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CHOICE OF COORDINATE SYSTEM IN ELLIPSOMETRY

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CHOICE OF COORDINATE SYSTEM IN ELLIPSOMETRY

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The choice of coordinate system made in two previous reports[1,2] for the reflection of light affects the results derived in an important way which deserves more comment. Due to the choice made for the positive direction of the reflected electric field component $\vec{E}_p''$ polarized parallel to the plane of incidence (see Fig. 1) the coordinate system $(\vec{E}_p \vec{E}_s \vec{k}_o)$ for the incident wave is right handed while that for the reflected wave $(\vec{E}_p'' \vec{E}_s'' \vec{k}_o'')$ is left handed and all the data derived pertain to this system. Therefore, if the usual x-y coordinate system is employed for the incident wave (looking into the beam) and p and s components are substituted for the case of reflection on a vertical surface (Fig. 2a), then the positive p direction appears reversed after reflection (Fig. 2b).

Since it is preferable to use a right-handed coordinate system as a laboratory frame of reference to analyze the elliptic polarization, and since the previous analysis of ellipse shape and orientation[3] has been based on the usual right hand orientation of the x and y coordinates, either of the following two equivalent coordinate transformations for incident or reflected light, respectively, may be made:

a. Change the coordinate system of the incident light to a left-handed system: For incident linearly polarized light this entails the simple replacement of the +45° azimuth by -45° in order to produce s and p components which are in phase and of equal amplitude. For the reflected wave, shape, orientation and sense of rotation of the ellipse can then be considered in the usual right-handed coordinate system (Figs. 5 and 6, ref. 3) with the values for ψ and Δ derived by the theory.

b. Change the coordinate system for the reflected light to a right-handed system: A reflection of the ellipse around the $\vec{E}_s''$ axis is necessary for this transformation. Since this operation changes orientation and sense of rotation of the ellipse in the laboratory frame of reference, the value of θ is replaced by $(180° - \theta)$ and the sign of γ is reversed (Fig. 3). Thus for given values of ψ (e.g. ~60°) and Δ (e.g. ~ -45°) the ellipse expected to be of the form of Fig. 3a from Fig. 5, ref. 3 is observed as in Fig. 3b and described by the right-hand system of Fig. 3c. Considered in the <u>same</u> coordinate system, the first and the last ellipse show the same amplitude ratio $\tan\psi$ but differ by π in phase difference Δ. Thus, this second coordinate transformation is equivalent to the addition (or subtraction) of π to the computed value of Δ in order to predict the correct ellipse orientation to be observed. Thus, the dependence of ellipse orientation on phase difference Δ shown in Fig. 4 is obtained from the previously derived Fig. 5 (Fig. 6, ref. 3).

Acknowledgment:

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REFERENCES

1. J. R. Mowat and R. H. Muller, Reflection of Polarized Light from Absorbing Media, UCRL-11813, August 1966).
2. J. R. Mowat and R. H. Muller, Reflection of Polarized Light from Film-Covered Surfaces, UCRL-17128, February 1967.
3. R. H. Muller and J. R. Mowat, Analysis of Elliptically Polarized Light, UCRL-11802, October 1965.

