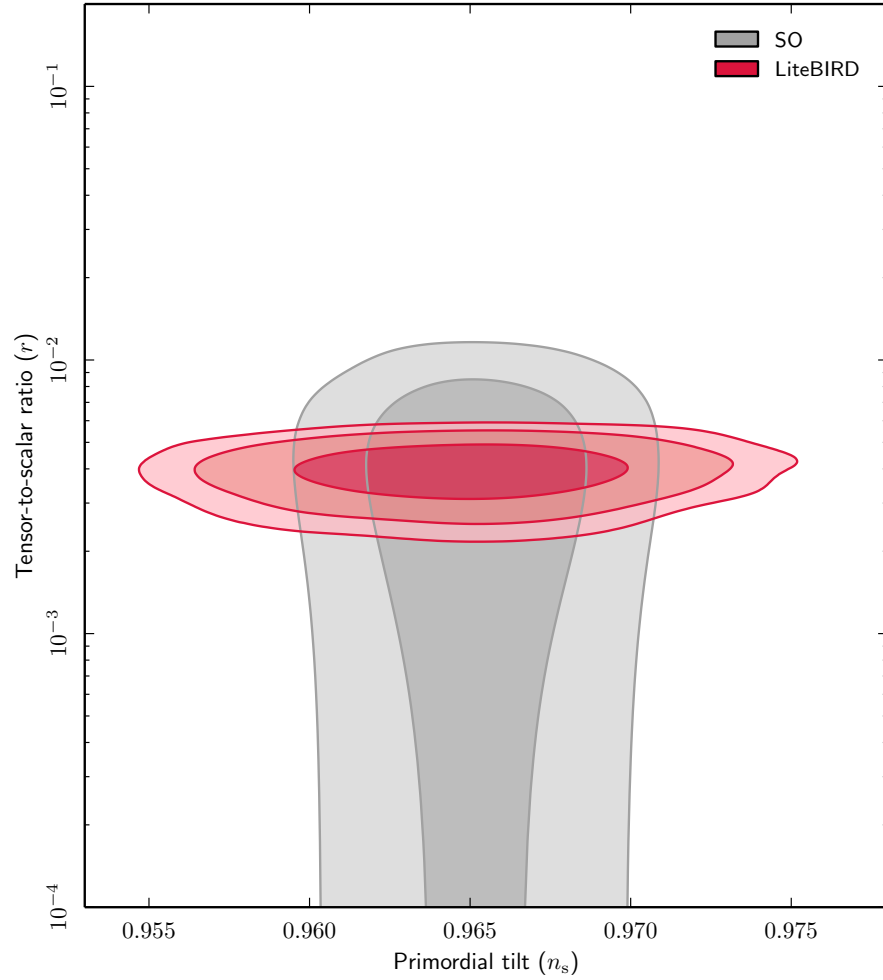


Figure 32.3: Sensitivity contours of LiteBIRD and the Simons Observatory. Systematic uncertainties are not included here. The plot assumes that the actual value of r is 0.004.³

the primary limitations in growing number of detectors is cryogenic cooling limitations, where multiplexing is an essential part of the solution. Hence, we are going to see increasingly larger mux factor, translated into the size of the crosstalk matrix. It makes the crosstalk systematics increasingly more important to be understood and mitigated.

32.4 Crosstalk between detectors

With so many detectors chained together, crosstalk between them can contaminate our signal.

For example, We would like to chain detectors across different observing frequency for fault tolerance such that if a whole SQUID is down, we are not knocking out a whole observing frequency or wafer. The presence of crosstalk across different frequency channels will mix different amount of foreground components into our resultant maps. Since we typically form individual frequency maps and from there we cross-correlate to estimate the foreground systematics, such as how it is analyzed in Planck. This mixing will breaks our model and systematics can bias our results.

To solve this problem, it is a joint effort between the hardware and software team. The experimentalists can reduce the crosstalk in the design, and provides realistic simulation on the amount of crosstalk expected from a design. The data scientists can take the crosstalk matrix as an input and simulates the amount of bias injected into the signal.

This feedbacks into the hardware design process which informs us if it can achieve our science goal.

Also, in terms of software development, we can devise mitigation strategies in how we can reduce the amount of bias.

Moreover, a given hardware design has a huge parameter space to explore. For example, in our current design for a certain SQUID, it is required for around 60 detectors to be chained across 3 wafers with around 20 detectors each. The exact ordering within a SQUID is totally free. This is then an optimization problem for us to simulate the amount of bias subjected to the same requirement and optimized the configuration to provide feedback to hardware design.

In chapter 33, we will see how to generate the crosstalk matrix from SQUID parameters. In chapter 34, a map-domain crosstalk model is presented together with a set of assumptions which is fulfilled approximately in the case of LiteBIRD. In chapter 35, an implementation of crosstalk simulation is presented in the framework of TOAST. In chapter 36, crosstalk systematics simulations in the example of LiteBIRD is performed, and a mitigation strategy is presented.

Using an Improved Noise and Crosstalk Model”.

Chapter 33

Modeling crosstalk from SQUID design

33.1 Introduction

Montgomery et al.¹ developed a theoretical model describing how the SQUID design generates the crosstalk matrix, and should be valid for digital frequency-domain multiplexing (DfMUX) with Digital Active Nulling (DAN), and is validated using SPT-3G data. See fig. 33.1 for the circuit model.

33.2 The crosstalk model and an exact solution

Montgomery et al.³ makes uses of 3 approximations, 2 mathematical and 1 computational,

1. Eq. 3 approximates the total (network) impedance by that at a leg, used in leakage power crosstalk;
2. Sec. 3.2 approximates the phase of the total signal by that of the detector signal; and
3. an implicit assumption in software that solve for resonance freq. by finding extrema on a grid.

Here, the same model is used without these 3 approximations.

¹“Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.

²Montgomery et al., “Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.

³“Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.

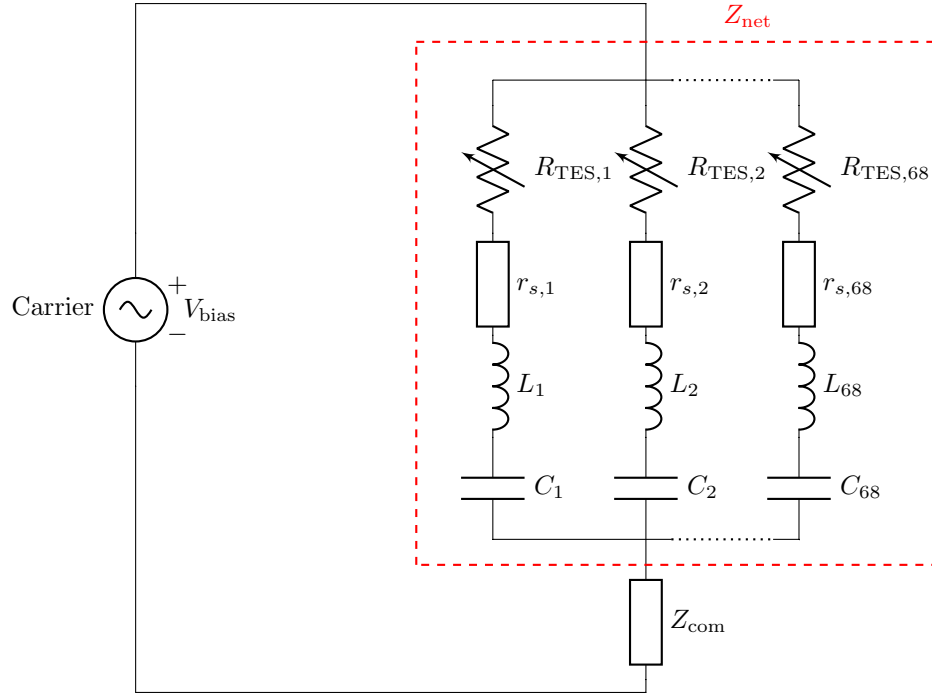


Figure 33.1: An example circuit diagram of the cryogenic network. This includes all relevant components used in the derivation of leakage current crosstalk and leakage power crosstalk.²

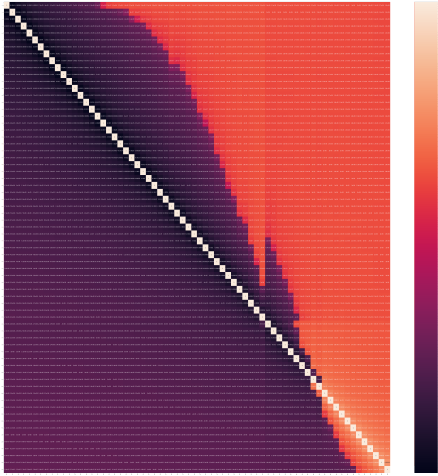
This is released in the util module in open-source library [coscon](#) which stands for Cosmological Convenient functions. The `total_crosstalk_matrix` function implements the model Montgomery2020 in Montgomery et al.⁴, whereas the `total_crosstalk_matrix_exact` function implements the same model without the 3 approximation, named as Montgomery2020Exact here.

33.3 Comparison

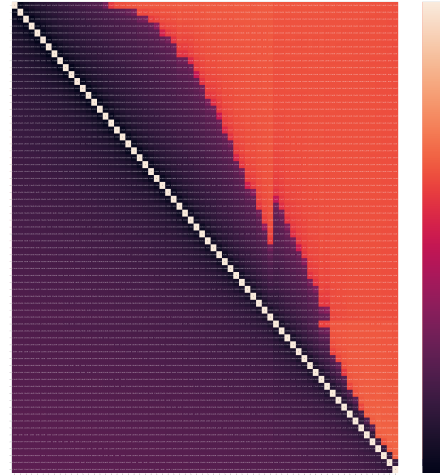
Using the same internal SPT-3G SQUID parameters as that in Montgomery et al.⁵, the crosstalk matrices using Montgomery2020 and Montgomery2020Exact models are compared in fig. 33.2.

⁴“Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.

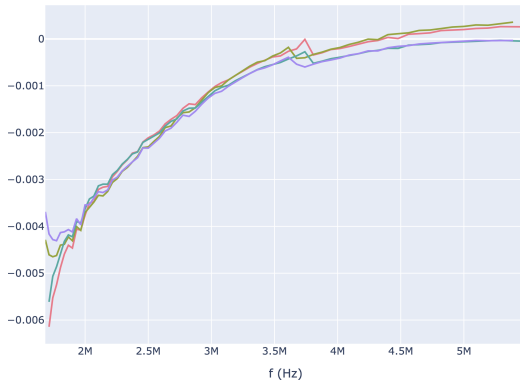
⁵“Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.



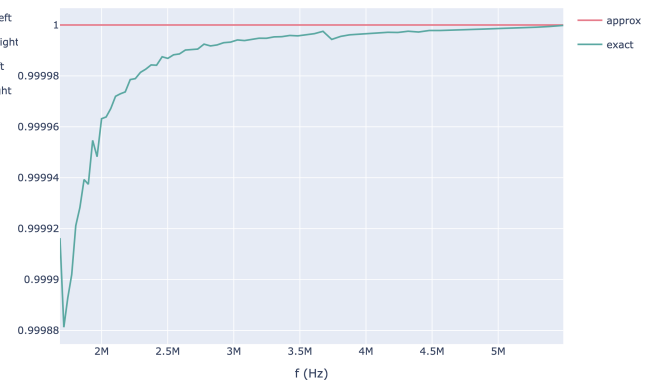
(a) Crosstalk matrix, Montgomery2020



(b) Crosstalk matrix, Montgomery2020Exact



(c) Nearest neighbor crosstalk comparison



(d) Diagonal “crosstalk” comparison

Figure 33.2: Comparison between Montgomery2020 and Montgomery2020Exact models.

These are tiny corrections and the experimental variations are much bigger⁶. The result from the Montgomery2020Exact model does has a few minor but interesting features:

1. all nearest neighbors has negative crosstalk in this specific example.
2. the diagonal is no longer exactly 1, with correction peaked on the order of the crosstalk strength ~ 0.01 which is negligible. This correction is originated from not using approximation (2). The change of phase contributed by crosstalk will change the magnitude slightly. In practice, the true sensitivity at calibration is measured with this contribution baked in⁷.

⁶Montgomery et al., “Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model,” fig. 7.

⁷So the raw sensitivity should be what is measured divided by this factor.

Chapter 34

A map domain model to predict crosstalk systematic effects on maps and power-spectra

Crosstalk systematics is a time-domain problem. It mixes the detector TOD according to¹,

$$\tilde{d}^i = \sum_j X^{ij} d^j.$$

However, it is equivalent to a map-domain problem if the following holds:

Assumption 1. Assumptions of treating crosstalk systematics as a map-domain problem.

1. In scanning strategy:
 - a. each detector has an uniform map coverage (i.e. the weight-map per detector is constant over the sphere);
 - b. (for fixed orientation) the focal plane has fixed orientation w.r.t. the sky locally;
 - c. (or for isotropic orientation) the scanning strategy is isotropic in focal plane orientation²;
2. The mapmaking operation:
 - a. is linear in the observed TOD, and
 - b. depends on the pointing matrix only;

¹Matrices are specified with explicit indices throughout unless stated otherwise. Also, upper indices and lower indices are used for detectors and the discrete sampling points respectively.

²i.e. (1a) assume the scan is $\mathbb{S}^2$ invariant, and (1c) in addition to (1a) assumes $\text{SO}(3)$ invariant.

- c. there is no noise such that the mapmaking operation can be considered as the inverse transform of the observation operation;
 - d. where the resultant map is a weighted sum of individual detector maps (independence of detectors);
 - e. each detector can resolve the I, Q, U components independently;
 - f. has infinite resolution, i.e. in the continuum limit;
3. In detector layout,
- a. all detectors are fixed on the focal plane;
 - b. the focal plane angular scale is $\ll 1$.

Proof. With Assumption 1, it can be proved that the crosstalk systematics is a map-domain systematics.

Observation and mapmaking is often written in discrete sum:

$$d_t = \sum_p A_{tp} m_p + n_t$$

$$\hat{m}_p = \sum_t W_{pt} d_t$$

Here, this is made abstract under the continuum limit (2f), and noise is ignored using assumption (2c):

$$\begin{aligned} \mathcal{O}[A]\{m\}(t) &\equiv d(t) &= \oint A(t, \mathbf{r}) m(\mathbf{r}) d^2\mathbf{r} \\ \mathcal{M}[A]\{d\}(\mathbf{r}) &\equiv m(\mathbf{r}) &= \int W[A](\mathbf{r}, t) d(t) dt \end{aligned}$$

where $\mathcal{O}[A], \mathcal{M}[A]$ ³ are linear transformation of m, d respectively (assumption 2a) that depends on A (assumption 2b). They are inverse transformation of each other (assumption 2c). Note that while $\mathcal{O}[A]$ is linear in A , $\mathcal{M}[A]$ is not required to be linear in A , and in practice they are not.

For the map m , from assumption (2e), We can consider either $m = T$ as a real scalar field, or $m = Q = Q + iU$ as a complex spin-2 field. Explicitly, we can write $A(t, \mathbf{r}) = e^{-si\alpha(t)} \delta^{(2)}(\mathbf{r} - \mathbf{r}(t))$ where α is the orientation of the detector, s is the spin of the field ($s^T = 0, s^Q = 2$). This can be used with assumption (3a) to rewrite the detector pointing w.r.t. the boresight pointing $A^i(t, \mathbf{r}) = e^{-si\Delta\alpha^i} A(t, R_i^{-1}\mathbf{r})$, where $R_i : \mathbf{r}(t) \mapsto \mathbf{r}^i(t)$, $\Delta\alpha^i \equiv \alpha^i(t) - \alpha(t)$ ⁴ are independent of t by assumption (3a)⁵. With this, note that we can rewrite an observation at A^i to an observation at A :

³ $\mathcal{O}[f]\{g\}(x)$ means an operator $\mathcal{O}$ is a higher-order function of f which transforms a function g to another function $\mathcal{O}\{g\}$ evaluated at x .

⁴Lower indices are used for the detector index in R and its variants for ease of typesetting.

⁵ R can depends on $\mathbf{r}$ by assumption (1b).

$$\begin{aligned}
 \mathcal{O}[A^i]\{m\}(t) &= \oint A^i(t, \mathbf{r}) m(\mathbf{r}) d^2\mathbf{r} \\
 &= e^{-si\Delta\alpha^i} \oint A(t, R_i^{-1}\mathbf{r}) m(\mathbf{r}) d^2\mathbf{r} \\
 &= e^{-si\Delta\alpha^i} \oint A(t, \mathbf{r}') m(R_i\mathbf{r}') d^2\mathbf{r}' \\
 &= \mathcal{O}[A]\{\mathcal{R}_i^{-1}\{m\}\}(t)
 \end{aligned}$$

where $\mathcal{R}\{m\}(\mathbf{r}) \equiv e^{si\Delta\alpha} m(R^{-1}\mathbf{r})$ is the active transform of the spin-s field m ⁶ (and hence $\mathcal{R}^{-1}\{m\}(\mathbf{r}) \equiv e^{-si\Delta\alpha} m(R\mathbf{r})$ is the passive transform.)

The resultant map per detector about the crosstalk in TOD would then be,

$$\begin{aligned}
 \tilde{m}^i &\equiv \mathcal{M}[A^i]\{\tilde{d}^i\} \\
 \mathcal{O}[A^i]\{\tilde{m}^i\} &= \tilde{d}^i = \sum_j X^{ij} \mathcal{O}[A^j]\{m\} \\
 \mathcal{O}[A]\{\mathcal{R}_i^{-1}\{\tilde{m}^i\}\} &= \sum_j X^{ij} \mathcal{O}[A]\{\mathcal{R}_j^{-1}\{m\}\} \\
 \mathcal{R}_i^{-1}\{\tilde{m}^i\} &= \sum_j X^{ij} \mathcal{R}_j^{-1}\{m\} \\
 \tilde{m}^i &= \sum_j X^{ij} \mathcal{R}_{i \rightarrow j}^{-1}\{m\}, \text{ or explicitly,} \tag{34.1} \\
 e^{-si\Delta\alpha^i} \tilde{m}^i(R_i\mathbf{r}) &= \sum_j X^{ij} e^{-si\Delta\alpha^j} m(R_j\mathbf{r}) \\
 \tilde{m}^i(\mathbf{r}') &= \sum_j X^{ij} e^{-si\alpha^{i \rightarrow j}} m(R_j R_i^{-1}\mathbf{r}') \\
 \tilde{m}^i(\mathbf{r}) &= \sum_j X^{ij} e^{-si\alpha^{i \rightarrow j}} m(R_{i \rightarrow j}\mathbf{r})
 \end{aligned}$$

where $\alpha^{i \rightarrow j} \equiv \Delta\alpha^j - \Delta\alpha^i$, $R_{i \rightarrow j} \equiv R_j R_i^{-1}$, $\mathcal{R}_{i \rightarrow j} \equiv \mathcal{R}_j \mathcal{R}_i^{-1}$. Finally, from assumption (2d) & (1a), the resultant map obtained in the mapmaking operation is,

$$\begin{aligned}
 m'(\mathbf{r}) &= \sum_i w^i \tilde{m}^i(\mathbf{r}), \quad \sum_i w^i = 1 \\
 &= \mathcal{X}m(\mathbf{r}) + \sum_{i,j \neq i} \mathcal{X}^{ij} \mathcal{R}_{i \rightarrow j}^{-1}\{m\}(\mathbf{r}) \tag{34.2}
 \end{aligned}$$

$$= \mathcal{X}m(\mathbf{r}) + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-si\alpha^{i \rightarrow j}} m(R_{i \rightarrow j}\mathbf{r}) \tag{34.3}$$

⁶This fixes the remaining d.o.f. of R in terms of $\Delta\alpha$.

where $\mathcal{X}^{ij} \equiv w^i X^{ij}$, $\mathcal{X} \equiv \text{tr}(\mathcal{X}^{ij})$. Note that in the limit that the crosstalk matrix is identity on the diagonal, $\mathcal{X} = 1$ by definition. i.e. The first term above recovers the original map, whereas the 2nd term is a first order correction as we normally have $X^{ij} \ll 1$. $\square$

This section is closed by observing when the assumptions are approximately true. Note that the assumption that mapmaking is linear in d is true for most mapmaking methods,⁷ while the assumption that it depends on A only are valid for mapmaking methods 1–3 of Tegmark⁸, which includes binned mapmaking (a.k.a. naïve mapmaking) and a maximal likelihood mapmaking.

