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Author

Stapp, Henry P.

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TOPOLOGICAL EXPANSION FOR BARYONS AND MESONS

For Reference

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Henry P. Stapp

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3. The Elementary Surfaces and Graphs

The elementary surfaces σ are the handleless, windowless surfaces with $b = b_{\min}$ and no baryon closed loops. They are defined by

$$(3.1) \quad h = w = \delta = \ell = 0.$$

The condition $\ell = 0$ representing the absence of baryon closed loops will no longer be explicitly exhibited: it holds throughout this work unless otherwise stated.

Each capped surface σ is associated with two other surfaces. The first is the uncapped version, which is constructed from the capped version by cutting off the capping regions. This procedure produces a set of κ separate surfaces, which are called the components of the uncapped surface. The second related surface is called the joined surface δ . It is formed from the uncapped version by joining together into one line, called the junction line, the three inner lines associated with each baryon-antibaryon pair. This junction line is analogous to the baryon line, and is likewise represented by a dotted line. The components C_i of a joined surface δ are the parts of δ that are connected when no crossing of junction lines is allowed. These components coincide with the components of the uncapped surface.

A component C_i is said to be elementary if and only if it is handleless and windowless, and has only one boundary:

($h_i = w_i = b_i - 1 = 0$). A joined surface δ is said to be elementary if and only if it is connected, has only elementary components, and has no closed sequence of components. This last requirement is that there be no way of numbering the components C_i of δ so that $(C_1, C_2, \dots, C_n)$ is a sequence of n distinct components that contains a closed path P that runs through them in the indicated order, but that cannot be continuously deformed in δ to a point.

A typical elementary component is shown in Fig. 5a. The vertices at which two solid (quark) lines meet are meson connections. The dotted lines are "inner lines", and a vertex where a dotted line meets a solid line is a baryon connection. To form a joined surface three such dotted lines, coming usually from different components, must be joined together to form a single junction line. A typical joined surface is shown in Fig. 6a. The vertices at which two solid (quark) lines meet are again meson connections, and the vertices at which three solid lines meet a dotted (junction) line are baryon connections.

Each surface can be represented by a graph. The graph G_i of a component C_i , like the graph of an uncapped surface, is formed by squeezing the surface first into narrow strips, and then into a graph, where each line of the graph represents two oppositely directed boundary sections of the surface. Since each elementary component C_i has no handles, no windows, and one boundary it is topologically equivalent to a disc, with particle connections on its boundary. Thus the graph G_i corresponding to any elementary component C_i is a tree graph. Conversely, the component C_i corresponding to any tree graph G_i is an

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elementary component, which is topologically equivalent to a disc with connection points on its boundary.

The graph G_i of Fig. 5b corresponds to the component C_i of Fig. 5a. Each exterior vertex of the graph G_i not marked with b represents a meson connection. Each b -vertex represents the contraction of an inner line.

To form the graph $\hat{G}$ corresponding to a joined surface $\hat{\sigma}$ one combines triples of b -vertices of component graphs G_i into single b -vertices of $\hat{G}$. Thus each b -vertex of $\hat{G}$ represents a contracted junction line, and also a pair of baryon connection points. It must have exactly three lines incident upon it, one from each of the three components G_i of $\hat{G}$ that corresponds to one of the three components C_i of $\hat{\sigma}$ joined at the junction line whose contraction is represented by it. The graph $\hat{G}$ corresponding to the joined surface $\hat{\sigma}$ of Fig. 6a is shown in Fig. 6b. The heavy dots on the ends of lines of $\hat{G}$ are meson connection points. Each b -vertex represents, as already stated, a pair of baryon connection points.

A connection between an elementary capped surface σ , the associated joined surface $\hat{\sigma}$, and the graph $\hat{G}$ that represents $\hat{\sigma}$ is given by the following theorem:

Theorem: Let σ be a capped surface. Let $\hat{\sigma}$ be the corresponding joined surface. Let $\hat{G}$ be the graph that represents $\hat{\sigma}$. Then the following three properties are equivalent:

- (1) σ is elementary ($h = w = \delta = 0$)
- (2) $\hat{\sigma}$ is elementary ($\hat{\sigma}$ is connected, has only elementary components, and has no closed sequence of components).
- (3) $\hat{G}$ is a connected tree graph. ($\hat{G}$ is connected and has no closed loops).

Proof: Suppose that the joined surface $\hat{\sigma}$ satisfies condition (2). Then $\hat{\sigma}$ is connected, has only elementary components, and no closed sequence of components. The fact that $\hat{\sigma}$ is connected implies that $\hat{G}$ is connected. The fact that all the components C_i of $\hat{\sigma}$ are elementary, implies that all the components G_i of $\hat{G}$ are tree graphs. The final condition that there be no closed sequence of components C_i implies that $\hat{G}$ is a tree graph. For if $\hat{G}$ were not a tree graph then it would contain a closed loop. This closed loop could not be contained in any single G_i , since each G_i is a tree graph. Thus this closed loop would have to pass through a sequence of G_i . If some G_i appeared several times in this loop then, because each G_i is connected, a smaller loop could be formed, and ultimately a loop could be formed in which no G_i appeared more than once. This loop would define a closed sequence $(G_1, G_2, \dots, G_n)$, and a sequence $(b_{12}, b_{23}, \dots, b_{n1})$ of b -vertices. By joining in G_2 the junction lines represented by b_{12} and b_{23} , etc., one would form a closed path P in $\hat{\sigma}$ that could not be continuously deformed in $\hat{\sigma}$ to a point, since its image in $\hat{G}$ cannot be so deformed. Thus $\hat{G}$ must be a connected tree graph, and (2) implies (3).

$$(3.29) \quad b_i = 1 \quad (\text{all } i)$$

Thus, conditions (1) imply that σ is connected [(3.28)], has only elementary components [(3.23), (3.25), (3.29)], and has no closed sequence of components. Hence (1) implies (2). This completes the proof of the theorem.

It is now easy to see geometrically that any elementary capped surface σ can be imbedded on a sphere. The structure shown in Fig. 8 can be imbedded on a small sphere in the manner shown in Fig. 10. Three tubes can be attached to this sphere, and the lines of G , or the surfaces of σ , that leave the structure can leave the small sphere along the surfaces of these tubes.

The various lines of G , or surfaces of σ , that are parts of any single component G_i , or C_i , can be placed on tubes that are joined together like the spokes of a wheel, with the parts of G_i or of C_i joined at the hub, in the manner shown in Fig. 11. The possibility of doing this is entailed by the tree structure of each G_i , or by the equivalent elementary character of each C_i .

Since the set of tubes upon which the full graph G or surface σ is attached can be connected together in the way specified by the graph $\hat{G}$, and the graph $\hat{G}$ is a connected tree graph, this structure of tubes can be deformed into a sphere. Thus any elementary σ , can be imbedded on a sphere, and so can the corresponding graph G .

It is also easy to visualize in this way more complicated cases. For example, each of the three cases shown in Fig. 1 are represented by using two spheres of the kind shown in Fig. 10. In general there is always one of these spheres for each dotted line of the baryonic quark diagrams. In the case of Fig. 1a these two spheres are connected by only one tube. Two spheres connected by a tube is equivalent to a sphere. Hence $h = 0$.

In this case $\kappa = 1$, $\kappa' = 5$ and $n_B = 4$. Each of the five components is free of handles. Thus (2.9) gives

$$(3.30) \quad h = \sum h_i + n_B + \kappa - \kappa' = 0,$$

in agreement with the result obtained above geometrically.

In the case represented by Fig. 1b the two spheres are joined by two tubes, and hence the structure is a torus with $h = 1$. On the other hand, $\kappa = 1$, and $\kappa' = 4$, hence (2.9) gives

$$(3.31) \quad h = \sum h_i + n_B + \kappa - \kappa' = 1,$$

in agreement with the result obtained geometrically.

