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dc SQUID: Current Noise

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ABSTRACT

The computer model of Tesche and Clarke used to calculate the voltage noise across the dc SQUID is extended to calculate the circulating current noise around the SQUID loop, and the correlation between the circulating current noise and the voltage noise across the SQUID. The parameters chosen are $\beta = 2LI_0/\Phi_0 = 1$, $\Gamma = 2\pi k_B T/I_0 \Phi_0 = 0.05$, and an applied flux of $\Phi_0/4$ (L is the SQUID inductance, $I_0$ is the critical current per junction, $T$ is the temperature, and $\Phi_0$ is the flux quantum). At frequencies well below the Josephson frequency and at the optimum current bias, the voltage power spectral density is approximately $16 k_B T R$, the current power spectral density is approximately $11 k_B T$, and the voltage-current correlation spectral density is approximately $12 k_B T$, where $R$ is the resistance per junction.
I. INTRODUCTION

The dc SQUID\textsuperscript{1,2} (Superconducting Quantum Interference Device) can be used either directly as a magnetometer, or in conjunction with various input circuits as a magnetometer, gradiometer, or voltmeter. In a previous paper,\textsuperscript{3} we developed a computer model for the isolated SQUID, and calculated the voltage noise at the SQUID output as a function of the device parameters. Thus, we determined the values of these parameters that optimized the energy resolution of the isolated SQUID. However, in addition to the voltage noise, the SQUID also generates a circulating current noise that induces noise into any input circuit coupled to it. To properly optimize the input circuit and SQUID, it is essential to include this additional noise. In this paper, we extend our computer model\textsuperscript{3} to calculate the current noise as a function of the appropriate SQUID parameters. In the following paper,\textsuperscript{4} the results will be applied to practical circuits used for magnetometers and voltmeters.
II. COMPUTER MODEL FOR CURRENT NOISE IN THE dc SQUID

Our model for the isolated dc SQUID consists of two resistively shunted Josephson tunnel junctions mounted on a superconducting ring (Fig. 1). This model was discussed in detail in an earlier paper; here, we restrict the discussion to a symmetric SQUID with a loop inductance \( L \), and with critical current \( I_0 \) and shunt resistance \( R \) per junction. As before, we neglect the junction capacitance. The SQUID is biased at a constant current \( I \) and threaded by an applied flux \( \Phi_o \). The voltage across the SQUID is \( V \), and the current around the SQUID is \( J \). The phase differences across the junctions are \( \delta_1 \) and \( \delta_2 \). The Johnson noise generated in the shunt resistors is modeled by the noise voltages \( V_{N1} \) and \( V_{N2} \). We use the following dimensionless units: voltage, \( v \), in units of \( I_0 R \); currents, \( i \) and \( j \), in units of \( I_0 \); flux \( \Phi \), in units of the flux quantum; and time, \( \theta \), in units of \( \Phi_0 / 2\pi I_0 R \). We define

\[ \beta = 2LI_0 / \Phi_0. \]

It is straightforward to show that the dimensionless SQUID equations are

\[ j = (\delta_1 - \delta_2 - 2\pi \phi_a) / \pi \beta, \]

(1)

\[ v = \frac{1}{2} \frac{d\delta_1}{d\theta} + \frac{1}{2} \frac{d\delta_2}{d\theta}, \]

(2)

\[ \frac{d\delta_1}{d\theta} = \frac{1}{2} - j - \sin \delta_1 + v_{N1}, \]

(3)

and

\[ \frac{d\delta_2}{d\theta} = \frac{1}{2} + j - \sin \delta_2 + v_{N2}. \]

(4)
We solve these equations numerically on a computer by integrating the phases in Eqs. (2) and (3) stepwise in time. For brevity we confine our calculations to the value $\beta = 1$ that was found to optimize the energy resolution, and to the case $\Gamma = 2\pi k_B T / I_0 \Phi_0 = 0.05$. The Johnson noise voltages, $v_{N1}$ and $v_{N2}$, are modeled by two uncorrelated trains of voltage pulses of constant duration. The random pulse heights are Gaussian distributed about a zero mean with voltage power spectral densities satisfying $S^N_v = 4\Gamma$. From the computed values of $\delta_1(\theta)$ and $\delta_2(\theta)$ we calculate $v(\theta)$ and $j(\theta)$ from Eqs. (1) and (2). The voltage power spectral density, $S_v(f)$, the circulating current power spectral density, $S_j(f)$, and the real part of the correlation power spectral density, $S_{vj}(f)$, are then computed as functions of the bias current, $i$, and applied flux, $\Phi_a$. The spectral densities have noise-broadened resonances at the Josephson frequency $f_J = \langle v \rangle / 2\pi$ and its harmonics ($\langle v \rangle$ is the time-averaged voltage). At frequencies below the resonance at $f_J$ (say, $f < f_J/10$), the spectral densities are white. We define $S^0_v$, $S^0_j$, and $S^0_{vj}$ to be the low frequency values of $S_v$, $S_j$, and $S_{vj}$.

