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Hardware Acceleration for Tensorized Neural Network

A Thesis submitted in partial satisfaction of the  
requirements for the degree Master of Science in Electrical and Computer Engineering

by

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March 2018

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March 2018

Hardware Acceleration for Tensorized Neural Network

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by

Yiming Gan

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## ABSTRACT

### Hardware Acceleration for Tensorized Neural Network

by

Yiming Gan

Machine learning has gained success in many application domains including medical data analysis, finance, computer vision, and so forth. However, many popular machine learning models (e.g., deep neural networks) are both data-intensive and computationally expensive: they require high-volume data samples to train the network, millions to billions of parameters to describe the model, and large-scale computations to complete the optimization or inference. Therefore, deep learning can cause unaffordable energy and run-time cost on a hardware platform. In this paper, we present a way of accelerating deep neural network as well as compressing weights used by designing hardware acceleration for tensor train decomposition layers in deep neural network. By utilizing hardware acceleration on tensorized neural network, we achieved massive memory saving on two fully -connected layers. Parameters shrink 4880644x and 3195660x separately. At the same time, we achieve speed up at 2600x and 2900x compared to original matrix multiplication process.

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# **I. Introduction and Related Work**

## ***A. Introduction***

Deep neural network <sup>1</sup> has gained great success at solving a wide range of practical tasks including vision and speech <sup>2</sup>, language modeling and translation <sup>3</sup> and autonomous driving <sup>4</sup>. These approaches usually use high-volume of variables to parametrize a neural network, and require high-volume data sets to train the model. For instance, the VGG-19 network needs 500M memory for image recognition <sup>5</sup>. The RNN model in <sup>6</sup> even involves more than 10G parameters. As a result, high-performance computing platforms (e.g., GPU and cloud computing) are usually required to handle the computation and memory expensive training and inference (i.e., prediction) tasks. Training and running a complex deep neural network is infeasible in many application scenarios, due to the limited data sets and computing resources (e.g., power budget and bandwidth constraints on a hardware platform, no access to GPUs or cloud computing) such as IoT (Internet of Things) devices, robotics and embedded platforms. Therefore, developing compact neural network models and designing high performance as well as energy efficient machine learning hardware have become a key enabler for many AI and IoT applications.

### **1. Efforts in Hardware Optimization.**

In the communities of computer architecture and VLSI design, many approaches have been reported to improve the performance of neural network hardware. On GPU and FPGA platforms and in ASIC designs, various techniques have been investigated to reduce both the amount and bit width of the data in weight matrices, neurons and gradients in order to save memory and computation resources <sup>7,8,9,10,11,12</sup>. Customized FPGA/ASIC neural network



accelerators have also been investigated by many groups. For instance, FPGA accelerators have achieved several to dozens of times speedup and energy-efficiency improvement by elaborately designing the parallel processing units, memory allocation and topology tiling. With on-chip DRAM for near-data processing, ASIC accelerators have gained great success. For instance, the DaDianNao<sup>13,14</sup> has gained 450x speedup and 150x energy reduction compared with GPUs by increasing the inner bandwidth of computing parallelism and by reducing the external communications. The performance and energy efficiency of ASIC accelerators have been further boosted by utilizing process-in-memory (PIM) and non-volatile memory-crossbars<sup>15,16,17,18</sup>. For instance, with PIM design, the speed and energy efficiency of the design PRIME<sup>19</sup> are 2360x and 895x better, respectively, compared with GPUs.

## 2. Efforts in Algorithm Innovation

Meanwhile, the computational math and machine learning communities have proposed some numerical approaches to compress neural networks, but these approaches have been rarely exploited in hardware design. It was first observed that the large weight matrices have low-rank properties, therefore, they could be largely compressed by low-rank matrix factorization<sup>20,21,22</sup>. In Sainish's work, the number of parameters of the network was reduced by 30-50% for a deep neural network for large vocabulary continuous speech recognition tasks. Recently, tensor decompositions were applied to compress neural networks with much higher compression ratios<sup>23,24,25,26</sup>. Tensors are high-dimensional generalization of matrices. While matrix decomposition only compress data in 2 dimensions, tensor decomposition can achieve much higher compression ratio by compressing data arrays in many dimensions. In

the work proposed by Novikov<sup>21</sup>, they first used tensor train decomposition<sup>27</sup> on fully connected(FC) layers and shrink the models for  $10^5$  times.