In the case represented by Fig. 1c three tubes are needed to carry the connecting lines or surfaces from one sphere to the other, and hence $h = 2$. On the other hand, $\kappa = 1$ and $\kappa' = 3$, and hence (2.9) gives

$$(3.32) \quad h = \sum h_i + n_B + \kappa - \kappa' = 2.$$

4. Discontinuity Formula

The "planar" amplitudes are those that correspond to $\ell = 0$ surfaces with the minimum number of boundaries and handles, namely those with $h = \delta = 0$. The term "planar" is appropriate in the sense that the associated surface can be imbedded on a sphere, and hence on a plane. However, the surface associated with a "planar" amplitude generally has more than one boundary and hence is not planar in the same sense as in the meson sector. Perhaps the name "primitive" or "ordered" would be better, particularly because the word planar is often used in the meson sector to denote a sum of primitive or ordered amplitudes analogous to ours over all cyclic orderings of the particles.

The elementary surfaces have $h = \delta = w = \ell = 0$. If the $w = 0$ condition is removed then each elementary surface can be filled with windows, each of which is formed by cutting out of the surface a small disc and then running a quark line around the newly formed circular edge of the surface. Baryon closed loops will be temporarily excluded.

The amplitude corresponding to a surface $\hat{\sigma}$ is assumed to have a normal-threshold discontinuity in a certain channel $A \rightarrow B$ only if $\hat{\sigma}$ can be cut into two disjoint parts each of which is connected and contains all of the particle connections corresponding to either A or B , but not both.

If an elementary surface $\hat{\sigma}$ is cut into two disjoint connected parts then each part is again an elementary surface $\hat{\sigma}$, provided the cut is tree-shaped. This is proved in the appendix.

This crucial property is the basis of the validity of the planar discontinuity formula (1.1).

The planar diagrams of (1.1) are the elementary surfaces $\hat{\sigma}$. There will usually be several such diagrams $P_i(A,B)$ associated with the processes $A \rightarrow B$. For each such diagram $P_i(A,B)$ one can consider all possible ways of cutting it into two disjoint parts in such a way that the connection points that represent the particles corresponding to A are separated from those corresponding to B . Two examples of typical cuts are shown in Fig. 12. The particle connections indicated by the small letters a and b belongs to A and B , respectively.

In Fig. 12 each cut has the form of a tree graph. Figure 13 shows the tree graph corresponding to the cut illustrated in Fig. 12b. In general, the tree graph corresponding to a cut has only three-line vertices, one for each time the cut in the surface $\hat{\sigma}$ cuts through a junction line. Each edge of the tree-diagram corresponds to a traversal of the cut through some component of the surface. Each such traversal is a connected line that begins at either a quark line or a junction and ends at either a quark line or a junction line.

For each $P_i(A,B)$ one must consider the corresponding surfaces with windows: $P_i(A,B)$ can be considered to represent the infinite sum of surfaces obtained by putting independently into each component of $P_i(A,B) = \hat{\sigma}$ each possible number of windows.

Each window in any component is equivalent to any other one; only the number of them matters. The cut through $P_i(A,B)$ (see Fig. 12) will cut through each component C_j a certain number $n_j \geq 0$ of times, and will split this component into $n_j + 1$ parts. On each section of the cut there will be a certain number of windows. And in each connected (after cutting) part of each component there will be a certain number of windows.

The left-hand side of (1.1) represents a multiple sum over each of the topologically distinct surfaces $P_i(A,B)$, over each topologically distinct tree-graph cutting of $P_i(A,B)$ into two disjoint connected parts, over each number of windows lying on each section of the cut, and over each number of windows in each connected (after cutting) part of each component. Each diagram must be given a topological weight factor of $1/n$, where n is its symmetry number, which is the number of permutations of its components C_i that change it into a diagram that is indistinguishable from itself⁶. Thus Fig. 14a has weight $1/3!$, whereas Fig. 14b has weight $1/2$ because it is indistinguishable from Fig. 14c.

The right-hand side of (1.1) is formed by first taking for each set C of intermediate particles all pairs of planar surfaces $P_i(A,C)$ and $P_j(C,B)$. The particles associated with C are paired off in a one-to-one fashion. This can be represented as in Fig. 15. There is a sum over each possible way of pairing the particles C of $P_i(A,C)$

to the particles C of $P_j(C,B)$. For each such term one must also sum independently over all possible numbers of windows in each component of both $P_i(A,C)$ and $P_j(C,B)$. Each meson connection is now replaced by a meson strip, and each baryon connection is replaced by a set of three connecting strips, which connect the three component-strips corresponding to the baryon on one end to the three component strips corresponding to the baryon on the other end. There is a sum over all $3!$ ways of connecting the three strips on one end to the three strips on the other. Then all terms that are not elementary surfaces with windows are eliminated. Equation (1.1) says that the result is equal to what was previously constructed on the left-hand side.

The validity of the planar discontinuity formula (1.1) is regarded here as the principal justification of this topological classification scheme. The scheme enjoys also the other features listed in section 1.

An added bonus is obtained if one considers the formula for powers of the coupling constant g . This constant appears in the form

$$(4.1) \quad g^{n_M - n_B + 2b + 2w + 4h - 4\kappa} = g^{n_M - n_B - 2} \times (g^2)^{2h + b - 1} (g^2)^w g^{4(1-\kappa)},$$

where closed baryon loops are now admitted. Contributions to scattering functions that correspond to surfaces without closed baryon loops have $\kappa = 1$. If in these contributions one incorporates

a factor of N for each window and assumes that $N = g^{-2}$ then the g dependence becomes

$$(4.2) \quad g^{n_M - n_B - 2} \times (g^2)^{2h+b-1}.$$

The only difference between this formula and the formula in the meson sector is the factor g^{-n_B} , which is the same for all contributions to any given scattering function.

Closed baryon loops are allowed only in higher order.

The simplest way to achieve this is to consider the number l of baryon closed loops as an additional expansion parameter. Konishi³ distinguishes three different types of baryon closed loops and introduces three separate numbers l_A , l_B , and l_C . That is also a possibility.

I would like to thank Professor G. F. Chew, Dr. F. J. Capra, Dr. J. P. Sursock, and Mr. G. Weissmann for valuable discussions related to this work.

TOPOLOGICAL EXPANSION FOR BARYONS AND MESONS*

Henry P. Stapp

Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

August 19, 1977

ABSTRACT

A method of including baryons in the topological classification scheme of Veneziano is described. Every amplitude is classified in terms of the number of handles and boundaries of an associated surface. This classification is purely topological. It does not refer to numbers of flavors N , or rely on an equality $N^{-1} \approx g^2$ that is supposed to be a consequence of the topological expansion. The lowest-order amplitudes are associated with surfaces that have a characteristic tree structure. The analog of Veneziano's planar discontinuity formula holds.

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1. Introduction

The aim of this paper is to describe a topological expansion scheme for baryons and mesons. The main idea was described briefly in ref. 1, and in modified form in ref. 2.

Recently, two closely related papers have appeared. In the first Rossi and Veneziano³ have been led, by considerations based on quantum chromodynamics, to study a Y-shaped model for baryons similar to the one introduced for purely topological reasons in ref. 1. Rossi and Veneziano incorporate into their baryonic quark diagrams certain extra "dotted" lines, called junction lines, that are similar in form and function to the dotted lines used in ref. 1. More recently Konishi⁴ has discussed the topological classification of models like the one of Rossi and Veneziano and has arrived at a classification that is, in effect, the same as the one of ref. 1 with the exception of the following three points:

- (1) Baryon closed loops are treated differently.
- (2) The scheme of ref. 1 is a purely topological classification scheme that does not rely on dynamical relations such as $g^2 \approx N^{-1}$, whereas Konishi's scheme, when baryon loops are absent, is essentially an expansion in $1/N$, under the dynamical condition that $g^2 \approx N^{-1}$.
- (3) The scheme of ref. 1 classifies amplitudes in accordance with two topological parameters, h and b ,

which specify the numbers of handles and boundaries of a certain surface, whereas Konishi introduces many topological numbers, but ultimately orders the contributions, in the case where no baryon closed loops are present, on the single parameter $1/N \approx g^2$. This $1/N$ expansion would be equivalent to our using only $(2h + b)$, instead of both h and b separately.