34.1 Flat sky, fixed orientation, isotropic map coverage

To apply (34.3) under flat-sky approximation with a fixed focal plane orientation, one substitutes $R_i \rightarrow T_i$, the translation operator, and reinterpret $\mathbf{r}$ as the 2-dimensional angle in radian. This implies $R_{i \rightarrow j} \mathbf{r} \rightarrow T_{i \rightarrow j} \mathbf{r} = \mathbf{r} + \mathbf{r}^{i \rightarrow j}$,

$$\begin{aligned} m'(\mathbf{r}) &= \mathcal{X} m(\mathbf{r}) + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-s i \alpha^{i \rightarrow j}} m(\mathbf{r} + \mathbf{r}^{i \rightarrow j}) \\ &= (m * P)(\mathbf{r}) \equiv \int_{\mathbb{R}^2} m(\mathbf{r}') P(\mathbf{r} - \mathbf{r}') d^2 \mathbf{r}, \text{ where} \\ P(\mathbf{r}) &= \mathcal{X} \delta^{(2)}(\mathbf{r}) + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-s i \alpha^{i \rightarrow j}} \delta^{(2)}(\mathbf{r} + \mathbf{r}^{i \rightarrow j}) \end{aligned}$$

is the point spread function⁹. By convolution theorem¹⁰, $m'(\ell) = P(\ell) m(\ell)$, where

$$\begin{aligned} \mathcal{F}\{P\}(\ell) &= \int_{\mathbb{R}^2} P(\mathbf{r}) e^{-i \mathbf{r} \cdot \ell} d^2 \mathbf{r} \\ {}_s P(\ell) &= \mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{i(\mathbf{r}^{i \rightarrow j} \cdot \ell - s \alpha^{i \rightarrow j})} \end{aligned}$$

⁷Methods 1–8 in Tegmark, “How to Make Maps from Cosmic Microwave Background Data Without Losing Information”, table 1.

⁸“How to Make Maps from Cosmic Microwave Background Data Without Losing Information”.

⁹Note that the point spread function convolution here is analogous to beam convolution with $P(\mathbf{r}) = B(-\mathbf{r})$.

¹⁰Physicists’ convention is followed here. The same symbol to represent the function before or after Fourier Transform depends on the argument, and the Fourier Transform convention used is standard in the CMB literature, namely, defined with angular frequency in the non-unitary gauge where only the inverse transform has a factor of 2π .

Hence, this is how the fields are transformed under crosstalk,

$$\begin{aligned} T'(\ell) &= {}_0P(\ell)T(\ell) \\ \mathcal{Q}'(\ell) &= {}_2P(\ell)\mathcal{Q}(\ell) \end{aligned}$$

With the following standard equations to propagate from $\mathcal{Q}$ to E, B , with $\mathcal{E} \equiv E + iB$,

$$\begin{aligned} \mathcal{E}(\ell) &= \mathcal{Q}(\ell)e^{-2i\phi_\ell} \\ E(\ell) &= \frac{1}{2} [\mathcal{E}(\ell) + \bar{\mathcal{E}}(-\ell)] \\ B(\ell) &= \frac{1}{2i} [\mathcal{E}(\ell) - \bar{\mathcal{E}}(-\ell)] \end{aligned}$$

With some derivation and simplification, and defining $\mathcal{A}^\top(\ell) = [T(\ell) \ E(\ell) \ B(\ell)]$, and expanding the sum inside P^{11} ,

$$\mathcal{A}'(\ell) = \begin{bmatrix} {}_0P(\ell) & 0 & 0 \\ 0 & \Re\{{}_2P(\ell)\} & -\Im\{{}_2P(\ell)\} \\ 0 & \Im\{{}_2P(\ell)\} & \Re\{{}_2P(\ell)\} \end{bmatrix} \mathcal{A}(\ell) \quad (34.4)$$

$$= \left[\mathcal{K} + \sum_{i,j \neq i} \mathcal{K}^{ij} \mathcal{I}^{ij}(\ell) \right] \mathcal{A}(\ell) \quad (34.5)$$

$$\begin{aligned} \mathcal{I}(\mathbf{r} \cdot \ell, \alpha) &\equiv \begin{bmatrix} e^{i\mathbf{r} \cdot \ell} & 0 & 0 \\ 0 & \cos(\mathbf{r} \cdot \ell - 2\alpha) & -\sin(\mathbf{r} \cdot \ell - 2\alpha) \\ 0 & \sin(\mathbf{r} \cdot \ell - 2\alpha) & \cos(\mathbf{r} \cdot \ell - 2\alpha) \end{bmatrix} \\ \mathcal{I}^{ij}(\ell) &\equiv \mathcal{I}(\mathbf{r}^{i \rightarrow j} \cdot \ell, \alpha^{i \rightarrow j}) \end{aligned}$$

Writing the usual power spectrum C_ℓ in terms of $\mathcal{A}$,

$$\begin{aligned} \mathbb{C}(\ell) &\equiv \mathcal{A}(\ell) \mathcal{A}^\dagger(\ell) \\ \mathbb{C}_\ell &\equiv \langle \mathbb{C}(\ell) \rangle_{\hat{\ell}} \equiv \langle \mathcal{A}(\ell) \mathcal{A}^\dagger(\ell) \rangle_{\hat{\ell}} \\ &= \begin{bmatrix} C_\ell^{TT} & C_\ell^{TE} & C_\ell^{TB} \\ C_\ell^{ET} & C_\ell^{EE} & C_\ell^{EB} \\ C_\ell^{BT} & C_\ell^{BE} & C_\ell^{BB} \end{bmatrix} \end{aligned}$$

Expanding using (34.5), we have¹²,

¹¹ $\Re, \Im$ below are the real part and imaginary part operators.

¹²h.c. stands for Hermitian Conjugate.

$$\begin{aligned}
 \mathbb{C}'_\ell &= \mathcal{X}^2 \mathbb{C}_\ell + \mathcal{X} \sum_{i,j \neq i} \mathcal{X}^{ij} \langle \mathcal{J}^{ij}(\ell) \mathbb{C}(\ell) + \text{h.c.} \rangle_{\hat{\ell}} + \dots \\
 &= \mathcal{X}^2 \mathbb{C}_\ell + \mathcal{X} \sum_{i,j \neq i} \mathcal{X}^{ij} (\mathcal{J}_\ell^{ij} \mathbb{C}_\ell + \text{h.c.}) + \dots
 \end{aligned} \tag{34.6}$$

$$\begin{aligned}
 \mathcal{J}_\ell^{ij} &\equiv \langle \mathcal{J}^{ij}(\ell) \rangle_{\hat{\ell}} \\
 &= J_0(r^{i \rightarrow j} \ell) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha^{i \rightarrow j} & \sin 2\alpha^{i \rightarrow j} \\ 0 & -\sin 2\alpha^{i \rightarrow j} & \cos 2\alpha^{i \rightarrow j} \end{bmatrix} \\
 &= J_0(r^{i \rightarrow j} \ell) R_T^{-1}(2\alpha^{i \rightarrow j}) \\
 \mathbb{C}'_\ell &= \mathcal{X}^2 \mathbb{C}_\ell + \mathcal{X} \sum_{i,j \neq i} \mathcal{X}^{ij} J_0(r^{i \rightarrow j} \ell) [R_T^{-1}(2\alpha^{i \rightarrow j}) \mathbb{C}_\ell + \text{h.c.}] + \dots
 \end{aligned} \tag{34.7}$$

where the 2nd line has the statistical average applied on the inner $\mathbb{C}(\ell)$, J_α is the Bessel functions of the first kind, R_T is the rotational matrix over the T -axis, and only up to first-order in $\mathcal{X}^{ij}$ is kept. Note that it means that to first order, each of the crosstalk i, j component are modulated by a Bessel function and rotated in the EB -plane. Unlike a global mis-calibration, this rotation is not global as the individual contributions are different.

34.2 Flat sky, isotropic orientation, isotropic map coverage

In the previous section, the focal plane is assumed to be fixed in orientation w.r.t. the (flat-)sky when we replace R by T . However, in a more realistic situation, it will have different orientations as it scans across the same point at sky. That effect can be considered analytically if the scanning strategy is assumed to be isotropic in the focal plane orientation (assumption (1c)). Due to this isotropic averaging effect, the effective PSF is now the azimuthal average of the original PSF,

$$\begin{aligned}
 P'(r) &\equiv \langle P(\mathbf{r}) \rangle_{\hat{\mathbf{r}}} = \frac{1}{2\pi} \int_0^{2\pi} P(\mathbf{r}) d\theta \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \mathcal{X} \frac{\delta(r)}{2\pi r} \\
 &\quad + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-i\alpha^{i \rightarrow j}} \frac{\delta(r + r^{i \rightarrow j}) \delta(\theta + \theta^{i \rightarrow j})}{r} d\theta \\
 &= \frac{1}{2\pi r} \left[\mathcal{X} \delta(r) + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-i\alpha^{i \rightarrow j}} \delta(r + r^{i \rightarrow j}) \right]
 \end{aligned}$$

$$\begin{aligned}\mathcal{F}\{P'\}(\ell) &= \int_0^{2\pi} \int_0^\infty P'(r) e^{-ir\ell \cos \theta} r dr d\theta \\ {}_sP'_\ell &= \mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} e^{-si\alpha^{i \rightarrow j}} J_0(\ell r^{i \rightarrow j})\end{aligned}\quad (34.8)$$

Note that as P is azimuthally symmetric, it depends on ℓ only. Substituting (34.8) into (34.4),

$$\mathcal{A}'(\ell) = \left[\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} \mathcal{J}_\ell^{ij} \right] \mathcal{A}(\ell) \quad (34.9)$$

Following the same procedure as in last section, it is found that the resultant power-spectra is the same as that in (34.7) up to first order.

Comparing the results of this section with the last, it is found that the 2d-power-spectra in the previous section (34.5) is not statistically isotropic, while here (34.9) is. Furthermore, while the resultant 1d-power-spectra is the same up to 1st order, the 2nd order term is different, where in the previous section it is an average of products of $\mathcal{J}^{ij}(\ell)$ here it is a product of $\mathcal{J}_\ell^{ij}$ which is already averaged. This means the 2nd order term can be written explicitly,

$$\begin{aligned}\mathbb{C}'_\ell &= \mathcal{X}^2 \mathbb{C}_\ell + \mathcal{X} \sum_{i,j \neq i} \mathcal{X}^{ij} J_0(r^{i \rightarrow j} \ell) [R_T^{-1}(2\alpha^{i \rightarrow j}) \mathbb{C}_\ell + \text{h.c.}] \\ &\quad + \sum_{i,j \neq i, m, n \neq m} \mathcal{X}^{ij} \mathcal{X}^{mn} J_0(r^{i \rightarrow j} \ell) J_0(r^{m \rightarrow n} \ell) \times \\ &\quad \quad \quad R_T^{-1}(2\alpha^{i \rightarrow j}) \mathbb{C}_\ell R_T(2\alpha^{m \rightarrow n})\end{aligned}$$

34.3 Curved sky, isotropic orientation, isotropic map coverage

To generalize the above results to curved sky, it is necessary to digress to some definitions and theorems.

First, CMB power-spectra in curved sky is expanded in spin-weighted spherical harmonics (SWSH). Using a view that is popularized by,¹³ defines

¹³Boyle, “How Should Spin-Weighted Spherical Functions Be Defined?”

$$\begin{aligned}
 {}_s\mathcal{Y}_{\ell m}(\phi, \theta, \psi) &\equiv e^{-is\psi} {}_sY_{\ell m}(\theta, \phi) \\
 {}_s\mathcal{Y}_{\ell m}(R) &\equiv e^{-is\psi} {}_sY_{\ell m}(R\hat{\mathbf{z}}), \quad R \equiv R(\phi, \theta, \psi) \\
 &= (-1)^m \sqrt{\frac{2\ell+1}{4\pi}} D_{-ms}^{(\ell)}(R)
 \end{aligned}$$

${}_sY_{\ell m}$ is a spin- s function on the 2-sphere, i.e. it changes upon a local transformation. ${}_s\mathcal{Y}_{\ell m}$ is defined instead as a scalar function s.t. ${}_s\mathcal{Y}_{\ell m}: \text{SO}(3) \rightarrow \mathbb{C}$ ¹⁴. R is a rotation parameterized by the Euler angles¹⁵, and D is the Wigner- D matrix in Wigner's original sign convention.

The key to derive a curve-sky solution is to define a delta function on this $\text{SO}(3)$ space. Rewriting the delta function in $\mathbb{R}^2$ as $\delta^{R'}(\hat{\mathbf{n}}) \equiv \delta^{(2)}(R'\hat{\mathbf{n}} - \hat{\mathbf{z}})$ where R' is the pointing. Define a 2-dimensional spin- s delta function in $\text{SO}(3)$,

$${}_s\delta^{R'}(R) \equiv \delta^{(2)}(R'R = R_z(\alpha)) e^{-is\psi}$$

Note that α is an arbitrary angle, and hence after an integral over $\text{SO}(3)$, there remains a 1-dimensional integral.

The convolution with this delta function will then be¹⁶,

$$\begin{aligned}
 ({}_sf * {}_s\delta^{R'}) (R) &= \frac{1}{2\pi} \int_{R'' \in \text{SO}(3)} {}_sf(R'') {}_s\delta^{R'}(R''^{-1}R) dR'' \\
 {}_s\delta^{R'}(R''^{-1}R) &= \delta^{(2)}(R'R''^{-1}R = R_z(\alpha)) e^{-is\psi'''} \\
 R''' &\equiv R''^{-1}R \\
 &= R'^{-1}R_z(\alpha) \\
 R(\phi, \theta, \psi) &= R_z(\phi)R_y(\theta)R_z(\psi) \Rightarrow \psi''' = -\phi' + \alpha \\
 R'' &= RR_z(-\alpha)R' \\
 ({}_sf * {}_s\delta^{R'}) (R) &= \frac{1}{2\pi} \oint {}_sf(RR_z(-\alpha)R') e^{is(\phi' - \alpha)} d\alpha \\
 &= \frac{1}{2\pi} \oint {}_sf(RR'(\phi' = \phi'')) e^{is\phi''} d\phi''
 \end{aligned}$$

Specializing at $R = I$,

¹⁴Spin(3) for half-integer spin.

¹⁵Using convention commonly used in quantum mechanics, namely, the z-y-z, righthand, active transformation.

¹⁶See sec. 38.6.

$$\begin{aligned} ({}_sf * {}_s\delta^{R'}) (I) &= \frac{1}{2\pi} \oint {}_sf(R'(\phi' = \phi'')) e^{is\phi''} d\phi'' \\ &= \frac{1}{2\pi} \oint {}_sf(R_z(\phi'')R'(\phi' = 0)) e^{is\phi''} d\phi'' \end{aligned}$$

The above shows that it is performing a spin-weighted azimuthal average around R' . i.e. with assumption (1c, 3b) and eq. (34.1), (34.3), the convolved map is, using the SO(3) notation,

$$\begin{aligned} {}_sm'(R) &= (m * {}_sP)(R), \text{ where} \\ {}_sP(R) &\equiv \mathcal{X}_s \delta^I(R) + \sum_{i,j \neq i} \mathcal{X}^{ij} {}_s\delta^{i \rightarrow j}(R) \\ {}_s\delta^{i \rightarrow j} &\equiv {}_s\delta^{R^{i \rightarrow j}} \end{aligned}$$

Assumption (1c) is implicit when the convolution is used which has an azimuthal averaging effect as explained above, corresponds to a scanning strategy with isotropic focal plane orientation.

Then, the harmonic expansion of this delta function can be derived,

$$\begin{aligned} {}_s\delta_{\ell m}^{R'} &= \oint {}_s\delta^{R'}(R) {}_s\bar{\mathcal{Y}}_{\ell m}(R) d\Omega \\ &= \oint \delta^{(2)}(R'R = R_z(\alpha)) {}_s\bar{Y}_{\ell m}(\theta, \phi) d\Omega \\ &= {}_s\bar{Y}_{\ell m}(R'^{-1}\hat{z}) = {}_s\bar{\mathcal{Y}}_{\ell m}(R'^{-1})|_{\phi'=0} \\ &= (-1)^{-m} \sqrt{\frac{2\ell+1}{4\pi}} d_{s,-m}^{\ell}(\theta') e^{-is\psi'} \end{aligned}$$

And that of P ,

$${}_s({}_sf * {}_sP)_{\ell m} = {}_sf_{\ell m} \left(\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} d_{ss}^{\ell}(\theta^{i \rightarrow j}) e^{-is\alpha^{i \rightarrow j}} \right)$$

where d is the Wigner d matrix.

Equip with this, the effect of crosstalk on the spherical harmonics can be derived,

$$\begin{aligned} a'_{\ell m} &\equiv {}_0(T * {}_0P)_{\ell m} = a_{\ell m} \left(\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} \mathcal{P}_\ell(\cos \theta^{i \rightarrow j}) \right) \\ {}_2a'_{\ell m} &\equiv {}_2(\mathcal{Q} * {}_2P)_{\ell m} = {}_2a_{\ell m} \left(\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} d_{22}^\ell(\theta^{i \rightarrow j}) e^{-2i\alpha^{i \rightarrow j}} \right) \\ -{}_2a'_{\ell m} &= -{}_2a_{\ell m} \left(\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} d_{22}^\ell(\theta^{i \rightarrow j}) e^{2i\alpha^{i \rightarrow j}} \right) \end{aligned}$$

Defining $\mathcal{A}_{\ell m}^\top = [a_{\ell m}^T \quad a_{\ell m}^E \quad a_{\ell m}^B]$, the effect of crosstalk on $\mathcal{A}$ would be,

$$\begin{aligned} \mathcal{A}'_{\ell m} &= \left[\mathcal{X} + \sum_{i,j \neq i} \mathcal{X}^{ij} \mathcal{J}_\ell^{ij} \right] \mathcal{A}_{\ell m} \\ \mathcal{J}_\ell(\theta, \alpha) &\equiv \begin{bmatrix} \mathcal{P}_\ell(\cos \theta) & 0 & 0 \\ 0 & d_{22}^\ell(\theta) \cos 2\alpha & d_{22}^\ell(\theta) \sin 2\alpha \\ 0 & -d_{22}^\ell(\theta) \sin 2\alpha & d_{22}^\ell(\theta) \cos 2\alpha \end{bmatrix} \\ \mathcal{J}_\ell^{ij} &\equiv \mathcal{J}(\theta^{i \rightarrow j}, \alpha^{i \rightarrow j}) \end{aligned}$$

Where $\mathcal{P}$ is the Legendre polynomials¹⁷. Defining the 2-point power-spectra as,

$$\begin{aligned} \mathbb{C}_\ell &\equiv \langle \mathcal{A}_{\ell m} \mathcal{A}_{\ell m}^\dagger \rangle_m \\ &\equiv \begin{bmatrix} C_\ell^{TT} & C_\ell^{TE} & C_\ell^{TB} \\ C_\ell^{ET} & C_\ell^{EE} & C_\ell^{EB} \\ C_\ell^{BT} & C_\ell^{BE} & C_\ell^{BB} \end{bmatrix} \end{aligned}$$

The resultant power-spectra with crosstalk is computed as,

$$\mathbb{C}'_\ell = \mathcal{X}^2 \mathbb{C}_\ell + \left[\mathcal{X} \sum_{i,j \neq i} \mathcal{X}^{ij} \mathcal{J}_\ell^{ij} \mathbb{C}_\ell + \text{h.c.} \right] + \sum_{i,j \neq i, m, n \neq m} \mathcal{X}^{ij} \mathcal{X}^{mn} \mathcal{J}_\ell^{ij} \mathbb{C}_\ell \mathcal{J}_\ell^{\dagger mn} \quad (34.10)$$

This can be compared with the derivation in sec. 34.1 and the result in sec. 34.2. Their relationship with each other is more apparent, given $\mathcal{P}_\ell(\cos \theta) = d_{00}^\ell(\theta)$ and¹⁸

$$\mathcal{P}_\ell(\cos \theta) = \sqrt{\frac{\theta}{\sin \theta}} J_0 \left(\left(\ell + \frac{1}{2} \right) \theta \right) \left(1 + \mathcal{O} \left(\frac{1}{\ell} \right) \right),$$

¹⁷ $\mathcal{P}$ is used instead of the usual P as it is already used for the PSF above.