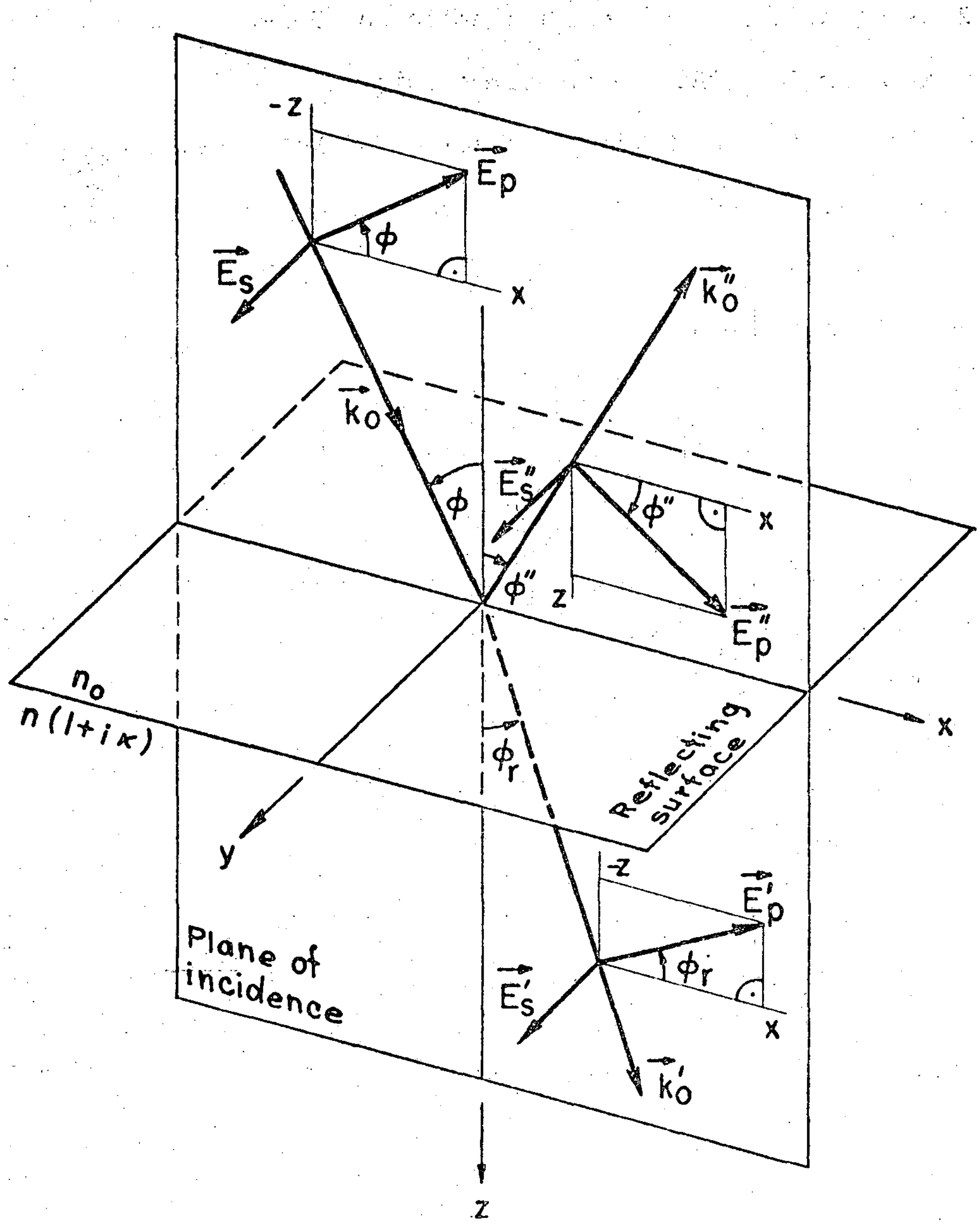

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Fig. 1 Sign conventions for the electric field (positive direction of $\vec{E}_s$ and $\vec{E}_p$ in incident, reflected and refracted waves indicated by arrows). The complex index of refraction of the absorbing medium is designated $n(1+i\kappa)$. The subscript p stands for polarization parallel and the subscript s for polarization normal to the plane of incidence.