The papers of Rossi and Veneziano and of Konishi make ample reference to physical applications. The present paper concentrates on purely theoretical aspects.

Our aim is to devise a topological expansion scheme that satisfies the following six requirements:

- (1) Each meson is represented by a pair of oppositely directed quark lines, whereas each baryon is associated with a triple of lines that run in the same direction.
- (2) Certain diagrams are classified as planar. These planar diagrams include the "obviously planar" diagrams, which are those that can be drawn on a plane with the external lines entering along the outer boundary.
- (3) The rules allow the classification of all diagrams that correspond to multiple unitarity-type products of amplitudes represented by planar diagrams.
- (4) For purely mesonic diagrams the topological classification scheme reduces to the topological classification

of Veneziano⁵ in terms of numbers of boundaries b and handles h .

- (5) The planar diagrams are the diagrams that are of lowest order in the topological scheme, namely those with $h = 0$ and $b = b_{\min}$, and no baryon closed loops.
- (6) The planar discontinuity formula holds in the following topological sense. Suppose A is a set of initial particles and B is a set of final particles. Let $P_i(A, B)$, $i = 1, 2, \dots$ be the set of planar diagrams that correspond to the process $A \rightarrow B$. Let $P_{ik}(A, B)$, $k = 1, 2, \dots$ be the set of diagrams that correspond to the different ways that $P_i(A, B)$ can be cut by a tree-shaped cut (see below) into two connected parts in a way such that each connected part of $P_{ik}(A, B)$ is connected to either A or B , but not to both A and B . Then

$$(1.1) \quad \sum_{ik} P_{ik}(A, B) = \left[\sum_{i, j, C} P_i(A, C) \otimes P_j(C, B) \right]_{\text{planar part}}$$

where the product $P_i(A, C) \otimes P_j(C, B)$ of the two diagrams $P_i(A, C)$ and $P_j(C, B)$ is the diagram obtained by joining the lines associated with the corresponding particles of C in $P_i(A, C)$ and $P_j(C, B)$.

On the right-hand side of (1.1) one must sum over all i, j , and C , and then discard all terms that are not planar.

These topological requirements can be satisfied if the baryon is regarded as a triad of orientable surfaces arranged like the feathers of an arrow (i. e., in a Y shape) with the usual three quark lines running in the same direction along the outer edges of these three surfaces. The inner edges of these three surfaces are required to remain always in close proximity to each other. The path through the diagram of each triad of inner boundary lines is represented by a single dotted line, called a baryon (or a dotted) line. These dotted lines must be added to the usual (solid) quark lines to give the complete topological structure of the baryonic quark diagram, for they show the paths through the diagram of the various triads of "inner" boundary lines.

Several baryonic quark diagrams for baryon-baryon scattering are shown in Fig. 1. The first of these, Fig. 1a, is the diagrammatic representation of a process that in lowest order corresponds to the exchange of a meson between the two baryons, as shown in Fig. 2a. The second diagram, Fig. 1b, represents a process that in lowest order corresponds to an exchange of two mesons, as shown in Fig. 2b. The third figure, Fig. 1c, represents a process that in lowest order corresponds to an exchange of three mesons, as shown in Fig. 2c.

The dotted line represents, as already stated, the path of three separate inner boundary lines. There is one of these inner boundary lines for each of the three outer (quark) lines associated with a baryon. Thus in Fig. 1a there are five separate surfaces.

The two uppermost quark lines and the two lowermost quark lines each lie on separate surfaces, but the two remaining quark lines lie on the boundary of a single surface. Similarly, in Fig. 2b there are four separate surfaces, and in Fig. 2c there are three separate surfaces. If the two baryons had not been connected at all there would have been six separate surfaces, three for each baryon.

To obtain the topological classification one proceeds as follows. For each incoming baryon and each outgoing baryon one joins the three corresponding orientable surfaces together to form a single orientable surface. The procedure for joining these surfaces together is called the capping operation, and is illustrated by the sequence of diagrams in Fig. 3. Fig. 3a shows the original arrangement of three surfaces with the three outer quark lines running from left to right, and the three inner boundary lines running from right to left along a small cylinder, which is the shaft of the arrow. Fig. 3b shows the same thing with the cylinder enlarged. Fig. 3c shows the same thing again, with the third surface now slid around to the front, where we can see it. Fig. 3d shows the three surfaces rotated so that they lie on the cylinder. The rotation is such as to place all three surfaces on the cylinder with clockwise orientation. Thus if one looks at the cylinder from the outside then the quark lines and boundary lines always run clockwise around the surfaces they bound. Fig. 3e

shows the capping operation: the end of the cylinder is added to the surface. This "cap" joins the three oriented strips together into a single oriented surface. This single surface is shown also in Fig. 3f, where the three original quark lines are shown as solid lines, and the three original inner boundary lines are shown as dotted lines. In Fig. 3f the parts of the boundary that were originally on the end of the cylinder have been contracted to points.

The capping procedure described above refers to an outgoing baryon. For an incoming baryon the procedure is completely analogous: instead of capping the right-hand end of the cylinder in Fig. 3d one caps the left-hand end, and obtains Fig. 3g in place of Fig. 3f. The cyclic order of the three inner lines is assumed to stay the same as they move along the cylinder from the incoming end to the outgoing end. Thus the capping procedure at the two ends produces a diagram of the form of Figure 4 (This rule replaces the color prescription used in ref. 1. The present rule is more general.). For each of our examples of Fig. 2, the capping procedure joins the various surfaces together into a single orientable surface. In general the capping procedure will join the various original uncapped surfaces together into a single surface unless the original surface contains a closed baryon loop (i. e., a closed dotted line).

Let κ represent the number of components of the capped surface. Let κ' represent the number of components of the original

uncapped surface. If capped surfaces with no boundaries are ignored, then the number of boundaries must be at least $b_{\min} \equiv \kappa' - \kappa - 1$. Let δ be defined by $\delta = b - b_{\min}$, and let ℓ be the number of baryon closed loops. Then each amplitude is classified in accordance with the triple of numbers (h, δ, ℓ) associated to the corresponding capped surface, and the planar amplitudes are those with $h = \delta = \ell = 0$.

The topological classification of a diagram is in terms of the capped surface σ described above. There is a related surface $\hat{\sigma}$ called the joined surface. The joined surface is formed by taking the components of the uncapped surface and joining them together along each baryon line, rather than capping them together at the ends of these lines. The joined surface is the one considered by Rossi and Veneziano, and by Konishi. The connection between the planar capped surfaces and the corresponding joined surfaces is examined in Section 3. The result is that the joined surfaces $\hat{\sigma}$ that correspond to planar surfaces σ are the joined surfaces that have no baryon closed loops and are simply connected when windows are ignored.

The planar diagrams $P_i(A,B)$, $P_i(A,C)$, and $P_j(C,B)$ of (1.1) are the joined surfaces $\hat{\sigma}$. These surfaces can contain an arbitrary number of windows. A cut that separates such a surface into two connected parts can pass through an arbitrary number of these windows. This cut is "tree shaped" if and only if it is simply connected when the gaps corresponding to the windows through which it passes are ignored. This cut has no superfluous parts; the parts of $\hat{\sigma}$ on opposite sides of any portion of the cut belong to the two different parts of that are separated by the cut. The cut intersects junction lines only at isolated points, and in the neighborhood of any such point the cut has a Y-shaped form, with each leg in a different component.