¹⁸ Technology (U.S.), *NIST Handbook of Mathematical Functions Hardback and CD-ROM*, eq. 14.15.11.

where $\theta \ll 1$ from assumption (3b).

Lastly, note that keeping C'_ℓ up to first order is easier to understand the crosstalk effect on the resultant power-spectra qualitatively, while $\mathcal{A}'_{\ell m}$ is more useful to evaluate the exact¹⁹ crosstalk effect. Notice how the crosstalk effect is independent of the signal and is completely captured by the hardware information only, which includes the crosstalk matrix, detector noise, focal plane configuration, etc. This means that it can be computed very fast without performing computationally expensive TOD simulations. This prediction can be used to design the hardware to minimize crosstalk effect with very fast iterations.

Below we will expand eq. (34.10) in a special case which will be useful later.

2-detectors with equal weights

If we take a 2-detector case with symmetric crosstalk matrix and equal weighting between them,

$$X = \begin{bmatrix} 1 & 2x \\ 2x & 1 \end{bmatrix} \Rightarrow \mathcal{X}^{ij} = \begin{bmatrix} \frac{1}{2} & x \\ x & \frac{1}{2} \end{bmatrix} \Rightarrow \mathcal{X} = 1.$$

Assuming $C_\ell^{TB} = C_\ell^{EB} = 0$, expanding eq. (34.10),

$$\begin{aligned} C_\ell'^{TT} &= C_\ell^{TT} (xP_\ell + 1)^2 \\ C_\ell'^{EE} &= d_{22}^2 x^2 \sin^2(2\alpha) C_\ell^{BB} + C_\ell^{EE} (d_{22} x \cos(2\alpha) + 1)^2 \\ C_\ell'^{BB} &= C_\ell^{BB} (d_{22} x \cos(2\alpha) + 1)^2 + d_{22}^2 x^2 \sin^2(2\alpha) C_\ell^{EE} \\ C_\ell'^{TE} &= C_\ell^{TE} (xP_\ell + 1) (d_{22} x \cos(2\alpha) + 1) \\ C_\ell'^{TB} &= -x d_{22} \sin(2\alpha) C_\ell^{TE} (xP_\ell + 1) \\ C_\ell'^{EB} &= d_{22} x \sin(2\alpha) (C_\ell^{BB} - C_\ell^{EE}) (d_{22} x \cos(2\alpha) + 1) \end{aligned} \tag{34.11}$$

with some of the functions suppressed, such as $P_\ell \equiv P_\ell(\cos \theta)$, $d_{22} \equiv d_{22}^\ell(\theta)$, where θ is the angular distance on sky between the 2 detectors, and α is the angle between the polarization of the 2 detectors. The expression is more simplified in the case of flat-sky approximation where $P_\ell(\cos \theta) \sim d_{22}^\ell(\theta) \sim J_0(\ell\theta)$ as mentioned above.

There are a few features worth pointing out here. α does not affect TT , as expected. For EE, BB , there is a symmetry in the equations in swapping between them. Lastly, the $E \rightarrow B$ leakage induced by crosstalk systematics is 2nd order in x , which can still be significant as $C_\ell^{EE} \gg C_\ell^{BB}$.

¹⁹Exact as long as the assumptions are valid. But it is still an approximation if the assumptions are only approximately true.

Chapter 35

Simulating crosstalk systematics in the time-domain

In this section, we will focus on how to simulate crosstalk systematics from time domain simulations.

35.1 Crosstalk as a computational problem

Fundamentally, crosstalk simulation is a matrix multiplication problem. So let us take a look in the computational details first.

The software of choice in our simulation is [Time Ordered Astrophysics Scalable Tools \(TOAST\)](#), a software framework simulating and processing TODs in CMB observations and perform data compression into maps. It is designed with High-Performance Computing (HPC) in mind and is [endorsed by NERSC](#), and for example, is used in the processing of the Planck satellite mission data.

Needless to say, TOAST is a massively parallel application. For example, [fig. 35.1](#) shows the data distribution across MPI groups and ranks. The main takeaway here is that per group, TODs from different detectors is distributed across different rank, where each rank may hold data from a different total number of detectors. Say if there are a total of 10 detectors and 4 ranks, some rank may hold data from 2 or 3 detectors.

Our task is to apply a matrix multiplication on all detectors under this data distribution pattern. And from the hardware simulation using SQUID parameters in [chapter 33](#), the crosstalk matrix is dense². We should note that block-diagonal structure can be broken down into separate dense matrix operations, as we can loop over each block (typically a wafer) and perform the dense matrix multiplication there.

¹Kisner, “TOAST Workshop”.

²While the n -th diagonal in the crosstalk matrix is roughly n -th order, later we will see that the first few leading order effects can be approximately canceling. Hence, Without Loss of Generality (WLOG), it should be treated as dense and can only be treated as sparse when justified by simulations.

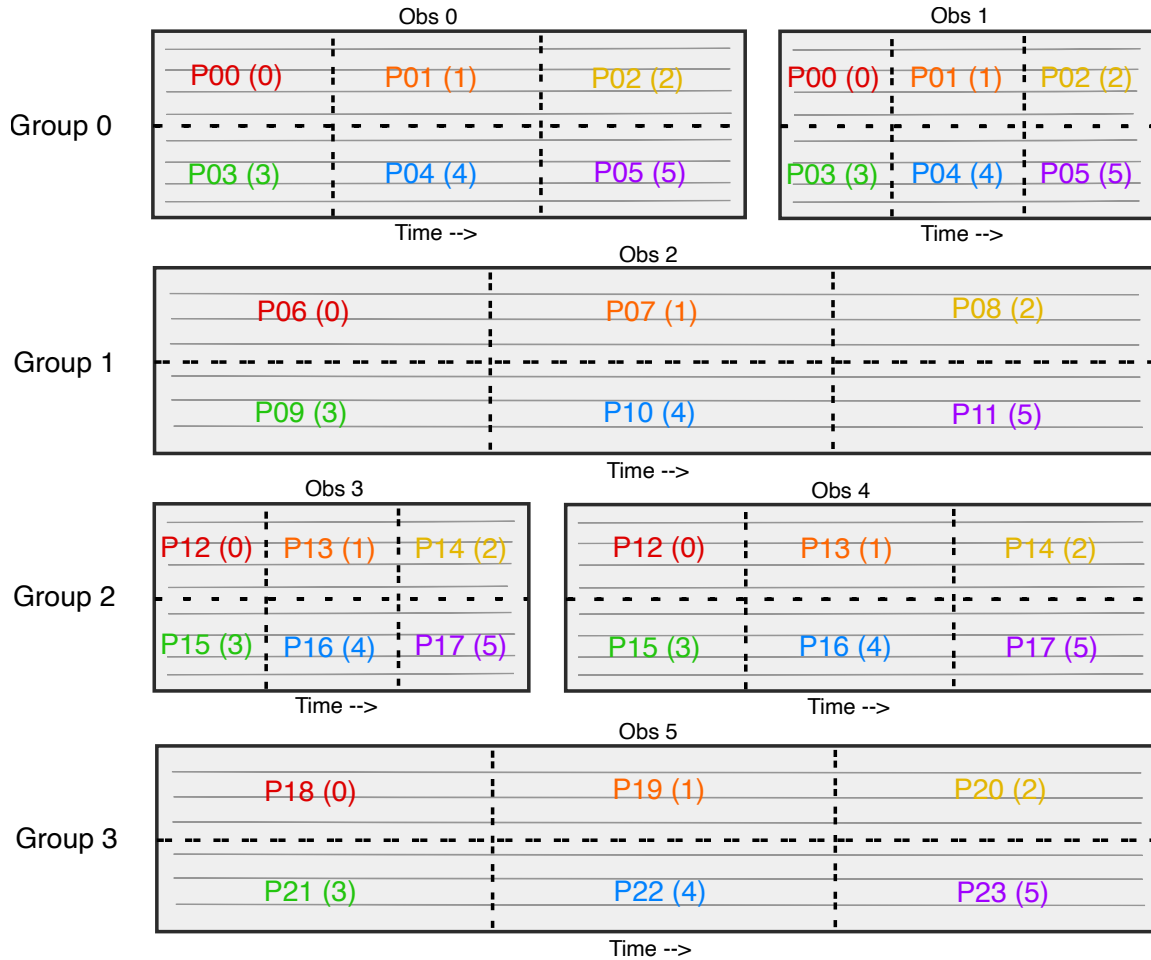


Figure 35.1: TOAST data distribution¹

Since we are having a massively parallel application, and the fact that TOAST is already memory limited for other operations such as the Madam maker, the goal is to minimize the amount of MPI communications needed and the memory overhead in computing the intermediate results.

35.2 OpCrosstalk

Each detector owner, the rank which holds the detector TOD locally, eventually needs the TODs from all other detectors and hence all other ranks within a group, as per detector this is just a weighted sum of all detectors. Because of this, we have to loop over each detector, corresponding to a row in the crosstalk matrix.

Per row, to minimize the MPI communication, a weighted sum is first performed

within a local rank who owns a couple of detectors already, and after that, a single MPI-Reduce is called to perform the global sum, which costs around $O(\log n_{\text{rank}})$ communication time.

Since we loop over the number of detectors, the total communication cost is $O(n_{\text{det}} \log n_{\text{rank}})$.

To minimize memory use, note that as someone once said, “the Python garbage collector is garbage.” We use an internal facility from TOAST to create new arrays. While it has a Numpy array interface, it is allocated in C++ directly where we can manually free them to avoid memory overhead.

With this, $(2n_{\text{local dets}} + 1) \times n_{\text{samples}}$ memory is needed, comparing to $n_{\text{local dets}} \times n_{\text{samples}}$ originally needed, this requires double memory as each rank need to hold the intermediate results during the whole matrix multiplication process.

However, with an implementation details in TOAST that typically n_{obs} observations are performed in a run, such as $n_{\text{obs}} = 365$ for a year of satellite mission, this doubling of memory requirements only occur within each observation, so the actual memory increase is negligible.

Currently, technological limit places n_{det} per SQUID to be around ~ 100 . And since $n_{\text{rank}} \leq n_{\text{det}}$, the computational aspect appears to be well under control. An example run below with $n_{\text{det}} \sim 60$ has negligible amount of compute time comparing to the total duration with other expensive operation such as Madam mapmaker.

OpCrosstalk is also general purpose and is not tied to a particular CMB experiments, and will benefits any future CMB experiments that uses TOAST such as SO and CMB-S4. The pull request has been merged to TOAST and is available in <https://github.com/hpc4cmb/toast/pull/380>.

Chapter 36

Crosstalk systematics simulations with hardware based mitigation

36.1 Introduction

This section put everything together and demonstrate the simulation of the crosstalk systematics using the LiteBIRD Instrument Model (IMo) v1 as an example.

Terminologies & conventions of detectors

Fig. 36.1 shows an example focal plane using the MFT central wafer at 140 GHz.

Table 36.1: Q/U -pixel vs. Top/Bottom-ness

	T	B
Q	0°	90°
U	45°	135°
color	light	dark

Table 36.2: Q/U -pixel & handedness colors

	A	B
Q	Blue	Red
U	Green	Orange

Q detectors are represented by arrows at 0° , 90° , and U detectors are represented by arrows at 45° , 135° . They are color-coded in brightness as summarized in tbl. 36.1.

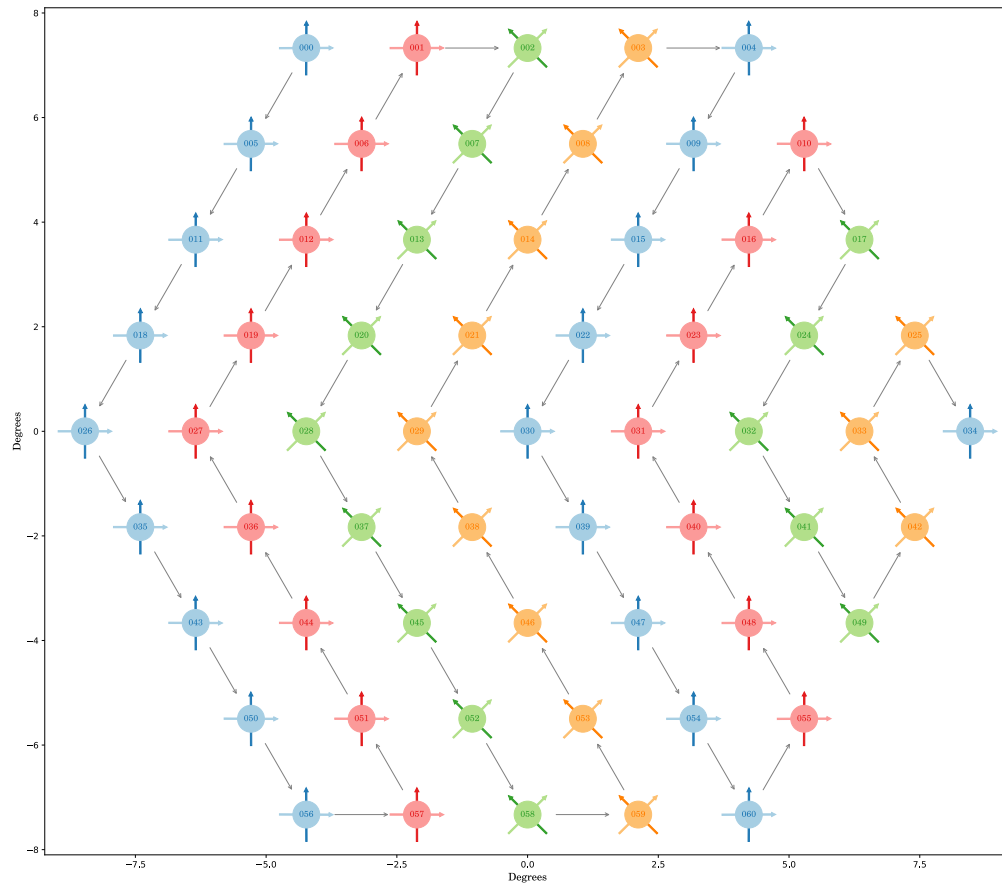


Figure 36.1: An example focal plane using the MFT central wafer at 140 GHz.

Then there are 2 kinds of detector handedness where one is a mirror image of another. They are simply named A, B . The 4 combinations of Q, U and A, B are color-coded according to tbl. 36.2. Hence, the color blue, red, green, orange corresponds to QA, QB, UA, UB.

Also note that n -th nearest neighbor can be in focal plane (angular distance) or in SQUID (frequency) depending on the context.

36.2 2-detectors simulations

Decomposition of crosstalk matrix

First, a constant crosstalk matrix with only non-zero elements in the nearest neighbor (in SQUID) is considered. In the limit of infinitely long SQUID, it can be decomposed in a superposition of 2-detectors crosstalk matrix like this:

$$\begin{bmatrix} 1/2 & x & 0 & \dots \\ x & 1/2 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots \\ 0 & 1/2 & x & \dots \\ 0 & x & 1/2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} + \dots = \begin{bmatrix} 1/2 & x & 0 & 0 & \dots & \dots \\ x & 1 & x & 0 & \dots & \dots \\ 0 & x & 1 & x & \dots & \dots \\ 0 & 0 & x & 1 & \ddots & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & x \\ \vdots & \vdots & \vdots & 0 & x & 1/2 \end{bmatrix}$$

The decomposition is exact in TOD It should be approximately true in map-domain and eventually power-spectra-domain as well if $x \ll 1$.

Moreover, in a realistic crosstalk matrix simulated from SQUID, it is approximately true that the next nearest neighbors (in SQUID) are of lower orders, whereas the nearest neighbor crosstalk is of same order of magnitude. (See more in Montgomery¹ or below.)

This means that if the 2-detector case is understood, it can be generalized to a long SQUID approximately.

2-detectors simulation results

Below are the systematic effects of a pair of detectors in $\frac{C_{\ell}^{\text{crosstalk}}}{C_{\ell}}$,

- Using IMO v1.0 parameters,
- Modified from the central pixel of MFT at 140 GHz,
- Scanning from a synfast input map using Planck 2018 best-fit Λ CDM cosmology with 183 days of observations,

¹“Statistical Scatter in LC Resonators and LiteBIRD Crosstalk Simulations”.

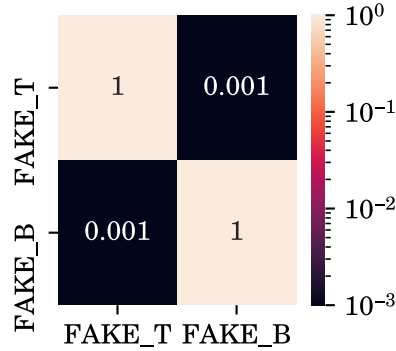


Figure 36.2: 2-detectors crosstalk matrix

- At an angular distance of 2.114° (corresponding to the mean lattice angular distance of the MFT hexagonal wafer),
- varying the polarization angles of the 1st and 2nd detectors.

Below, 1st number is 1st detector polarization angle, last number is 2nd detector polarization angle.

fig. 36.3 shows the result of the simulations, presented as a transfer function of the power-spectra. These can be compared to the analytic formula in sec. 34.3 shown in the plot. There is a high level of agreement, with the exception of damping towards high- ℓ , which is a effect similar to the PWF as the analytical model assumes a delta function, while in mapmaking, finite size pixels are used.

Lesson learnt—rule of thumb of wafer ordering

Combining the empirical simulation results and the analytical behavior expected from (34.11), the crosstalk systematics have minimal effects on the resultant EE, EB, BB power-spectra in these special cases,

1. $\theta = 0$, or
2. for α to be either

$$\begin{aligned}
 &1. \alpha = 45^\circ \text{ or } 135^\circ \rightarrow \frac{C_\ell^{\text{crosstalk}}}{C_\ell} = 0 \\
 &2. \frac{C_\ell^{\text{crosstalk}}(\alpha=0^\circ)}{C_\ell} = -\frac{C_\ell^{\text{crosstalk}}(\alpha=90^\circ)}{C_\ell}
 \end{aligned}$$

Now, we can apply what we learnt in wafer & SQUID *design* using the following empirical rules. *Design* here means that given a wafer, there are considerable degree of freedom to how to wire them together (ordering in SQUID) and how to *walk* the wafer. Here is an empirical algorithm in doing so.