Figures 2 to 4 show $S^0_v$, $S^0_j$, and $S^0_{vj}$ as functions of $i$ for several values of $\Phi_a$. (Figure 3 duplicates Fig. 14(a) of the previous paper.) The parameters $\beta = 1$ and $\Gamma = 0.05$ correspond to $L \approx 0.3\text{nH}$, $I_0 \approx 3\mu\text{A}$, and $T \approx 4\text{K}$. All of the SQUID responses are periodic in $\Phi_a$ with unity period. In addition, for the symmetric SQUID the spectral densities are unchanged if $\Phi_a - \Phi_a$ and $j - j$. For each value of $\Phi_a$, the spectral densities peak approximately at the corresponding noise-free critical current, $i_c$, which decreases as $\Phi_a$ increases. For $i >> i_c$,
where the junctions contribute negligibly to the noise spectral densities, $S^0_v$ and $S^0_j$ tend to the Johnson noise values: $S^0_v \rightarrow 2\Gamma = 2k_BTR/(I_0R\Phi_0/2\pi)$, and $S^0_j \rightarrow 2\Gamma = (2k_BTR)/(I_0\Phi_0/2\pi R)$. Furthermore, since the voltage and circulating current noises produced by the Johnson noise in two parallel resistors are uncorrelated, $S_{vj} \rightarrow 0$ for $i \gg i_c$. Near $i_c$, the effect of the junctions and loop is to increase both $S^0_v$ and $S^0_j$ above the value for the resistors, $2\Gamma$, and, in addition, to correlate the voltage and current noises. The correlation can be understood qualitatively from the following argument. The total flux through the SQUID is $\phi_T = \phi_0 + \beta j/2$. Thus, Johnson noise superimposed on $j$ produces a noise in the total flux that is similar to an externally applied flux noise. The SQUID transfer function, $\partial\bar{v}/\partial\phi_0$, relates changes in the time averaged voltage $\bar{v}$ to changes in $\phi_0$, and thus relates the effective flux noise due to noise in $j$ to the total voltage noise. For $i \sim i_c$, $\partial\bar{v}/\partial\phi_0 \neq 0$ and the voltage noise is correlated with the current noise. For $i \gg i_c$, $\partial\bar{v}/\partial\phi_0 \rightarrow 0$ and no correlation is introduced between the voltage and current noises. Note that in the special case $\phi_0 = 0, 0.5$, $\partial\bar{v}/\partial\phi_0 = 0$, and $S^0_{vj} = 0$ for all values of bias current.

As $\phi_0$ is increased from 0 to 0.5, $S^0_v$ decreases in a way that is consistent with the corresponding decrease in the dynamic resistance of
the SQUID. On the other hand, the maximum value of $S_j^0$ increases as $\phi_a$ is increased from 0 to 0.5. In addition, for $\phi_a = 0.5$, $S_j^0$ rises rapidly as $i$ is lowered below $i_c$. This behavior can be understood by examining the stable configurations of the noise-free SQUID below $i_c$. For the case $\phi_a = 0.5$, two such states exist, one with $j < 0$ and the other with $j > 0$. The Johnson noise generated in the shunts not only produces small excursions of the circulating current about the noise-free value, but also induces transitions between the two states at random times. This switching behavior is illustrated in Fig. 5 for $i = 0.9$, $\phi_a = 0.5$, $\beta = 1.0$, and $\Gamma = 0.05$. As the current switches (Fig. 5a), a corresponding voltage pulse appears across the SQUID (Fig. 5b). Both $S_v^0$ and $S_j^0$ are dominated by the switching noise at bias currents well below $i_c$. In most applications, the SQUID is biased at $i = i_c$ and $\phi_a \sim 0.25$, and the large current noise at $i \ll i_c$, $\phi_a \sim 0.5$ is not observed.
III. SUMMARY

We have computed low frequency voltage power spectral densities, \( S_V^0 \), current power spectral densities, \( S_J^0 \), and correlation power spectral densities, \( S_{VJ}^0 \), vs. bias current for the dc SQUID operated at \( \beta = 1 \) and \( \Gamma = 0.05 \). At the operating bias current, we find in dimensioned units
\[ S_V^0 \approx 16 k_B T R, \quad S_J^0 \approx 11 k_B T / T, \quad \text{and} \quad S_{VJ}^0 \approx 12 k_B T. \]
These values are used in the following paper\(^4\) to calculate the noise temperature of SQUID voltmeters, and the energy resolution of SQUID magnetometers.

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REFERENCES


5. W. C. Stewart, Appl. Phys. Lett. 12, 277 (1968);
FIGURE CAPTIONS

Fig. 1. Model for the isolated symmetric dc SQUID.

Fig. 2. Normalized low frequency voltage power spectral density, $S_v^0/2\Gamma$, vs. bias current, $i$, as a function of applied flux, $\phi_a$.

Fig. 3. Normalized low frequency current power spectral density, $S_j^0/2\Gamma$, vs. bias current, $i$, as a function of applied flux, $\phi_a$.

Fig. 4. Normalized low frequency correlation power spectral density $S_{vj}^0/2\Gamma$ vs. bias current, $i$, as a function of applied flux, $\phi_a$.

Fig. 5(a). Circulating current, $j$, and (b) voltage, $v$, as functions of time $\theta$ for bias current $i = 0.9$, and applied flux $\phi_a = 0.5$. 
Figure 1
Figure 3

\[ \beta = 1.0 \]
\[ \Gamma = 0.05 \]

Normalized current power spectral density, \( s_i^0/2\Gamma \)

\( \phi_a = 0.5 \)
\( \phi_a = 0.4 \)
\( \phi_a = 0.25 \)
\( \phi_a = 0 \)

Bias current, \( i \)

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Figure 4

\[ \beta = 1.0 \]
\[ \Gamma = 0.05 \]
\[ \phi_a = 0.4 \]
\[ \phi_a = 0.25 \]
Figure 5
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