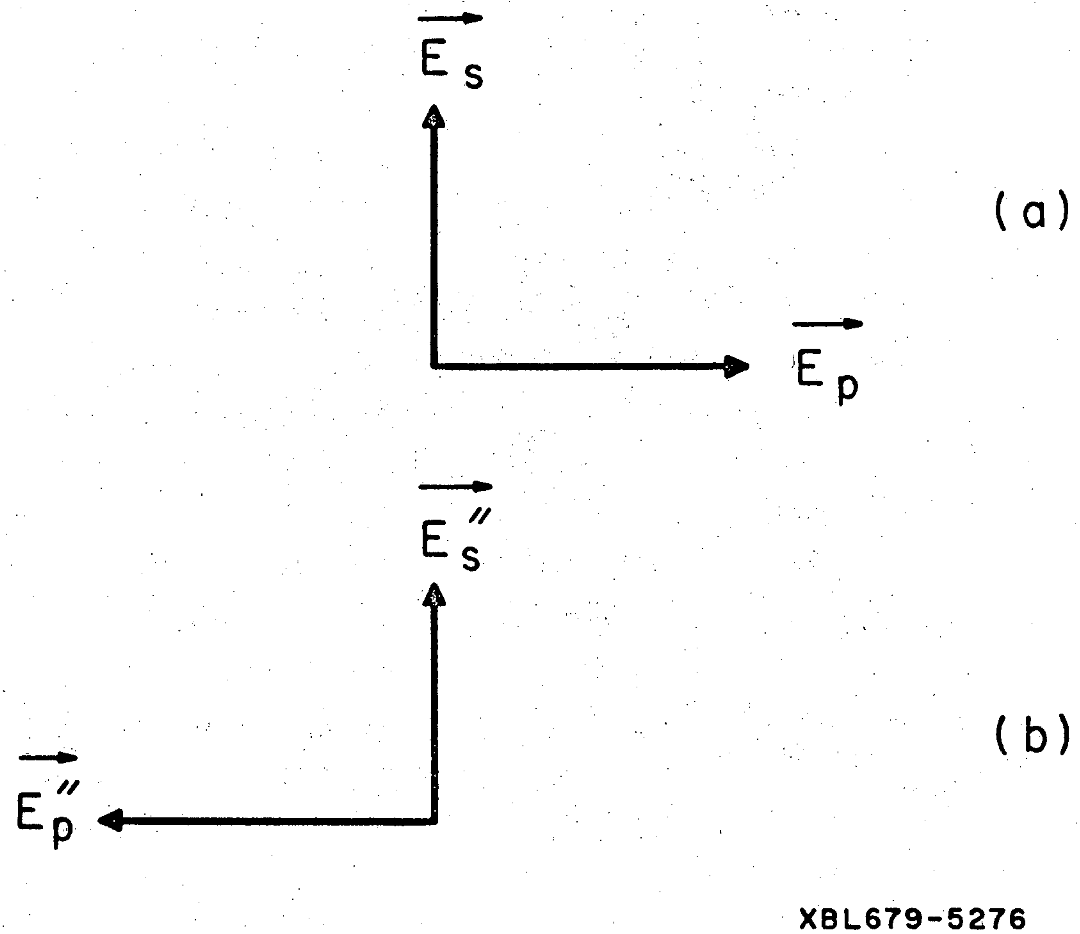

Fig. 2 Coordinate system for incident (a) and reflected (b) waves as seen looking toward the source for reflection on a vertical surface.

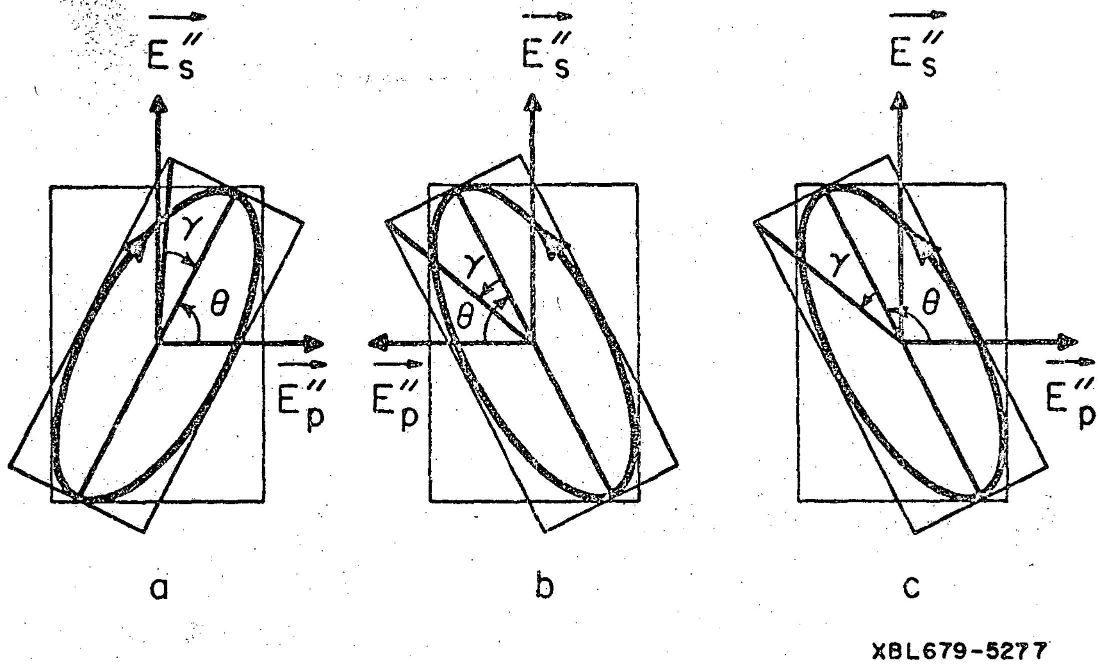

Fig. 3 Coordinate systems for reflected light. Derivation of ellipse from computed values of ψ and Δ (assumed here approximately 60° and -45° respectively).