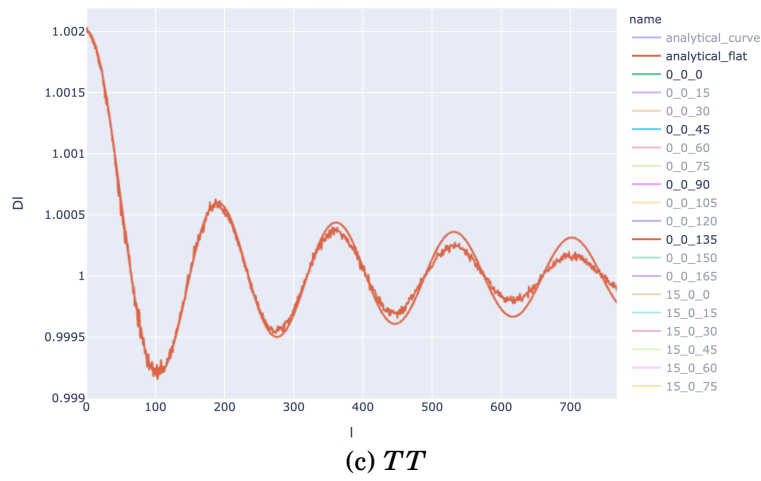
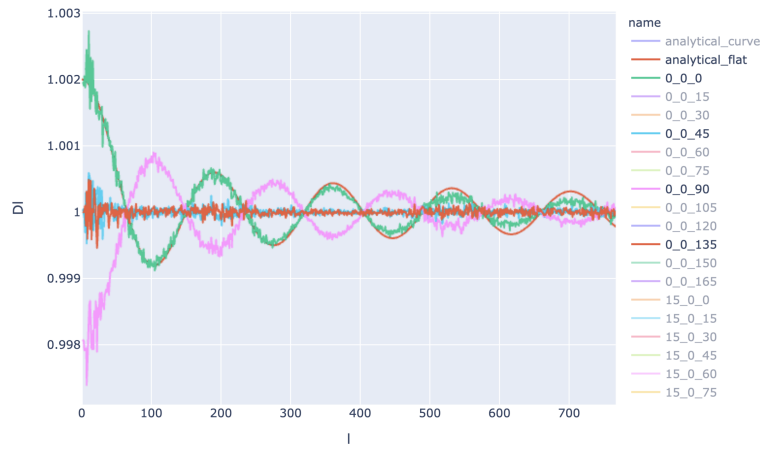
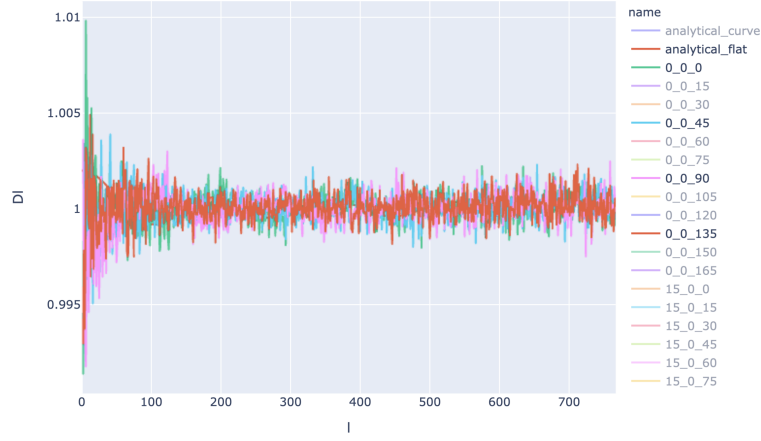


Figure 36.3: 2-detectors crosstalk systematics expressed as $\frac{C_{\ell}^{\text{crosstalk}}}{C_{\ell}}$.

Corollary 2. *Rule-of-thumb to design the wafer to minimize crosstalk systematics.*

1. *Walk within the same pixel first, as this has no bias ($\theta = 0$).*
2. *Then walk to the nearest neighbors on the focal plane (note that in hexagonal wafer this is always injecting systematics in the same angular scale θ , but for rectangular one it has 2 angular scales). This rule is from limitation in how they can be wired physically.*

Calculate α , the angle turned when walking from one pixel to another, following the following preferences:

- a) *Prefer 45° or 135° , as (34.11) shows this has no bias up to 1st order², otherwise,*
- b) *if a pair has a rotation of 0° ($\cos 0 = 1$), balance it by another pair that rotates by 90° ($\cos 90^\circ = -1$). Essentially, for the whole SQUID, $\sum_i \cos(2\alpha_i) = 0$ is desired as a rule of thumb³. If there are multiple angular scale in walking the focal plane (for example, in LFT), balance this for each angular scale individually.*

36.3 Full wafer simulations—Realistic wafer simulation with the example of central wafer of MFT at 140 GHz with 2 SQUIDs each having 61 detectors

Wafer mockups

In fig. 36.4, fig. 36.5, fig. 36.6, we look at 3 different wafer mockups ArrowTopLeftFirst, AlternateQU, & SpiralCounterclockwise_QUAB.

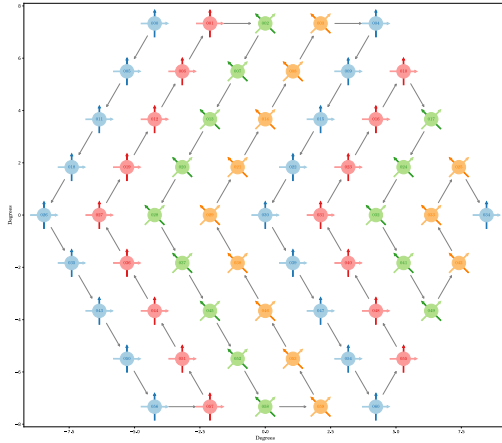
In the first 2 cases, ArrowTopLeftFirst, AlternateQU, the IMO v1 MFT central wafer at 140 GHz is taken as is.

In ArrowTopLeftFirst, it walks the focal plane by following the same QA line first, etc., i.e. most of the walking across pixels happens at 0° or 90° , this follows Corollary 2 2b.

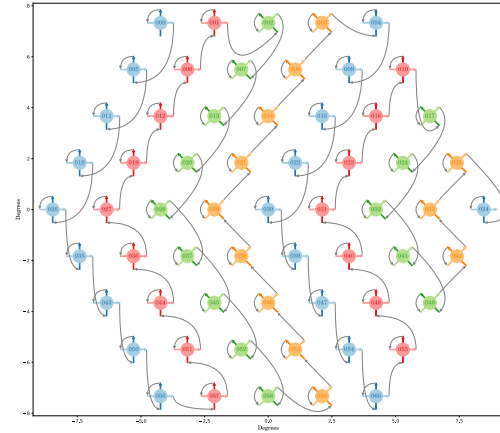
In AlternateQU, when crossing between pixels, it tries to go from a Q-pixel to a U-pixel as much as possible. This follows Corollary 2 2a. Note that since the wafer has more Q-pixels than U-pixels, the last few detectors cannot fulfill this.

²Note that however this maximize the $E \rightarrow B$ leakage at 2nd order in x . In practice, which rules is more advantageous depends on the separation of scales comparing higher-order crosstalk effects to E to B hierarchy.

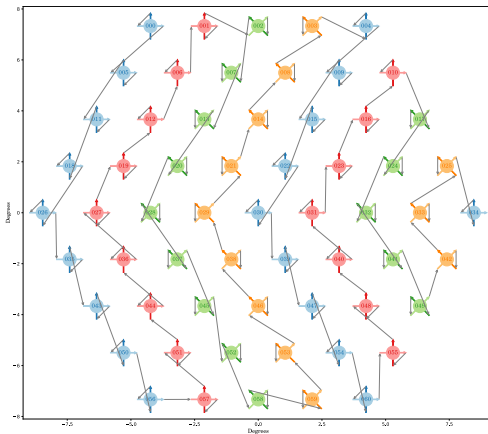
³Assuming 1st order crosstalk is roughly constant.



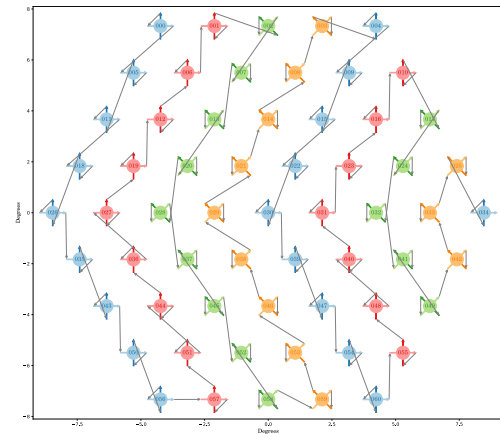
(a) ArrowTopLeftFirst



(b) ArrowTopLeftFirst-wire_TB

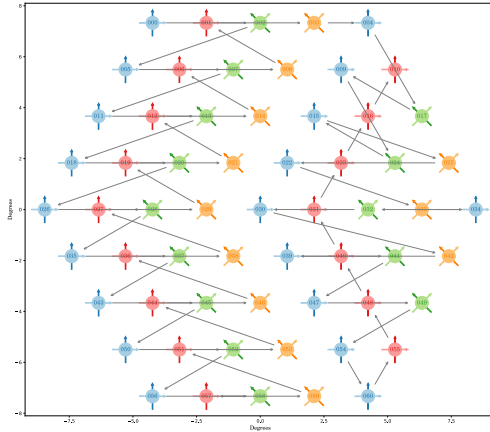


(c) ArrowTopLeftFirst_TB-alternate-anti-
alignment-wire_TB

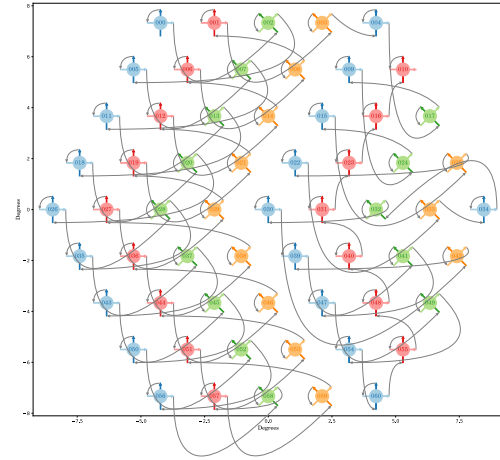


(d) ArrowTopLeftFirst_TB-anti-alignment-
wire_TB

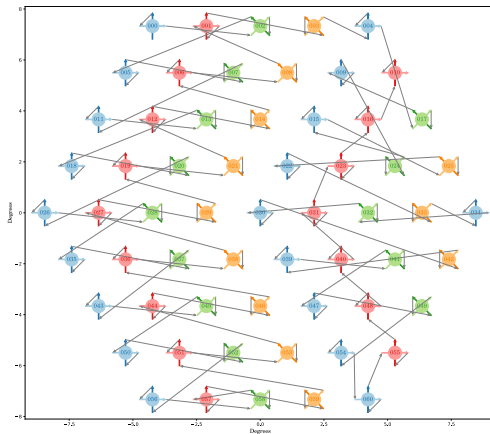
Figure 36.4: ArrowTopLeftFirst: a wafer design to walk the wafer. Top-left corner plot shows the ordering in pixel orders only. The other 3 plots shows the T/B -wiring explicitly, each with a different T/B ordering.



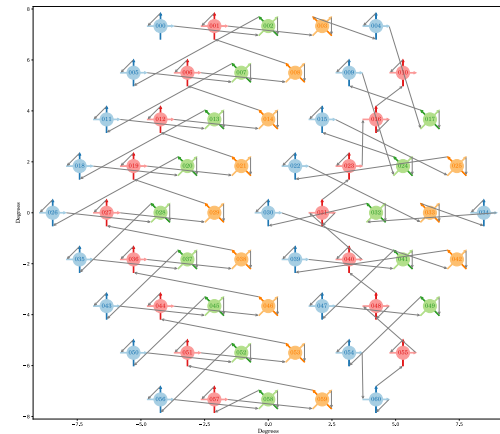
(a) AlternateQU



(b) AlternateQU-wire_TB

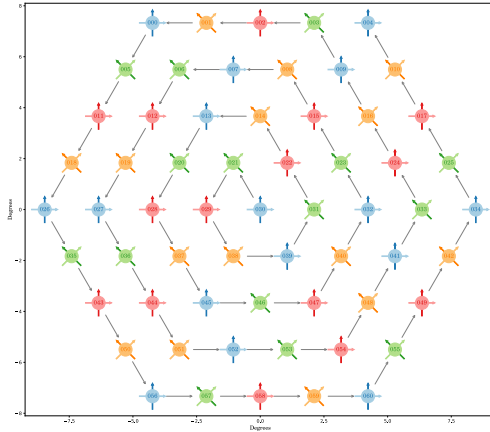


(c) AlternateQU_TB-alternate-anti-alignment-
wire_TB

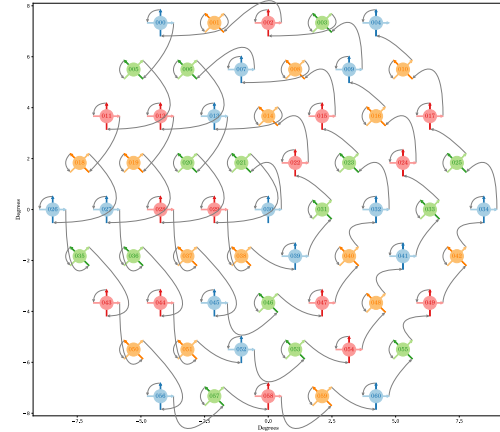


(d) AlternateQU_TB-anti-alignment-wire_TB

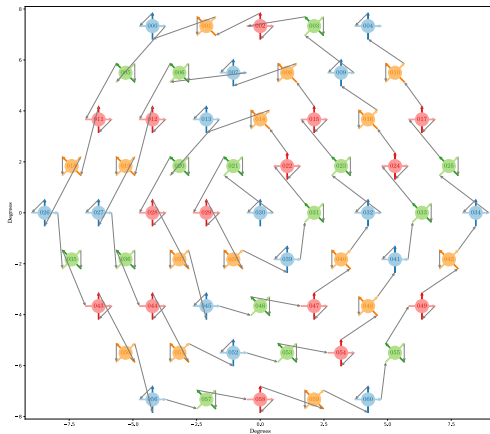
Figure 36.5: AlternateQU: a wafer design to walk the wafer. Top-left corner plot shows the ordering in pixel orders only. The other 3 plots shows the T/B -wiring explicitly, each with a different T/B ordering.



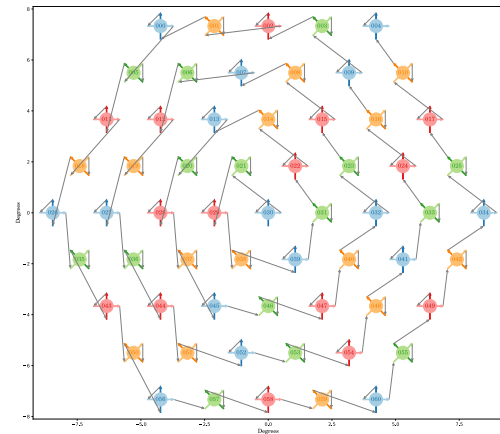
(a) SpiralCounterclockwise_QUAB



(b) SpiralCounterclockwise_QUAB-wire_TB



(c) SpiralCounterclockwise_QUAB_TB-alternate-
anti-alignment-wire_TB



(d) SpiralCounterclockwise_QUAB_TB-anti-
alignment-wire_TB

Figure 36.6: SpiralCounterclockwise_QUAB: a wafer design to walk the wafer. Top-left corner plot shows the ordering in pixel orders only. The other 3 plots shows the T/B -wiring explicitly, each with a different T/B ordering.

In the last case `SpiralCounterclockwise_QUAB`, the wafer is redesigned in a way that given a SQUID-ordering on the focal-plane, the ordering is alternated between Q and U -pixels. This follows Corollary 2 2a exactly by construction. Note that this has considerable degree of freedom and the only requirement is the number of Q and U -pixels are balanced.

“TB-alignment” represents when walking to the next detector, how to wire between the Top & Bottom detectors.

normal It always walk from a Bottom detector to a Top detector in a new location.

$$0T \rightarrow 0B \rightarrow 1T \rightarrow 1B \rightarrow 2T \rightarrow 2B \rightarrow 3T \rightarrow 3B \dots$$

This results in $\sum_i \cos(2\alpha_i)$ maximally negative.

TB-anti-alignment It always walk from a Bottom detector to a Bottom detector in a new location, similarly for Top to Top.

$$0T \rightarrow 0B \rightarrow 1B \rightarrow 1T \rightarrow 2T \rightarrow 2B \rightarrow 3B \rightarrow 3T \dots$$

This results in $\sum_i \cos(2\alpha_i)$ maximally positive.

TB-alternate-anti-alignment It mixes the above 2 strategies alternatively.

$$0T \rightarrow 0B \rightarrow 1B \rightarrow 1T \rightarrow 2B \rightarrow 2T \rightarrow 3T \rightarrow 3B \dots$$

This results in $\sum_i \cos(2\alpha_i) \approx 0$ by alternating between the 2.

These serve as examples. Other ordering following similar schemes are explored, which seems to indicate the general characteristics in the final power-spectra does not depend on the exact details of how they are ordered, as long as Corollary 2 is followed.

Full wafer simulation results

Similar to the 2-detectors case, but now with the wafer defined above, with 122 detectors and 4 days of observations (with a sky coverage around 60–70%).

For the crosstalk matrix, the following SQUID parameters are used, this is similar to the SPT-3G design⁴, slightly modified for the LiteBIRD case provided by Joshua Montgomery, with L_{com} chosen with the expectation of using $\mu\text{K-SQUID}$.

$$\begin{aligned} R_{\text{TES}} &= 0.6\Omega \\ r_s &= 0.3\Omega \\ L &= 60\mu\text{H} \\ L_{\text{com}} &= 9\text{nH} \\ f_{\text{min}} &= 1.69\text{MHz} \\ f_{\text{max}} &= 5.49\text{MHz} \end{aligned}$$

⁴Montgomery et al., “Performance and Characterization of the SPT-3G Digital Frequency Multiplexed Readout System Using an Improved Noise and Crosstalk Model”.

with 68 detectors with SQUID frequencies spacing linearly between its min/max.
Then the highest 7 frequencies are removed to have a crosstalk matrix of 61 detectors,
as shown in fig. 36.7.

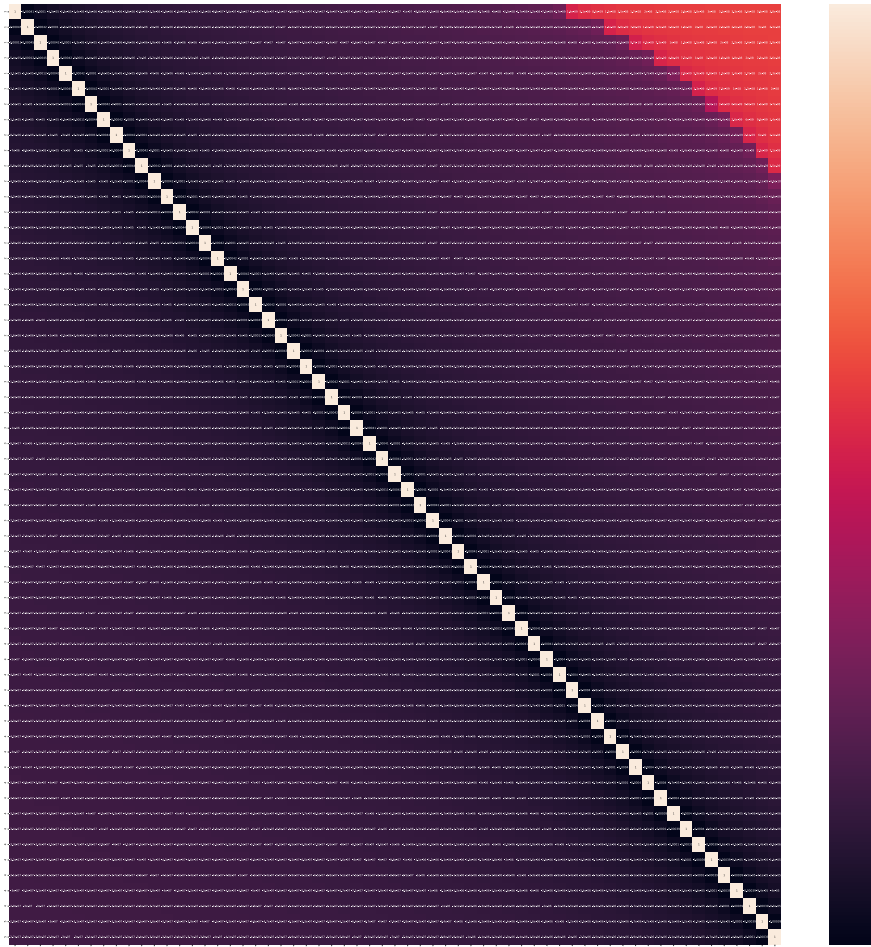


Figure 36.7: Analytical crosstalk matrix simulated from SQUID parameters

And 2 such SQUIDs are put in the focal plane in the previous sections. i.e. the first 61 detectors belong to the 1st SQUID, last 61 to the 2nd SQUID.