### ***B. Related Work***

Plenty of works have focused on accelerating deep neural network from both algorithm perspective and hardware perspective as shown in Fig. 1. In the first work bring tensor train decomposition into deep neural network, they compressed two FC layers into tensor train format. In the following work, they tried the same method on convolutional layers and get further compression rate. Since fewer parameters are used in presenting layers, the compute complexity is reduced also. The first work designing hardware accelerator for tensor train based deep neural network is proposed by Huang<sup>27</sup>, in which they describe a special 3D RRAM and CMOS combined architecture. 3 layers of circuits were used to form the accelerator. The first layer is RRAM-crossbar implemented as a buffer to store weights. Since each tensor core is a 3-D tensor consisted of several slices of matrix, all the matrices are stored in the order of different tensor core. The second layer is another RRAM-crossbar used for computing. It is designed for performing logic operations such as matrix vector multiplication and vector addition. The second layer uses the values stored in the first layer through through-silicon-via(TSV). Layer 3 is designed to perform overall synchronization of the tensorized neural network.

## II. Tensor Train Decomposition in Neural Network

### A. Tensor Train Decomposition

Tensors are natural multidimensional generalizations of matrices and have attracted tremendous interest in recent years. Working with  $d$ -dimensional problems with  $d$  being as high as 10, 100 or even 1000 has always been nontrivial. Problems of such size cannot be handled by standard numerical methods due to the curse of dimensionality since parameters grows exponentially with  $d$ . An efficient representation of a tensor by a small of parameters may give people an opportunity to work with high dimensional problems. There are several widely used tensor decomposition method including canonical decomposition, Tucker decomposition and tensor train decomposition. A  $d$ -dimensional  $n_1 \times n_2 \times \dots \times n_d$  tensor  $\mathbf{A}$  is said to be in the TT-format with cores  $G_k$  of size  $r_{k-1} \times n_k \times r_k, k = 1, 2, \dots, d$  and  $r_0 = r_1 = 1$ , if its elements are defined by formula(2.1).

$$(2.1) \quad A(i_1, \dots, i_d) = \sum_{\alpha_0, \dots, \alpha_d} G_1(\alpha_0, i_1, \alpha_1) G_2(\alpha_1, i_2, \alpha_2) \dots G_d(\alpha_{d-1}, i_d, \alpha_d)$$

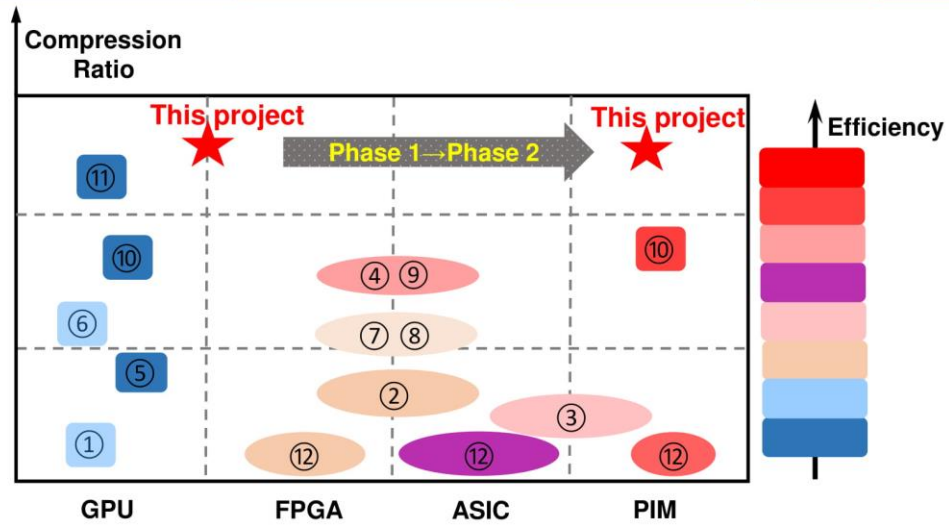
In which three-dimensional tensors  $G_k$  are called cores of the TT-decomposition and are all needed to be stored in memory. The ranks  $r_k$  are called compression ranks or TT-ranks. By decomposing tensors into tensor train format, basic tensor operations such as addition and Hadamard product can be performed based on tensor cores shown in table 1.