a. Ellipse expected in right-hand system on the basis of ψ and Δ values computed for left hand system

b. observed ellipse due to left hand system used in computation

c. ascribed orientation of observed ellipse in right hand laboratory system

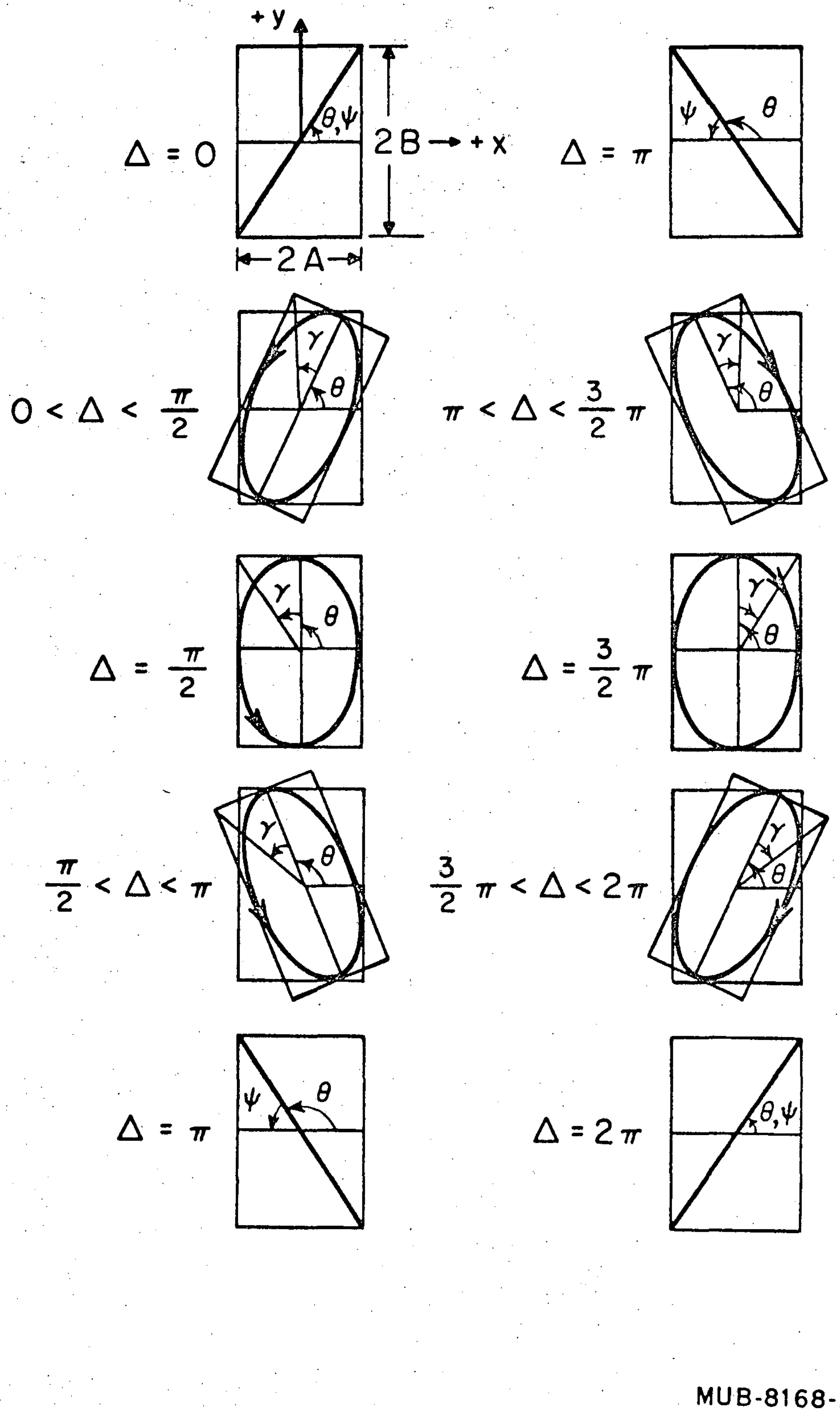

Fig. 4 Ellipse orientation observed in right-hand laboratory coordinate system with phase difference Δ computed for left-hand coordinate system.