From fig. 36.8, one can observe,

- in polarizations,
 - the B -mode power-spectra is noisier comparing to the E -mode, but they follow a similar trend.
 - For walking order, `SpiralCounterclockwise_QUAB` > `AlternateQU` > `ArrowTopLeftFirst`. It is basically determined by the amount of Q - U pixel crossing when walking the wafer.
 - for TB -alignment, `TB-alternate-anti-alignment` is as bad as normal-ordering, and `TB-alternate-anti-alignment-ordering` is the best. As $\sum_i \cos(2\alpha_i) \approx 0$.
- in TT ,
 - `AlternateQU` is better. As systematics are injected at different angular scales, so the Legendre polynomial term partially cancelled between different contributions. In fact, if we walk the wafer randomly, this effect would be maximized.

36.4 Conclusions

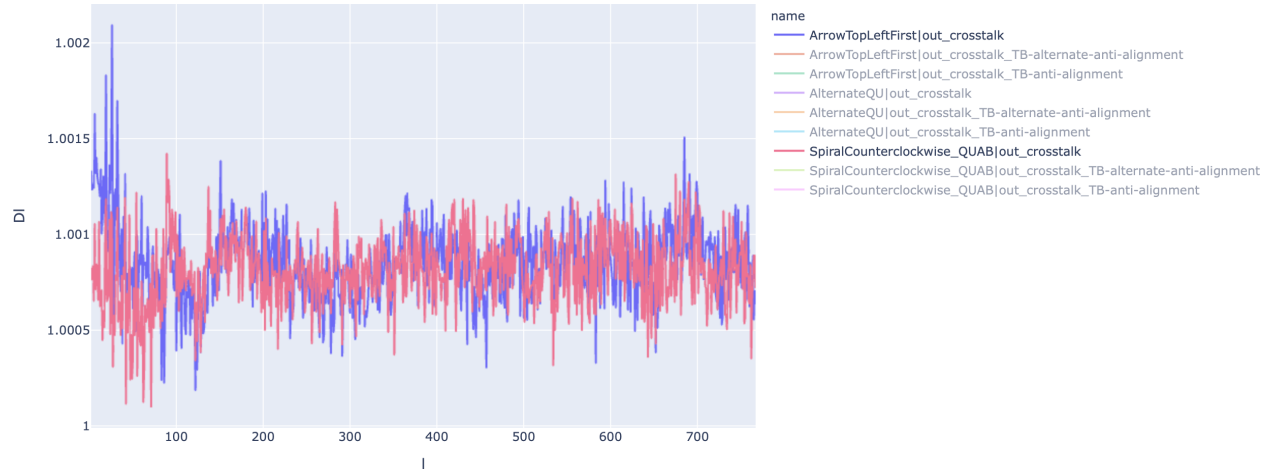
The QU -ordering optimization effectively removes crosstalk systematic bias in the resultant E -mode and B -mode power-spectra.

One reason that `SpiralCounterclockwise_QUAB` family is so effective, is that we can consider the ordering in a SQUID (below we use `SpiralCounterclockwise_QUAB_TB-alternate-anti-alignment-wire_TB` as an explicit example),

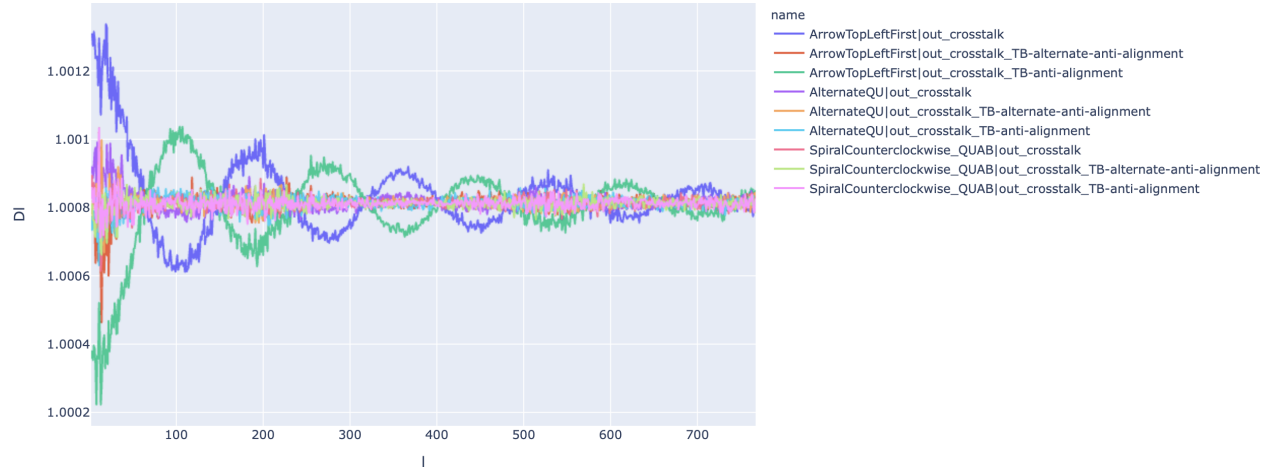
$$1QAT \rightarrow 1QAB \rightarrow 2UAB \rightarrow 2UAT \rightarrow 3QBT \rightarrow 3QBB \rightarrow 4UBB \rightarrow 4UBT \dots$$

Note that not only the nearest neighbor (in SQUID) is a nearest neighbor (in focal plane) crossing between Q & U pixels (such as $1QAB$ - $2UAB$), but it is true for the next nearest neighbor too (such as $1QAT$ - $2UAB$). So effectively this has mitigated the leading 2 orders of crosstalk in the matrices.

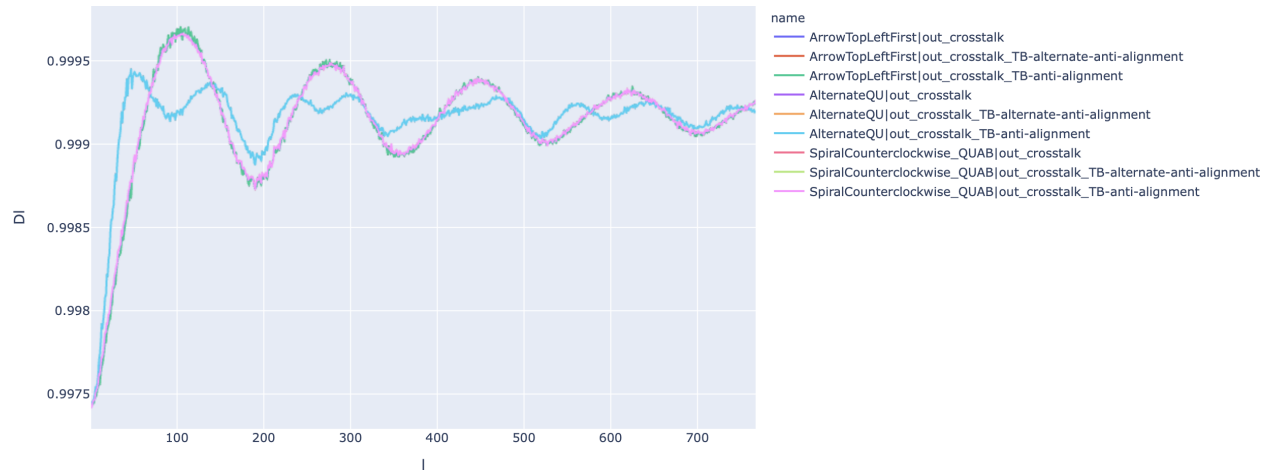
It is also superior in terms of physically wiring the wafer. For example, `ArrowTopLeftFirst_TB-alternate-anti-alignment-wire_TB` also has small bias due to $\sum_i \cos(2\alpha_i) \approx 0$, but it is considerably harder to wire because it requires a lot of crossing. Not to mention the other case `AlternateQU_TB-alternate-anti-alignment-wire_TB` which needs to go back and forth.



(a) *BB*



(b) *EE*



(c) *TT*

Figure 36.8: Full wafer crosstalk systematics expressed as $\frac{C_{\ell}^{\text{crosstalk}}}{C_{\ell}}$.

Instead, `SpiralCounterclockwise_QUAB` designs the wafer starting from how one wants to wire/walk them on the focal plane first, once that is done, Q/U -ness of the detectors is assigned accordingly. As the T/B -ordering does not matter in this design, one is free to choose the TB -ordering due to physical limitation (minimize crossing.)

We should emphasize that this mitigation technique depends on the presence of CRHWP, as its design is based on Corollary 2, which is derived from eq. (34.10), a consequence of an analytic model with a set of assumptions listed in Assumption 1. How well it is to generalize to other CMB experiments is remained to be explored. It is likely that any CMB experiments with CRHWP with a fairly large sky coverage and isotropic scanning strategy (which are typical design requirements) such as the SAT in SO can take advantage of this, because they fulfill Assumption 1 approximately.

If this mitigation technique is applied, the crosstalk requirements may, for example, be relaxed (which may allow larger multiplexing factor and in turn lower the payload & cost) by using this mitigation method, but the BB systematics have to be studied more in detail at large crosstalk limit.

Note that a popular technique to mitigate crosstalk systematics is to perform matrix inversion to remove the crosstalk in the TOD. This requires an accurate measurement of the crosstalk matrix, and the systematic uncertainties will be dominated by how accurate the crosstalk matrix can be measured. The proposed technique here can be used together with the matrix inversion technique, such that any residue crosstalk in the TOD can be mitigated by this wafer ordering technique.

There remains more crosstalk systematics to be explored, such as time-dependent crosstalk, and cross-frequency crosstalk.

Lastly, another possible exploration of the analytical structure seen here is to perform likelihood analysis using the power-spectra with the $P_\ell(\cos\theta)$, $d_{22}^\ell(\theta)$ as templates with θ at some of the characteristics angular scales of the focal plane on sky. This gives an advantage to remove residue crosstalk systematics, but reduces the statistical power of the experiment.

Part VII

Conclusion

Chapter 37

Summary

In part [I](#), a brief overview of the standard model of Big Bang cosmology is given. Unresolved problems such as fine-tuning problems can be resolved by introducing cosmic inflation. Inflation is generally believed to be true but has not been experimentally or observationally verified. Due to the extreme high energy involved, observation of the CMB radiation offers one of the best chance to falsify inflationary models.

In part [II](#), the physics of the CMB is briefly presented, showing the anisotropies in the intensity and polarization of the CMB hold a wealth of information to be extracted to better categorize and understand the universe. In particular, the primordial *B*-mode holds the key to falsify inflationary models.

In part [III](#), an overview of CMB observations is given. It ranges from the intrinsic properties of the CMB that affects experimental designs, to the mathematics of CMB data analysis, to the computational aspects including simulations and systematics mitigations, to the statistical aspects which enable us to extract information of the universe from the data. It concludes with the current measurements of the CMB and what to expect next.

In part [IV](#), the POLARBEAR experiment is introduced. Some of the key details on existing data analysis in POLARBEAR are given, such as the null-test framework, mapmaking method including CRHWP demodulation, and power-spectra estimation method.

Building on top of that, part [V](#) shows an original research that builds upon the observed data and existing data analysis framework. In particular, a measurement of the high- ℓ *B*-mode in the range of 600–3000 using 3rd–5th seasons of POLARBEAR data is presented. It started with the expected sensitivity that should have been known before taking the observation. It also pointed out the major challenges experienced in this research including computational efficiency, reproducible research, and null-test failures. Among other details, the solutions to these major obstacles are presented. It also presented improved statistical techniques, including the definition of null-statistics and its combined statistics, a full-Bayesian likelihood procedure, and an improved formula to calculate the analytical statistical uncertainty. The limitation of

the null-test framework is documented where some are general and some are specific to POLARBEAR. In the end, the lensing amplitude is detected with $P(A_{\text{lens}} > 0) = 99.6\%$.

In part VI, the crosstalk systematic is studied in detailed. An analytical model on the map-domain is developed to predict the effect of crosstalk on the power-spectra given a set of conditions. A TOAST operator is developed to enable time-domain simulations of crosstalk systematics in the framework of the software Time Ordered Astrophysics Scalable Tools (TOAST). The two predictions are compared and resulted in a high degree of agreement in the example of the LiteBIRD experiment. Exploiting the analytical behavior in the model, a hardware based mitigation technique is developed to minimize its effect on the 2pt-power-spectra. This mitigation technique is general as long as the set of conditions are fulfilled, and a simple rule of thumb is presented that is easy to use for experimentalists. Given a new focal plane design, the effect can be predicted without time-consuming time-domain simulations for fast prototyping. One limitation of this model and mitigation technique is the requirement of the HWP.

Chapter 38

Discussions and future prospects

38.1 Use of HWP in CMB observations

Incidentally, the HWP is central to both part of my research in parts [V](#) and [VI](#). The HWP enables the POLARBEAR experiments to measure a wide ℓ -range using the same instrument, and some core analysis techniques are built around the presence of the CRHWP such as demodulation. The HWP in part [VI](#) plays a central role that the analytical model is only valid with its presence, which is a prerequisite for the proposed hardware-based mitigation strategy to work.

The HWP should continue to play an important role in CMB experiments in the near future. Important upcoming CMB experiments including the LiteBIRD experiment, the Small Aperture Telescopes (SATs) of the Simons Observatory^{[1](#)}, etc. will include the HWP.

However, it should be stressed that while the HWP eliminates a class of systematics, it introduces some of its own. And having moving parts such as the HWP on mission critical experiments such as the LiteBIRD is a significant risk that warrants careful considerations.

38.2 Control of systematics and null-test framework in CMB experiments

For any scientific research worth publishing, a high degree of assurance of the lack of biases is an essential ingredient. Among these biases are systematic effects, observer bias, and confirmation bias.

To control for systematic effects, a systematic effect has to be identified, and such effects have to be measured and/or simulated to ensure it is subdominant to the signal. Mitigation strategies can also be devised to suppress it.

¹[Kiuchi et al., “The Simons Observatory Small Aperture Telescope Overview”.](#)

The null-test framework provides a way to discover systematic effects. As long as there is a physical variable that can be used to split the data into two-halves, a null-test can be performed.

While my research has shown examples of how it can be achieved in current generations of CMB experiments, it will be an ongoing challenge for future generations of CMB experiments. It follows simply from the fact that CMB datasets are growing exponentially, and our target signal is fainter than ever as we march towards the uncharted territory. With fainter signals, any previously subdominant systematics can become dominant. This means that the number of systematic effects we need to consider also grow in size. And for well-known systematics, better analysis and mitigation techniques may be required.

This exploding size of datasets also makes the number of possible ways to cut the data into two-halves grows exponentially. While the naïve mapmaker is very suitable for running null-tests as TOD processing can potentially be shared for all null-tests. Memory and disk requirements to coadd and process the null-split maps will grow accordingly, on top of the already growing dataset size.

This may mean a more carefully selected set of null-splits is needed. And this diminishes the usefulness of the null-test framework to discover unknown systematics. This goes back to the previous point that identifying, measuring and/or simulating systematics are going to be more important.

38.3 CMB data analysis as a statistical problem

As a random example, people studying network topology used to be able to just look at the graph and discover patterns about it. As networks goes bigger, visualizing the networks becomes increasingly difficult to impossible, and abstract, formal statistical analysis was developed to address this problem.

Similarly, for dataset as small as the POLARBEAR observation, there are many things that can still be done with the intuition of a researcher. To name a few, we can look at each of the maps by brute force and see if there is any abnormal patterns. Since there are 38 map bundles, 21 null-splits, there are 1535 null-split maps². If it is converted into a video at 24fps, it will takes slightly more than 1 minute to watch all of them³. But what if we want to look at each individual CES? There are 3391 of them, which becomes around $3391 \times 21 \times 2 = 142,422$. This equates to more than an hour of video, making it virtually impossible to complete.

Another example would be null-power-spectra and many of its associated visualization such as chi-square plots. Again some researchers can eyeball them and use

²See sec. 25.2.

³Do not try this at home if you have photosensitive epilepsy! Even for normal people, they may experience dizziness, nausea, and vomiting.

intuition to discover problems. But the number of null-power-spectra increases with the number of null-splits chosen, possibly making the task intractable by humans.

Some solutions of problems like this have been proposed in my research, for example by using the Holm–Bonferroni method to decide objectively. But there are more areas that can benefit from proper statistical procedure to automate some analyses which rely on human intuition, such as data selection.

One potential avenue to tackle these problems (such as data selection and visualizing maps) that is on very high demand right now is statistical machine learning. They can be well-suited to these sort of problems as they are good at pattern recognition, and they are only making discoveries on potential biases that can be further investigated using more stringent methods. Application of machine learning in science and in CMB experiments, in particular, is already actively explored by many researchers.

38.4 CMB data analysis as a computation problem

As have been shown briefly in chapter 17 with fig. 17.1, and also in the above context, CMB data analysis will continue to be challenging computationally. Even for existing algorithms, they need to be updated constantly to keep up with state-of-the-art HPC systems. This includes adaption to continually changing paradigms such as different CPU architectures and/or topologies, adoption of accelerators such as GPUs; challenges in scaling to increasingly more number of nodes and/or heterogeneous computing; and energy-constrained computing. On top of this, more systematic simulations and more sophisticated analysis techniques are required.

Moreover, as the complexity of these softwares increase, wide-scale adoption of good software engineering skills is needed in the CMB community. This includes proper maintenance of the code base, proper collaboration skills in programming to scale beyond a few contributors, and proper structuring to nurture grow in complexity while maintaining correctness. From the limited observation of the author, only a handful of researchers do this right. Of all the skills needed in CMB data analysis—physics, mathematics, statistics, and software engineering—the last is the most neglected and underrated.

Education may be part of the answer. It is safe to say no physicist can enter graduate school without a proper training in mathematics and some basic training in statistics. This may not be true in programming, and in general computer science and software engineering. Often these are self-taught, yours truly included. With no quality control and no requirement, it is easy for it to be swept under the rug.

It can also be an attitude problem. While no physicist will gloss over the important of mathematics perhaps because a mathematical model can only be right or wrong. But it is easy for them to gloss over the importance of statistics and software engineering. Often they have a mindset to avoid complexity and only opt for things simple to understand. One solution may be to let specialized people handle specialized areas. For

example, in the industry, it is not uncommon for data scientists to prototype models and successful models will be productionized by professional programmers.

38.5 Reproducible research

The problem of reproducible research is briefly went through in chapter 24. The author makes an effort to move closer in this direction, but it is far from completely reproducible.

I expect this continue to be an important area to explore and a challenge to overcome.

Paradigm shift exists all the time in science, and while it is too early to tell, we may be experiencing a paradigm shift in reproducible research. Here, the paradigm is not how we understand the world but how we carry out science.

Today, we find it unacceptable to call something science without publishing its methodology and results. If indeed, we are going through such a paradigm shift, then in the future we may find it equally unacceptable to call something science without being reproducible (in the strict sense of what computational reproducible research means.)

But as the CMB datasets are growing exponentially, we are facing a challenge in making these available. And even if an individual can access all the data and softwares, few can have the computational resource to reproduce it. May be because of this, the paradigm shift of reproducible research would be restricted in science with smaller datasets such as biology. Or may be a smaller subset of reproducibility is defined to make it more manageable. Or may be similar to how mathematicians accept formal proofs, we develop an “in-principle” reproducible research method where at least one independent party is required to reproduce the result. Suffice it to say that reproducible research in CMB will look very different from the current reproducible research practicing elsewhere nowadays.

38.6 Crosstalk systematics

The crosstalk simulation method presented here is general, while computationally it may have room for improvements to handle future CMB experiments where the crosstalk matrix can be much larger.

The analytical model comes with a set of assumptions, where the hardware-based mitigation method relies on its validity. The biggest limitation would be the requirement of the HWP so that each individual detector can resolves the Q , U components by itself.

There are other assumptions that are only approximately fulfilled. For example, isotropy in both scanning strategy and attack angle is unrealistic, while a realistic one may be a good approximation of. It will be interesting to see how well this model holds in other CMB experiments. A natural next step would be to compare time-domain

simulation to the prediction of this model using the specification of CMB experiments with HWP and a very different scanning strategy such as the SO SATs.

There are also other aspects of crosstalk systematics not covered in this research, for example, time dependent crosstalk, and crosstalk between detectors observing at different frequencies. These will be explored in the future.