**Table 1. Tensor operations in tensor train format**

Operation	Formula	Tensor Cores
Addition	$C_{i_1 \dots i_d} = A_{i_1 \dots i_d} + B_{i_1 \dots i_d}$	$C_k = \begin{pmatrix} A_k & 0 \\ 0 & B_k \end{pmatrix}$
Hadamard product	$C_{i_1 \dots i_d} = A_{i_1 \dots i_d} B_{i_1 \dots i_d}$	$C_k = (A_k \otimes) B_k$

Figure 1. Existing software and hardware solutions for deep neural networks

Index	Existing Implementations	Compression Method	Hardware Platforms	Inference	Training
①	Mixed-precision	Quantization	GPU	✓	✓
②	ReLU-Zeros	Zero Compression	FPGA, ASIC	✓	×
③	16b/8b fixed-point	Quantization	ASIC, PIM	✓	×
④	Deep Compression	Pruning	FPGA, ASIC	✓	×
⑤	Structural Sparsity	Pruning	GPU	✓	×
⑥	Hashing Compression	Pruning	GPU	✓	✓
⑦	SVD Decomposition	Approximation	FPGA, ASIC	✓	×
⑧	Huffman Encoding	Approximation	FPGA, ASIC	✓	×
⑨	Weight Share	Quantization	FPGA, ASIC	✓	×
⑩	Binary/Ternary NNs	Quantization	GPU, PIM	✓	×
⑪	Partial Tensorizing	Approximation	GPU (no-optimization)	✓	✓
⑫	Accelerators	No-compression	FPGA, ASIC, PIM	✓	×/✓



## B. Tensor Train Decomposition in Neural Network

### 1. TT-layer

The direct application of the TT-decomposition to a matrix (2-dimensional tensor) coincides with the low-rank matrix format. A matrix in the TT-format is called TT-matrix and a TT-matrix is not restricted to be square. All operations available for TT-tensors are

applicable to the TT-matrices as well. TT-matrices can perform matrix-by-vector(matrix-by-matrix) easily which will be described later. If one of the operands is in TT-format, the result would be a vector(matrix). If both operands are in the TT-format, the result would be in TT-format with increasing ranks on TT-cores.

When bringing tensor train decomposition into neural networks, TT-layer is introduced. In this work, fully-connected layers are used as an example to explain how tensor train decomposition are used in neural network. As for other layers such as convolutional layers, they can be transformed into the same compute format with FC layers. Fully connected layers apply a linear transformation to an N-dimensional input  $\mathbf{x}$ :

$$(2.2) \quad y = Wx + b$$

in which the weight matrix  $W$  and the bias vector defines the transformation.

A TT-layer is the fully-connected layer in which weight matrix  $W$  is stored in TT-format allowing to store only tensor train cores instead of original weight matrix which save the parameters in a low rank way. The actual parameters stored can be controlled by changing tensor core size. The linear transformation shown in (2.2) can be expressed in the tensor form:

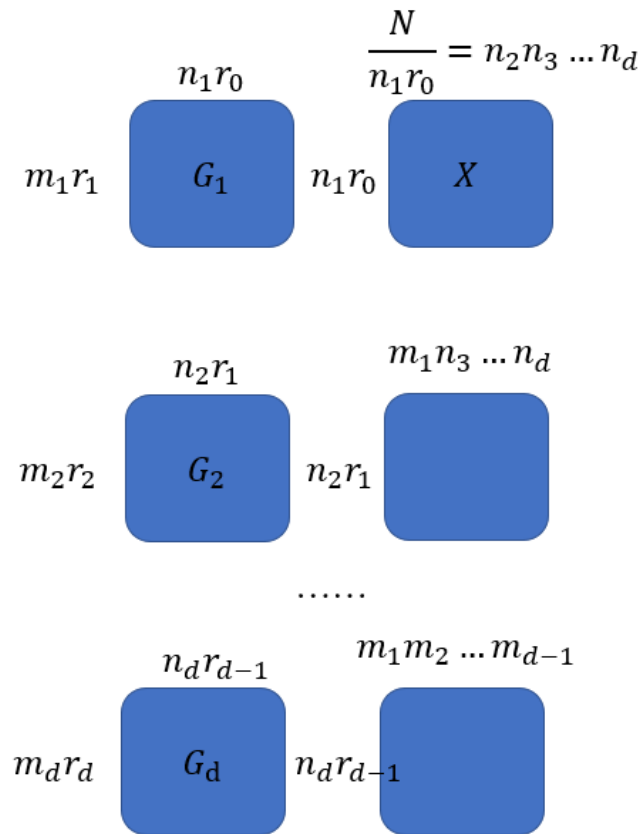
$$(2.3) \quad y(i_1, i_2, \dots, i_d) = \sum_{j_1, j_2, \dots, j_d} G_1 G_2 G_3 \dots G_d \times x(j_1, j_2, \dots, j_d) + b(i_1, i_2, \dots, i_d)$$

Since the process is transformed from matrix multiplication to matrix times tensor cores, the dimensional matching becomes a problem. How data is represented and multiplied will be shown in next part.

## 2. Compute Complexity

When the weight matrix is represented in TT-format, original matrix multiplication become tensor matrix multiplication. The process is shown in figure 2 in which  $N = n_1 n_2 n_3 \dots n_d$  and  $M = m_1 m_2 m_3 \dots m_d$ .

**Figure 2. TT-layer multiplication process**



The first tensor core will be unfolded to a matrix and reshaped to the matching size with reshaped input matrix  $x$ . The first sub-process is a matrix multiplication with the complexity of  $C = m_1 r_1 n_1 r_0 n_2 \dots n_d = r^2 m_1 n_1 \dots n_d$  resulting an output matrix in size of  $n_2 r_1 \times m_1 n_3 \dots n_d$ . The following tensor core is then reshaped to the matching size with previous output and performs the second matrix multiplication with the complexity of  $C = m_2 r_2 n_2 r_1 n_3 \dots n_d = r^2 m_1 m_2 n_2 \dots n_d$ . There will be  $d$  iterations in total and the final compute complexity is  $O(dr^2 m \max\{M, N\})$ . Compared to the previous fully-connected layer, TT-layer doesn't only save memory on storing parameters but also reduce the compute complexity from  $O(MN)$  to  $O(dr^2 m \max\{M, N\})$ .

### III. Hardware Acceleration for Tensorized Neural Network

#### A. Motivation

Although it has been proved that using tensor train decomposition on compressing deep neural networks will not only achieve obviously compression rate, but also have better compute complexity since weights represented in tensor core format are much smaller than before. However, from the data of our profiling, the real performance of tensor based deep neural network seems worse. As shown in Fig.3, we did profiling based on comparison of the baseline which is VGG-16 and compressed networks, another comparison has been made between two simple networks with one TT layer plus one FC layer and two FC layers. The result showing in Table 2 and 3 prove that although theoretically compute complexity

will be reduced, tensorized neural networks don't show actual performance improvement. The evaluation platform is Intel® Xeon® E5-2603 CPU with a Nvidia® Titan XP GPU.