2. Formula for h

Consider any graph G with V vertices, E edges, and κ components (connected parts). Each edge begins at a vertex and ends at a vertex. Suppose G is imbedded on an orientable two-dimensional surface S such that each face (i. e., connected part) of $S - G$ is topologically equivalent to a disc. Then Euler's formula says that the number of handles h of S is related to the number of faces F by

$$(2.1) \quad 2h = 2\kappa + E - V - F.$$

If one writes

$$(2.2) \quad F = b + w$$

where b is the number of boundaries and w is the number of windows then (2.1) becomes

$$(2.3) \quad 2h = 2\kappa + E - V - b - w.$$

In order to use this formula each surface bounded by quark lines (and by the "inner lines" into which they continue) is converted into a graph. This is done by first distorting the surface into a set of narrow strips, and then shrinking each strip to a line. Points where external lines connect are placed at ends of strips and become vertices of the graph, as do places where several strips meet.

In addition to the full surface (or graph) produced by the capping procedure it is often convenient to consider the set of surfaces (or graphs) obtained if one does not perform the capping operation. These surfaces (graphs) will be labelled by i , which runs from 1 to κ' . For each i one has the analog of (2.3):

$$(2.4) \quad 2h_i = 2\kappa_i + E_i - V_i - b_i - w_i$$

where $\kappa_i = 1$.

The component graphs G_i are joined together by the capping procedure to form the full graph G . Before capping each external baryon line is associated with three edges, each ending at a vertex. The capping operation joins these edges together at one single vertex. Thus the capping operation replaces three vertices by one at each external baryon. Thus

$$(2.5) \quad V = \sum_{i=1}^{\kappa'} V_i - 2n_B,$$

where n_B is the total number of initial and final baryons. On the other hand,

$$(2.6) \quad E = \sum_{i=1}^{\kappa'} E_i$$

and

$$(7.7) \quad w = \sum_{i=1}^{\kappa'} w_i .$$

Moreover, as explained below,

$$(2.8) \quad b = \sum_{i=1}^{\kappa'} b_i .$$

Combining these formulas one obtains

$$(2.9) \quad h = \sum_{i=1}^{\kappa'} h_i + n_B + \kappa - \kappa' .$$

Boundaries and windows are identified by either following the quark lines around the diagram, or equivalently, for graphs, by tracing out lines that stay next to the lines of a graph, but that never intersect the graph. These latter lines are called face lines. Those that encounter external particle lines are called boundaries. Those that do not are called windows. Clearly the number of windows w of the full graph is the same as the sum of the number of windows of the components, since any face or quark line that encounters an external baryon line is not a window, and only those lines that encounter a baryon line can be affected by the capping operation.

Each quark line that enters an external baryon vertex v (see Fig. 3f) continues along a dotted line to the associated antibaryon vertex $\bar{v}$, and then out along one of the outgoing quark lines at $\bar{v}$ (Fig. 4). Figure 4 shows that the association between incoming quark lines at v and outgoing quark lines at $\bar{v}$ is not changed by the capping procedure: the quark lines continue in the same way in the two cases. For this reason the number of boundaries b is equal to $\sum b_i$, as asserted by Eq. (2.8).

3. The Elementary Surfaces and Graphs

The elementary surfaces are the handleless, windowless surfaces $\hat{\sigma}$ with $b = b_{\min}$. They are defined by

$$(3.1) \quad h = \delta = w = 0.$$

The condition $w = 0$ implies the absence of baryon closed loops ($\ell = 0$). This condition is assumed throughout this section.

Each capped surface σ is associated with two other surfaces. The first is the uncapped version, which is constructed from the capped version by cutting off the capping regions. This procedure produces a set of κ separate surfaces, which are called the components of the uncapped surface. The second related surface is called the joined surface $\hat{\sigma}$. It is formed from the uncapped version by joining together into one line, called the junction line, the three inner lines associated with each baryon-antibaryon pair. This junction line is analogous to the baryon line, and is likewise represented by a dotted line. The components C_i of a joined surface $\hat{\sigma}$ are the parts of $\hat{\sigma}$ that are connected when no crossing of junction lines is allowed. These components coincide with the components of the uncapped surface.

A component C_i is said to be elementary if and only if it is handleless and windowless, and has only one boundary

($h_i = w_i = b_i - 1 = 0$). A joined surface $\hat{\sigma}$ is said to be elementary if and only if it is connected, has only elementary components, and has no closed sequence of components. This last requirement is that there be no way of numbering the components C_i of $\hat{\sigma}$ so that $(C_1, C_2, \dots, C_n)$ is a sequence of n distinct components that contains a closed path P that runs through them in the indicated order, but that cannot be continuously deformed in $\hat{\sigma}$ to a point.

A typical elementary component is shown in Fig. 5a. The vertices at which two solid (quark) lines meet are meson connections. The dotted lines are "inner lines", and a vertex where a dotted line meets a solid line is a baryon connection. To form a joined surface three such dotted lines, coming usually from different components, must be joined together to form a single junction line. A typical joined surface is shown in Fig. 6a. The vertices at which two solid (quark) lines meet are again meson connections, and the vertices at which three solid lines meet a dotted (junction) line are baryon connections.

Each surface can be represented by a graph. The graph G_i of a component C_i , like the graph of an uncapped surface, is formed by squeezing the surface first into narrow strips, and then into a graph, where each line of the graph represents two oppositely directed boundary sections of the surface. Since each elementary component C_i has no handles, no windows, and one boundary it is topologically equivalent to a disc, with particle connections on its boundary. Thus the graph G_i corresponding to any elementary component C_i is a tree graph. Conversely, the component C_i corresponding to any tree graph G_i is an

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elementary component, which is topologically equivalent to a disc with connection points on its boundary.

The graph G_1 of Fig. 5b corresponds to the component C_1 of Fig. 5a. Each exterior vertex of the graph G_1 not marked with b represents a meson connection. Each b -vertex represents the contraction of an inner line.

To form the graph $\hat{G}$ corresponding to a joined surface $\hat{\sigma}$ one combines triples of b -vertices of component graphs G_1 into single b -vertices of $\hat{G}$. Thus each b -vertex of $\hat{G}$ represents a contracted junction line, and also a pair of baryon connection points. It must have exactly three lines incident upon it, one from each of the three components G_1 of $\hat{G}$ that corresponds to one of the three components C_1 of $\hat{\sigma}$ joined at the junction line whose contraction is represented by it. The graph $\hat{G}$ corresponding to the joined surface $\hat{\sigma}$ of Fig. 6a is shown in Fig. 6b. The heavy dots on the ends of lines of $\hat{G}$ are meson connection points. Each b -vertex represents, as already stated, a pair of baryon connection points.

A connection between an elementary capped surface σ , the associated joined surface $\hat{\sigma}$, and the graph $\hat{G}$ that represents $\hat{\sigma}$ is given by the following theorem:

Theorem: Let σ be a capped surface. Let $\hat{\sigma}$ be the corresponding joined surface. Let $\hat{G}$ be the graph that represents $\hat{\sigma}$. Then the following three properties are equivalent:

- (1) σ is elementary ($h = w = \delta = 0$)
- (2) $\hat{\sigma}$ is elementary ($\hat{\sigma}$ is connected, has only elementary components, and has no closed sequence of components).
- (3) $\hat{G}$ is a connected tree graph. ($\hat{G}$ is connected and has no closed loops).

Proof: Suppose that the joined surface $\hat{\sigma}$ satisfies condition (2). Then $\hat{\sigma}$ is connected, has only elementary components, and no closed sequence of components. The fact that $\hat{\sigma}$ is connected implies that $\hat{G}$ is connected. The fact that all the components C_1 of $\hat{\sigma}$ are elementary, implies that all the components G_1 of $\hat{G}$ are tree graphs. The final condition that there be no closed sequence of components C_1 implies that $\hat{G}$ is a tree graph. For if $\hat{G}$ were not a tree graph then it would contain a closed loop. This closed loop could not be contained in any single G_1 , since each G_1 is a tree graph. Thus this closed loop would have to pass through a sequence of G_1 . If some G_1 appeared several times in this loop then, because each G_1 is connected, a smaller loop could be formed, and ultimately a loop could be formed in which no G_1 appeared more than once. This loop would define a closed sequence $(G_1, G_2, \dots, G_n)$, and a sequence $(b_{12}, b_{23}, \dots, b_{n1})$ of b -vertices. By joining in C_2 the junction lines represented by b_{12} and b_{23} , etc., one would form a closed path P in $\hat{\sigma}$ that could not be continuously deformed in $\hat{\sigma}$ to a point, since its image in $\hat{G}$ cannot be so deformed. Thus $\hat{G}$ must be a connected tree graph, and (2) implies (3).