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These figures are circulated internally in different presentations and the actual sources are unknown to the author: fig. 9.1, fig. 9.2, fig. 9.4, fig. 19.1, fig. 22.1.

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Appendix

The Mathematics of CMB flat sky approximation

Flat-sky approximation

The followings are derived from,⁴ including its sign conventions.

Also, we follow the physicists' convention that the same symbol with a different argument actually indicates a different function after a transform. For example, $f(x), f(k)$.

Definitions

$$\begin{aligned}\mathcal{Q}(\boldsymbol{\theta}) &\equiv Q(\boldsymbol{\theta}) + iU(\boldsymbol{\theta}) \\ \mathcal{E}(\boldsymbol{\ell}) &\equiv E(\boldsymbol{\ell}) + iB(\boldsymbol{\ell})\end{aligned}$$

Note that Q, U are real fields, $\mathcal{Q}, \mathcal{E}, E, B$ are complex fields.

From curved sky to flat sky

We replace

$$T(\hat{\mathbf{n}}) = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) \rightarrow \int T(\boldsymbol{\ell}) e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} d^2\boldsymbol{\ell}$$

i.e., using the angular frequency, non-unitary convention of Fourier transform,

$$\begin{aligned}\begin{cases} T(\boldsymbol{\ell}) &\equiv \int T(\boldsymbol{\theta}) e^{-i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} d^2\boldsymbol{\theta} \\ \mathcal{Q}(\boldsymbol{\ell}) &\equiv \int \mathcal{Q}(\boldsymbol{\theta}) e^{-i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} d^2\boldsymbol{\theta} \end{cases} \\ \begin{cases} T(\boldsymbol{\theta}) &= \frac{1}{(2\pi)^2} \int T(\boldsymbol{\ell}) e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} d^2\boldsymbol{\ell} \\ \mathcal{Q}(\boldsymbol{\theta}) &= \frac{1}{(2\pi)^2} \int \mathcal{Q}(\boldsymbol{\ell}) e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} d^2\boldsymbol{\ell} \end{cases}\end{aligned}\tag{38.1}$$

⁴Zaldarriaga and Seljak, “All-Sky Analysis of Polarization in the Microwave Background,” sec. V.

Note that $\mathcal{Q}(\ell)$ is defined above, without giving a Physical meaning.

$\mathcal{Q}$ as spin-2 field

As $\mathcal{Q}(\theta)$ is spin-2 fields (vice versa for $\bar{\mathcal{Q}}(\theta)$),

$$\mathcal{Q}(\theta) = \frac{1}{(2\pi)^2} \int [E(\ell) + iB(\ell)] \frac{-1}{\ell^2} \delta^2 e^{i\ell \cdot \theta} d^2 \ell \quad (38.2)$$

$$\bar{\mathcal{Q}}(\theta) = \frac{1}{(2\pi)^2} \int [E(\ell) - iB(\ell)] \frac{-1}{\ell^2} \bar{\delta}^2 e^{i\ell \cdot \theta} d^2 \ell \quad (38.3)$$

In the small scale limit,

$$\begin{aligned} \frac{-1}{\ell^2} \delta^2 e^{i\ell \cdot \theta} &\approx e^{-2i(\phi - \phi_\ell)} e^{i\ell \cdot \theta} \\ \frac{-1}{\ell^2} \bar{\delta}^2 e^{i\ell \cdot \theta} &\approx e^{2i(\phi - \phi_\ell)} e^{i\ell \cdot \theta} \end{aligned}$$

i.e.

$$\mathcal{Q}(\theta) \approx \frac{1}{(2\pi)^2} \int [E(\ell) + iB(\ell)] e^{-2i(\phi - \phi_\ell)} e^{i\ell \cdot \theta} d^2 \ell \quad (38.4)$$

$$\bar{\mathcal{Q}}(\theta) \approx \frac{1}{(2\pi)^2} \int [E(\ell) - iB(\ell)] e^{2i(\phi - \phi_\ell)} e^{i\ell \cdot \theta} d^2 \ell \quad (38.5)$$

Comparing (38.1), (38.4), we have

$$\mathcal{Q}(\ell) \approx \mathcal{E}(\ell) e^{-2i(\phi - \phi_\ell)}$$

Further approximation

Where ϕ_ℓ is the complex phase of ℓ (when we interpret $\ell \in \mathbb{R}^2 \cong \mathbb{C}$).

We can further set $\phi = 0$ by rotating from $(\hat{\theta}, \hat{\phi})$ to $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ on the $\hat{\mathbf{z}}$ -tangent plane. Hence,

$$\begin{cases} \mathcal{Q}(\ell) &\approx \mathcal{E}(\ell) e^{2i\phi_\ell} \\ \mathcal{E}(\ell) &\approx \mathcal{Q}(\ell) e^{-2i\phi_\ell} \end{cases} \quad (38.6)$$

Decomposing $\mathcal{E}$ in terms of E, B

Starting from (38.5), take the c.c., and $\ell \mapsto -\ell$, we have

$$\mathcal{Q}(\boldsymbol{\theta}) \approx \frac{1}{(2\pi)^2} \int [\bar{E}(-\ell) + i\bar{B}(-\ell)] e^{-2i(\phi - \phi_\ell)} e^{i\ell \cdot \boldsymbol{\theta}} d^2\ell \quad (38.7)$$

Comparing (38.7), (38.4), we have

$$\begin{cases} \mathcal{E}(\ell) & \equiv E(\ell) + iB(\ell) \approx \bar{E}(-\ell) + i\bar{B}(-\ell) \\ \mathcal{E}(-\ell) & \approx E(\ell) - iB(\ell) \end{cases}$$

i.e.

$$\begin{cases} E(\ell) & \approx \frac{1}{2} [\mathcal{E}(\ell) + \bar{\mathcal{E}}(-\ell)] \\ B(\ell) & \approx \frac{1}{2i} [\mathcal{E}(\ell) - \bar{\mathcal{E}}(-\ell)] \end{cases} \quad (38.8)$$

Note that E, B has the following property accordingly,

$$\begin{aligned} \bar{E}(-\ell) &= E(\ell) \\ \bar{B}(-\ell) &= B(\ell) \end{aligned}$$

i.e. the field corresponding to $E(\ell), B(\ell)$, namely, $E(\boldsymbol{\theta}), B(\boldsymbol{\theta})$, are real, as they fulfill the reality condition.

Also, using (38.8), we can derive the relationships directly between the real-fields ⁵:

$$\begin{aligned} \mathbb{E}(\ell) &\equiv \begin{bmatrix} E(\ell) \\ B(\ell) \end{bmatrix} \\ \mathbb{Q}(\ell) &\equiv \begin{bmatrix} Q(\ell) \\ U(\ell) \end{bmatrix} \\ R(\phi) &\equiv \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \\ \mathbb{E}(\ell) &\approx R(-2\phi_\ell) \mathbb{Q}(\ell) \\ \mathbb{Q}(\ell) &\approx R(2\phi_\ell) \mathbb{E}(\ell) \end{aligned}$$

which can be compared to (38.6).

⁵using the reality condition on real fields. See proof in the companion file flat-sky-approximation.nb

Conclusion

i.e. we can go between

$$(Q(\boldsymbol{\theta}), U(\boldsymbol{\theta})) \leftrightarrow \mathcal{Q}(\boldsymbol{\theta}) \leftrightarrow \mathcal{Q}(\boldsymbol{\ell}) \leftrightarrow \mathcal{E}(\boldsymbol{\ell}) \leftrightarrow (E(\boldsymbol{\ell}), B(\boldsymbol{\ell}))$$

Summary of convolution and cross-correlation

$\mathbb{R}^n$

See [Convolution](#):

$$(f * g)(x) = \int_{\mathbb{R}^d} f(y)g(x-y) dy = \int_{\mathbb{R}^d} f(x-y)g(y) dy,$$

Convolution theorem on $\mathbb{R}^n$ convolution

$$\mathcal{F}\{f * g\} = k \cdot \mathcal{F}\{f\} \cdot \mathcal{F}\{g\},$$

where k depends on the gauge. See [Fourier transform](#). $k = 1$ for the 2 commonly used convention, unitary with ordinary frequency & non-unitary with angular frequency. See also in [Fourier](#) page.

Group

See [Convolution](#).

$$(f * g)(x) = \int_G f(y)g(y^{-1}x) d\lambda(y).$$

where G is a locally compact Hausdorff topological group and λ is a (left-) Haar measure.

It has the property of

$$L_h(f * g) = (L_h f) * g.$$

where L is the left translation of the group.

If G is commutative, then

$$\tau_x(f * g) = (\tau_x f) * g = f * (\tau_x g)$$

Specializing in translational group in $\mathbb{R}^n$, we recovered the above special case of $\mathbb{R}^n$. Here, the translation group is defined as

$$(\tau_x f)(y) = f(y - x).$$

Properties

A few useful properties are

$$f * \delta = f$$

$$\tau_x f = f * \tau_x \delta$$

with some conditions, where δ is the delta distribution. See [Convolution](#).

References to Fourier series and transform

Conventions

[Fourier transform - Wikipedia](#) has a nice table summarizing (non-)unitary and ordinary/angular frequency definition.

The unitary, ordinary frequency definition is the simplest, and most commonly used in Mathematics.

In Quantum mechanics, angular frequency with the non-unitary definition is used. The reason for angular frequency is probably for the canonical momentum relationship.

In [Fourier transform definition conventions and formulas](#), John Cook unifies all different conventions in a parameterized definition, which can have 8 combinations. Mathematica has a different parametrization in [FourierTransform—Wolfram Language Documentation](#). He pointed out the relationship between the 2 parameterization in [Fourier transforms and conventions in Mathematica](#).

Definitions & formula

Copied from Wikipedia for completeness:

ordinary frequency ξ (Hz), unitary:

$$\begin{aligned}\hat{f}_1(\xi) &\stackrel{\text{def}}{=} \int_{\mathbb{R}^n} f(x) e^{-2\pi i x \cdot \xi} dx = (2\pi)^{\frac{n}{2}} \hat{f}_2(2\pi \xi) = \hat{f}_3(2\pi \xi) \\ f(x) &= \int_{\mathbb{R}^n} \hat{f}_1(\xi) e^{2\pi i x \cdot \xi} d\xi\end{aligned}$$

angular frequency ω (rad/s), unitary:

$$\begin{aligned}\hat{f}_2(\omega) &\stackrel{\text{def}}{=} \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n} f(x) e^{-i\omega \cdot x} dx = \frac{1}{(2\pi)^{\frac{n}{2}}} \hat{f}_1\left(\frac{\omega}{2\pi}\right) = \frac{1}{(2\pi)^{\frac{n}{2}}} \hat{f}_3(\omega) \\ f(x) &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n} \hat{f}_2(\omega) e^{i\omega \cdot x} d\omega\end{aligned}$$

angular frequency ω (rad/s), non-unitary:

$$\begin{aligned}\hat{f}_3(\omega) &\stackrel{\text{def}}{=} \int_{\mathbb{R}^n} f(x) e^{-i\omega \cdot x} dx = \hat{f}_1\left(\frac{\omega}{2\pi}\right) = (2\pi)^{\frac{n}{2}} \hat{f}_2(\omega) \\ f(x) &= \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} \hat{f}_3(\omega) e^{i\omega \cdot x} d\omega\end{aligned}$$

The Mathematics of telescope beam

Intro

Beam convolution and definitions are confusing.

First of all, how beam should be defined? Is it the Point Spread Function (PSF)?

2 different pictures of possible definitions of beam can be related by

Curved sky:

$$B(\theta, \phi) = P(\theta, \pi + \phi) \Rightarrow B(\hat{\mathbf{n}}) = P\left(R_z\left(\frac{\pi}{2}\right)\hat{\mathbf{n}}\right)$$

Flat sky:

$$B(\mathbf{r}) = P(-\mathbf{r})$$

In the later case, the relationship in Fourier domain is simple:

$$\mathcal{F}\{\pi f\}(\mathbf{k}) = \mathcal{F}\{f\}(-\mathbf{k})$$

where

$$(\pi f)(x) \equiv f(-x)$$

In the case of symmetric beam (which means that the beam depends only on the “distance” to the center, not the angular direction), this subtlety does not exist, which may explain why the definition is not clear in the literature.

In integral form, in flat sky,

$$\begin{aligned} m_{\text{obs}}(\mathbf{x}) &= \int_{\mathbb{R}^2} P(\mathbf{x} - \mathbf{y}) m(\mathbf{y}) d^2\mathbf{y}, & \mathbf{x}, \mathbf{y} \in \mathbb{R}^2 \\ &= \int_{\mathbb{R}^2} B(\mathbf{y} - \mathbf{x}) m(\mathbf{y}) d^2\mathbf{y} \end{aligned}$$

The Mathematics of spin-weighted spherical harmonics and related concepts

Legendre polynomials

Condon-Shortley phase Convention

The following convention agrees with [Associated Legendre polynomials - Wikipedia](#), and is also the choice in Gorski et al.⁶, revision 3.80.

Also compare to Technology (U.S.)⁷, 14.7.8, 14.7.10, 14.7.13 which shares the same sign convention here.

Mathematica also shares the same convention in [LegendreP—Wolfram Language Documentation](#).

Note that P_ℓ^m below is sometimes denotes as $P_{\ell m}$, for example in CMB.

Definition

The general legendre equation (which reduced to the Legendre equation when $m = 0$),

$$(1 - x^2) \frac{d^2}{dx^2} P_\ell^m(x) - 2x \frac{d}{dx} P_\ell^m(x) + \left[\ell(\ell + 1) - \frac{m^2}{1 - x^2} \right] P_\ell^m(x) = 0$$

$$x \in [-1, 1], \ell, m \in \mathbb{N} \cup \{0\}, -\ell \leq m \leq \ell$$

Note that the equation is invariant under $\ell \rightarrow -\ell - 1$, hence negative values are ignored WOLOG.

It has a solution when $m = 0$ (Rodrigues' formula),

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell} [(x^2 - 1)^\ell]$$

⁶“The HEALPix Primer”.

⁷[NIST Handbook of Mathematical Functions Hardback and CD-ROM](#).

And for general m , the solution is the associated Legendre polynomials, with the Condon-Shortley phase,

$$P_{\ell}^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} (P_{\ell}(x)), m \geq 0$$

Note that if m is odd, the associated Legendre “polynomials” is not a polynomial despite the name.

$$P_{\ell}^m(x) = \frac{(-1)^m}{2^{\ell} \ell!} (1-x^2)^{m/2} \frac{d^{\ell+m}}{dx^{\ell+m}} (x^2-1)^{\ell}, -\ell \leq m \leq \ell$$

Note that the differential equation is invariant under $m \rightarrow -m$, hence the 2 sets of solutions are proportional to each other:

$$P_{\ell}^{-m}(x) = (-1)^m \frac{(\ell-m)!}{(\ell+m)!} P_{\ell}^m(x)$$

Close form

$$P_{\ell}^m(x) = (-1)^m \cdot 2^{\ell} \cdot (1-x^2)^{m/2} \cdot \sum_{k=m}^{\ell} \frac{k!}{(k-m)!} \cdot x^{k-m} \cdot \binom{\ell}{k} \binom{\frac{\ell+k-1}{2}}{\ell}$$

Here, we can see that

$$\lim_{x \rightarrow 1^-} \text{sgn}(P_{\ell}^m(x)) \rightarrow (-1)^m$$

Spherical harmonics

Condon-Shortley phase Convention

The following convention is from the “quantum mechanics” convention in [Spherical harmonics - Wikipedia](#). Note that the formula looks different, but is identical as we absorb the Condon-Shortley phase into the Associated Legendre polynomials.

It also shares the same convention in Gorski et al.⁸, revision 3.80 after simplifications, albeit it does not define the Associated Legendre polynomials when $m < 0$.

Mathematica also shares the same convention in [SphericalHarmonicY—Wolfram Language Documentation](#).

Note that Y_{ℓ}^m below is sometimes denoted as $Y_{\ell m}$, for example in CMB.

⁸“The HEALPix Primer”.

Definition

$$Y_{\ell}^m(\theta, \varphi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_{\ell}^m(\cos \theta) e^{im\varphi}$$

Here, we can see that

$$\lim_{\theta \rightarrow 0^+} \operatorname{sgn}(Y_{\ell}^m(\theta, 0)) \rightarrow (-1)^m$$

Spin-weighted spherical harmonics (SWSH)

See [Spin-weighted spherical harmonics - Wikipedia](#) for details.

Spin-weighted functions

A function is called spin-weighted s if it transforms under rotation on the North pole by angle ψ ,

$$\eta_s \rightarrow e^{is\psi} \eta_s$$

Operator $\eth$

There exists an operator that can raise the spin-weight of a function.

Explicitly,

$$\begin{aligned} \eth \eta &= -(\sin \theta)^s \left\{ \frac{\partial}{\partial \theta} + \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right\} [(\sin \theta)^{-s} \eta] \\ \bar{\eth} \eta &= -(\sin \theta)^{-s} \left\{ \frac{\partial}{\partial \theta} - \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right\} [(\sin \theta)^s \eta] \end{aligned}$$

Note that

$$\bar{\eth}^n g_s = \overline{\eth^n \bar{g}_s}$$

as $\bar{g}_s$ is of spin-weight $-s$.

pf. by Mathematical Induction.

Definition

$${}_sY_{lm} = \begin{cases} \sqrt{\frac{(l-s)!}{(l+s)!}} \bar{\partial}^s Y_{lm}, & 0 \leq s \leq l; \\ \sqrt{\frac{(l+s)!}{(l-s)!}} (-1)^s \bar{\partial}^{-s} Y_{lm}, & -l \leq s \leq 0; \\ 0, & l < |s|. \end{cases}$$

Close form

Spin-weighted Spherical Harmonics w/ Condon-Shortley phase. See [Spin-weighted spherical harmonics - Wikipedia](#).

$${}_sY_{lm}(\theta, \phi) = (-1)^m \sqrt{\frac{(l+m)!(l-m)!(2l+1)}{4\pi(l+s)!(l-s)!}} \sin^{2l} \left(\frac{\theta}{2} \right) \\ \times \sum_{r=0}^{l-s} \binom{l-s}{r} \binom{l+s}{r+s-m} (-1)^{l-r-s} e^{im\phi} \cot^{2r+s-m} \left(\frac{\theta}{2} \right).$$

Note on Condon-Shortley phase Convention

Note that there exists different sign conventions. Healpix does not explicitly declare the sign convention used in their documentation/Primer, but

1. since its definition of Spherical Harmonics agrees with above,
2. we consistently include the Condon-Shortley phase here in both the associated Legendre polynomials and spherical harmonics, and
3. Spin-weighted spherical harmonics defined through eth of spherical harmonics does not seem to have different sign convention (for example compare Zaldarriaga and Seljak⁹, no. A2 to the eth defined above.)
4. Zaldarriaga and Seljak¹⁰, no. A6 agrees with the close form formula above.

We should be confident that the above definition should agrees with Healpix and the CMB literature.

⁹“All-Sky Analysis of Polarization in the Microwave Background”.

¹⁰“All-Sky Analysis of Polarization in the Microwave Background”.

Wigner matrices

Conventions

Note that different authors use different conventions, sometimes without any notes or explanations.

The main source of confusion probably comes from active vs. passive transformation. Below we use the active transformation commonly found in Quantum Mechanics. See citations below.

Definitions

With the convention of z-y-z convention, right-handed frame, right-hand screw rule, active interpretation

$$D_{m'm}^j(\alpha, \beta, \gamma) \equiv \langle jm' | \mathcal{R}(\alpha, \beta, \gamma) | jm \rangle = e^{-im'\alpha} d_{m'm}^j(\beta) e^{-im\gamma}$$

See proofs in Sakurai and Napolitano¹¹, p. 175, 177, 196, 198.