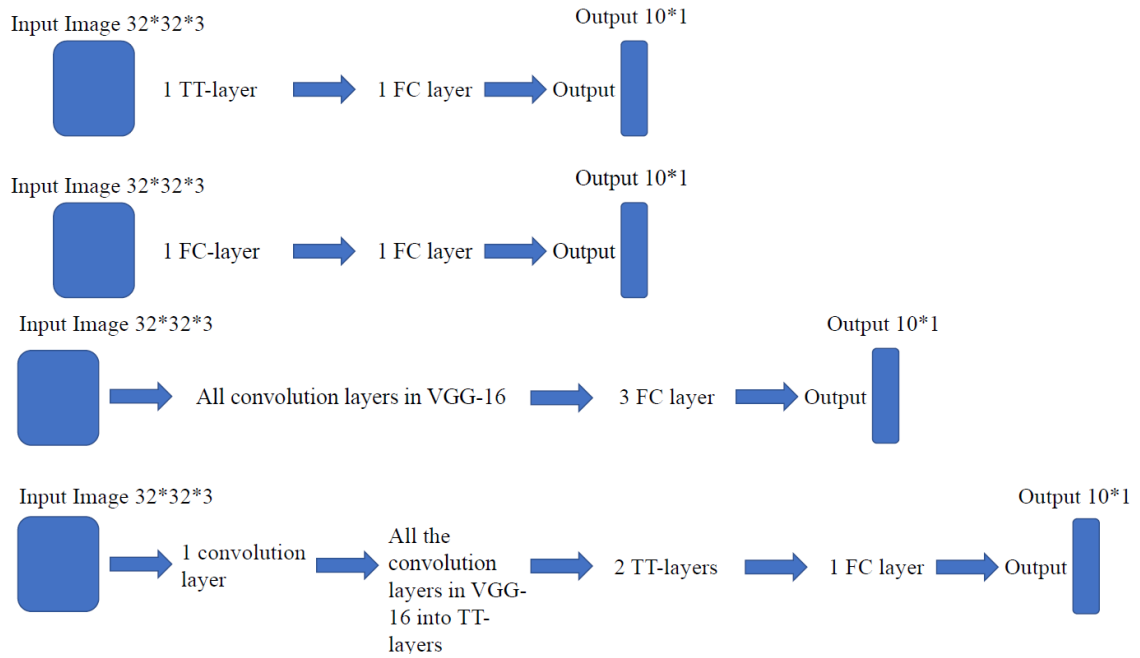
The reasons for the profiling results are not hard to explain. In actual operations, weights are stored in memory. Each time one matrix multiplication is processed, the weights will be taken from memory and be put into GPU for computation. In tensor train computation, each tensor core will be multiplied by a matrix. To complete this multiplication, some reshape procedure will be required as mentioned in the previous section. Although compute complexity is largely reduced, the time that reshape is happening is not taken into consideration. That's the reason that tensorized neural network will take longer time than it supposed to and why hardware acceleration is needed for this kind of network.

### ***B. FPGA Optimization for TT-layer***

FPGA is a popular option of existing accelerators, and it is widely used in embedded environments and in cloud servers. FPGA consists of hundreds of thousands of programmable logic blocks and programmable interconnections, which can provide reconfigurable functions; FPGA also consumes less power than GPU, therefore, it can be utilized in embedded scenarios. Four methods have been used in this paper on FPGA optimization for TT-layer.

**Figure 3. Profiling on networks including TT-layer**





**Table 2. Profiling Results for TT-layer**

Networks	2 FC layers	1 FC layer+1 TT-layer
Runtime(ms)	11	57

**Table 3. Profiling Results for Tensorized VGG-16**

Networks	VGG-16	Tensorized VGG-16
Runtime(ms)	132	127

### 1. Separate Circuit Design

As shown in the previous section, every iteration dealing with TT-layer consists of several different matrix multiplication and reshaping. Since once circuits are designed and build, the data set size it can deal with (i.e. matrix size) can not change, we design separate

circuits for different parts in one iteration. Each circuit will be designed to handle one matrix multiplication and its related reshape process.

## 2. LUT Design for Matrix Reshape

Look-up table method is used when performing matrix reshape. All the elements are stored in a 1-d array and each element will have a corresponding index after reshaping under the reshaping rules (i.e.  $4 \times 4$  to  $8 \times 2$ ). This is a perfect look-up table problem that can easily be solved on FPGA. Corresponding calculation is the only time-consuming part that can be done in parallel since each element's position after reshaping is independent.