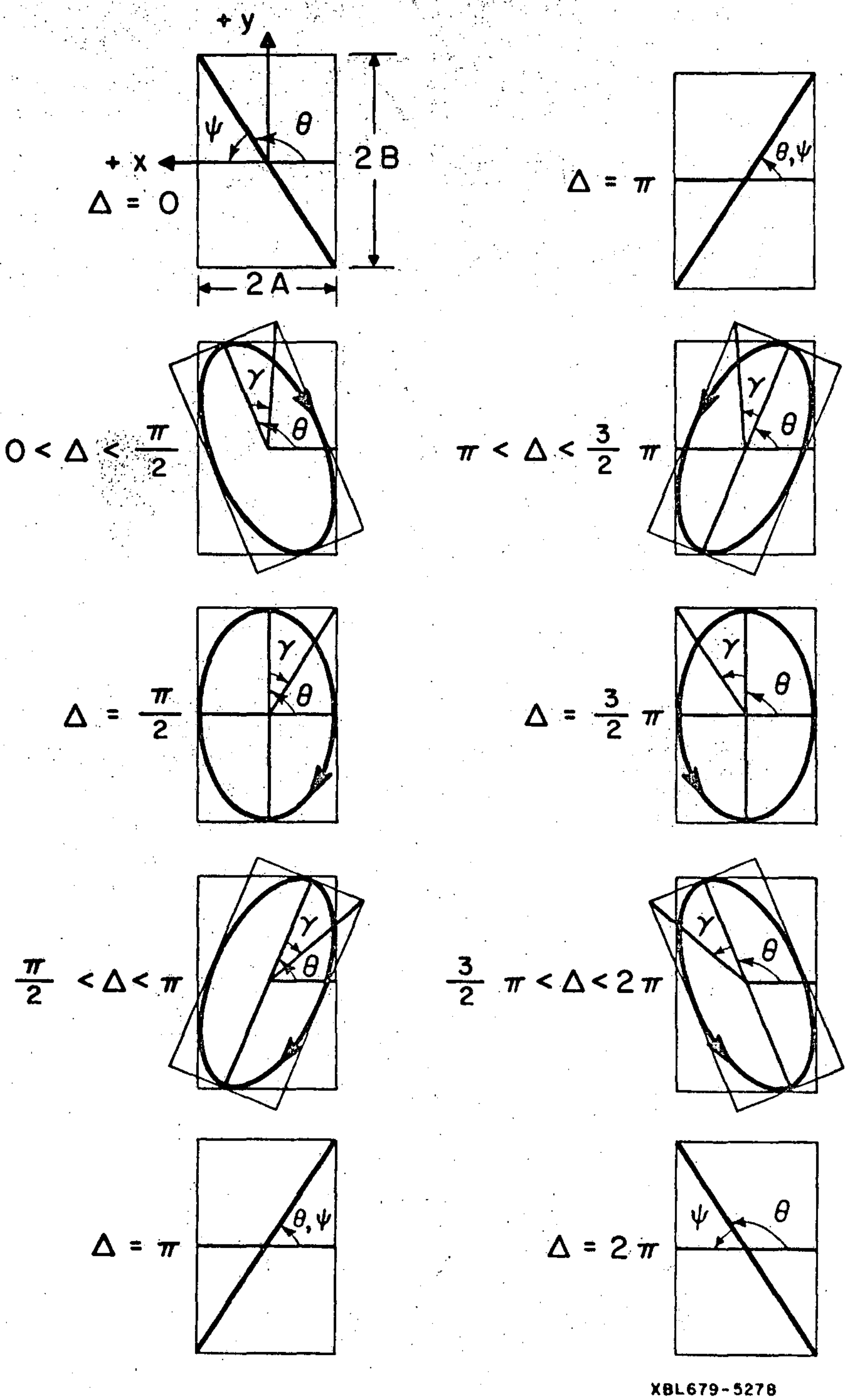

Fig. 5 Ellipse orientation in right-hand coordinate system with phase difference Δ for right-hand system.