Note that this has a different sign convention from Mathematica.¹²

The Wigner d-matrix has closed form formula below, with the sign convention given by Wigner¹³, which is also used in Sakurai and Napolitano¹⁴, p. 238. See [Wigner D-matrix - Wikipedia](#).

$$d_{m'm}^j(\beta) = [(j+m')!(j-m')!(j+m)!(j-m)!]^{\frac{1}{2}} \\ \times \sum_{s=s_{\min}}^{s_{\max}} \left[\frac{(-1)^{m'-m+s} \left(\cos \frac{\beta}{2}\right)^{2j+m-m'-2s} \left(\sin \frac{\beta}{2}\right)^{m'-m+2s}}{(j+m-s)!s!(m'-m+s)!(j-m'-s)!} \right]$$

where

$$s_{\min} = \max(0, m - m') \\ s_{\max} = \min(j + m, j - m')$$

¹¹[Modern quantum mechanics](#).

¹²[WignerD—Wolfram Language Documentation](#)

¹³[Group theory and its application to the quantum mechanics of atomic spectra](#).

¹⁴[Modern quantum mechanics](#).

Note on Convention

Note that mathematica uses a different convention

1. in terms of the sign

```
WignerDPhysics[{j_, m1_, m2_}, _] = (-1)^(m1 - m2) WignerD[{j, m1,
↪ m2}, _]
```

2. and also in label convention: the 3 argument form (ψ, θ, φ) are the 3 Euler angles, and is commonly defined as (α, β, γ) , or (φ, θ, ψ) instead.

Relationship with SWSH

In Mathematica,

```
SpinWeightedSphericalHarmonicY[s_, l_, m_, _, _] = \
(-1)^m Sqrt[(2 l + 1) / (4 )] WignerDPhysics[{l, -m, s}, _] E^(I m )
```

(with Mathematica's sign convention: *)*

```
SpinWeightedSphericalHarmonicY[s_, l_, m_, _, _] = \
(-1)^(-s) Sqrt[(2 l + 1) / (4 )] WignerD[{l, -m, s}, _] E^(I m )
```

Mathematically,

$$\begin{aligned} {}_s Y_{\ell m}(\theta, \phi) &= (-1)^m \sqrt{\frac{2\ell + 1}{4\pi}} d_{-ms}^{\ell}(\theta) e^{im\phi} \\ &= (-1)^m \sqrt{\frac{2\ell + 1}{4\pi}} D_{-ms}^{\ell}(\phi, \theta, 0) \end{aligned}$$

Generalized convolution theorem on $\text{SO}(3)$

Definition 4.

$$({}_s f * {}_{s'} g)(R) \equiv \frac{1}{2\pi} \int_{R' \in \text{SO}(3)} {}_s f(R') {}_{s'} g(R'^{-1}R) \, dR'$$

Theorem 3.

$${}_{s'} ({}_s f * {}_{s'} g)_{\ell m} = (-1)^{-s} \sqrt{\frac{4\pi}{2\ell+1}} {}_s f_{\ell m} {}_{s'} g_{\ell, -s}$$

Proof.

$$\begin{aligned}
(sf *_{s'} g)(R) &\equiv \frac{1}{2\pi} \int_{R' \in SO(3)} sf(R')_{s'} g(R'^{-1}R) dR' \\
&= \sum_{\ell m \ell' m'} \frac{1}{2\pi} \int_{R' \in SO(3)} sf_{\ell m s} Y_{\ell m}(R')_{s'} g_{\ell' m' s'} Y_{\ell' m'}(R'^{-1}R) dR' \\
&= \sum_{\ell m \ell' m'} \frac{1}{2\pi} \int_{R' \in SO(3)} sf_{\ell m s} g_{\ell' m'} \\
&\quad \times (-1)^m \sqrt{\frac{2\ell+1}{4\pi}} D_{-ms}^{(\ell)}(R') \sum_{m''} \left[D_{m' m''}^{(\ell')}(R'^{-1}) \right]^*_{s'} Y_{\ell' m''}(R) dR' \\
\left[D_{m' m''}^{(\ell')}(R'^{-1}) \right]^* &= D_{m'' m'}^{(\ell')}(R') = (-1)^{m'-m''} \left[D_{-m'', -m'}^{(\ell')}(R') \right]^* \\
(sf *_{s'} g)(R) &= \sum_{\ell m \ell' m' m''} \frac{1}{2\pi} sf_{\ell m s} g_{\ell' m'} (-1)^{m+m'-m''} \sqrt{\frac{2\ell+1}{4\pi}}_{s'} Y_{\ell' m''}(R) \\
&\quad \times \int_{R' \in SO(3)} D_{-ms}^{(\ell)}(R') \left[D_{-m'', -m'}^{(\ell')}(R') \right]^* dR' \\
&= \sum_{\ell m \ell' m' m''} \frac{1}{2\pi} sf_{\ell m s} g_{\ell' m'} (-1)^{m+m'-m''} \sqrt{\frac{2\ell+1}{4\pi}}_{s'} Y_{\ell' m''}(R) \\
&\quad \times \frac{8\pi^2}{2\ell+1} \delta_{\ell, \ell'} \delta_{-m, -m''} \delta_{s, -m'} \\
&= (-1)^{-s} \sqrt{\frac{4\pi}{2\ell+1}} \sum_{\ell, m} sf_{\ell m s} g_{\ell, -s s'} Y_{\ell m}(R)
\end{aligned}$$

□

Definition 5.

$$(sf \odot_{s'} g)(R) \equiv \oint sf(R\hat{n})_{s'} g^*(\hat{n}) d\hat{n}$$

Theorem 4.

$$sf \odot_{s'} g(\phi, \theta, \psi) = \sum_{\ell m m'} sf_{\ell m} (-1)^{-m'} \sqrt{\frac{4\pi}{2\ell+1}}_{-m'} Y_{\ell m}(\theta, \phi) e^{im'\psi} g^*_{s' \ell m'}$$

Proof.

$$\begin{aligned}
(sf \odot {}_s g)(R) &\equiv \oint sf(R\hat{\mathbf{n}}) {}_s g^*(\hat{\mathbf{n}}) d\hat{\mathbf{n}} \\
&= \sum_{\ell m} \int sf_{\ell m} Y_{\ell m}(R\hat{\mathbf{n}}) {}_s g^*(\hat{\mathbf{n}}) d\hat{\mathbf{n}} \\
&= \sum_{\ell m m'} \int sf_{\ell m} \left[D_{mm'}^{(\ell)}(R) \right]^* {}_s Y_{\ell m'}(\hat{\mathbf{n}}) {}_s g^*(\hat{\mathbf{n}}) d\hat{\mathbf{n}} \\
&= \sum_{\ell \ell' m m' m''} \int sf_{\ell m} \left[D_{mm'}^{(\ell)}(R) \right]^* {}_s Y_{\ell m'}(\hat{\mathbf{n}}) {}_s g_{\ell' m''}^* {}_s Y_{\ell' m''}^*(\hat{\mathbf{n}}) d\hat{\mathbf{n}} \\
&= \sum_{\ell \ell' m m' m''} sf_{\ell m} \left[D_{mm'}^{(\ell)}(R) \right]^* {}_s g_{\ell' m''}^* \delta_{\ell \ell'} \delta_{m' m''} \\
&= \sum_{\ell m m'} sf_{\ell m} \left[D_{mm'}^{(\ell)}(R) \right]^* {}_s g_{\ell m'}^* \\
sf \odot {}_s g(\phi, \theta, \psi) &= \sum_{\ell m m'} sf_{\ell m} (-1)^{-m'} \sqrt{\frac{4\pi}{2\ell+1}} {}_s Y_{\ell m}(\theta, \phi) e^{im'\psi} {}_s g_{\ell m'}^*
\end{aligned}$$

□

Final configuration of the null-test v1.0.2 used in POLARBEAR large-patch high- ℓ analysis

The configuration used in the final analysis in chapter [31](#) is presented below.

Configuration for real maps

```
# "variables" to be reuse later
common:
```

```
mapsplits: &mapsplits
  mapsplits:
    - FIRST_SECOND
    # - FIFTH_SEPARATE
    # - FOURTH_SEPARATE
    # - THIRD_SEPARATE
    - RS_MIDDLE
    - RM_SET
    - SM_RISE
    - LR_SUBSCAN
    - PWV
    - GAIN_BY_CES
    - QU_PIXEL
    - LHS_RHS
    - SUN_DIST
    - SUN_HORIZON
    - MOON_DIST
    - MOON_HORIZON
    - LEAK_BY_CES
    - LEAK_BY_BOLO
    - AMP_2F_BY_CES
```

```

- AMP_2F_BY_BOLO
- AMP_4F_BY_CES
- AMP_4F_BY_BOLO
# - RANDOM_BY_BOLO1
# - RANDOM_BY_BOLO2
# - RANDOM_BY_BOLO3
# - RANDOM_BY_BOLO4
# - RANDOM_BY_BOLO5
# - RANDOM_BY_BOLO6
# - RANDOM_BY_BOLO7
# - RANDOM_BY_BOLO8
# - RANDOM_BY_CES1
# - RANDOM_BY_CES2
# - RANDOM_BY_CES3
# - RANDOM_BY_CES4
# - RANDOM_BY_CES5
# - RANDOM_BY_CES6
# - RANDOM_BY_CES7
# - RANDOM_BY_CES8
- THS_BHS
- TOP_BOTTOM
# - RANDOM_BUT_PAIR_BY_BOLO1

# mapmaking sim iter

mapbolo_sim: &boloargs
  gain_folder: $SCRATCH/data/largepatch4/gain_181224_renorm
  psd_folder: $SCRATCH/data/largepatch4/psd_iota_poleff_181224_renorm
  filterfolder: $SCRATCH/data/largepatch4/filterdb_iota_final
  alt_flag_folder:
    ↪ $SCRATCH/data/largepatch4/iota_dataselection_final_highell_corr
  # projection args
  width: 45
  fixed_offset: BICEP
  pixelsize: 2
  hwppos: 45
  projection: OLEA
  # filter args
  detrend: 2
  ? groundlr
  groundtemplate: 0.24

```

```
poly: 9
polpoly: 9
? commonmode
? notch_lines
notch_line_width: 0.01
mask_radius: 3.5
# null only
null_folder:
↪ $SCRATCH/data/largepatch4/null_test_flags_iota_highell_corr_pair
<<: *mapsplits

mapbolo_sim_iter: &boloinput
f: $SCRATCH/data/largepatch4/hdf5_ces_dmids

# coadd

coadd:
basedir: $SCRATCH/largepatch_null/high_1_0_2/maps
# outsubdir: coadd
<<: *mapsplits

# for coadd_mpi_iter.py
coadd_sim:
# outsubdir: coadd
<<: *mapsplits

# pseudo

pseudo:
# weight_name: realmap
lmax: 4901
pixel_size: 2

selectionpath: $SCRATCH/largepatch_null/high_1_0_2/weight_nauto.hdf5
maskpath: $SCRATCH/largepatch_null/high_1_0_2/masks.hdf5
weightdir: $SCRATCH/largepatch_null/high_1_0_2/coadd/null_realmaps

pseudo_iter:
<<: *mapsplits

# null map to full map
```

```
null2full:
  basedir: $SCRATCH/largepatch_null/high_1_0_2/coadd
  outbasedir: $SCRATCH/largepatch_null/high_1_0_2/coadd_full
# cat args-common.yml before this

# "variables" to be reuse later
template:

  iter: &iter
  mapcase:
    - null_realmaps
  nreal:
    - null
  save_weight:
    - True

# mapmaking-real

mapbolo_whwp:
  <<: *boloargs
  ? removeleakage_pca
  leak_fmax: 0.4
  correct_poleff:
    ↪ $SCRATCH/data/largepatch4/analysis_data_iota/cal/lp/combined_poleff_20181221.pkl
  ? deconvolve
mapbolo_whwp_iter:
  <<: *boloinput
  tssim:
    - null
  whitenoise:
    - False
  <<: *iter

# coadd-real

coadd_iter: &coadd_iter
  name:
    - realmap
  <<: *iter

# coadd-real-final
```

```
coadd_final:
  basedir: $SCRATCH/largepatch_null/high_1_0_2/coadd
  # outsubdir: coadd
  <<: *mapsplits
coadd_final_iter:
  <<: *coadd_iter

# coadd-sign flip

coadd_sign:
  basedir: $SCRATCH/largepatch_null/high_1_0_2/maps/null_realmaps
  signpath:
    ↪ $SCRATCH/data/largepatch_high/obs_paths_by_dir_0_13_sign_flip_512.hdf5
  name: realmap
  nreal: 512
  # outsubdir: coadd
  <<: *mapsplits

# null map to full map

null2full_iter:
  mapcase:
    - null_realmaps
  name:
    - realmap
  nreal:
    - null
  final:
    - True
  rawmaps:
    - - d0
    - - d4r
    - - d4i
    - - w0
    - - w4
```

Configuration for simulated maps

```
# "variables" to be reuse later
common:
```

```
mapsplits: &mapsplits
mapsplits:
- FIRST_SECOND
# - FIFTH_SEPARATE
# - FOURTH_SEPARATE
# - THIRD_SEPARATE
- RS_MIDDLE
- RM_SET
- SM_RISE
- LR_SUBSCAN
- PWV
- GAIN_BY_CES
- QU_PIXEL
- LHS_RHS
- SUN_DIST
- SUN_HORIZON
- MOON_DIST
- MOON_HORIZON
- LEAK_BY_CES
- LEAK_BY_BOLO
- AMP_2F_BY_CES
- AMP_2F_BY_BOLO
- AMP_4F_BY_CES
- AMP_4F_BY_BOLO
# - RANDOM_BY_BOLO1
# - RANDOM_BY_BOLO2
# - RANDOM_BY_BOLO3
# - RANDOM_BY_BOLO4
# - RANDOM_BY_BOLO5
# - RANDOM_BY_BOLO6
# - RANDOM_BY_BOLO7
# - RANDOM_BY_BOLO8
# - RANDOM_BY_CES1
# - RANDOM_BY_CES2
# - RANDOM_BY_CES3
# - RANDOM_BY_CES4
# - RANDOM_BY_CES5
# - RANDOM_BY_CES6
# - RANDOM_BY_CES7
# - RANDOM_BY_CES8
- THS_BHS
- TOP_BOTTOM
```

```
# - RANDOM_BUT_PAIR_BY_BOLO1

# mapmaking sim iter

mapbolo_sim: &boloargs
  gain_folder: $SCRATCH/data/largepatch4/gain_181224_renorm
  psd_folder: $SCRATCH/data/largepatch4/psd_iota_poleff_181224_renorm
  filterfolder: $SCRATCH/data/largepatch4/filterdb_iota_final
  alt_flag_folder:
    ↪ $SCRATCH/data/largepatch4/iota_dataselection_final_highell_corr
  # projection args
  width: 45
  fixed_offset: BICEP
  pixelsize: 2
  hwppos: 45
  projection: OLEA
  # filter args
  detrend: 2
  ? groundlr
  groundtemplate: 0.24
  poly: 9
  polpoly: 9
  ? commonmode
  ? notch_lines
  notch_line_width: 0.01
  mask_radius: 3.5
  # null only
  null_folder:
    ↪ $SCRATCH/data/largepatch4/null_test_flags_iota_highell_corr_pair
    <<: *mapsplits

mapbolo_sim_iter: &boloinput
  f: $SCRATCH/data/largepatch4/hdf5_ces_dmds

# coadd

coadd:
  basedir: $SCRATCH/largepatch_null/high_1/maps
  # outsubdir: coadd
  <<: *mapsplits
```



```
# for coadd_mpi_iter.py
coadd_sim:
  # outsubdir: coadd
  <<: *mapsplits

# pseudo

pseudo:
  # weight_name: realmap
  lmax: 4901
  pixel_size: 2

  selectionpath: $SCRATCH/largepatch_null/high_1/weight_nauto.hdf5
  maskpath: $SCRATCH/largepatch_null/high_1/masks.hdf5
  weightdir: $SCRATCH/largepatch_null/high_1/coadd/null_realmaps

pseudo_iter:
  <<: *mapsplits

# null map to full map

null2full:
  basedir: $SCRATCH/largepatch_null/high_1/coadd
  outbasedir: $SCRATCH/largepatch_null/high_1/coadd_full
  # cat args-common.yml before this

# "variables" to be reuse later
template:

  iter: &iter
  mapcase:
    - null_simmmaps_TBB
    - null_simmmaps_TTEE
    - null_simmmaps_TTEEBB
  nreal:
    - 128
    - 128
    - 512
  save_weight:
    - False
    - False
    - False
```

mapmaking-sim

mapbolo_whwp:

<<: *boloargs

mapbolo_whwp_iter:

<<: *boloinput

tssim:

-

↪ \$SCRATCH/data/largepatch_high/simmaps_base_2018_plikHM_TTTEEE_lowl_lowE_lensing_cl_1

-

↪ \$SCRATCH/data/largepatch_high/simmaps_base_2018_plikHM_TTTEEE_lowl_lowE_lensing_cl_1

-

↪ \$SCRATCH/data/largepatch_high/simmaps_base_2018_plikHM_TTTEEE_lowl_lowE_lensing_cl_1

whitenoise:

- False

- False

- False

<<: *iter

coadd-sim

coadd_iter:

name:

- sim

- sim

- sim

<<: *iter

null2full_iter:

mapcase:

- null_simmaps_TB

- null_simmaps_TEE

- null_simmaps_TTEEBB_noise

name:

- sim

- sim

- sim

nreal:

- 128

- 128

- 512

```
final:
- False
- False
- False
rawmaps:
- - d0
  - d4r
  - d4i
- - d0
  - d4r
  - d4i
- - d0
  - d4r
  - d4i
```

Changelog of null-tests performed in POLARBEAR large-patch analysis

Below is the changelog of all null-test runs performed in the POLARBEAR analysis in part V. They are reproduced here for reference.

Note:

- Semantic versioning is used: `major.minor.bugfix`. Major versions all share the same map-making, for example v0.6 through v0.6.6. Minor versions differ in post-mapmaking, for example how they are grouped to coadd, how the spectra are generated, etc.
- Since v0.13.2.1: revert to plotting C_l instead.
- Since v0.6.2, $C_l B_l^2$ is plotted instead.
- Since v0.2.1, the spectra is plot *without* the dof factor of $\frac{l(l+1)}{2\pi}$. i.e. the previous spectra are so called D_l .
- If you see $a + bj$, interpret a as the value and b as the error-bar. It is just a shortcut to represent elements of $\mathbb{R}^2$ as elements of $\mathbb{C}$.
- Changelog:
- [v1.0.2](#): The only bug fix is the patch that fix ground template mentioned below:
 - Dominic found that tweaking how AnalysisBackend construct the ground template can remove the stripes. I applied a patch at AnalysisBackend commit 07b6610. Merge to cleanup?
 - What the patch does:
 - * before the patch, a point-source masked TOD is used to construct the ground template
 - * with the patch, TOD without point-source mask is used to construct the ground template
- v1.1.1: (superseded by fix in v1.0.2) Only real-mapmaking for deugging strips in I-maps: change group template from 0.5 to 1.
- v1.1: (superseded by fix in v1.0.2) Only real-mapmaking for deugging strips in I-maps: change group template from 0.24 to 0.5.