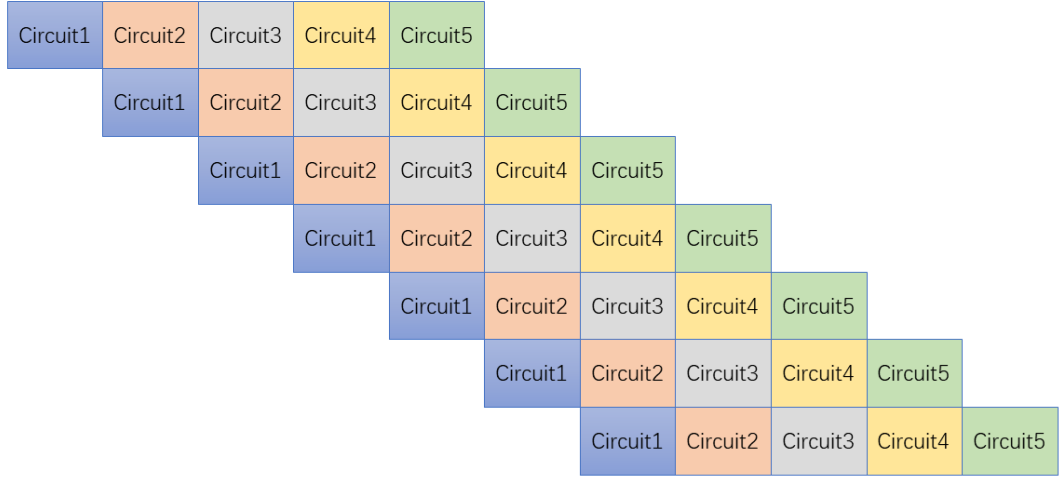
## 3. Parallel Matrix Multiplication

Matrix multiplication parallelization has been studied a lot on both FPGA and GPU. We also included some basic parallel optimization on matrix multiplication. Assuming a matrix multiplication happens between two matrices of size  $n \times n$ , each resulting element requires  $n$  times multiplication and addition which can be done in 2 clock cycles in the most extreme parallelization case. As a result, it requires  $n$  multiplier and a  $n$ -input adder. We also included block matrix multiplication in the process.

## 4. Pipeline Different Circuits.

This optimization is for multiple iterations. Since there will be no dependencies between second iteration and first iteration, a pipeline design shown in Figure 4 can be achieved.