Conversely, if (3) holds, and $\hat{G}$ is a connected tree graph, then $\hat{\sigma}$ must be connected as required by (2). Since $\hat{G}$ is a tree graph each of its components G_i must be a tree graph. Hence each component C_i of the surface $\hat{\sigma}$ must be elementary, as required by (2). Finally, $\hat{\sigma}$ can have no closed sequence of components. For if there were such a closed sequence $(C_1, C_2, \dots, C_n)$ then a closed loop could be formed in the graph $\hat{G}$ by first identifying the sequence of junction points $(b_{12}, b_{23}, \dots, b_{n1})$ associated with the sequence $(C_1, \dots, C_n)$ of components, and then tracing in the graph G a closed loop by connecting b_{n1} to b_{12} within G_1 , b_{12} to b_{23} within G_2 , etc. and finally $b_{n-1,n}$ to b_{n1} within G_n . The existence of such a path is contrary to the assumed property that $\hat{G}$ is a tree graph. Thus the assumption that a closed sequence of components exists is ruled out, and hence (3) implies (2).

The graph $\hat{G}$ that represents the joined surface $\hat{\sigma}$ is different from the graph G that represents the capped surface σ . The connection is simple: each b-vertex of $\hat{G}$ is replaced in G by a structure of six lines and five vertices, as shown in Fig. 7. The connection between the part of G shown in Fig. 7 and the corresponding portion of the capped surface σ is shown in Fig. 8.

The rule for computing the numbers of faces, boundaries, and windows of a capped surface is that one follows the quark lines, and traces out closed loops. Each quark-line loop represents one face.

If this quark line encounters any (meson or baryon) connection point it is a boundary. Otherwise it is a window.

The equivalent rule for the graph G is based on the convention that each vertex of the graph should be drawn on the page with the lines coming into the vertex ordered cyclically so that the associated surface near the vertex would be planar with clockwise orientation (i. e., so that locally the lines that bound the surface go clockwise around the surfaces they bound). Then the face lines of the graph can be drawn according to the rule that they remain always adjacent to the graph, but never cross it. The number F of faces of G is the number of face-lines of G . Face lines that run past (particle) connection points are boundaries, and the others are windows. A face line of a typical graph is shown in Fig. 9. The graph of that figure has three other face lines.

Each face line that enters the structure shown in Fig. 8 is a boundary, since it encounters, or passes by, two baryon connection points. This boundary leaves the structure by the same line (Fig. 8a) or pair of lines (Fig. 8b) by which it enters, but on the other side. Using these two facts one can construct a rule for assigning to each graph $\hat{G}$ a number $\hat{b}$ of boundaries and a number $\hat{w}$ of windows that necessarily agree with the numbers b and w of the associated graph G . The rule is that in counting boundaries and windows one considers each b-vertex of $\hat{G}$ to be separated into three vertices, one for each incident line. Each of these vertices is considered to be a connection point. This rule ensures that

$$(3.2) \quad \hat{b} = b$$

and

$$(3.3) \quad \hat{w} = w.$$

If κ , V , and E are the numbers of components, vertices, and edges of G , and $\hat{\kappa}$, $\hat{V}$, and $\hat{E}$ are the numbers of components, vertices and edges of the corresponding graph $\hat{G}$ then examination of Fig. 7 shows that

$$(3.4) \quad \hat{\kappa} = \kappa$$

$$(3.5) \quad \hat{V} = V - 2n_B$$

and

$$(3.6) \quad \hat{E} = E - 3n_B,$$

where n_B is, as before, the total number of initial and final baryon and antibaryons.

Suppose now that condition (3) holds. Then $\hat{G}$ is a connected tree diagram, and hence

$$(3.6) \quad \hat{\kappa} = 1.$$

Thus (3.4) gives

$$(3.7) \quad \kappa = 1.$$

Since $\hat{G}$ is a tree diagram each component G_1 of $\hat{G}$ is a tree diagram. Thus the rule for computing $\hat{b}$ gives

$$(3.8) \quad \hat{b} = \kappa'.$$

This result and (3.2) give

$$(3.9) \quad b = \kappa'.$$

This result and (3.7) give

$$(3.10) \quad b = b_{\min} = \kappa' - \kappa + 1$$

which is the condition $\delta = 0$ demanded by (1).

Condition (3) asserts that G is a tree graph. Hence it has no windows: $\hat{w} = 0$. Then (3.3) gives

$$(3.11) \quad w = 0,$$

as demanded by (1).

Condition (3) asserts that $\hat{G}$ is a connected tree diagram. Thus its Betti number vanishes:

$$(3.12) \quad \hat{\beta} = 1 + \hat{E} - \hat{V} = 0$$

(The Betti number of a graph is the number of independent closed loops.) Inserting (3.5) and (3.6) into (3.12) one obtains

$$(3.13) \quad 1 + E - V - n_B = 0$$

which, combined with the equality $\kappa' = n_B + 1$ proved below, and (3.9), gives

$$(3.14) \quad 2 + E - V - b = 0.$$

Since $F = b + w$, and $w = 0$, one obtains

$$(3.15) \quad 2 + E - V - F = 2h = 0.$$

as required by (1). Thus (3) implies (1). It is also easy to show directly that (2) implies (1).

The equality $\kappa' = n_B + 1$ is derived by noting that either the tree-graph structure of $\hat{G}$, or equivalently the no closed sequence property of $\hat{\sigma}$, allows one to uniquely define for any given G_i or C_i , a set of G_i or C_i that are joined directly to it,

and next on additional set of G_i or C_i that are joined to these, and so on. At each addition of graphs G_i at a b-vertex or of components C_i at a junction line one adds a baryon-antibaryon pair, which increases n_B by two. One also adds two new components G_i or C_i , which increases κ' by two. Thus for any connected tree graph $\hat{G}$, or any connected joined surface $\hat{\sigma}$ with no closed sequence of C_i ,

$$(3.16) \quad \kappa' = n_B + 1.$$

If the connectedness conditions are relaxed then one obtains instead of (3.16)

$$(3.17) \quad \kappa' = n_B + \kappa$$

This same argument shows that for any joined surface $\hat{\sigma}$ the maximum possible κ' satisfies (3.17):

$$(3.18) \quad \kappa'_{\max} = n_B + \kappa.$$

This is because a general joined surface can be formed by starting from a joined surface with no close sequence and then making additional connections. These added connections necessarily reduce the number of components, since they join together formerly distinct components. (See Fig. 6a.)

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Suppose, finally, that condition (1) holds. Then

$$(3.19) \quad h = \sum h_i + n_B + \kappa - \kappa' = 0,$$

$$(3.20) \quad b = b_{\min} \equiv \kappa' - \kappa + 1,$$

and

$$(3.21) \quad w = 0.$$

Equation (3.18) gives

$$(3.22) \quad n_B + \kappa - \kappa' \geq 0.$$

This combined with (3.19) and the conditions $h_i \geq 0$ gives the conditions

$$(3.23) \quad h_i = 0 \quad (\text{all } i)$$

demanding by (2), and also

$$(3.24) \quad \kappa' = n_B + \kappa = \kappa'_{\max}.$$

This maximum condition $\kappa' = \kappa'_{\max}$ can be achieved only if the joined surface satisfies the no closed sequence condition demanded by (2).

The condition $w = 0$ of (1) together with $w_i \geq 0$, gives the conditions

$$(3.25) \quad w_i = 0 \quad (\text{all } i)$$

demanding by (2).