- [v1.0.1](#): add `--deconvolve` to real mapmaking, everything else identical. i.e. sign-flip-noise from real map is not regenerated.
- [v1.0.0](#): identical to v1.0.0-beta except no. of TTEEBB simulation increased from 358 to 512.
- [v1.0.0-beta](#):
 - updated theory spectra input using Planck 2018 values and CLASS.
 - point source mask:
 - * boolean mask in mapmaking changed to 3.5 arcmin (from default 5 arcmin)
 - * map to pseudo-spectra mask changed to Gaussian (from C^∞ taper with inner and outer radius) for mode-coupling niceness, with FWHM at 10.5 arcmin.
 - removed null tests that involve random splits, reducing from 24 to 21 null tests.
 - 128 TTEE, TTBB simulations; 512 TTEEBB simulations with 358 only included in this report.
- [v0.13.4](#):
 - zero-padding on maps changed from 1.3x to 2x
 - “naïve” Wiener filter on temperature weights removed
- [v0.13.3](#): use sign-flip noise in TT/EE/BB/noise simulations
- [v0.13.2.1](#): null-spectra plotting changing from $C_l B_l^2$ to C_l
- v0.13.2: Fixed a bug that causes v0.13.1 wrong. i.e. this one has the correct output from `--correct_poleff`
- v0.13.1: `--correct_poleff` is applied to real map to correct for the polarization efficiency which effectively rescale the TOD.
- v0.13:
 - add point sources: `fixed_sources_pccsinlp` from `pointing/sources.py`
 - OLEA projection
 - completely new pipeline for post-mapmaking
 - * no intermediate binning, only 1 single binning step is used in the final step
 - * mode-coupling:
 - bug fixed
 - with and without point-source mask are compared
 - * pseudo-spectra:
 - fully implement `no_diag` variant for every case (also the cross cases has been unified to avoid bugs)

- should be numerically more stable (due to how the old one binning things)
 - * new filter transfer function definition: $\tilde{C}_l = \sum_{\ell'} F_{\ell} M_{\ell\ell'} B_{\ell'}^2 C_{\ell'}$
 - * there's only 1 single mask (per temperature/polarization, i.e. the final mask) used across all observations ("10"-day cycles). Previously, each observation has their own mask, where as the mode-coupling matrices are calculated once using the final mask, which is what's done in the low- ℓ pipeline. But since the low- ℓ pipeline calculated the mode-coupling by simulation rather than analytical like the high- ℓ pipeline, this shouldn't be a problem for low- ℓ .
 - * post mapmaking was more expensive than mapmaking, and it is now much cheaper. Debug QoS can almost accommodate the whole post-mapmaking calculation.
 - binning is done slightly differently in null-spectra
 - v0.12:
 - use 1 arcmin.
 - pseudo-spectra pipeline requires more memory than Cori I has available per node. So only maps are produced but no spectra.
 - v0.11:
 - largepatch4 datasets
 - merged AnalysisBackend/cleanup/4254c87 to the high-l post-making pipelines
 - applied the high-l data selection on top of it: No. of sub-scans: before: 50902628; after: 47995975. ~94% efficiency (requiring pairing would result in only 40405768 sub-scans. ~84% efficiency.)
 - v0.10: Same as v0.9, except:
 - regenerate null-flags *after* data-selection to ensure its correctness (on splitting the available data in a half)
 - no data-selection that requires top-bottom pairing.
 - 24 simulations for TTEE/TTBB; 48 simulations for TTEEBB. This is primarily for better p-values, and also as a scaling test.
 - No THS_BHS null-test because of some unknown bug.
 - v0.9.1: same as v0.9, except not using nodiag variant mentioned in v0.6.5.
 - v0.9: similar to v0.8, but apply an intelligent data-selection on correlations, and also add the pairing data-selection from v0.6.
 - v0.8.1: same as v0.8, except not using nodiag variant mentioned in v0.6.5.
 - v0.8: apply data selection to cut half amount of data by relations between detectors.
- Caveats:

- this aggressive data-cut is for showing if it does move the null-spectra or not. Different data-selection schemes will be tested to preserve more data.
 - null flags isn't regenerated (i.e. this additional data selection is done after the original data-selection and null-flag generations)
 - LEAK_BY_BOLO, LHS_RHS, THS_BHS cannot be generated (probably just a data volume problem between null-splits)
 - the last l-bin can be very large (again perhaps just data volume problem)
 - QU_PIXEL with fine bin is very bad (perhaps it's the same problem with the mismatch between fine-bin and large-bin result that we can't believe the fine-bin spectra)
- v0.7.1: same as v0.7, except not using nodiag variant mentioned in v0.6.5.
 - v0.7: turn off common-mode filter, otherwise same as v0.6.5.
 - v0.6.6: same as v0.6.4, except not using nodiag variant mentioned in v0.6.5.
 - v0.6.5:
 - back to v0.6.2 config, except
 - in real-maps, replace form_2d_cross_spectra with nodiag variant. If the noise between 2 null-splits are correlated, this would improve it. See Neil's "Expanding out the QUIET cross null spectrum".
 - v0.6.4: use roughly quarter volume of data to generate null-spectra, to test if some of the null-spectra is really noise-like (THS_BHS is missing due to an error)
 - v0.6.3: Coadd by season (instead of by 10-day cycle), i.e. from 37 observations down to 3. Note that the vertical axis is still $C_l B_l^2$.
 - v0.6.2: plot $C_l B_l^2$ instead.
 - v0.6.1: add noise to TTEEBB sim.
 - v0.6:
 - back to v0.3's configuration, except
 - with most null-tests in v0.2 plus new ones
 - new gain from Anh
 - add data-selection that requires t/b-pairing
 - add a null-test RANDOM_BUT_PAIR_BY_BOLO1 that is random but both t/b-pair always come together.
 - --width changed to 45 (between 45 to 70.3 there's only tiny bit of signals that's not worth saving, considering the extra computation requirements that would impose.)
 - changed definition of get_cross_weight in my pseudo-spectra pipeline from $\frac{1}{w} = \frac{1}{2} \left(\frac{1}{w_1} + \frac{1}{w_2} \right)$ to $\frac{1}{w} = \frac{1}{w_1} + \frac{1}{w_2}$
 - (accidentally) no white-noise in the TTEEBB simulations
 - backward incompatible HDF5 maps: new-ish mode-coupling and pseudo-spectra pipelines with a different HDF5 maps structure (for example maps are saved in 2d array.)

- v0.5:
 - applies heavy data-selection on T, in the hopes that it suppress the I to Q/U leakage in high- l .
 - add TOP_BOTTOM split.
 - (--mask-cross hasn't been applied due to a misunderstanding of what the code does. But the diff. is at ~4% level in the apodization masks a.k.a. tpweight/tpmask which should not make too much difference in the perceived severe failing of the null-tests.)
- v0.4: identical with v0.3 except for --notch_line_width: 0.04, to see if it helps the null spectra at all.
- v0.3:
 - config:
 - * null-tests covers only the 4 worst failing null-test from v0.2: QU_PIXEL, LEAK_BY_BOLO, AMP_2F_BY_BOLO, AMP_4F_BY_BOLO
 - add RANDOM_BY_BOLO1
 - * --groundlr to separately subtract the ground template on L/R scan
 - * --notch_lines --notch_line_width: 0.01 instead of --nofft: perform FFT and then applies notch filtering
 - data:
 - * Satoru's new HDF5
 - * Anh's new patch center (should be affecting the output maps only but not the spectra)
 - * Anh's new gain
 - * Neil's notch filter
 - * Kolen's new coadd (then doesn't save the I/Q/U and have a bunch of computational improvements)
- v0.2.3.: "gain matched" between the two splits for each spectra
- v0.2.2:
 - large bin at 300, fine bin at 30 (just above correlation length)
 - add EE χ -statistics.

Data released internally in POLARBEAR

Data Release—Pseudo-spectra

Introduction

The data version uniquely specified the beam spectra, pseudo spectra, mode-coupling matrices, and theory spectra. The content of these files has never been changed since the simulation runs at NERSC. (While in earlier internal versioning, it is in a different container.)

3 versions are released, v1.0.0, v1.0.1, v1.0.2, together with the masks:

```
high_1.hdf5  
high_1_0_1.hdf5  
high_1_0_2.hdf5  
masks.hdf5
```

It is released at NERSC in `/global/cfs/cdirs/polar/www/khcheung/largepatch_new/`.

`high_1_0_1.hdf5` and `high_1_0_2.hdf5` uses HDF5 external symlinks to point to data in `high_1.hdf5`. In practice it means both `high_1.hdf5` and `high_1_0_2.hdf5` have to be in the same directory in order to read v1.0.2 data.

Configurations

- The mapmaking is done using the AnalysisBackend khcheung branch
- The post-mapmaking pipeline (masks, pseudo-spectra, mode-coupling) is done using the software TAIL below
- The configuration of the softwares is summarized in sec. [38.6](#) to reproduce v1.0.0. Sec. [38.6](#) has recipe to reproduce v1.0.1 and v1.0.2.

All 1 starts at 0, and no binning. i.e. if a shape is 4901 then it spans the l-range from 0-4900.

p is the polarization mask. t is the temperature mask. Both in map-domain.

Specifications—Beam and theory spectra

This session should be self-explanatory, the datasets within each group are all one-dimensional with the same size. 1 gives the l-value and for example TT gives the TT spectrum at those l-values.

```
$> h5ls -r high_1.hdf5
...
/beam_spectra                                Group
/beam_spectra/10.1                          Dataset {293}
/beam_spectra/10.2                          Dataset {293}
/beam_spectra/10.3                          Dataset {293}
/beam_spectra/10.4                          Dataset {293}
/beam_spectra/10.5                          Dataset {293}
/beam_spectra/8.2.0                         Dataset {293}
/beam_spectra/9.4                           Dataset {293}
/beam_spectra/all                           Dataset {293}
/beam_spectra/l                             Dataset {293}
...
/theory_spectra                              Group
/theory_spectra/BB                          Dataset {4901}
/theory_spectra/EE                          Dataset {4901}
/theory_spectra/TE                          Dataset {4901}
/theory_spectra/TT                          Dataset {4901}
/theory_spectra/dE                          Dataset {4901}
/theory_spectra/dT                          Dataset {4901}
/theory_spectra/dd                          Dataset {4901}
/theory spectra/dens[1]-dens[1]            Dataset {4901}
```

```

/theory_spectra/dens[1]-lens[1] Dataset {4901}
/theory_spectra/l          Dataset {4901}
/theory_spectra/lens[1]-lens[1] Dataset {4901}
/theory_spectra/phi-dens[1] Dataset {4901}
...

```

Specifications—Mode-coupling matrices

The names should be self-explanatory. The difference between EBEB and EBEB_pseudo follows AnalysisBackend (although generated in TAIL.)

```

$> h5ls -r high_1.hdf5
...
/mode_coupling          Group
/mode_coupling/BBBBB    Dataset {4901, 4901}
/mode_coupling/EBEB     Dataset {4901, 4901}
/mode_coupling/EBEB_pseudo Dataset {4901, 4901}
/mode_coupling/EEEE     Dataset {4901, 4901}
/mode_coupling/TBTB     Dataset {4901, 4901}
/mode_coupling/TETE     Dataset {4901, 4901}
/mode_coupling/TTTT     Dataset {4901, 4901}
...

```

Specifications—pseudo-spectra

- full/null indicates it is from the full map data or null-split data.
- realmaps is from real map data, simmaps is from simulated map data.
- TTBB/TTEE/TTEEBB represent the power included in the input map.
- noise: generated from random sign-flip-noise from realmaps
- Each of these datasets are of 6 dimensions. In this order:
 - spectra: the 6 different spectra: TT , TE , ...
 - null-split: the 21 null-split: AMP_2F_BY_BOLO, ...
 - sub-split: 0/1/2. 0 is the 1st-half, 1 is the 2nd-half, 2 is the cross-spectrum between the 1st- and 2nd-half.
 - l: self-explanatory
 - sub-spectra: Cl, Nl. Cl is the signal spectra (using the cross-spectrum estimator), Nl is the noise spectra.
 - n: n-th realization

See more on meta below.

```
$> h5ls -r high_1.hdf5
...
/full                                     Group
/full/pseudo_spectra                     Group
/full/pseudo_spectra/realmaps Dataset {6, 1, 1, 4901, 2, 1}
/full/pseudo_spectra/simmaps_TTBb Dataset {6, 1, 1, 4901, 2, 128}
/full/pseudo_spectra/simmaps_TTEE Dataset {6, 1, 1, 4901, 2, 128}
/full/pseudo_spectra/simmaps_TTEEBB_noise Dataset {6, 1, 1, 4901, 2, 512}
...
/null                                     Group
/null/pseudo_spectra                     Group
/null/pseudo_spectra/realmaps Dataset {6, 21, 3, 4901, 2, 1}
/null/pseudo_spectra/simmaps_TTBb Dataset {6, 21, 3, 4901, 2, 128}
/null/pseudo_spectra/simmaps_TTEE Dataset {6, 21, 3, 4901, 2, 128}
/null/pseudo_spectra/simmaps_TTEEBB_noise Dataset {6, 21, 3, 4901, 2, 512}
...
```

Specifications—meta

- `null_split`: the names of the 21 null-split
- `spectra`: the names of the 6 spectra
- `sub_spectra`: the names as Cl, NI

```
$> h5ls -r high_1.hdf5
...
/meta                                     Group
/meta/null_split                         Dataset {21}
/meta/spectra                           Dataset {6}
/meta/sub_spectra                       Dataset {2}
...
```

Data Release—Spectra—final freeze

Final freeze configuration

This data is released as `high_1_0_2_spectra_final.hdf5`. It contains both bin-width of 50 and 100.

Using TAIL,

```
solve_spectra high_1_0_2.hdf5 -o high_1_0_2_spectra_final.hdf5 --scaling
↪ --sim-noise-matching -b 50 100 --return-w
```

```
iterative_map high_1_0_2.hdf5 masks.hdf5 -o high_1_0_2_spectra_final.hdf5
↪ --scaling --sim-noise-matching -b 100 --l-T-cutoff 1600 -n 100
iterative_map high_1_0_2.hdf5 masks.hdf5 -o high_1_0_2_spectra_final.hdf5
↪ --scaling --sim-noise-matching -b 50 --l-T-cutoff 1550 -n 100
```

Whenever MC error-bar is calculated, leave Spectra at default attributes:
`ev_est='signal', ddof=None.`

Note: difference between final freeze and stage 2 unblinding spectra

- all options mentioned in [Changes and configurable options] is changed here comparing to those in [Reproducing null-spectra and full-spectra at unblinding].
- although between now and unblinding, sometimes bin-width of 50 is used in presentations, here bin-width of 100 is used to be consistent with the unblinding configuration. For the reason of optimizing bin-width, see [Unblinding Proposal—Stage 1—Choosing \$\ell\$ -min, \$\ell\$ -max, bin-width](#)

Specifications—Filter Transfer Functions

- full/null indicates it is from the full map data or null-split data.
- `/full/filter_transfer/l` and `/null/filter_transfer/l` specify the filter l-range. Here the 2 elements are 50, 4300, meaning the l-range is from 50–4300 (no binning, i.e. `dl=1`)
- `/null/filter_transfer/F` and `/full/filter_transfer/F` are 4d-array, same as first 4 dimensions from pseudo-spectra. See more in [Specifications—pseudo-spectra](#).
- The first dimension (spectra) order is specified in `/meta/filter_transfer`. For example, `[..., BB, EE, TT]` specifies the 3 auto-spectra cases.

```
$> h5ls -r high_1_0_2_spectra_final.hdf5
/                               Group
/full                           Group
/full/filter_transfer           Group
/full/filter_transfer/F         Dataset {3, 1, 1, 4250}
/full/filter_transfer/l         Dataset {2}
...
/meta                           Group
/meta/filter_transfer           Dataset {5}
...
/null                           Group
/null/filter_transfer           Group
/null/filter_transfer/F         Dataset {3, 21, 3, 4250}
/null/filter_transfer/l         Dataset {2}
...
```

Specifications—spectra

- similar to pseudo-spectra, see more in [Specifications—pseudo-spectra](#). Same 6 dimensions except that l becomes b (bin). meta is also the same.
- there's one more sub-level, lmin_lmax_binwidth such as 50_4300_50, indicating the l-min, l-max and bin-width used when solving for spectra.
- `*/spectra/*/thetas_map` are the maximum a posteriori thetas found from the likelihood code that transform the spectra (including absolute gain and angle.)
- `*/spectra/*/w_bl`: band pass window function. 5 dimension: spectra, null-split, sub-split, b (bin), l.

```
$> h5ls -r high_1_0_2_spectra_final.hdf5
...
/full/spectra          Group
/full/spectra/100_4300_100 Group
/full/spectra/100_4300_100/map_cases Group
/full/spectra/100_4300_100/map_cases/realmaps Dataset {6, 1, 1, 42, 2, 1}
/full/spectra/100_4300_100/map_cases/simmaps_TTBB Dataset {6, 1, 1, 42, 2,
↪ 128}
/full/spectra/100_4300_100/map_cases/simmaps_TTEE Dataset {6, 1, 1, 42, 2,
↪ 128}
/full/spectra/100_4300_100/map_cases/simmaps_TTEEBB_noise Dataset {6, 1, 1,
↪ 42, 2, 512}
/full/spectra/100_4300_100/thetas_map Dataset {6}
/full/spectra/100_4300_100/w_bl Dataset {6, 1, 1, 42, 4200}
/full/spectra/50_4300_50 Group
/full/spectra/50_4300_50/map_cases Group
/full/spectra/50_4300_50/map_cases/realmaps Dataset {6, 1, 1, 85, 2, 1}
/full/spectra/50_4300_50/map_cases/simmaps_TTBB Dataset {6, 1, 1, 85, 2,
↪ 128}
/full/spectra/50_4300_50/map_cases/simmaps_TTEE Dataset {6, 1, 1, 85, 2,
↪ 128}
/full/spectra/50_4300_50/map_cases/simmaps_TTEEBB_noise Dataset {6, 1, 1,
↪ 85, 2, 512}
/full/spectra/50_4300_50/thetas_map Dataset {6}
/full/spectra/50_4300_50/w_bl Dataset {6, 1, 1, 85, 4250}
/meta          Group
...
/meta/null_split Dataset {21}
/meta/spectra    Dataset {6}
/meta/sub_spectra Dataset {2}
...
```

```

/null/spectra          Group
/null/spectra/100_4300_100 Group
/null/spectra/100_4300_100/map_cases Group
/null/spectra/100_4300_100/map_cases/realmaps Dataset {6, 21, 3, 42, 2, 1}
/null/spectra/100_4300_100/map_cases/simmaps_TTBB Dataset {6, 21, 3, 42, 2,
↪ 128}
/null/spectra/100_4300_100/map_cases/simmaps_TTEE Dataset {6, 21, 3, 42, 2,
↪ 128}
/null/spectra/100_4300_100/map_cases/simmaps_TTEEBB_noise Dataset {6, 21, 3,
↪ 42, 2, 512}
/null/spectra/100_4300_100/w_bl Dataset {6, 21, 3, 42, 4200}
/null/spectra/50_4300_50 Group
/null/spectra/50_4300_50/map_cases Group
/null/spectra/50_4300_50/map_cases/realmaps Dataset {6, 21, 3, 85, 2, 1}
/null/spectra/50_4300_50/map_cases/simmaps_TTBB Dataset {6, 21, 3, 85, 2,
↪ 128}
/null/spectra/50_4300_50/map_cases/simmaps_TTEE Dataset {6, 21, 3, 85, 2,
↪ 128}
/null/spectra/50_4300_50/map_cases/simmaps_TTEEBB_noise Dataset {6, 21, 3,
↪ 85, 2, 512}
/null/spectra/50_4300_50/w_bl Dataset {6, 21, 3, 85, 4250}

```

Data Release—Real maps

It is released at NERSC in `/global/cfs/cdirs/polar/www/khcheung/largepatch_null/high_1_0_2/`.

It contains all the maps created from the real data (but none of the simulated data.) For example, to find the final coadded maps, it is located in `/global/cfs/cdirs/polar/www/khcheung/largepatch_null/high_1_0_2/coadd_full/null_realmaps` at NERSC.

Specifications

```

$> h5ls -r
↪ /global/cfs/cdirs/polar/www/khcheung/largepatch_null/high_1_0_2/coadd_full/null_real
/
/d0          Dataset {1351, 1351}
/d4i         Dataset {1351, 1351}
/d4r         Dataset {1351, 1351}
/w0          Dataset {1351, 1351}
/w4          Dataset {1351, 1351}

```

They are all 2D-array, with w indicating the total weights, and d indicating the coadded signal. 0 is the 0-f component, 4 is the 4-f component, r is the real part, i is the imaginary part.

For example,

$$I = \frac{d_0}{w_0}$$

$$Q = \frac{d_4^r}{w_4}$$

$$U = \frac{d_4^i}{w_4}$$

These maps are projected using the OLEA projection, as requested from the high- ℓ *EE* pipeline. An example implementation of the projector is in <https://github.com/ickc/TAILO/blob/master/tail/pointsource.py#L10-L22>.

Also, for null-maps, there may be additional HDF5 group level with the name of respective null-split, which would be self-explanatory.