**Figure 4. Pipeline for multiple iterations**

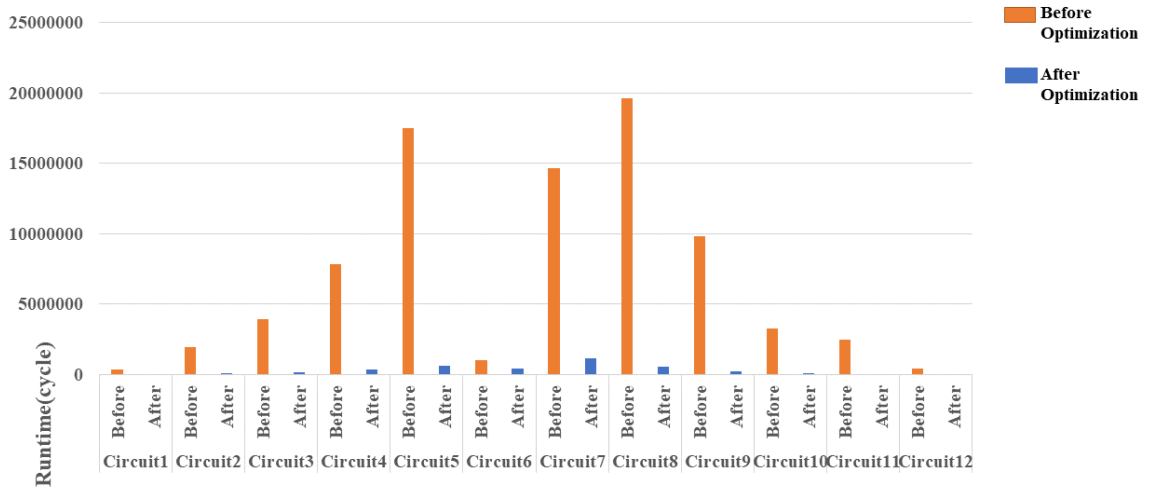


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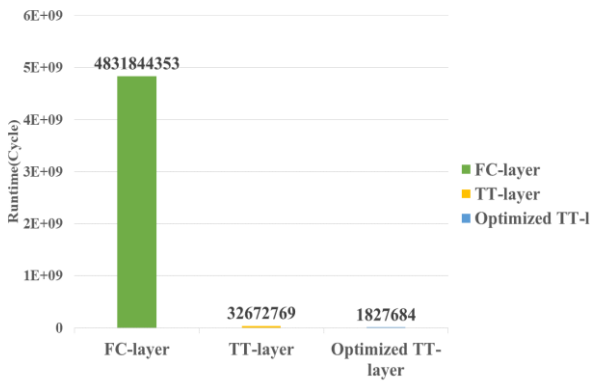
### C. Evaluation Results

We made the evaluation using High-Level-Synthesis(HSL) solution from Xilinx. We implemented on Xilinx Vivado HLS software and our targeting platform is Zynq xc7z100 FPGA. We implemented two very large fully-connected layers in tensor train format, the first weight matrix is in the size of  $3072 \times 262144$  and the second one is  $262144 \times 4096$ , the input vector size is  $1 \times 3072$ . Each matrix is represented by 6 tensor cores, thus 12 circuits are needed for these two TT-layers. As shown in Figure 5, orange bars represent each circuit's runtime performance before optimization, blue bars represent the result after optimization. On average 24x speed up can be achieved due to our FPGA optimized methods. Figure 6.a and Figure 6.b shows the overall comparison between optimized TT-layer with original matrix multiplication in fully-connected layer. TT-layers have shown 148x and 128x speed up already, when fpga optimization was used, the overall speedup have increased to 2643x and 2944x separately. The total resource used in 12 circuits are shown in Table 4. The memory usage are reduced 4880644x and 3195660x separately.

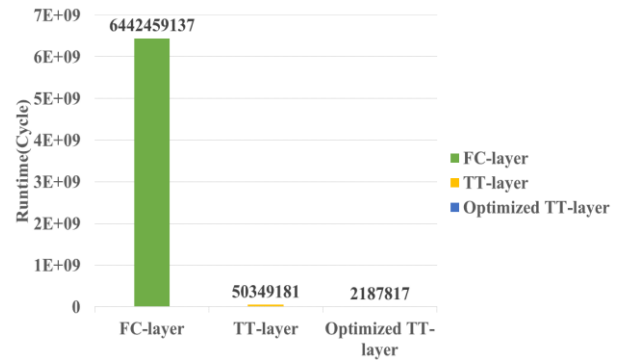
**Figure 5. FPGA optimization results**



**Figure 6. Overall Optimization Results**



**Figure 6.a**



**Figure 6.b**

### III. Discussion and Future Work

#### A. Back-Propagation

Neural networks are usually trained with the stochastic gradient descent algorithm where the gradient is computed using the back-propagation procedure. Back-propagation allows to

compute the gradient of a loss-function  $L$  with respect to all the parameters of the network. Our method optimized the performance of forward pass, but the backward pass compute complexity is also improved because of TT-layer. Back-propagation requires different operations which means different fpga optimization are needed to accelerate it. A complete hardware accelerator needs to include both forward-propagation and back-propagation.

### ***B. 3D Processing-in-memory***

Processing-in-memory(PIM) was first proposed in 1990s to address the memory-wall issue on von-Neumann architecture. Emerging technology of 3D die stacking has mitigated the manufacturing issues of PIM to some extent. The vertical 3D die-stacking with TSVs allows stacking multiple memory dies directly on top of the logic die to achieve high memory bandwidth. Since tensor train decomposition represents data in 3-dimensional tensor cores, it perfect matches the characteristic of 3D PIM which leaves room of imagination that using 3D PIM can solve the redundant unfolding and reshaping in neural network process.

**Table 4. FPGA Resource Utilization**

Resource	DSP48E	Flip Flop	LUT
Utilization	33	36625	88723

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