The conditions (3.20) and (3.24) give

$$(3.26) \quad b = n_B + 1.$$

On the other hand, each of the κ' components must have at least one boundary. Hence (3.24) gives

$$(3.27) \quad b \geq n_B + \kappa.$$

This result and (3.26) imply that

$$(3.28) \quad \kappa = 1$$

and that every component has exactly one boundary:

$$(3.29) \quad b_i = 1 \quad (\text{all } i)$$

Thus, conditions (1) imply that σ is connected $[(3.28)]$, has only elementary components $[(3.23), (3.25), (3.29)]$, and has no closed sequence of components. Hence (1) implies (2). This completes the proof of the theorem.

It is now easy to see geometrically that any elementary capped surface σ can be imbedded on a sphere. The structure shown in Fig. 8 can be imbedded on a small sphere in the manner shown in Fig. 10. Three tubes can be attached to this sphere, and the lines of G , or the surfaces of σ , that leave the structure can leave the small sphere along the surfaces of these tubes.

The various lines of G , or surfaces of σ , that are parts of any single component G_i , or C_i , can be placed on tubes that are joined together like the spokes of a wheel, with the parts of G_i or of C_i joined at the hub, in the manner shown in Fig. 11. The possibility of doing this is entailed by the tree structure of each G_i , or by the equivalent elementary character of each C_i .

Since the set of tubes upon which the full graph G or surface σ is attached can be connected together in the way specified by the graph $\hat{G}$, and the graph $\hat{G}$ is a connected tree graph, this structure of tubes can be deformed into a sphere. Thus any elementary σ , can be imbedded on a sphere, and so can the corresponding graph G .

It is also easy to visualize in this way more complicated cases. For example, each of the three cases shown in Fig. 1 are represented by using two spheres of the kind shown in Fig. 10. In general there is always one of these spheres for each dotted line of the baryonic quark diagrams. In the case of Fig. 1a these two spheres are connected by only one tube. Two spheres connected by a tube is equivalent to a sphere. Hence $h = 0$.

In this case $\kappa = 1$, $\kappa' = 5$ and $n_B = 4$. Each of the five components is free of handles. Thus (2.9) gives

$$(3.30) \quad h = \sum h_i + n_B + \kappa - \kappa' = 0,$$

in agreement with the result obtained above geometrically.

In the case represented by Fig. 1b the two spheres are joined by two tubes, and hence the structure is a torus with $h = 1$. On the other hand, $\kappa = 1$, and $\kappa' = 4$, hence (2.9) gives

$$(3.31) \quad h = \sum h_i + n_B + \kappa - \kappa' = 1,$$

in agreement with the result obtained geometrically.

In the case represented by Fig. 1c three tubes are needed to carry the connecting lines or surfaces from one sphere to the other, and hence $h = 2$. On the other hand, $\kappa = 1$ and $\kappa' = 3$, and hence (2.9) gives

$$(3.32) \quad h = \sum h_i + n_B + \kappa - \kappa' = 2.$$

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4. Discontinuity Formula

The "planar" amplitudes are those that correspond to $\ell = 0$ surfaces with the minimum number of boundaries and handles, namely those with $h = \delta = 0$. The term "planar" is appropriate in the sense that the associated surface can be imbedded on a sphere, and hence on a plane. However, the surface associated with a "planar" amplitude generally has more than one boundary and hence is not planar in the same sense as in the meson sector. Perhaps the name "primitive" or "ordered" would be better, particularly because the word planar is often used in the meson sector to denote a sum of primitive or ordered amplitudes analogous to ours over all cyclic orderings of the particles.

The elementary surfaces have $h = \delta = \omega = \ell = 0$. If the $w = 0$ condition is removed then each elementary surface can be filled with windows, each of which is formed by cutting out of the surface a small disc and then running a quark line around the newly formed circular edge of the surface. Baryon closed loops will be temporarily excluded.

The amplitude corresponding to a surface $\hat{\sigma}$ is assumed to have a normal-threshold discontinuity in a certain channel $A \rightarrow B$ only if $\hat{\sigma}$ can be cut into two disjoint parts each of which is connected and contains all of the particle connections corresponding to either A or B , but not both.

If an elementary surface $\hat{\sigma}$ is cut into two disjoint connected parts then each part is again an elementary surface $\hat{\sigma}$,

provided the cut is tree shaped (see Appendix). The crucial property basis of the validity planar discontinuity formula is (1.1).

The planar diagrams of (1.1) are the elementary surfaces $\hat{\sigma}$. There will usually be several such diagrams $P_i(A,B)$ associated with the processes $A \rightarrow B$. For each such diagram $P_i(A,B)$ one can consider all possible ways of cutting it into two disjoint parts in such a way that the connection points that represent the particles corresponding to A are separated from those corresponding to B . Two examples of typical cuts are shown in Fig. 12. The particle connections indicated by the small letters a and b belongs to A and B , respectively.

In Fig. 12 each cut has the form of a tree graph. Figure 13 shows the tree graph corresponding to the cut illustrated in Fig. 12b. In general, the tree graph corresponding to a cut has only three-line vertices, one for each time the cut in the surface $\hat{\sigma}$ cuts through a junction line. Each edge of the tree-diagram corresponds to a traversal of the cut through some component of the surface. Each such traversal is a connected line that begins at either a quark line or a junction and ends at either a quark line or a junction line.

For each $P_i(A,B)$ one must consider the corresponding surfaces with windows: $P_i(A,B)$ can be considered to represent the infinite sum of surfaces obtained by putting independently into each component of $P_i(A,B) = \hat{\sigma}$ each possible number of windows.

Each window in any component is equivalent to any other one; only the number of them matters. The cut through $P_i(A,B)$ (see Fig. 12) will cut through each component C_j a certain number $n_j \geq 0$ of times, and will split this component into $n_j + 1$ parts. On each section of the cut there will be a certain number of windows. And in each connected (after cutting) part of each component there will be a certain number of windows.

The left-hand side of (1.1) represents a multiple sum over each of the topologically distinct surfaces $P_i(A,B)$, over each topologically distinct tree-graph cutting of $P_i(A,B)$ into two disjoint connected parts, over each number of windows lying on each section of the cut, and over each number of windows in each connected (after cutting) part of each component. Each diagram must be given a topological weight factor of $1/n$, where n is its symmetry number, which is the number of permutations of its components C_i that change it into a diagram that is indistinguishable from itself⁶. Thus Fig. 14a has weight $1/3!$, whereas Fig. 14b has weight $1/2$ because it is indistinguishable from Fig. 14c.

The right-hand side of (1.1) is formed by first taking for each set C of intermediate particles all pairs of planar surfaces $P_i(A,C)$ and $P_j(C,B)$. The particles associated with C are paired off in a one-to-one fashion. This can be represented as in Fig. 15. There is a sum over each possible way of pairing the particles C of $P_i(A,C)$

to the particles C of $P_j(C,B)$. For each such term one must also sum independently over all possible numbers of windows in each component of both $P_i(A,C)$ and $P_j(C,B)$. Each meson connection is now replaced by a meson strip, and each baryon connection is replaced by a set of three connecting strips, which connect the three component-strips corresponding to the baryon on one end to the three component strips corresponding to the baryon on the other end. There is a sum over all $3!$ ways of connecting the three strips on one end to the three strips on the other. Then all terms that are not elementary surfaces with windows are eliminated. Equation (1.1) says that the result is equal to what was previously constructed on the left-hand side.

Baryon closed loops were excluded in the above discussion of the discontinuity formula. If they are allowed then one must introduce into each elementary surface not only arbitrary numbers of windows but also arbitrary numbers of baryon closed loops of each of the three kinds discussed by Konishi.³ These are the untwisted, the twisted, and the half-twisted baryon closed loops. The conditions $h = \delta = 0$ entail that in the untwisted case each added part be a window joined to two $h_i = b_i = 0$ components. In the half-twisted case there must be one $h_i = b_i = 0$ component. Thus in all cases the $h = \delta = 0$ conditions entail that there be one "principal" component of σ , which contains all the boundaries, plus a set of $\kappa - 1$ components that have no boundaries. One can add windows and baryon closed loops on each of these components and then on each of the components associated with the new closed

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loops etc. Thus each component is separated from the principal component by a well-defined number of junction lines. The cutting of such a surface gives another surface of the same kind, and the planar discontinuity formula still holds.

The validity of the planar discontinuity formula (1.1) is regarded here as the principal justification of this topological classification scheme. The scheme enjoys also the other features listed in section 1.

An added bonus is obtained if one considers the formula for powers of the coupling constant g . This constant appears in the form

$$(4.1) \quad g^{n_M - n_B + 2b + 2w + 4h - 4\kappa} = g^{n_M - n_B - 2} \times (g^2)^{2h+b-1} (g^2)^w g^{4(1-\kappa)},$$

where closed baryon loops are now admitted. Contributions to scattering functions that correspond to surfaces without closed baryon loops have $\kappa = 1$. If in these contributions one incorporates a factor of N for each window and assumes that $N = g^{-2}$ then the g dependence becomes

$$(4.2) \quad g^{n_M - n_B - 2} (g^2)^{2h+b-1}.$$

The only difference between this formula and the formula in the meson sector is the factor g^{-n_B} , which is the same for all contributions to any given scattering function.

The expansion scheme discussed above allows baryon closed loops. However, it is also possible to regard the number ℓ of baryon closed loops as an expansion parameter, and define the lowest-order term by $h = \delta = \ell = 0$. Or one can take the numbers A' , B' and C of untwisted, twisted, and half-twisted baryon closed loops to be different expansion parameters. This last procedure is the one adopted by Konishi³.

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APPENDIX

It is shown here that a tree-shaped cut separates an elementary joined surface into two elementary joined surfaces. The arguments are informal; use is made of geometric intuition.

Each elementary joined surface $\hat{\sigma}$ is the union of a set of disjoint components σ_i and a set of junction lines J_j . Each component σ_i is represented as the interior of a planar polygon. Each junction line J_j is a finite open line segment that lies on the boundary of three different components. The boundary of $\hat{\sigma}$ is the union of the boundaries of the components σ_i , minus the union of the junction lines.

A closed path in $\hat{\sigma}$ is a closed curve in $\hat{\sigma}$ that can be represented as a finite sequence of closed, directed, straight-line segments connected end-to-end to form a single closed undirected line.

Let $\hat{\sigma}$ be a joined surface and let P be a closed path in $\hat{\sigma}$. Then P separates $\hat{\sigma}$ into two disjoint parts. The first consists of the points outside P , and the second is the complement in $\hat{\sigma}$ of the first.

The points outside P are defined with the aid of tree-cuts.

A tree-cut in $\hat{\sigma}$ is a connected subset T of $\hat{\sigma}$ that enjoys the following three properties: i) T is a union of a finite set of disjoint open straight-line segments and a finite set of vertices, each of which lies at the end of at least two lines; ii) T contains no closed loop; and iii) T locally separates $\hat{\sigma}$, in the sense that

for every point p of T and every sufficiently small $\hat{\sigma}$ neighborhood $N(p)$ of p the set $N(p)-T$ consists of precisely two disjoint parts. The end points of T are the points that lie in the closure of T but not in T . They are the points that lie at the ends of lines of T , but not at the vertices.

Let $\hat{\sigma}$ be an elementary joined surface and let P be a closed path in $\hat{\sigma}$. A point p is said to lie outside of P if p lies in $\hat{\sigma}-P$ and lies at an end point of a tree-cut in $\hat{\sigma}$ that does not intersect P and that has all of its other end points in the boundary of $\hat{\sigma}$. The complement in $\hat{\sigma}$ of the points outside P is called $\hat{\sigma}(P)$. It includes all points of P .

If $\hat{\sigma}$ is an elementary joined surface then every closed path in $\hat{\sigma}$ can be deformed continuously in $\hat{\sigma}$ to a point. The original definition of an elementary joined surface demands this property only if the sequence of components through which P passes contains elements that are all different. However, from this original property the more general property follows easily. Thus an elementary joined surface $\hat{\sigma}$ is "simply connected".

Lemma 1. Let $\hat{\sigma}$ be an elementary joined surface. Let T be a tree-cut in $\hat{\sigma}$ such that each end point of T lies in the boundary of $\hat{\sigma}$. Then $\hat{\sigma}-T$ consists of two disjoint parts, each of which is a connected set.

Proof. For some sufficiently small $\hat{\sigma}$ -neighborhood $N(T)$ of T the set $N(T)-T$ consists of two disjoint sets each of which is connected. The intersection of $N(T)$ with each component of $\hat{\sigma}$ is equivalent to a set of disjoint thin ribbons, with T running longitudinally down the middle. $N(T)$ is formed by joining the ends of three different ribbons together at each junction line crossed by T . The boundary of $N(T)$ consists of two disjoint parts $\partial_1 N(T)$ and $\partial_2 N(T)$ that lie inside $\hat{\sigma}$ and that have the same topological structure as T , plus a strip in the boundary of $\hat{\sigma}$ for each end point of T . The disjointness of the two parts of $N(T)-T$ is ensured by taking $N(T)$ to be sufficiently close to the tree-cut T . The connectedness of each of the two parts of $N(T)-T$ is ensured by the connectedness of T .

Suppose the two parts of $N(T)-T$ were connected together by a path in $\hat{\sigma}-N(T)$. If this were possible then one could construct a closed path P in $\hat{\sigma}$ that cuts each of the two sets $\partial_1 N(T)$ and $\partial_2 N(T)$ exactly once. This path P could not be deformed continuously in $\hat{\sigma}$ to a point, because continuous deformations cannot alter the fact that P crosses each of the two disjoint sets $\partial_1 N(T)$ and $\partial_2 N(T)$ an odd number of times. But every closed path P in $\hat{\sigma}$ can be deformed continuously within $\hat{\sigma}$ to a point by virtue of the fact that $\hat{\sigma}$ is simply connected. Thus the two parts of $N(T)-T$ cannot be connected by a path in $\hat{\sigma}-N(T)$. Hence, since $\hat{\sigma}$ is connected, every point in $\hat{\sigma}-T$ must be connected in $\hat{\sigma}-T$ to precisely one of the two disjoint parts of $N(T)-T$.

Lemma 2. Let $\hat{\sigma}$ be an elementary joined surface. Let P be a closed path in $\hat{\sigma}$. Consider some triangulization of $\hat{\sigma}$ such that P lies in the union of the sides of the triangles. Then each open triangle will lie wholly outside of P or wholly inside of $\hat{\sigma}(P)$. Let t_1 and t_2 be two disjoint open triangles that have a finite portion P' of P as a common boundary. If t_1 and t_2 both lie outside P then any point p on P' must lie on an even number of segments of P . And if these segments are ordered in the same way as they are ordered in P , then successive segments touching p will have alternating directions.

Proof. Since P does not enter the interior of any triangle the whole interior must lie outside P if any point in it does. Suppose the two open triangles t_1 and t_2 both lie outside P , and share a finite portion P' of P as a common boundary. Then one can form a tree-cut T that cuts P nowhere but at a single arbitrary point p in P' , and that has all of its end points in the boundary of $\hat{\sigma}$. By Lemma 1 this tree-cut T separates $\hat{\sigma}$ into two disjoint sets. But then the closed path P , which crosses T only at p , must pass through p an even number of times in alternating directions, as specified in the lemma.

Lemma 3. Let $\hat{\sigma}$ be an elementary joined surface. Let P be a closed path in $\hat{\sigma}$. Then P can be deformed continuously within $\hat{\sigma}(P)$ to a point.

Proof. The closed set $\hat{\sigma}(P)$ includes no point in the boundary of $\hat{\sigma}$. It has a boundary $\hat{\sigma}(P)$ that is the intersection of P with the edges of the triangles whose interiors lie outside P . If any open triangle lies inside $\hat{\sigma}(P)$ then there must be some triangle t that lies inside of $\hat{\sigma}(P)$ and has an edge e that lies in $\partial\hat{\sigma}(P)$ but not in any junction line of $\hat{\sigma}$. The triangle (or triangles) on the other side of the part of the boundary $\hat{\sigma}(P) \cup e$ have interior(s) lying outside P . Thus P can be deformed continuously within t to a new closed path P_1 in $\hat{\sigma}$ such that the interior of t lies outside P_1 . If there is a triangle inside $\hat{\sigma}(P_1)$ then a new path P_2 can be introduced, until finally a path P_n is obtained such that $\hat{\sigma}(P_n)$ contains no open triangles. Then for each part P' of P_n the triangles on both sides of P' must have their interiors outside P_n . But then Lemma 2 entails that P runs past each point of its image in $\hat{\sigma}$ an even number of times in alternating directions.

Given any point p on P_n , and a direction in P_n , there will be a finite strip $L(p)$ of P_n between p and the next point p' on P_n that has the same image in $\hat{\sigma}$. Moving p along P_n in the positive direction shortens $L(p)$. One must finally reach a turning point, where $L(p) = 0$. These turning points can occur only at ends of the finite number of segments that make up P_n . Thus by a continuous deformation one can remove a section of P_n that consists of coincident, oppositely directed parts. This section is at least as long as the shortest segment of P_n . Proceeding in this way one finally,

after a finite number of steps, deforms the path P a point, staying always within $\hat{\sigma}(P)$ as required.

Lemma 4. Let $\hat{\sigma}$ be an elementary joined surface. Let T be a tree-cut each of whose end points lies in the boundary of $\hat{\sigma}$. Then $\hat{\sigma}-T$ consists of two elementary joined surfaces.

Proof. Lemma 1 asserts that $\hat{\sigma}-T$ consists of two disjoint parts, $\hat{\sigma}_1$ and $\hat{\sigma}_2$, each of which is a connected set. Each component of $\hat{\sigma}$ is represented by the interior of a planar polygon. Each of these components is separated by T into sections that are again interiors of planar polygons. These sections are identified as the components of $\hat{\sigma}_1$ and $\hat{\sigma}_2$. Thus it is sufficient to show that any closed path P in $\hat{\sigma}_1$ that lies wholly in one part, say $\hat{\sigma}_1$, can be deformed continuously within $\hat{\sigma}$ to a point.

Suppose P lies in $\hat{\sigma}_1$. Since T lies outside $\hat{\sigma}_1$ one has $T \cap P = \emptyset$.

Each point of T lies at the end point of a subset T' of T that is a tree-cut in $\hat{\sigma}$ that has every other end point in the boundary of $\hat{\sigma}$. Thus each point of T lies outside P : $T \cap \hat{\sigma}(P) = \emptyset$. But then the distortion of P within $\hat{\sigma}(P)$ must stay within $\hat{\sigma}_1$, and $\hat{\sigma}_1$ is an elementary joined surface. An analogous argument holds for $\hat{\sigma}_2$. Q.E.D.

REFERENCES

1. H. P. Stapp, Topological Expansion for Baryons and Mesons, Lawrence Berkeley Laboratory Report LBL 6118, Jan. 1977.
2. F. J. Capra, Physics Letters, 68B (1977) 93.
3. G. C. Rossi and G. Veneziano, CERN preprint, TH-2287-CERN, March 1977.
4. K. Konishi, Baryon and Dual Unitarization, Rutherford Laboratory Report RL-77-062/A T. 169, May 1977.
5. G. Veneziano, Nucl. Phys. B74, 365 (1974); Phys. Lett. 52B, 220 (1974).
6. J. Coster and H. P. Stapp, J. Math. Phys. 10, 371 (1969), Appendix A.

FIGURE CAPTIONS

Fig. 1: Three baryonic quark diagrams are shown in figures (a), (b) and (c). Each baryonic quark diagram has four lines corresponding to each baryon. One is dotted and runs from some incoming baryon (with arrows pointing in) to some outgoing baryon (with arrows pointing out). It is convenient to join together at a vertex the four lines corresponding to a baryon. This gives a graph. Graphs corresponding to figures (a), (b), and (c) are shown in figures (a'), (b'), and (c'). Vertices at which solid lines meet dotted lines are called baryon connection points.

Fig. 2: A form of the graphs of Figs. 1, (a'), (b'), and (c') that can be interpreted as mesons exchanged between baryons.

Fig. 3: The capping operation for an outgoing baryon is shown in figures (a) through (f). For incoming baryons (f) is replaced by (g).

Fig. 4: The structure of the surface associated with an incoming baryon and the outgoing baryon that is joined to it by a dotted baryon line. This dotted baryon line represents a triad of dotted lines, which join onto the quark lines in the way shown. This structure has six baryon connection points.

Fig. 5: Figure (a) shows a typical elementary component C_1 of a joined or uncapped surface. An elementary component has one boundary, no handles, and no windows. Thus it is

equivalent to a disc, with particle connection vertices along its periphery. Figure (b) shows the corresponding graph G_1 . It is a tree graph. Each dotted line is, by convention, contracted to a point vertex in G_1 . This point vertex is called a b-vertex. Three such b-vertices are combined together to give a single b-vertex of the graph $\hat{G}$ that represents the joined surface $\hat{\sigma}$.

Fig. 6a: A joined surface $\hat{\sigma}$. Three components C_1 of $\hat{\sigma}$ are joined together at each junction line. The surface $\hat{\sigma}$ in this example is elementary. If any two of the thirteen components were joined together, say by a meson strip, then the surface would no longer be elementary.

Fig. 6b: The graph $\hat{G}$ that represents the surface $\hat{\sigma}$ of Fig. 6. Each b-vertex represents a contracted junction line, and a pair of baryon connection points. The three lines that enter any b-vertex are lines from the three graphs G_1 that represent the three components C_1 that join at the junction line associated with the b-vertex.

Fig. 7: This figure shows the connection between a graph $\hat{G}$ that represents $\hat{\sigma}$ and the graph G that represents the capped surface σ that is associated with $\hat{\sigma}$. Each b-vertex of $\hat{G}$ is replaced by a graph with two baryon connection points and three other vertices.

Fig. 8: The part of the surface σ that corresponds to the graph of Fig. 7b is shown. Vertices where solid quark lines change to dotted lines are baryon connection points.

Fig. 9: A graph and a face line of that graph.

Fig. 10: The structure shown in Fig. 8 can be placed on a small sphere, with the parts of G or σ that leave the structure placed on tubes that protrude from the sphere.

Fig. 11: A tree graph G_1 , or the equivalent elementary component C_1 , can be placed on the surface of a set of tubes that join together like the spokes of wheel.

Fig. 12: This figure shows typical cuts that separate elementary surfaces $\hat{\sigma}$ into two disjoint parts each of which is connected, and connected to one of the two sets A and B , but not both. The particles of A and B are represented by the connection points labelled with the letters a and b , respectively.

Fig. 13: The cut of an elementary surface $\hat{\sigma}$ into two disjoint parts, each of which is connected, always has the form of a tree graph. The cut of Fig. 12 (b) is shown here.

Fig. 14: Diagram (a) has symmetry number $3!$. Diagram (b) has symmetry number 2. The symmetry number of a diagram is the order of its automorphism group. In other words, it is the number of independent permutations of the set $\{C_1\}$ of components of the diagram that give a diagram that

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is topologically indistinguishable from the original diagram. The particle connection points are considered to be distinguishable: each must be mapped onto itself.

Fig. 15: A product of two graphs corresponds to one-to-one association of some particle connections of one graph with some particle connections of the other graph. A particular correspondence is represented by connecting the corresponding vertices of the two graphs by a wavy line.

(a)

(b)

(c)

(a')

(b')

(c')

Fig. 1

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(a)

(b)

(c)

Fig. 2

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Fig. 3

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Fig. 4

(a)

(b)

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Fig. 6

Fig. 7

(b)

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Fig. 8

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Fig. 9

Fig. 10

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Fig. 11

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Fig. 12

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Fig. 13

Fig. 14

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Fig. 15

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720